Characteristics and Properties of Chimera States in a Ensemble of Nonlocally Coupled Chaotic Oscillators

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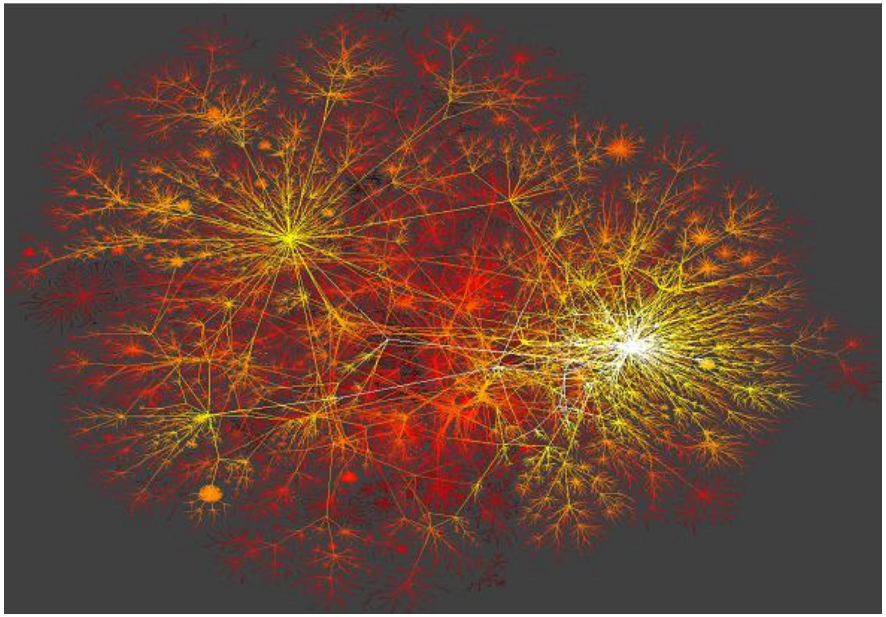
1. Introduction. Technological Complex Networks

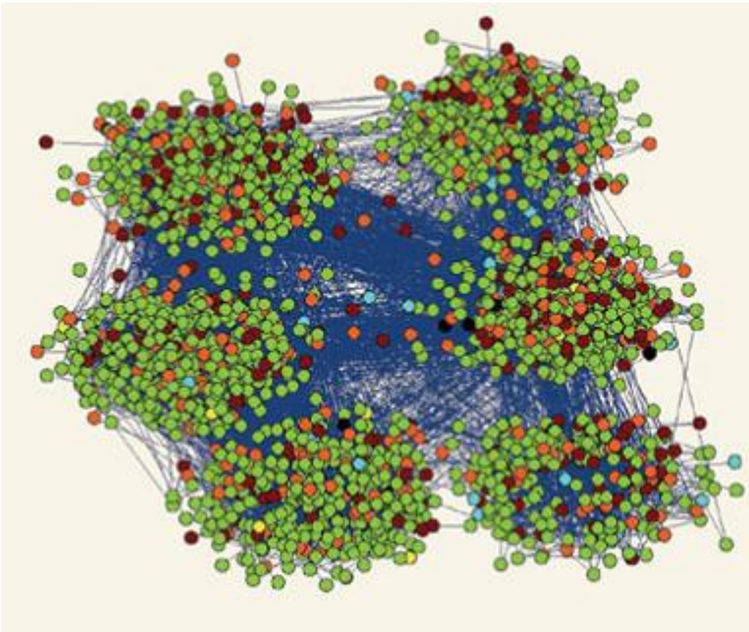


Traditionally the analysis of **the Internet structure** is made by means of ***traceroutes.*** That is to say, by exploring all the paths from a given point to all the possible destinations.

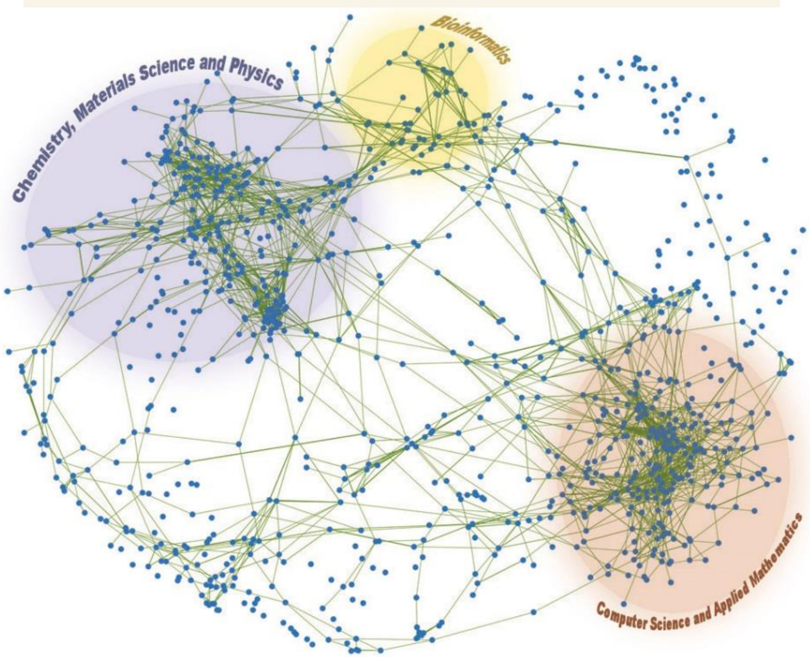
G. Caldarelli
CNR-INFM Centre SMC Dep. Physics University "Sapienza" Rome, Italy

Power grid network in the North America

Complex Social Networks

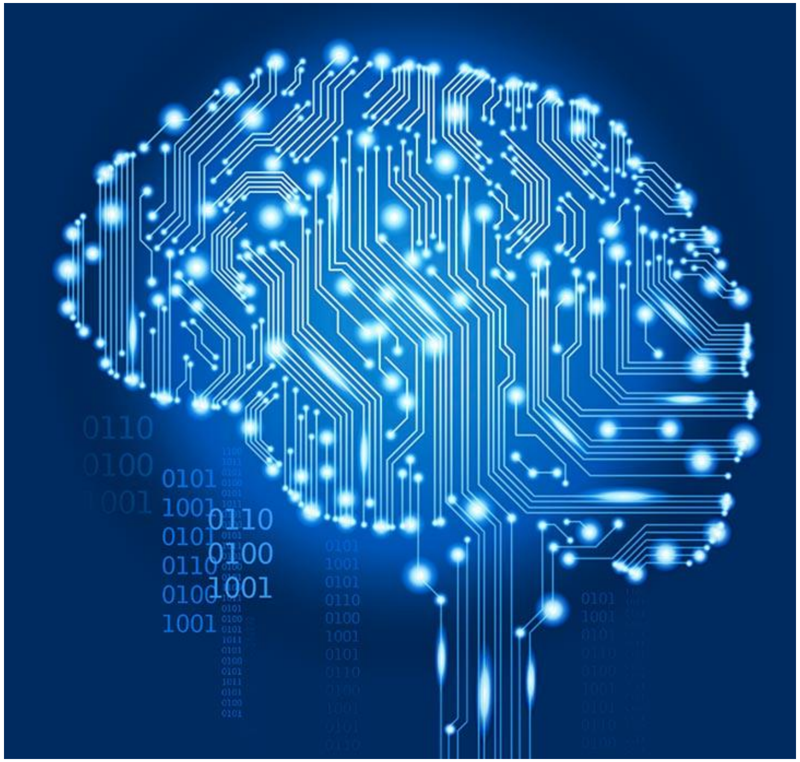


Friendship links in a school in the United States (from G. Caldarelli)



Knowledge Networks: networks to capture the collective knowledge of the communities of users of online resources, such as the scientific literature and digital libraries, Wikipedia, as well as social media such as Twitter and Instagram.

Complex Networks in Nature



Complex structure of neuron links in the human brain

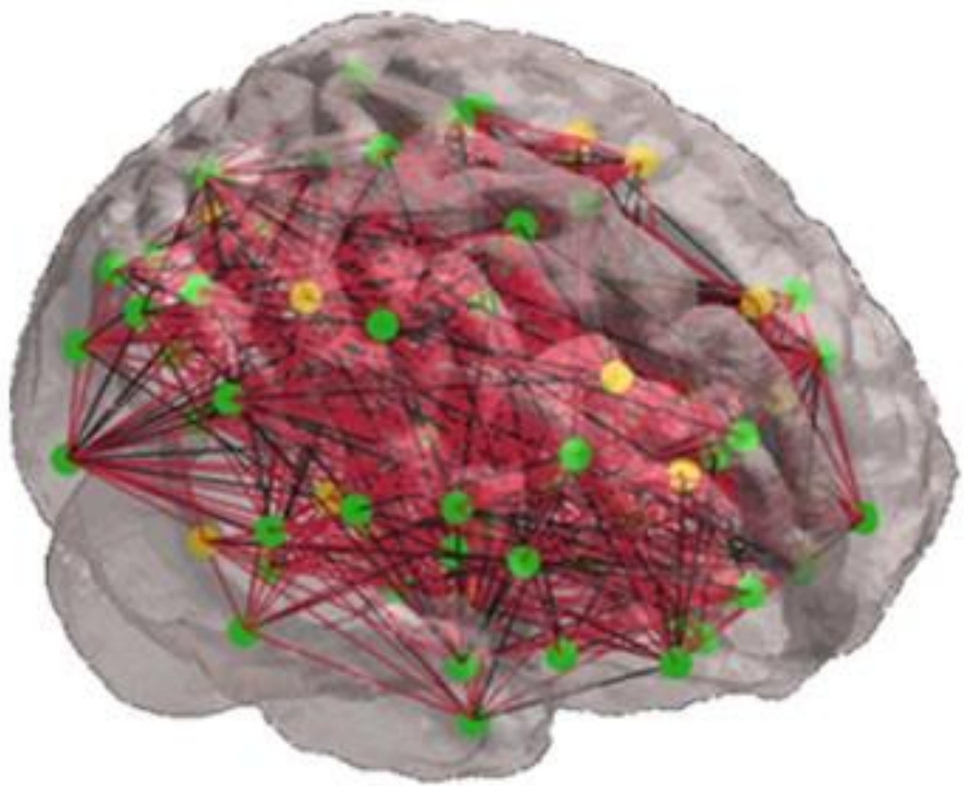


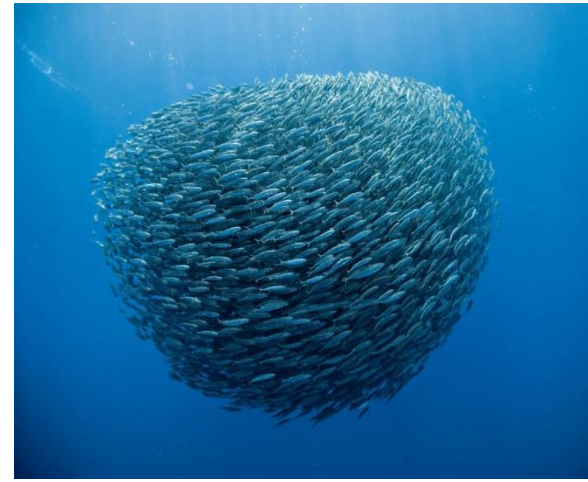
Figure 1. Agent based brain model. Each of 90 gray matter brain regions is represented by a node. Lines indicate functional connections, defined by correlated functional activity measured using fMRI. Nodes may either be on (green) or off (yellow) and connections indicate positive (red) or negative (black) correlations.

Examples of structure formation in nature

1. Fish shoals

“Chaotic” swarm

Structure formation around a sea lion



Dynamical regular structure

2. Bird swarm

“Chaotic” swarm

Structure formation process

Ordered structure: wedge

3. Pengiuns

“Chaotic” crowd (mob)

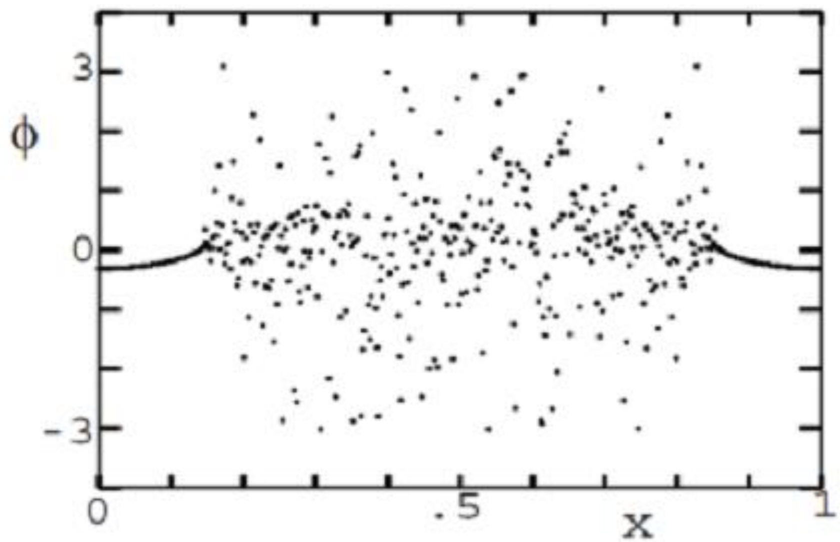
Stationary structure to protect the babies

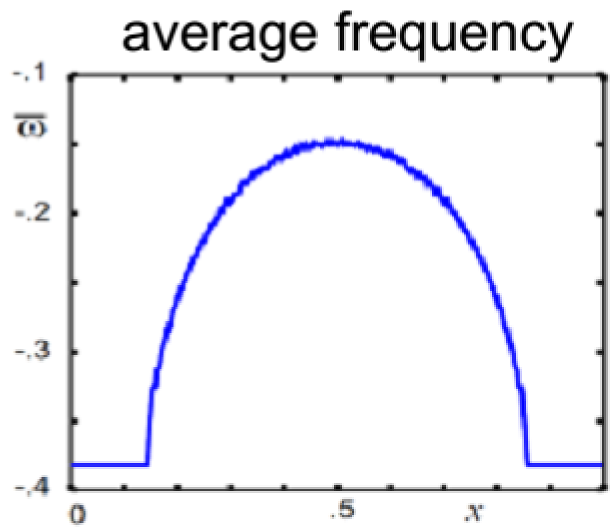
Chimera states in networks of nonlocally coupled identical oscillators

Kuramoto phase oscillator model:

$$\frac{\partial \psi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x-x')\sin[\psi(x,t)-\psi(x',t)+\alpha]dx' \quad \text{with} \quad \underbrace{G_{\exp}(x) = \frac{\kappa}{2}e^{-\kappa|x|}}_{\text{Exponential coupling function}}$$

Snapshot of chimera state





Coherent domains of periodic in-phase oscillations coexist with incoherent domains, characterized by a chaotic behavior in time and in space.

Y. Kuramoto and D. Battogtokh, Nonlinear Phenom. in Complex Syst. **5,** 380 (2002).

VOLUME 93, NUMBER 17 PHYSICAL REVIEW LETTERS week ending 22 OCTOBER 2004

Chimera States for Coupled Oscillators

Daniel M. Abrams* and Steven H. Strogatz†
Department of Theoretical and Applied Mechanics, Cornell University, 212 Kimball Hall, Ithaca, New York 14853-1503, USA

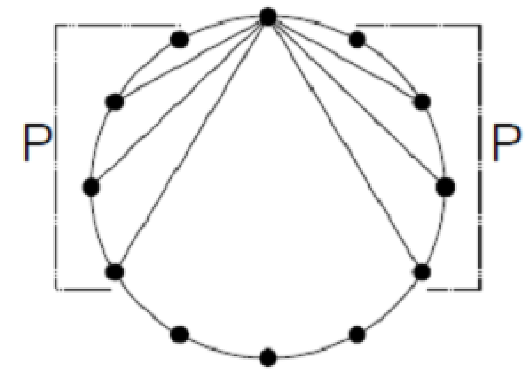
A chimera state was defined as a **spatio-temporal pattern** in which an array of identical oscillators is split into **coexisting regions** of **coherent** (phase and frequency locked) and **incoherent** (drifting) oscillations.

“In Greek mythology***, the chimera*** was a fire-breathing monster having a lion’s head, a goat’s body, and a serpent’s tail.
Today the word refers to anything composed of incongruent parts, or anything that seems fantastic.”

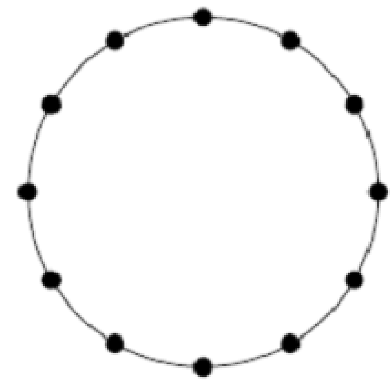
Chimera of Arezzo

Numerical observation of chimera states:
Network of nonlocally coupled identical systems

$$\dot{x}_i(t) = f(x_i(t)) + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}[f(x_j(t)) - f(x_i(t))]$$

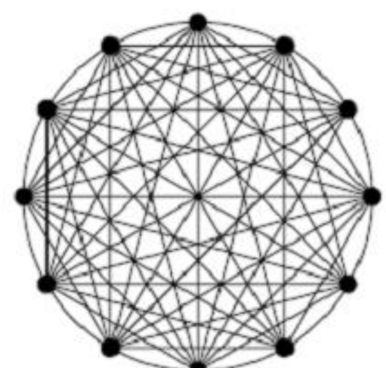


x_i are real dynamic variables ($i = 1, \ldots, N, N \gg 1$ and the index i is periodic $mod\ N$);
t denotes time;
σ is the coupling strength;
P specifies the number of neighbors in each direction coupled with the ith element;
r is the coupling radius (range), $r = P/N$;
$f(x)$ defines dynamics of individual element:



P=1

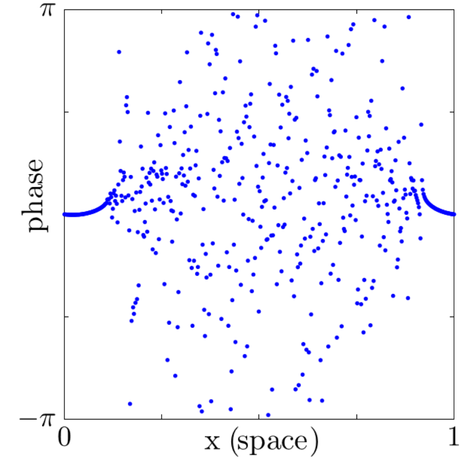
- Stuart-Landau harmonic self-sustained oscillators;
- discrete-time systems (maps);
- continuous-time chaotic models;
- FitzHugh-Nagumo neural systems;
- Van der Pol oscillators;
- quantum oscillator systems;
- …..



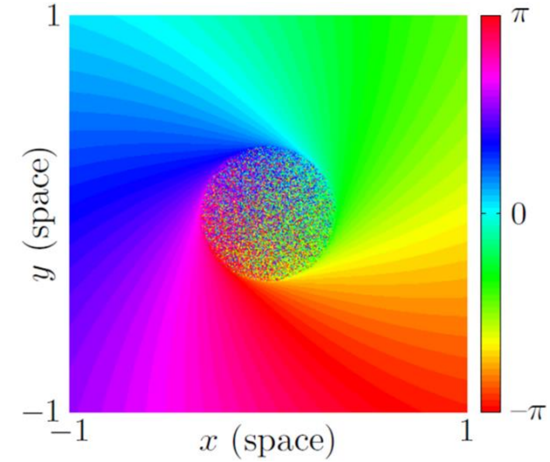
P=N/2

Examples of chimera states

1. Chimera states in a 1D periodic space (ring of phase oscillators).



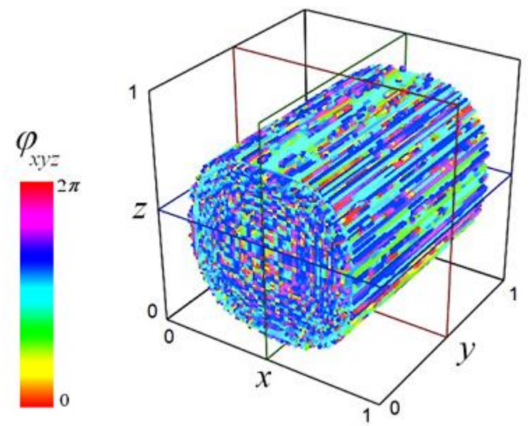
2. The incoherent region for a spiral chimera state on a 2D infinite space.



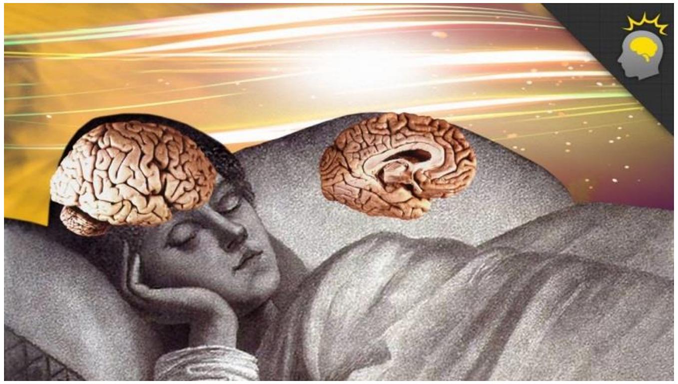
Mark J. Panaggio and Daniel M. Abrams. Nonlinearity 28 (2015) R67-R87.

3. 3D chimera states in a network of phase oscillators (incoherent tube).

Yu. Maistrenko et al. New Journal of Physics 17 (2015) 073037.



Chimera in Nature:
Unihemispheric Sleep



N.C. Rattenborg et al. / Neuroscience and Biobehavioral Reviews 24 (2000) 817–842

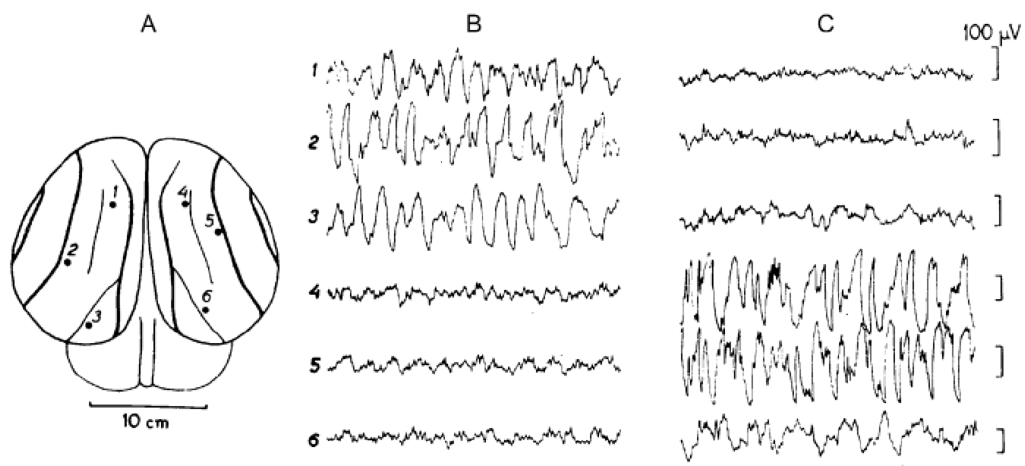
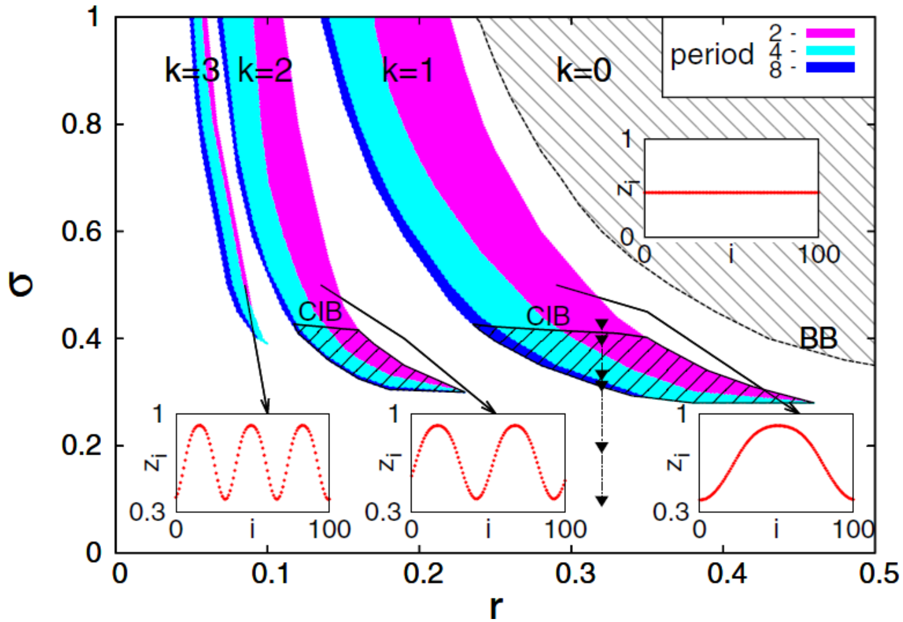


Fig. 1. EEG recorded from the parieto-occipital cortex (A) of a bottlenose dolphin during unihemispheric slow-wave sleep with either the left (B) or right (C) hemisphere asleep. Note the high-amplitude, low-frequency EEG activity in the sleeping hemisphere and the low-amplitude, high-frequency EEG activity in the awake hemisphere. Reprinted from Brain Research, Vol 134, Mukhametov, L.M., Supin, A.Y., Polyakova, I.G., Interhemispheric asymmetry of the electroencephalographic sleep patterns in dolphins, 581–584, 1977, with permission from Elsevier Science.

2. Role of Hyperbolicity in Realizing Chimera States in Networks of Chaotic Systems

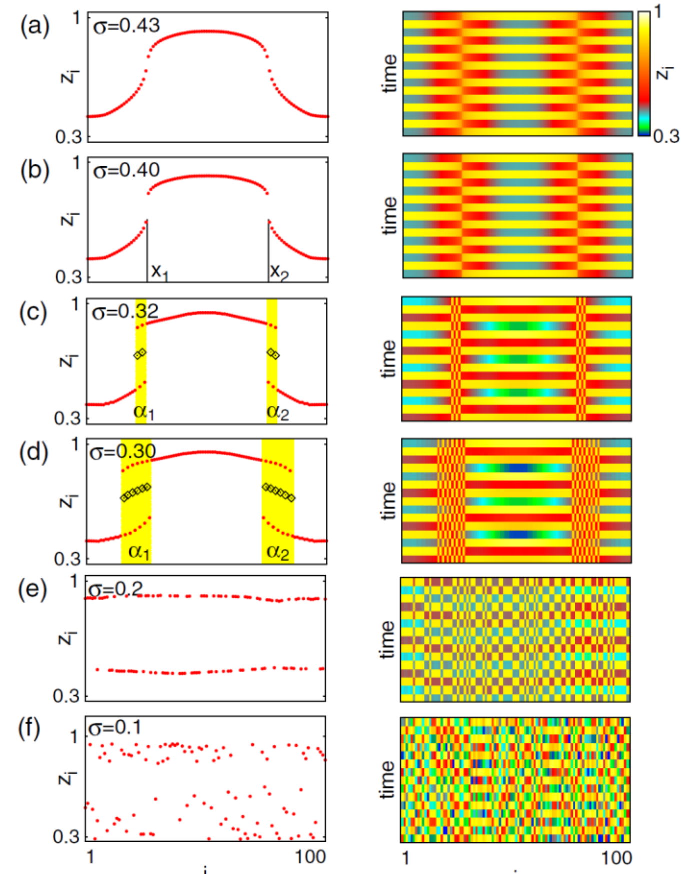
Known results:

Ring of coupled logistic maps $z_{n+1} = \lambda z_n (1 - z_n)$



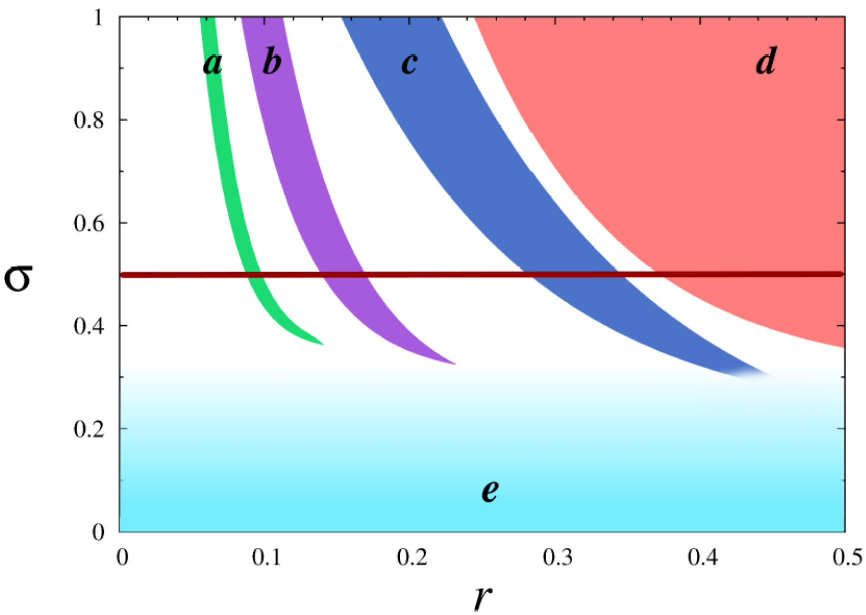
Regions of coherence for the system in the parameter plane (σ, r) with wave numbers $k = 1, 2, \text{and } 3$. Snapshots of typical coherent states z_i are shown in the insets. The color code inside the regions distinguishes different time periods of the states.

Iryna Omelchenko, et al.,
Phys. Rev. Lett. 106 (2011) 234102



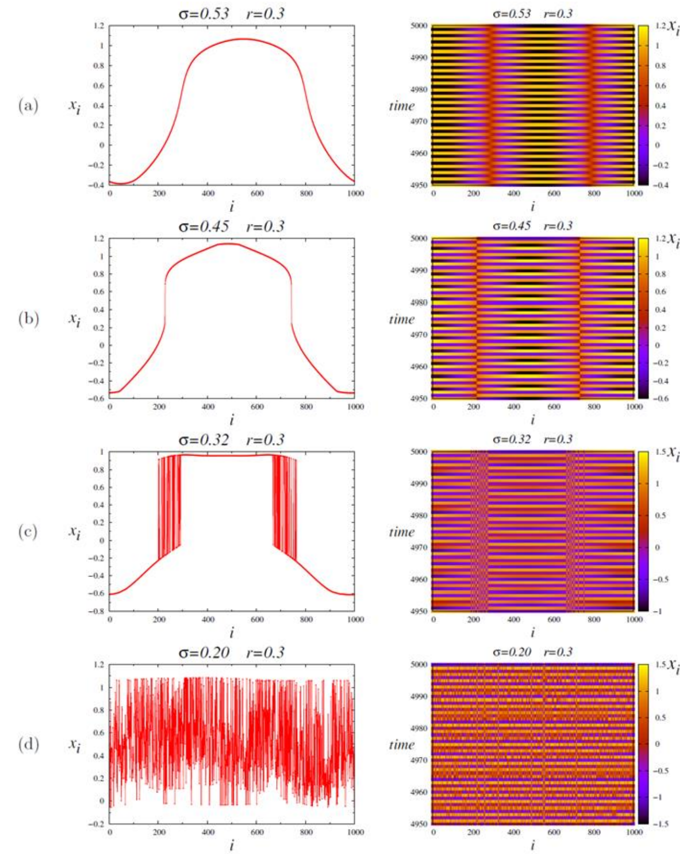
Coherence-incoherence bifurcation for coupled chaotic logistic maps for fixed coupling radius r=0.32. Snapshots (left columns) and space-time plots (right columns) for different values of coupling strength are shown.

Results for the network of Henon maps



Regions of coherence for non-locally coupled Henon maps (1) in the (r,σ) parameter plane with wave numbers $k = 1, 2$ and 3 (regions $(\boldsymbol{c})$, $(\boldsymbol{b})$ and $(\boldsymbol{a})$, respectively). $(\boldsymbol{d})$ is the area of completely synchronized chaotic states, $(\boldsymbol{e})$ is the region of complete incoherent states. Chimera states are observed for $\sigma < 0.5$. Parameters: $\alpha = 1.4$, $\beta = 0.3$, $N = 1000$.

N. Semenova, 2015.



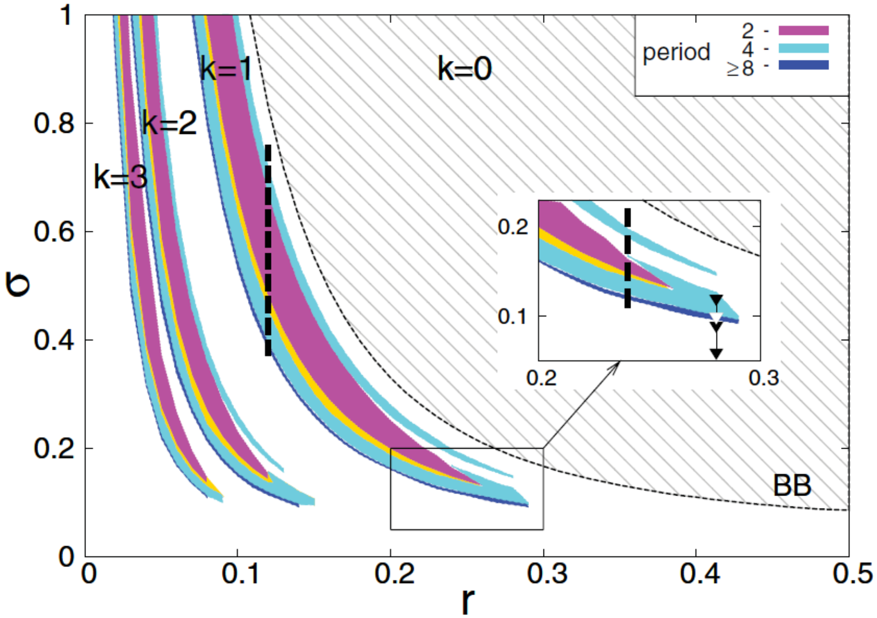
Coherence-incoherence bifurcation for coupled Henon maps with fixed coupling radius $r = 0.3$. For each value of the coupling parameter (decreasing from top to bottom, $\sigma = 0.53$, $\sigma = 0.45$, $\sigma = 0.32$ and $\sigma = 0.2$), snapshots in time $t = 5000$ (left columns) and space-time plots (right columns) are shown.

Results for the network of Rössler systems

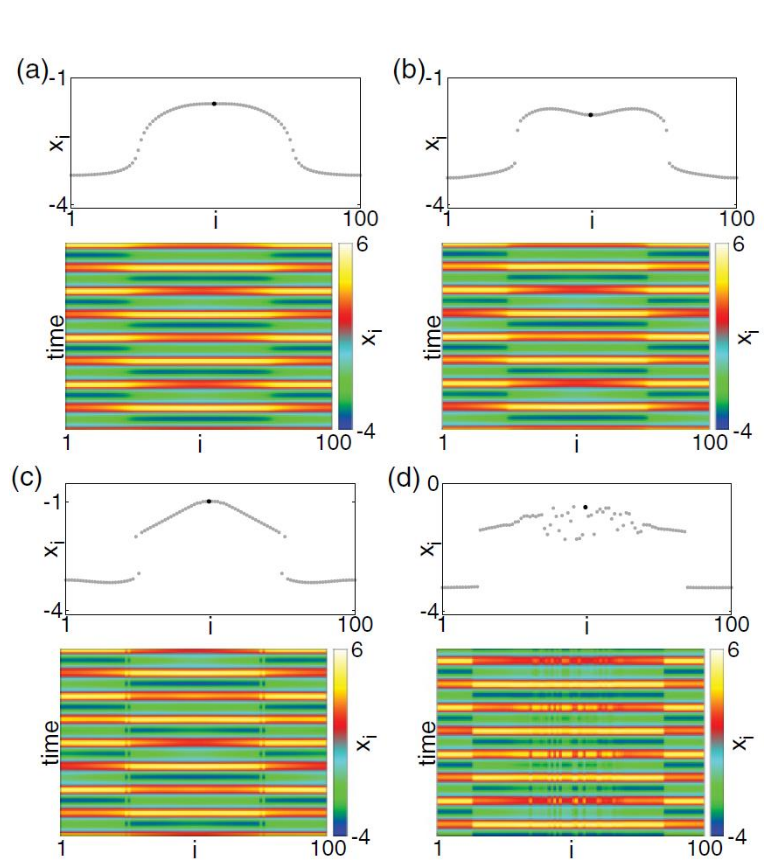
$$\dot{x}_i = -y_i - z_i + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(x_j - x_i),$$

$$\dot{y}_i = x_i + a y_i + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(y_j - y_i),$$

$$\dot{z}_i = b + z_i(x_i - c) + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(z_j - z_i),$$



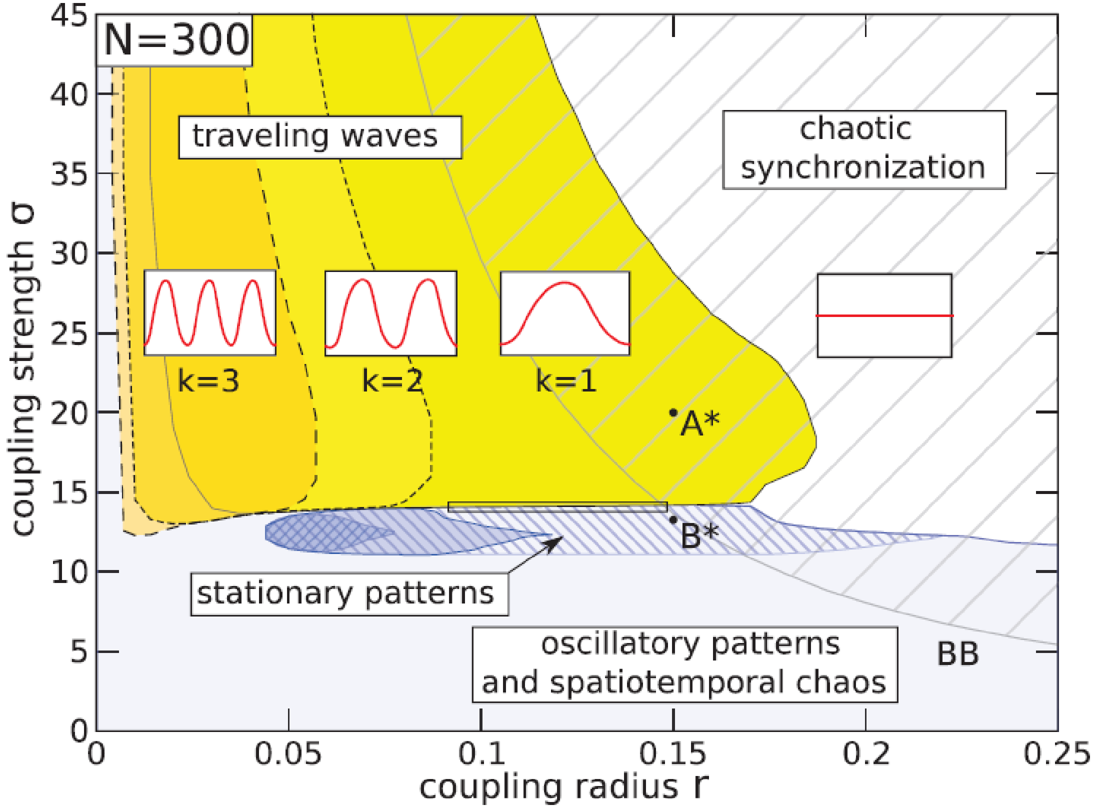
Bifurcation diagram



Coherence-incoherence bifurcation

Iryna Omelchenko, et al. Phys. Rev. E 85 (2012)

Lorenz system (with a quasi-hyperbolic attractor) as a partial element of a network



Volodymyr Dziubak, et al. //
Phys. Rev. E, 2013, 87

No chimera states!!!

Hypothesis

Chimera states can be obtained in rings of systems with period-doubling bifurcation (*non-hyperbolic systems*). The parameter planes (r, σ) of networks with such partial elements are topologically equivalent.

The chimera state has not been found in a network of Lorenz systems. The Lorenz system is *quasi-hyperbolic*.

We propose the following hypothesis:

Chimera states can be obtained only in networks of chaotic non-hyperbolic systems and cannot be found in networks of hyperbolic (quasi-hyperbolic) systems.

As the basic models of these two main types, we consider the non-hyperbolic Henon map and the quasi-hyperbolic Lozi map.

Basic models of chaotic systems as a partial element of a ring

It has been shown that chimera states can be obtained only in networks of chaotic systems with **non-hyperbolic attractors** and cannot be found in networks of chaotic systems with **hyperbolic** (**singular-hyperbolic**) attractors.

N. Semenova, A. Zakharova, E. Schöll, V. S. Anishchenko, Europhys. Lett. 112 (2015) 40002.

Discrete-time system with a non-hyperbolic attractor:

Henon map

$$x_{n+1} = 1 - \alpha x_n^2 + y_n$$
$$y_{n+1} = \beta x_n$$

It describes the properties of chaotic attractors in the Poincare section for the spiral chaos systems: the Rössler oscillator, the Anishchenko-Astakhov oscillator etc.

Discrete-time system with a singular-hyperbolic attractor:

Lozi map

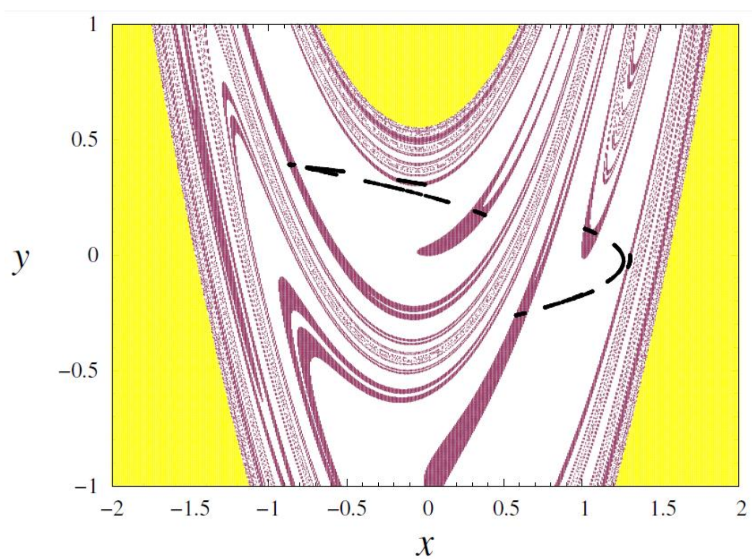
$$x_{n+1} = 1 - \alpha |x_n| + y_n$$
$$y_{n+1} = \beta x_n$$

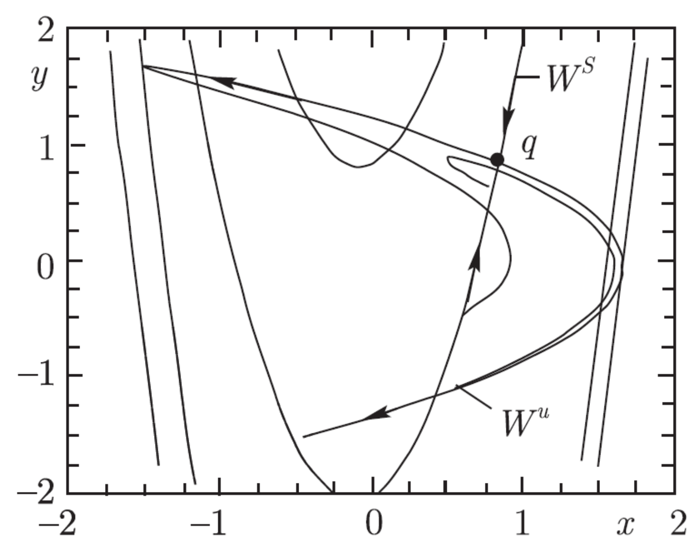
It represents the properties of the Lorenz-type attractors in the Poincare section.

The Henon map as a partial element of a network

$$x_{n+1} = 1 - \alpha x_n^2 + y_n$$
$$y_{n+1} = \beta x_n \qquad (1)$$

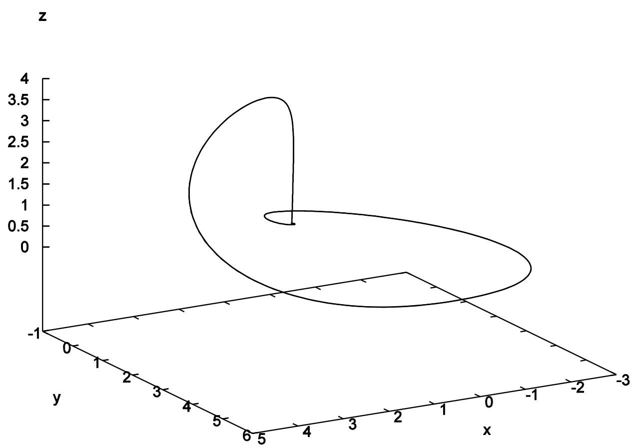
The Henon map can be used to describe qualitatively the bifurcational phenomena in three-dimensional time-continuous systems which demonstrate a spiral-chaos regime due to the existence of a saddle-focus separatrix loop. It has been shown that in this case, the Poincare section of a three-dimensional time-continuous system gives a two-dimensional map of type (1). The Henon map (1) is an example of a non-hyperbolic dynamical system with the bifurcation of homoclinic tangency of stable and unstable manifolds of a saddle point. The Henon map is the simplest model of the maps which can be obtained in the Poincare section of spiral-type chaotic attractors.



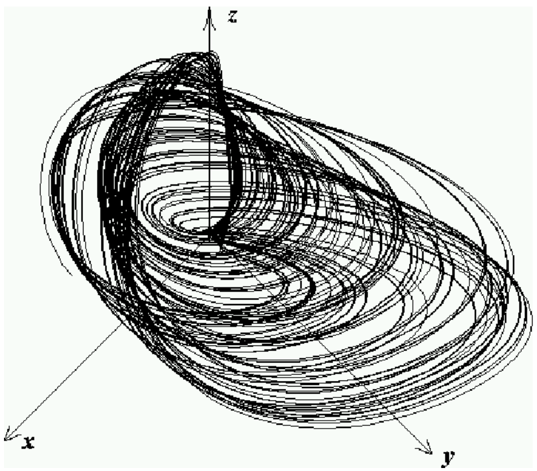


Example: Three-dimensional chaotic system – the Anishchenko-Astakhov oscillator

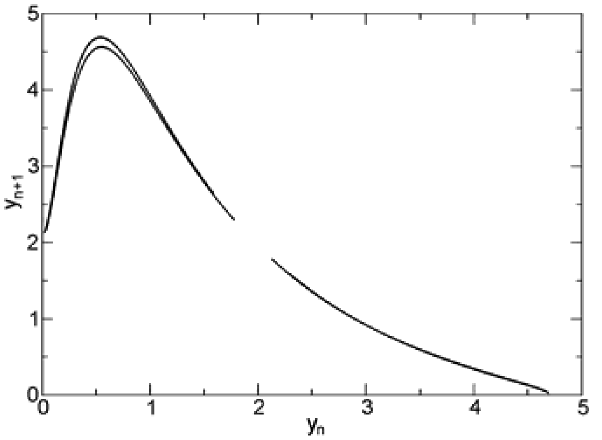
$$\dot{x} = mx + y - xz,$$
$$\dot{y} = -x + \gamma,$$
$$\dot{z} = -gz + gF(x)x^2.$$



Saddle-focus separatrix loop



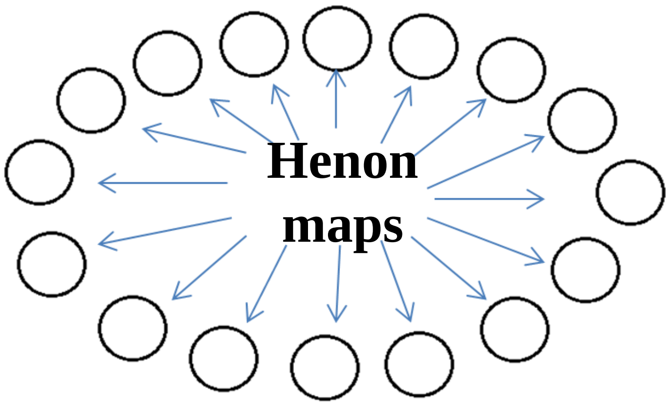
Chaotic attractor for
m =1.5 and *g*=0.2



The map for the chaotic attractor in the secant plane *x*=0

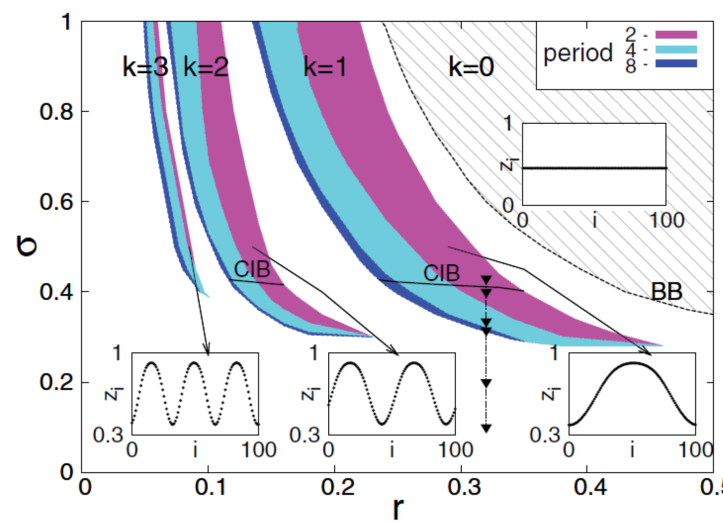
V.S. Anishchenko, 1987 – 1990.

Results for non-hyperbolic partial elements



Chimera states

Logistic map, cubic map, Rössler system, Anishchenko-Astakhov oscillator

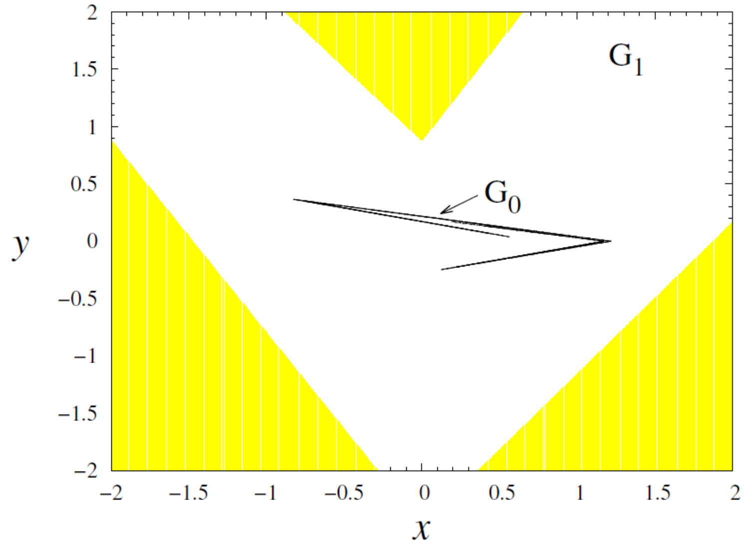


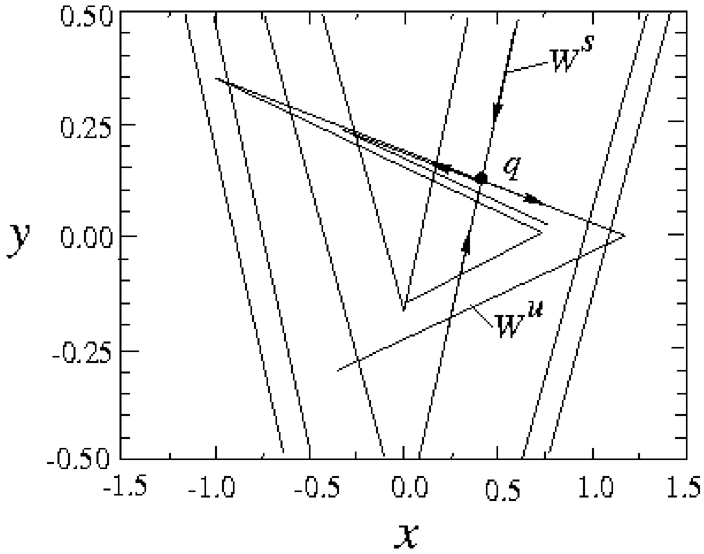
The Lozi map as a partial element of a network

$$x_{n+1} = 1 - \alpha|x_n| + y_n \qquad (2)$$
$$y_{n+1} = \beta x_n$$

The Lozi map is a quasi-hyperbolic system. It is a general discrete model of Lorenz-type chaotic attractors in the Poincaré section.

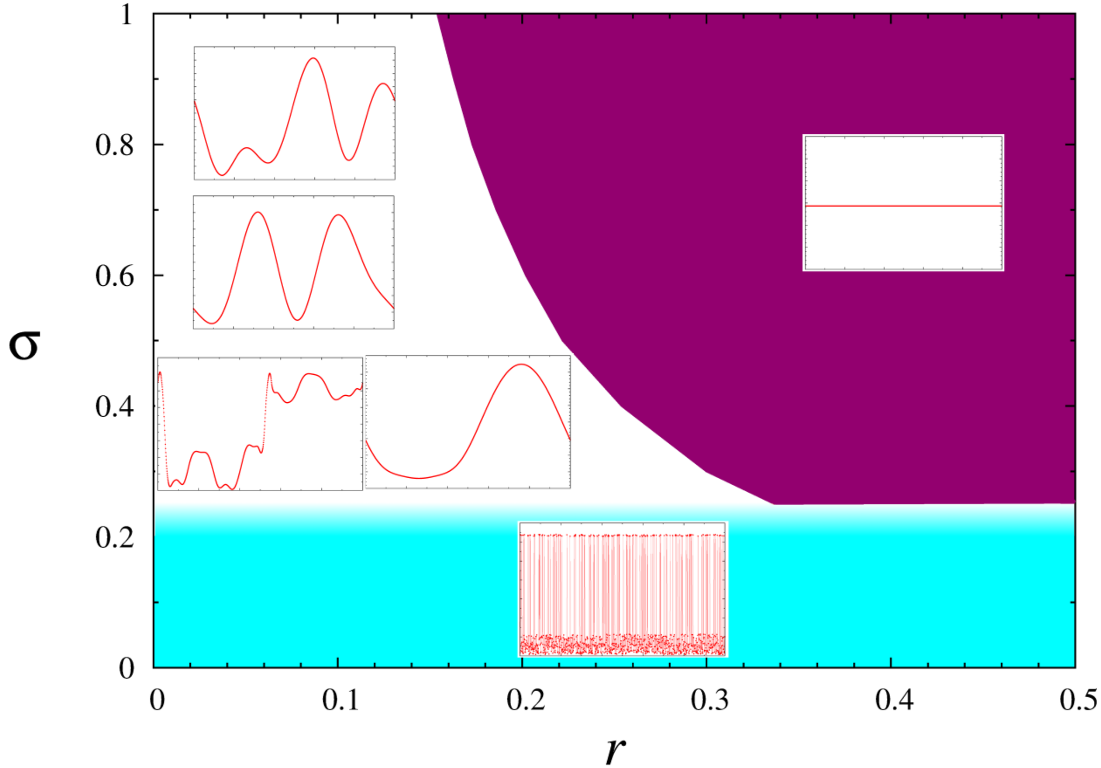
If our hypothesis is true, then the results for the network of Henon maps must be similar to those for the rings of logistic maps, cubic maps, and Rössler systems. For the network of Lozi maps, the observations must be similar to the network of Lorenz systems.





Results for the ring of Lozi maps

Standing or travelling waves



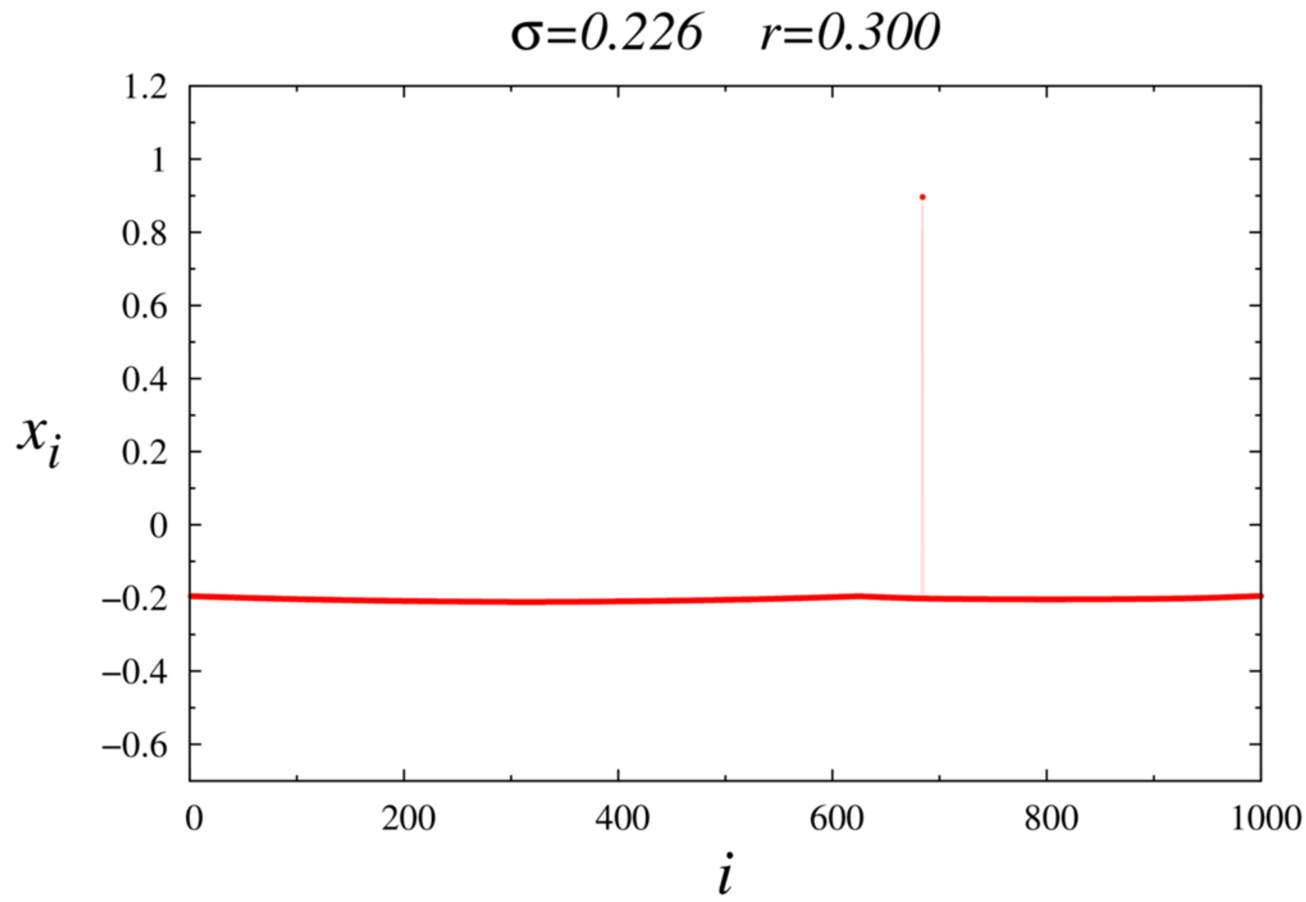
Complete chaotic synchronization

Spatial incoherence

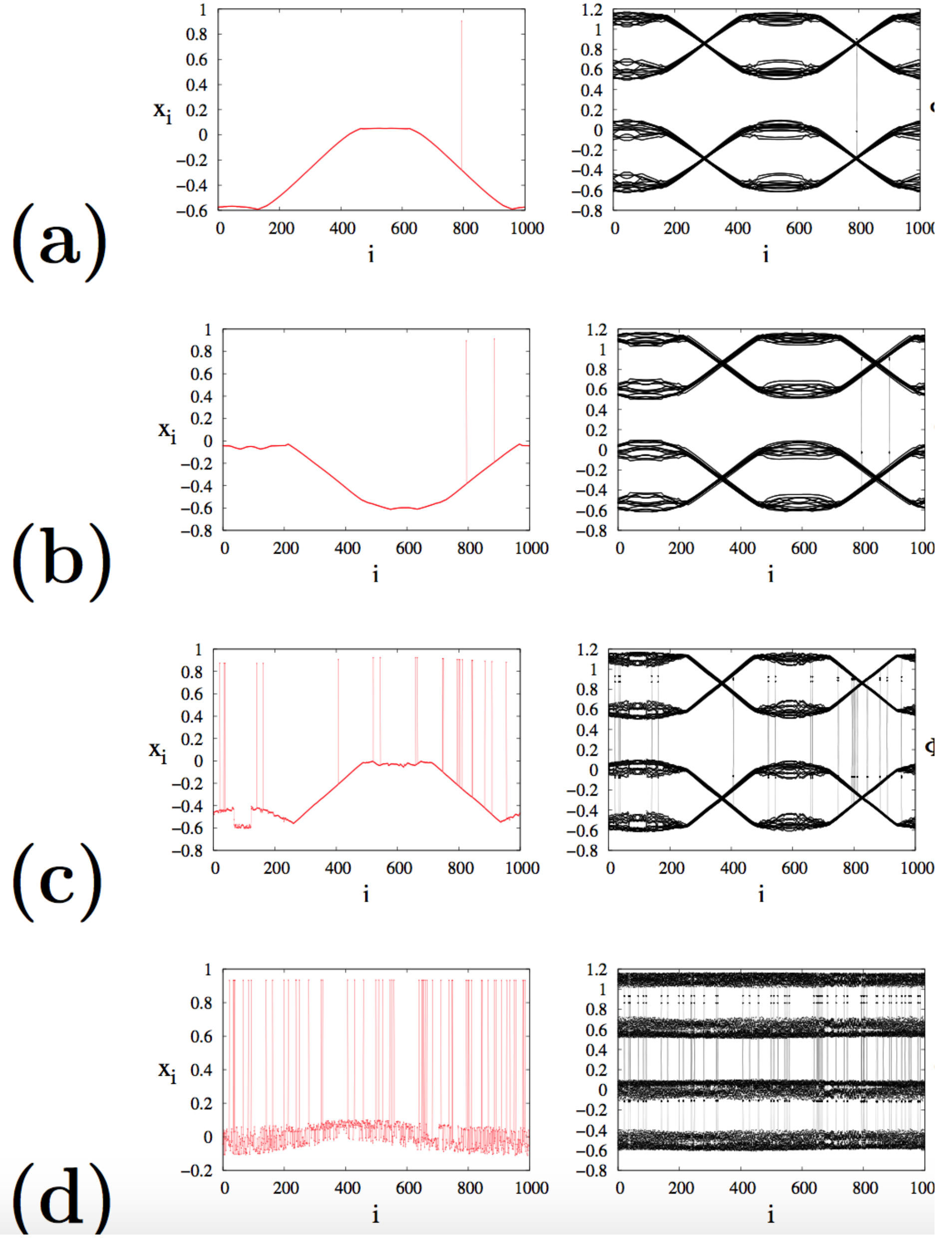
The structure of the parameter plane (r, σ) for the ring of non-locally coupled Lozi maps.

The parameters are $\alpha = 1.4, \quad \beta = 0.3, \quad N = 1000.$

Appearance of a single solitary state in the ring of Lozi maps



Transition to incoherence through solitary states in the network of Lozi maps for *r* = 0.193



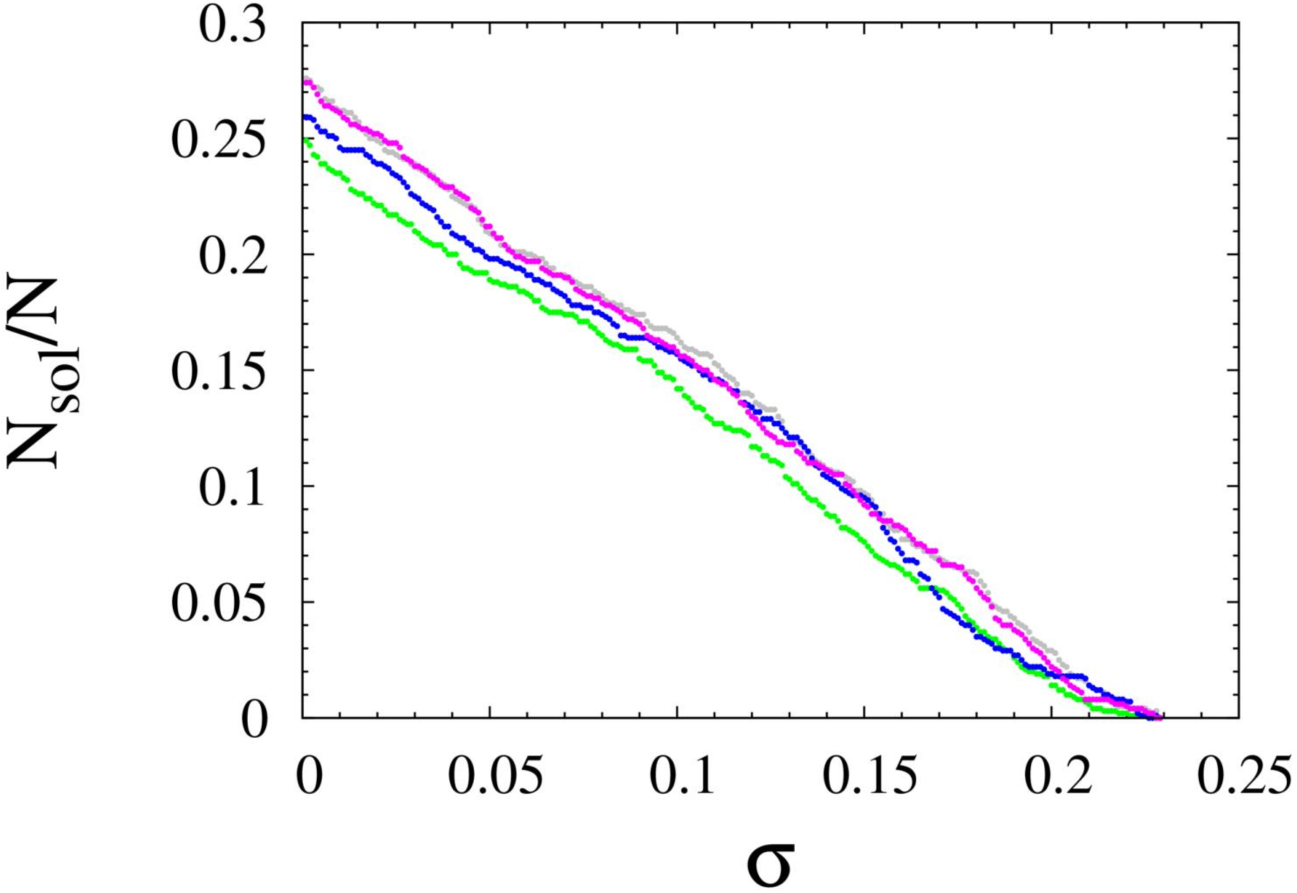
σ = 0.226

σ = 0.219

σ = 0.193

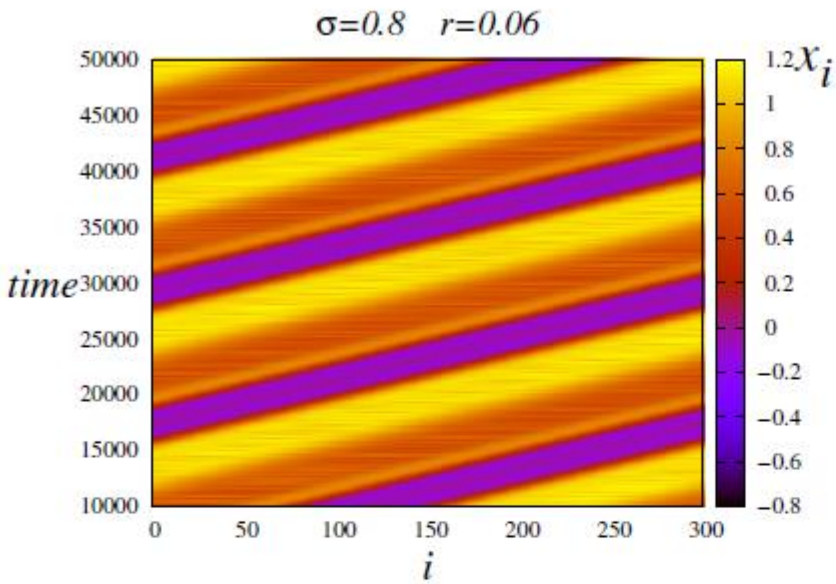
σ = 0.165

Number of solitary states versus the coupling coefficient for different initial conditions

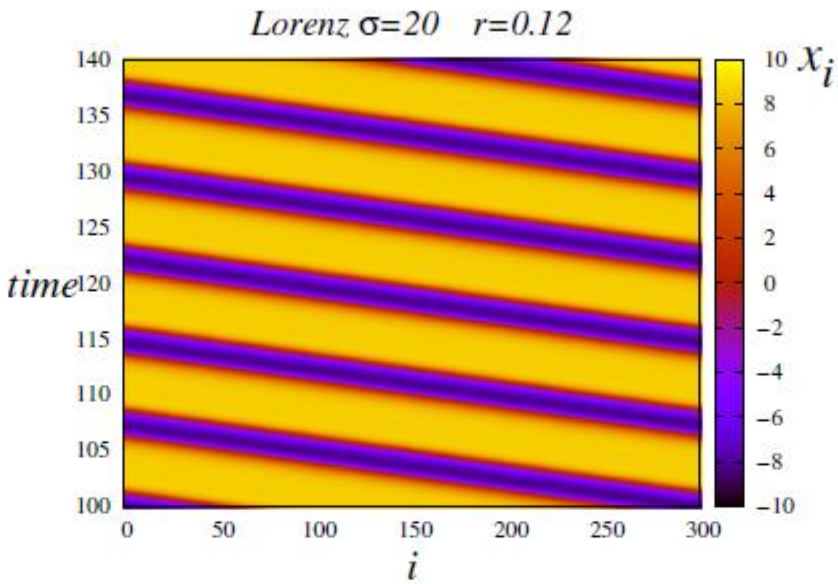


The law is approximately linear!

Travelling waves in the networks of Lozi maps and Lorenz systems

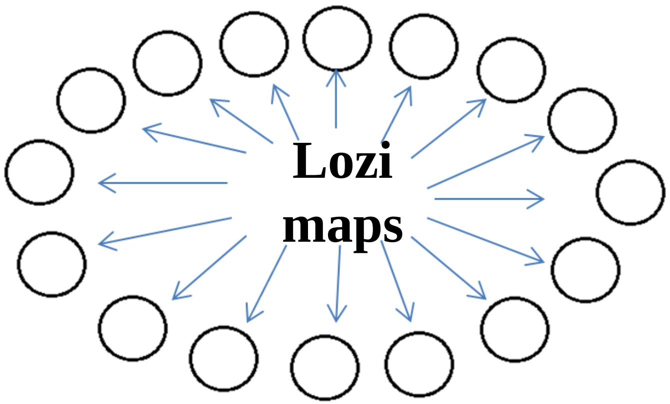


Ring of Lozi maps



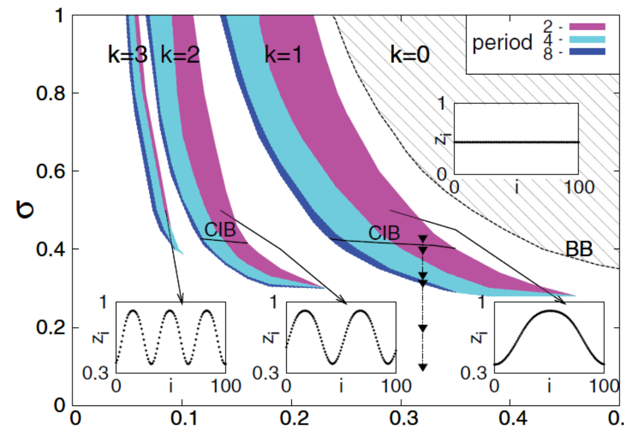
Ring of Lorenz systems

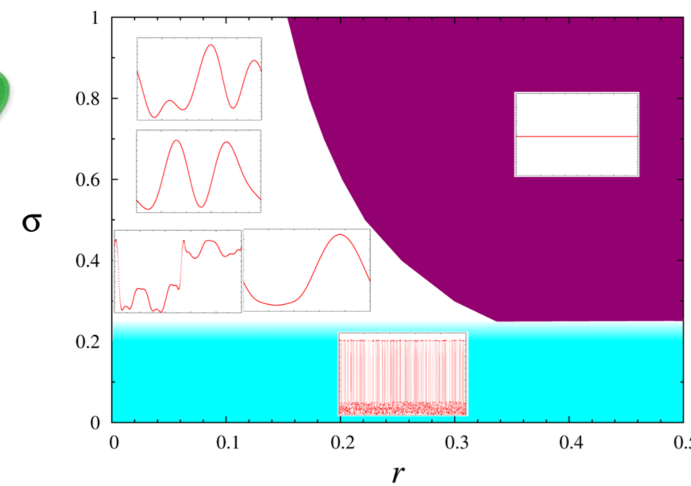
Results for hyperbolic (quasi-hyperbolic) partial elements



Lorenz-type systems

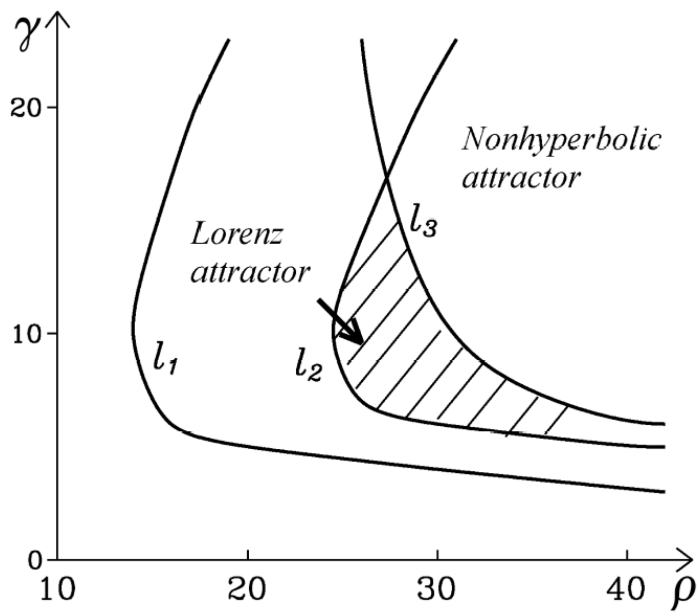
Chimera states



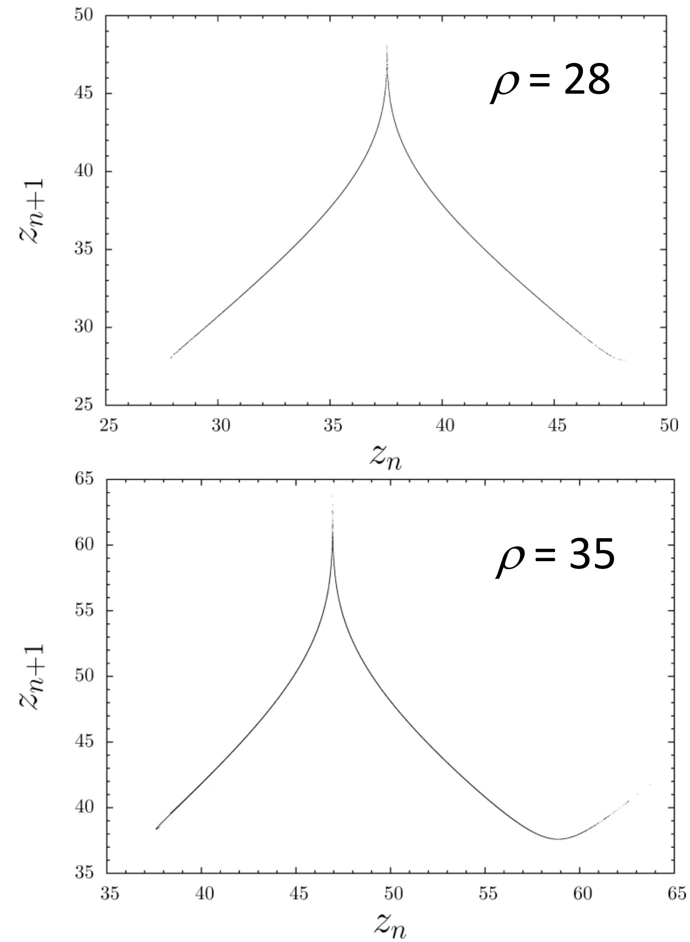


Lorenz attractor ⟶ Nonhyperbolic attractor in the Lorenz system

$$\dot{x} = -\gamma(x - y),$$
$$\dot{y} = -xz + \rho x - y, \qquad (3)$$
$$\dot{z} = -bz + xy.$$

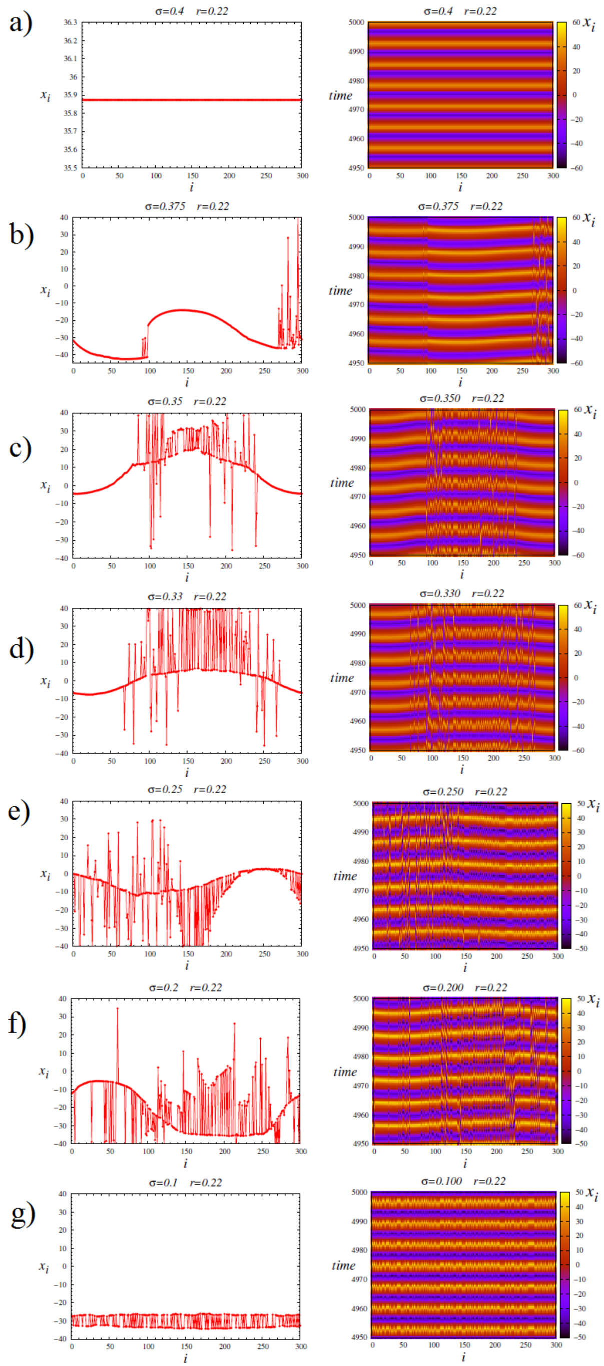


Bifurcation diagram of the Lorenz system (3) on the (ρ, γ) parameter plane for $b = 8/3$. The transition from the Lorenz attractor to a non-hyperbolic attractor is observed when one intersects the line l_3.



Map of the quasi-hyperbolic and non-hyperbolic Lorenz attractors in the secant plane z = const

V.V. Bykov, A.L. Shilnikov, About the boundaries of the Lorenz attractor existence. In: Methods of the Qualitative Theory and Bifurcation Theory. Gorky, 1989. P. 151-159 (in Russian).



Chimera states in a ring of non-locally coupled Lorenz systems (non-hyperbolic case)

$$\dot{x}_i = \zeta(y_i - x_i) + \frac{\sigma_x}{2P}\sum_{j=i-P}^{i+P}(x_j - x_i),$$

$$\dot{y}_i = x_i(\rho - z_i) - y_i + \frac{\sigma_y}{2P}\sum_{j=i-P}^{i+P}(y_j - y_i),$$

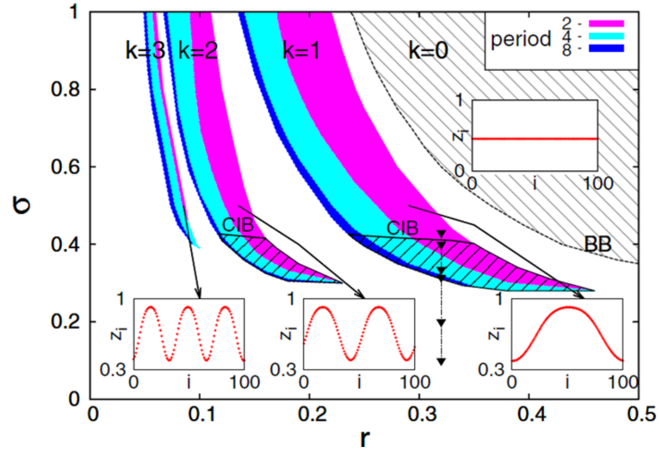
$$\dot{z}_i = x_i y_i - \beta z_i + \frac{\sigma_z}{2P}\sum_{j=i-P}^{i+P}(z_j - z_i),$$

$\boldsymbol{\sigma_x = \sigma_y = \sigma;}$
$\boldsymbol{\sigma_z = 0}$

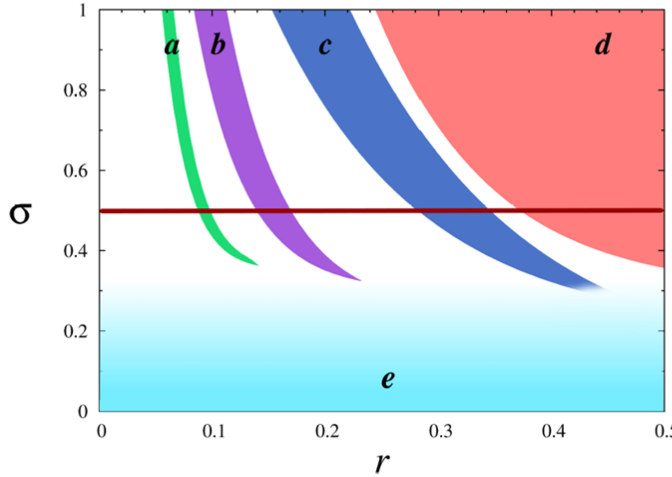
Chimera states in the ring of $N = 300$ non-locally coupled Lorenz systems (3). Snapshots in the time $t = 5000$ (left column) and space-time plots (right column) are shown. The parameters are $b = 8/3$, $\gamma = 10$, $\boldsymbol{\rho = 220}$, $r = 0.22$. Decreasing from top to bottom, $\sigma = 0.4 \div 0.1$.

(a) - complete synchronization, (b-e) - chimera states, (f),(g) - spatial incoherence.

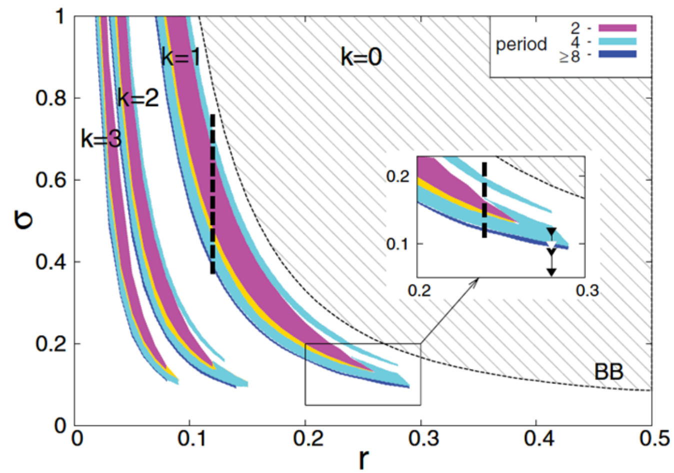
3. Transition from coherence to incoherence in a network of nonlocally coupled logistic maps



Logistic map



Henon map



Rössler oscillator

The bifurcation diagrams show the topological equivalence.

Thus, for simplicity, we consider **the dynamics of the ring of non-locally coupled logistic maps.**

System under study

$$x_i^{t+1} = f(x_i^t) + \underbrace{\frac{\sigma}{2P}\sum_{j=i-P}^{i+P}\left[f(x_j^t) - f(x_i^t)\right]} \qquad (1)$$

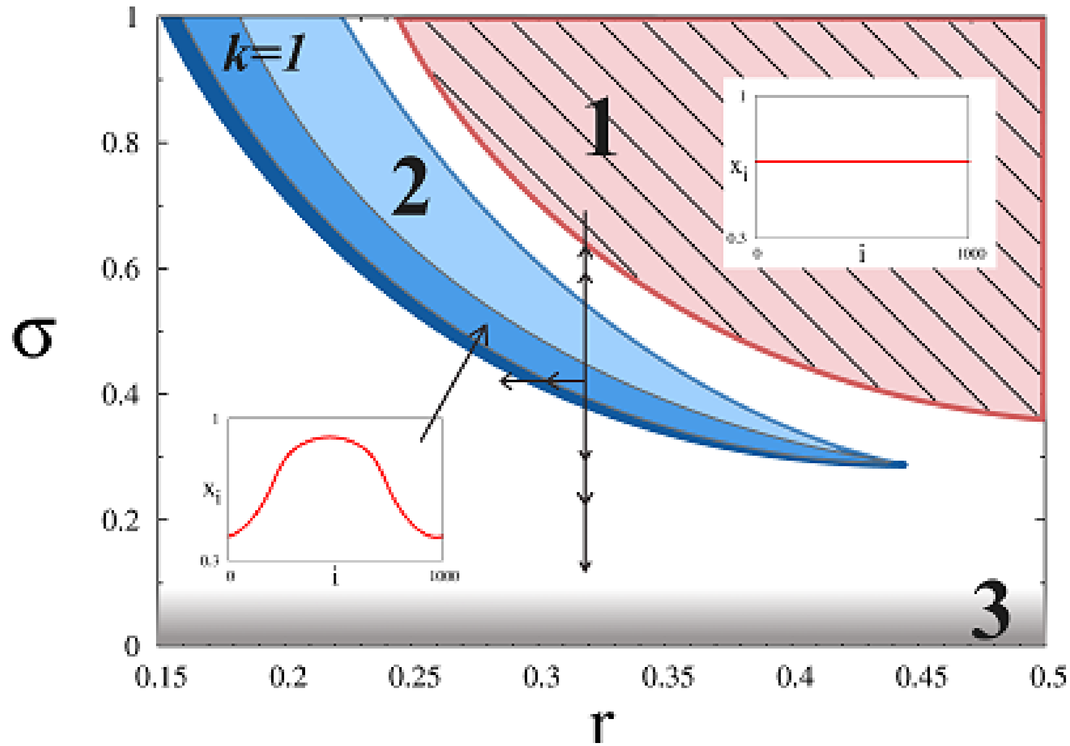
$F(\sigma, r, x_i^t)$ - **coupling function**

- x_i are real dynamic variables (i = 1,…,N, N = 1000, and the index i is periodic $mod\ N$);
- t denotes the discrete time;
- σ is the coupling strength;
- P specifies the number of neighbors on the left and right of the ith element;
- $f(x)$ is the logistic map with $a = 3.8$:

  $$f(x_i) = ax_i(1 - x_i)$$

- r is the coupling radius, $r = P/N$, $r = 0.32$.
- The initial conditions x_i^0 are chosen to be randomly distributed in the interval $0 < x_i^0 < 1$.

Fragment of the bifurcation diagram for the ring of logistic maps



We fix $a = 3.8$, $r = 0.32$ and set random initial conditions.
σ changes along the lines with arrows shown in the diagram.

The aim of the research is

- to study numerically the transition from complete chaotic synchronization to spatiotemporal chaos in the ring of non-locally coupled logistic maps when the coupling strength is varied (**$0 < \sigma < 1$**).

We plot snapshots of the system dynamics (a spatial distribution of the instantaneous values of dynamical variables x_i) and space-time profiles (the last 100 iterations of the dynamical variables for each network element).

- to analyze the spatial correlation in the ensemble by using **the cross-correlation coefficient (CCC)** of the oscillations of different elements.

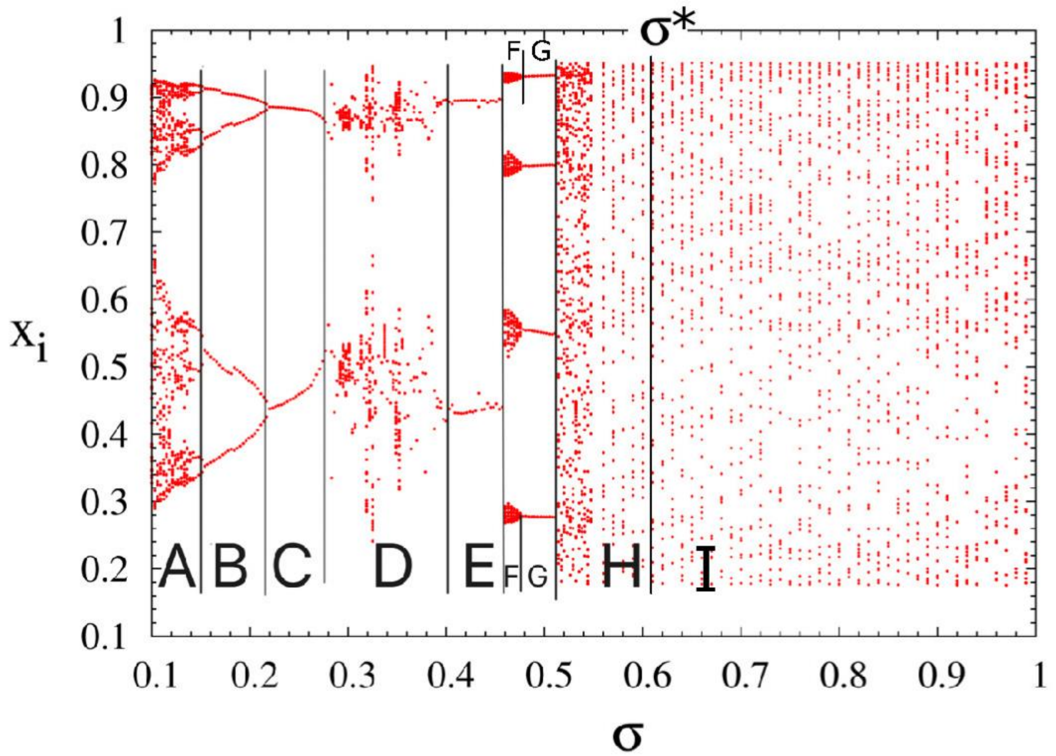
We consider the first and the ith elements at the same time and the CCC is defined as follows:

$$\Psi_{1,i} = \frac{\langle \tilde{x}_1(t)\tilde{x}_i(t)\rangle}{\sqrt{\langle \tilde{x}_1^2(t)\rangle\langle \tilde{x}_i^2(t)\rangle}}, \quad \tilde{x}(t) = x(t) - \langle x(t)\rangle$$

is a fluctuation around the average value. The brackets mean time averaging.

$$i = 2, 3, \ldots, N$$

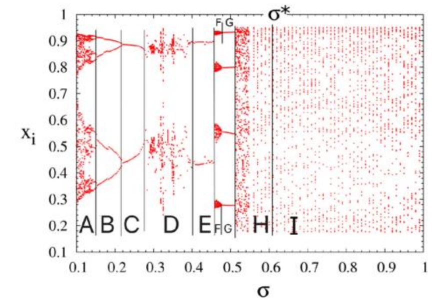
Phase-parametric diagram for the ring of logistic maps



$i = 500$
$r = 0.32$

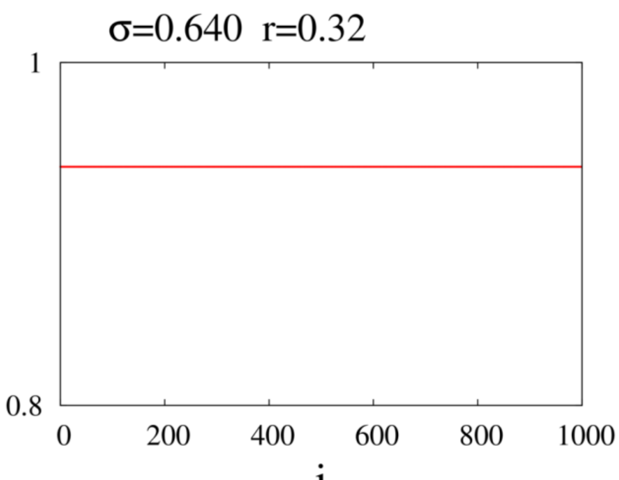
A – spatio-temporal chaos in the ensemble;
B, C, E, and G – periodic regimes;
F – quasiperiodic oscillations;
D – region of chimera states;
H – partial spatio-temporal chaotic synchronization;
I – complete spatio-temporal chaotic synchronization (σ^* is the blowout bifurcation point).

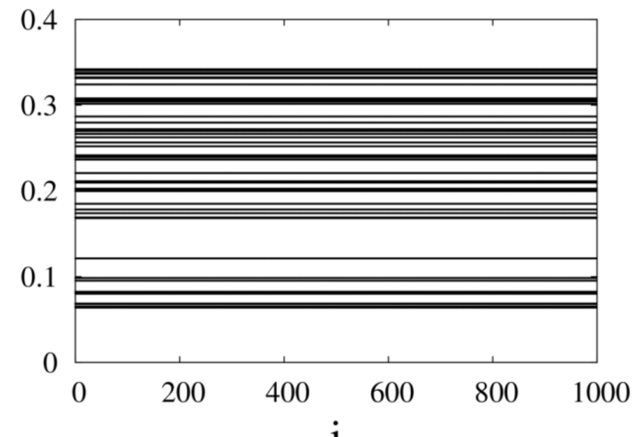
Transition from complete chaotic synchronization to partial synchronization regime (I → H)



Region I – complete chaotic synchronization

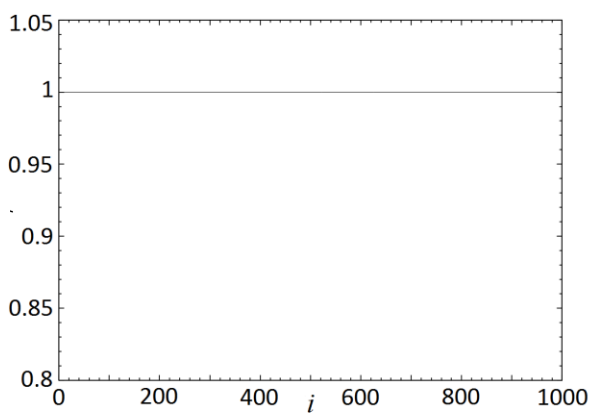
Snapshot x_i

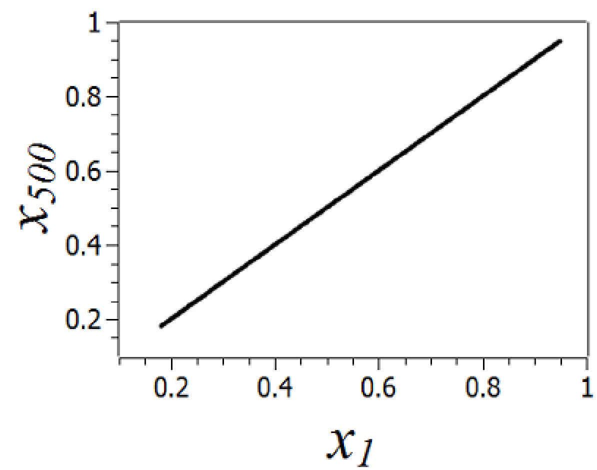




Space-time profile

Cross-correlation $\Psi_{1,i}$



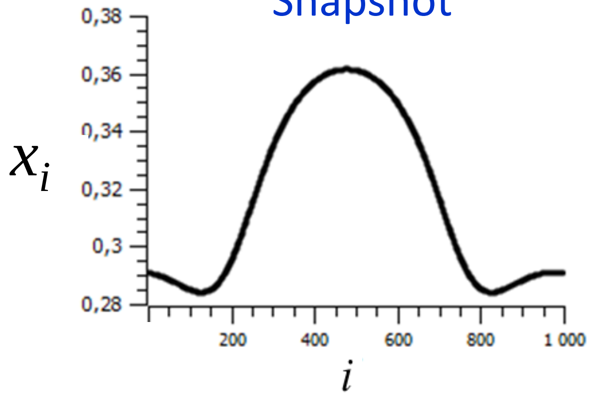


Phase trajectory projection

Although the oscillations in the elements are chaotic, the instantaneous values of all the variables x_i coincide at any time, the CCC $\Psi_{1,i}$ is strongly equal to 1, and the points of the chaotic trajectory is located in the symmetric subspace for all $i = 1, 2, 3, \ldots, 1000$.

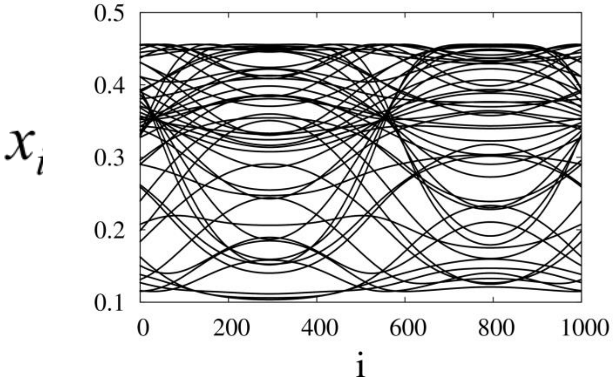
Regime of oscillating or partial chaotic synchronization (region H): $0.5 < \sigma < 0.63$

Snapshot



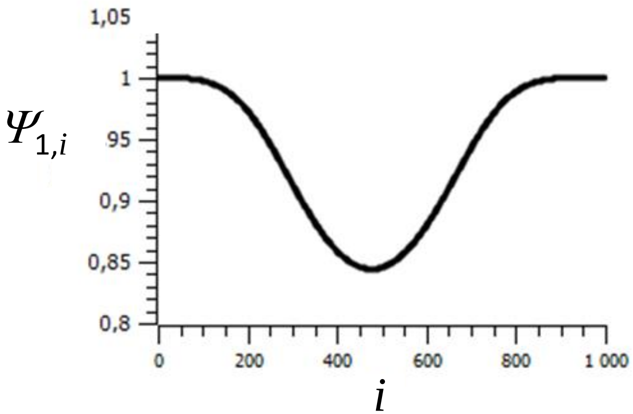
$\sigma = 0.55$

Space-time profile

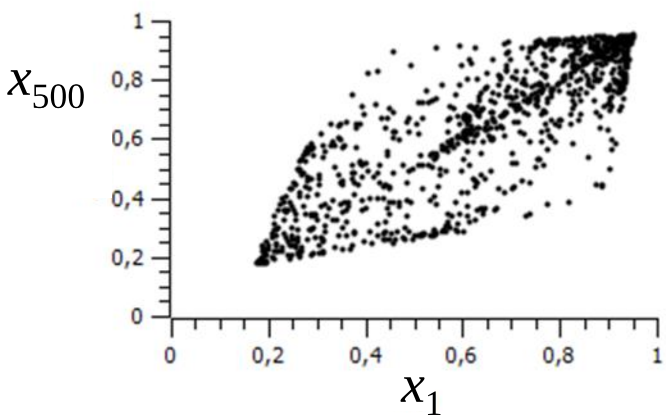


This regime is coherent according to the condition: $|x_i^t - x_{i+1}^t| < \delta, \quad \delta << 1, \quad t = const >> 1, \quad i = 1,2,\dots,N$

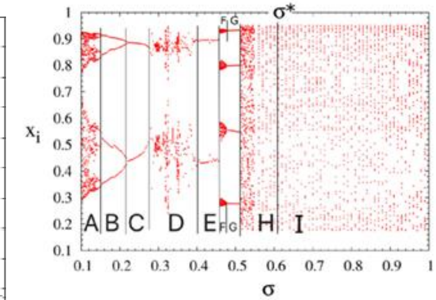
Cross-correlation



Phase trajectory projection



The wave-like profile oscillates randomly without losing its smoothness but the degree of cross-correlation decreases as *i* increases, and the phase trajectories no longer lie in the symmetric subspace.

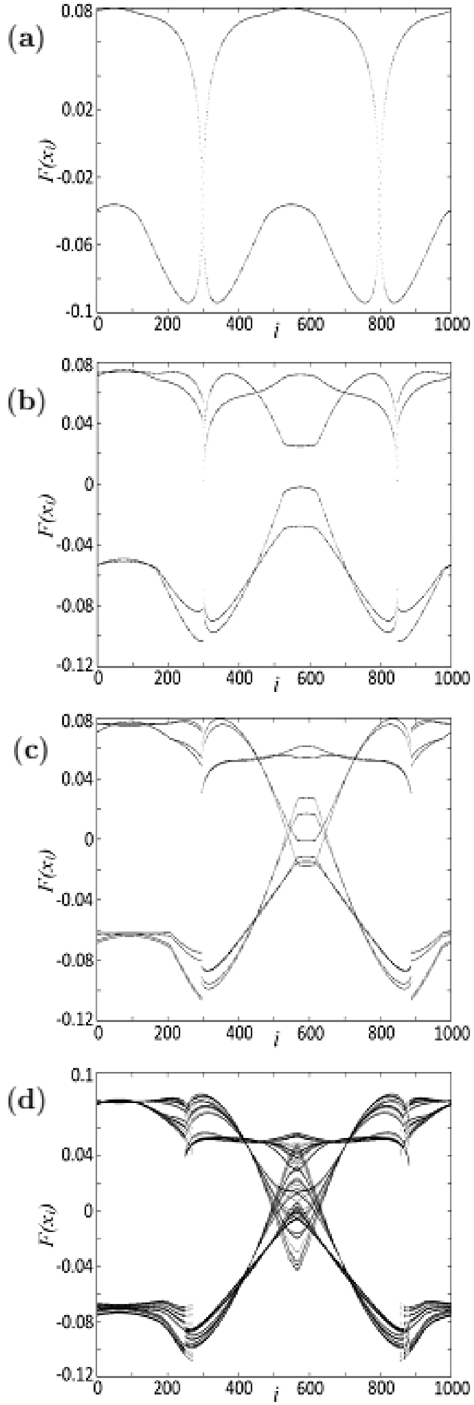


Transition to chimera regime

(E → D)

$0.25 < \sigma < 0.35$

Space-time profiles for the coupling function

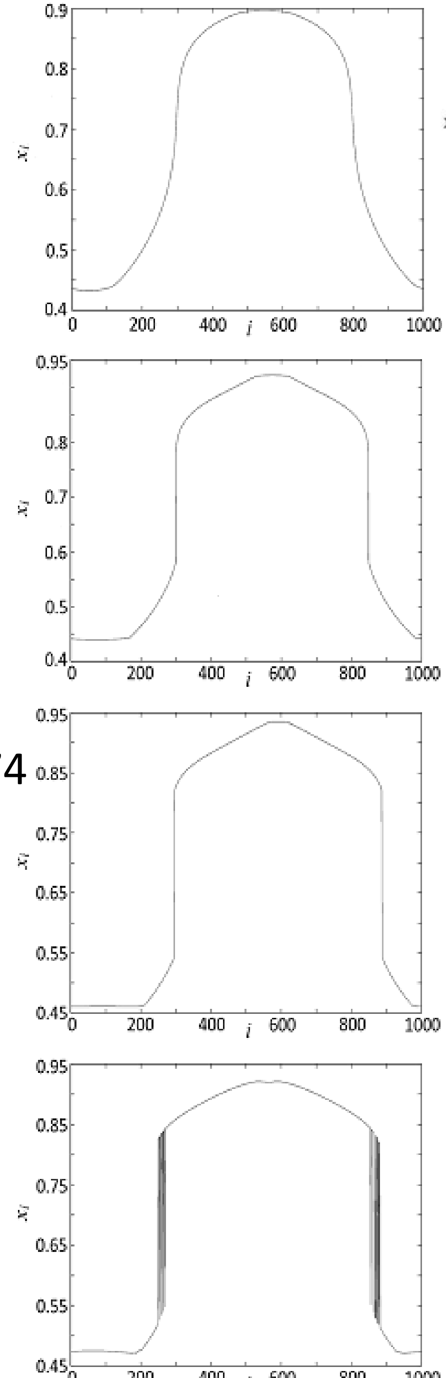


σ = 0.43
2-cycle

σ = 0.38
4-cycle

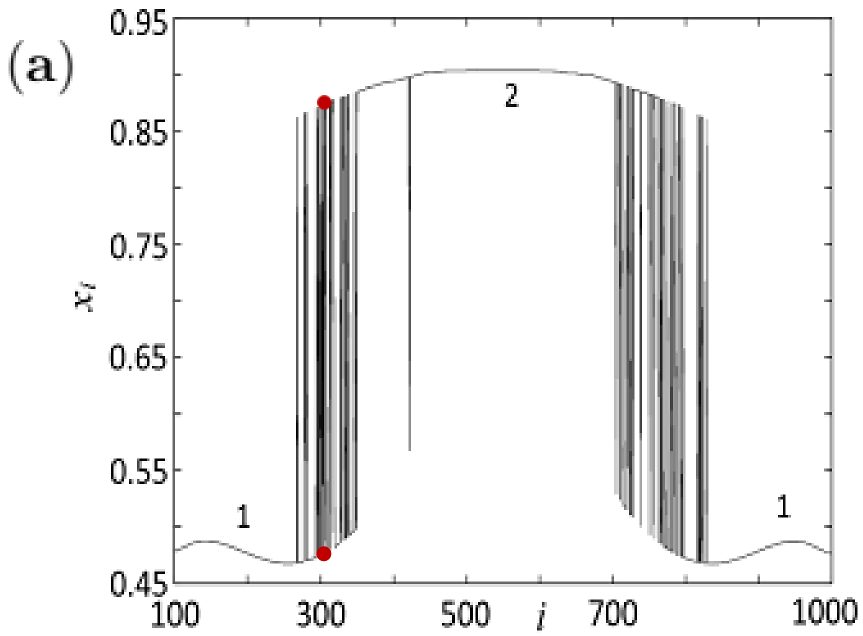
σ = 0.3574
8-cycle

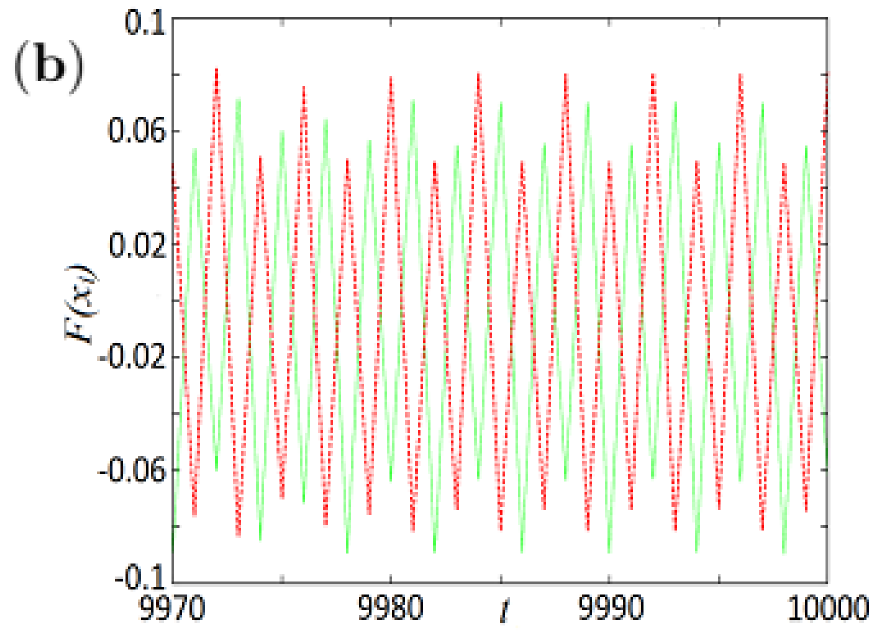
σ = 0.35
chaos origin



Snapshots of the network dynamics

Phase chimera state





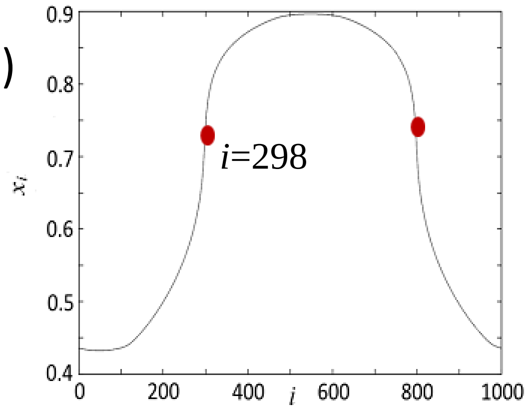
Snapshot (a) of the dynamics of the coupled logistic maps (1) and time series of the coupling function $F(x_i)$ for the 318th element (solid green line) and the 319th element (dotted red line) of the ring (b). The coupling strength is chosen as $\sigma = 0.27$.

Elements in regions 1 and 2 demonstrate **weakly chaotic oscillations which are close to the 2-cycle and shifted in phase by one iteration.**

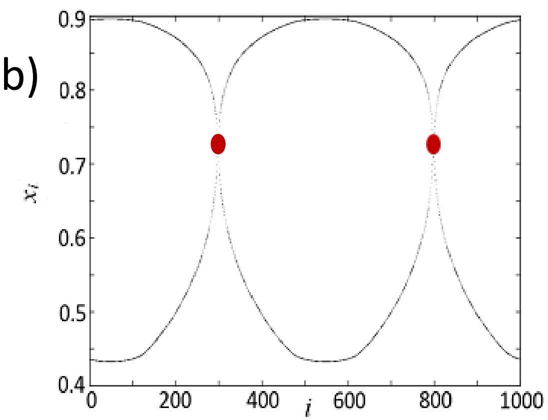
Phase chimera state: **random switchings** between the in-phase (region 1) and anti-phase (region 2) oscillations.

Phase shift along the ring

(a)

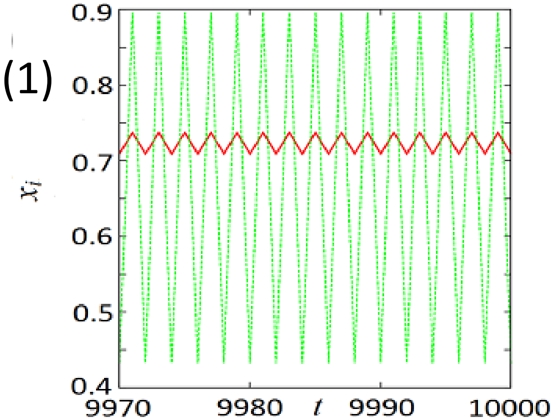


(b)

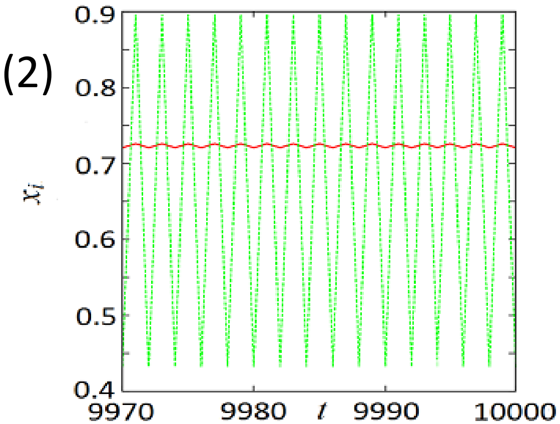


Snapshot (a) and space-time profile (b) for σ = 0.43 (2-periodic oscillation regime in the ring elements)

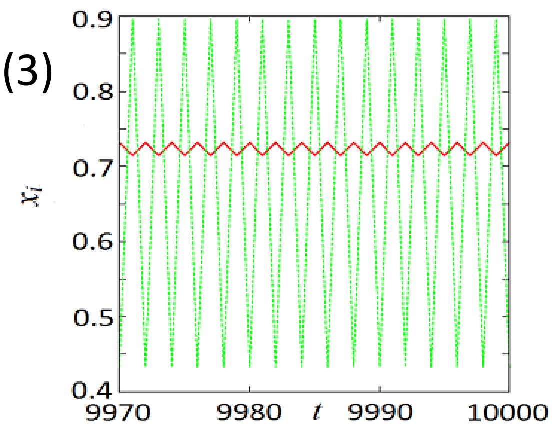
(1)



(2)



(3)



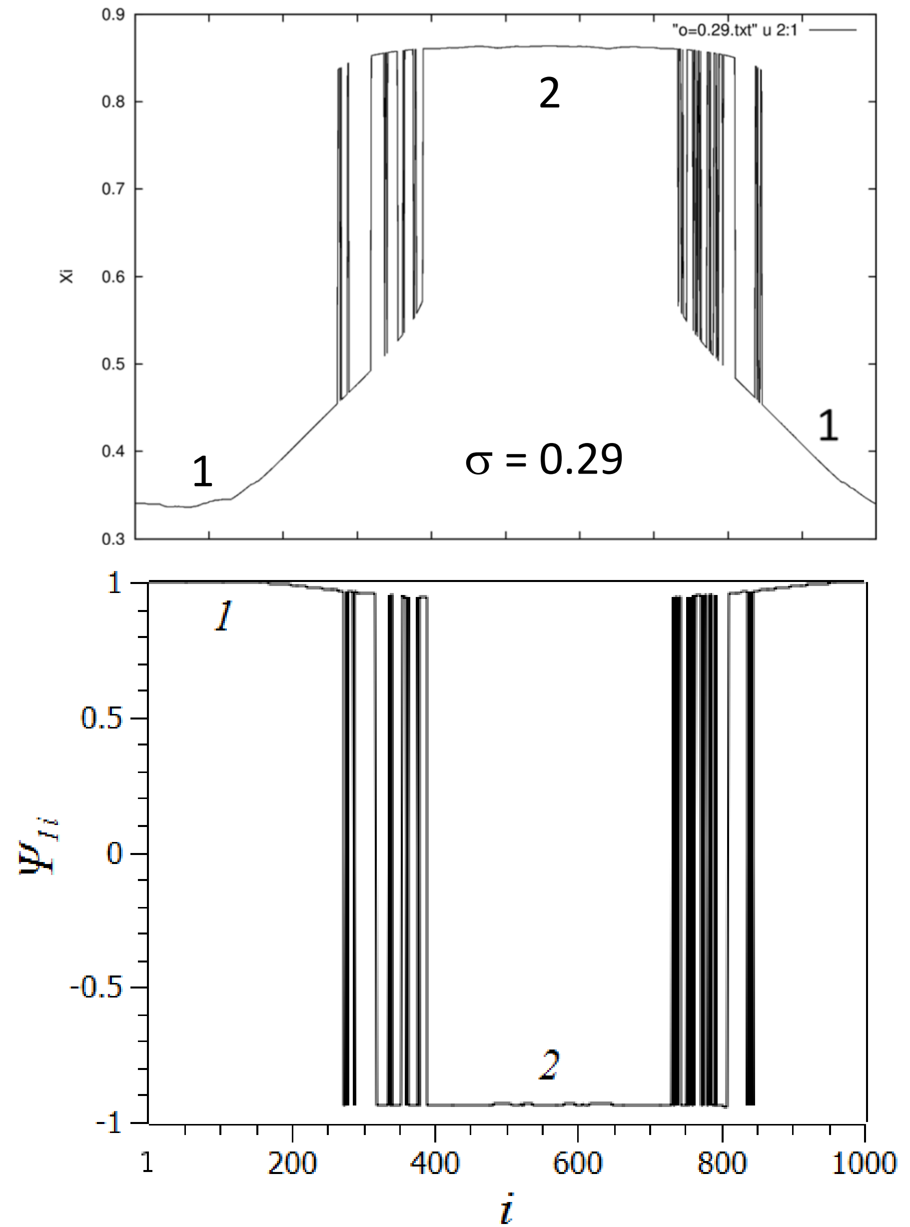
Time series for
***i* =48**

***i* =297** (1)
***i* =298** (2)
***i* =299** (3)

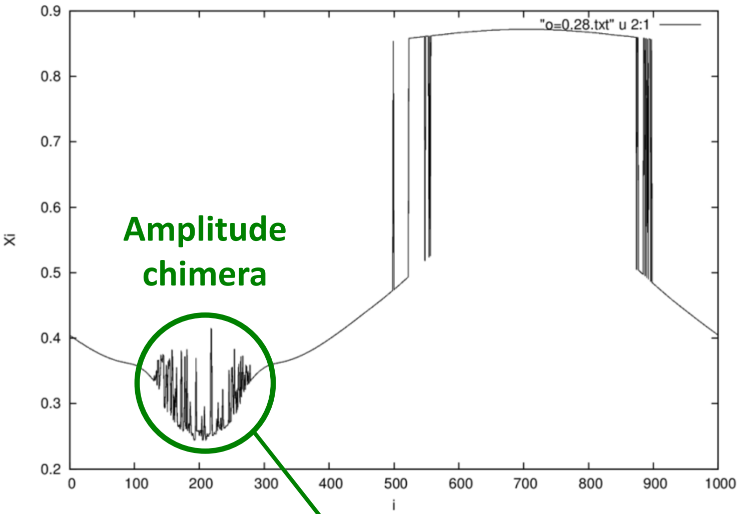
Phase is shifted when passing from *i* =297 to *i* =299

Cross-correlation in the phase chimera regime

The CCC in the regime of phase chimera changes between either +1 or -1 and characterizes random switchings between the in-phase (+1) and anti-phase (-1) oscillations.

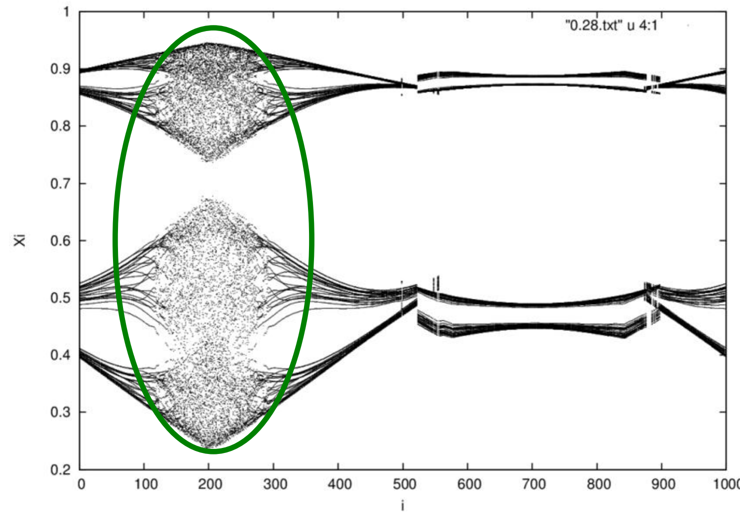


Amplitude chimera state (region D): $0.25 < \sigma < 0.29$

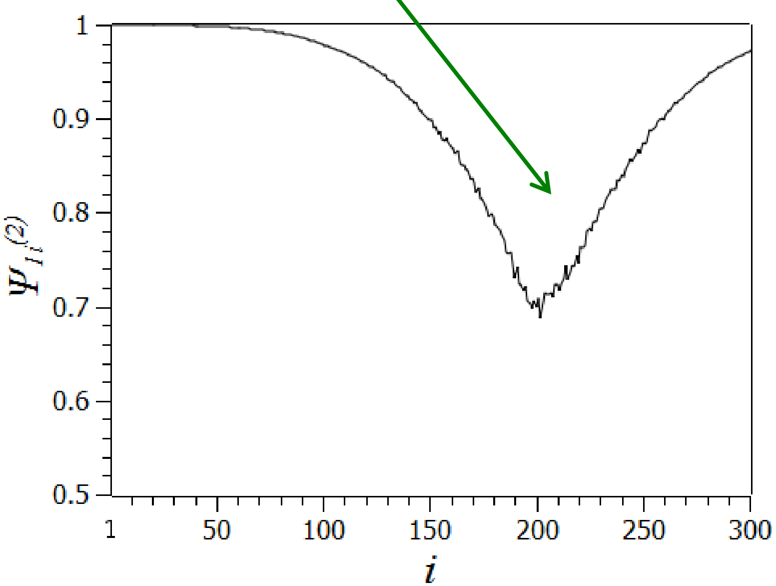


Snapshot

σ = 0.28



Space-time profile



Cross-correlation

Amplitude chimera state appears when the cluster of elements (**$120 < i < 290$**) demonstrates developed chaotic behavior.

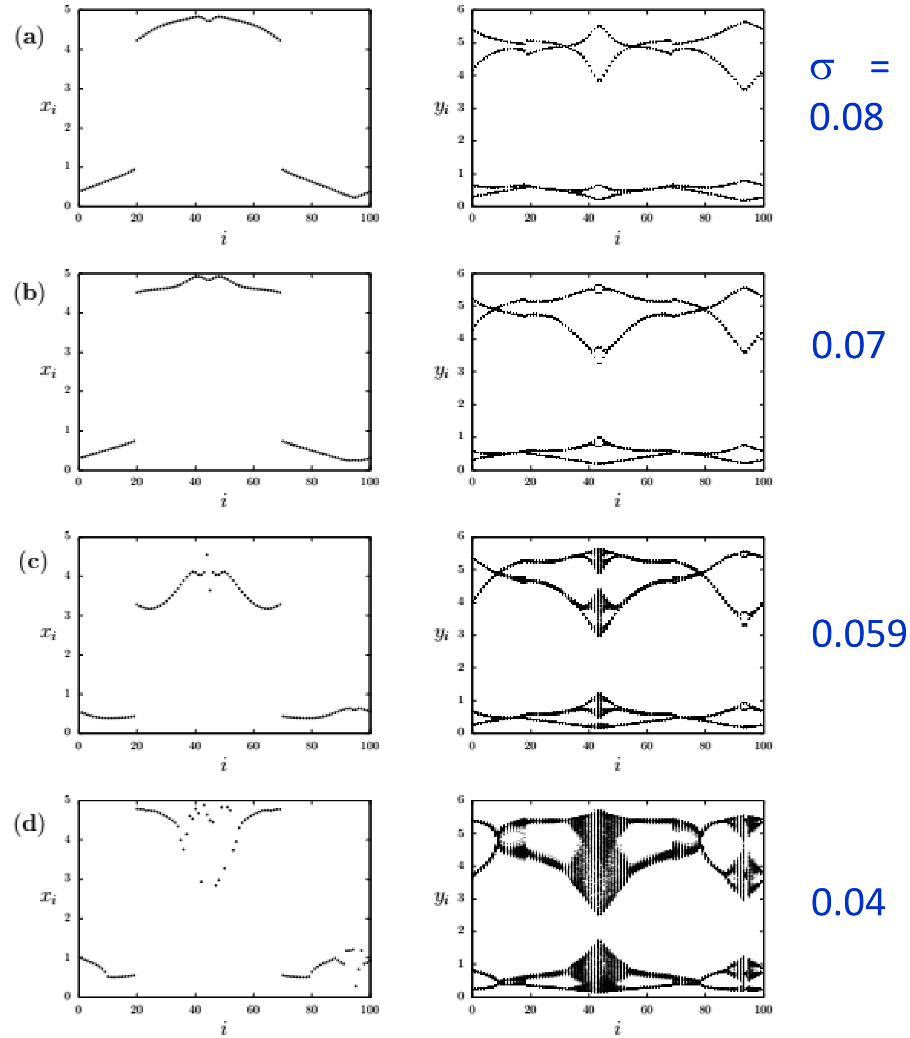
Amplitude chimera in the ensemble of Anishchenko-Astakhov oscillators

$$\frac{dx_i}{dt} = mx_i + y_i - x_i z_i + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(x_j - x_i),$$

$$\frac{dy_i}{dt} = -x_i + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(y_j - y_i),$$

$$\frac{dz_i}{dt} = g[\Phi(x_i) - z_i], \quad \Phi(x_i) = \frac{x_i}{2}(x_i + |x_i|) .$$

m = 1.49, g = 0.2, r = 0.25, N = 100.

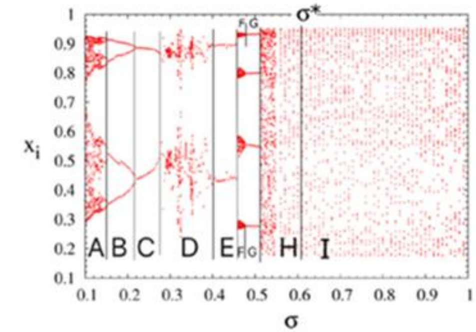


Amplitude chimera birth.

Snapshots x_i (left column) and space-time profiles (right column) for y_i in the Poincare section ($x_i = 0$) for decreasing coupling strength.

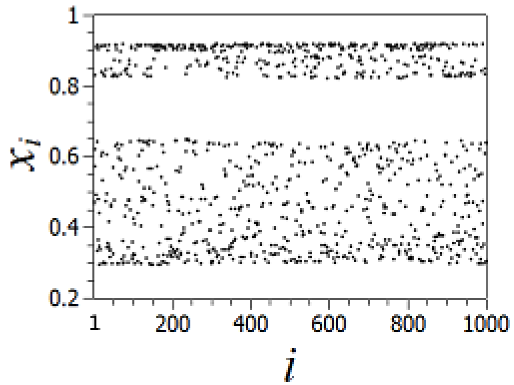
S.A. Bogomolov, **A.V. Slepnev**, G.I. Strelkova, E. Schöll, and V.S. Anishchenko. Commun. Nonlinear Sci. Numer. Simulat. **43**, 25-36 (2017)

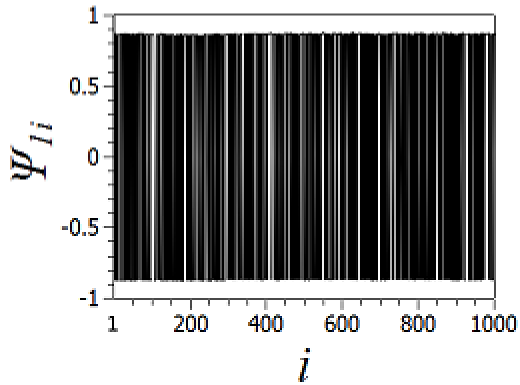
Transition to spatio-temporal chaos
(region A)



σ < 0.13

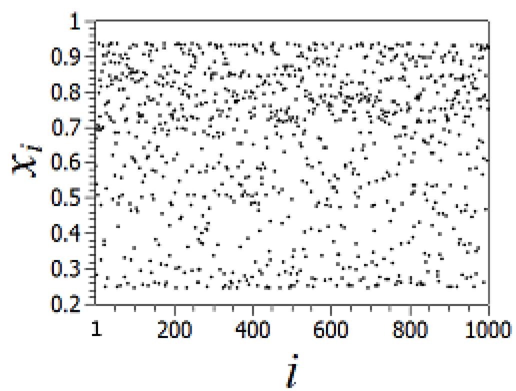
The individual oscillators behave chaotically in time and are completely desynchronized. The spatial behavior is fully irregular (incoherent).
When decreasing coupling strength the chaotic dynamics develops as a result of merging bifurcations.

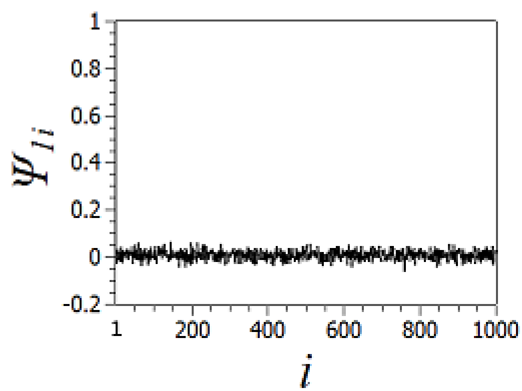




2-band chaotic set (σ =0.1)
The phase trajectory switches between the parts regularly in time and the CCC oscillates at the level ±0.87.

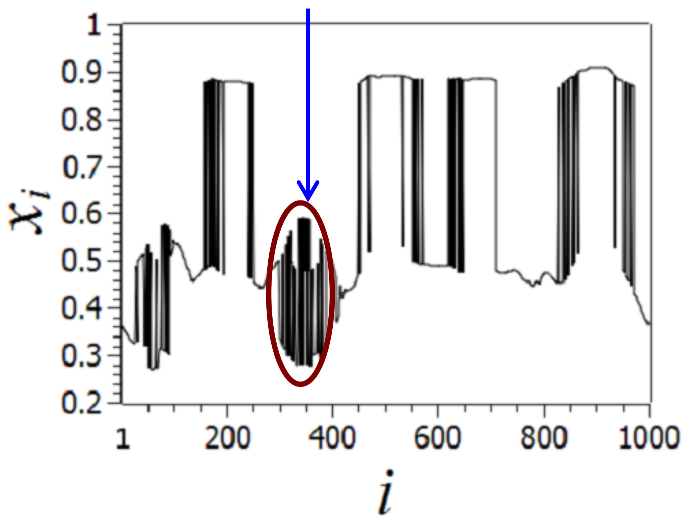
Snapshot **Cross-correlation**



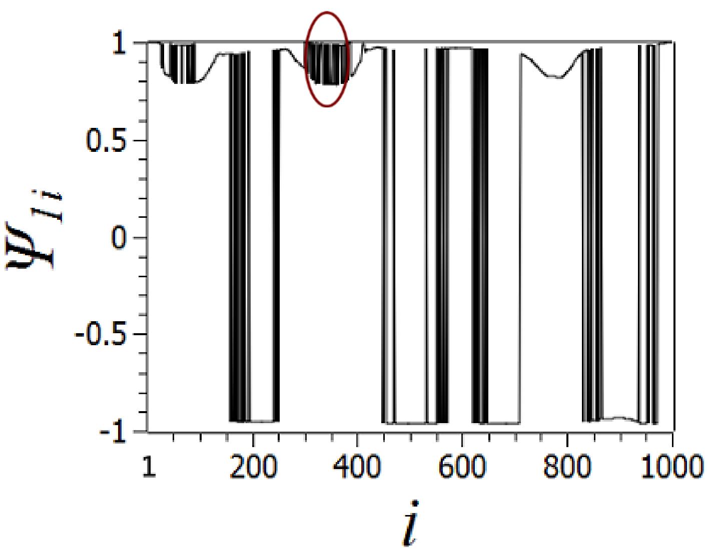


Single-band (developed) chaotic set (σ = 0.05)
The CCC almost vanishes.

Temporally intermittent chimera structure

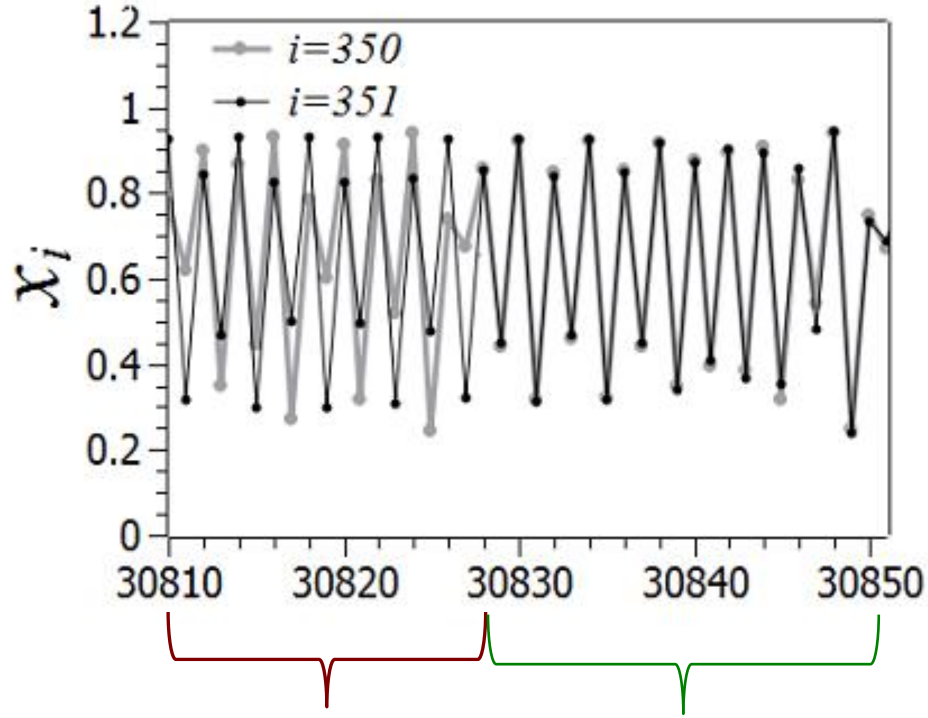


Snapshot



Cross-correlation

Amplitude-phase chimera intermittency for
r = 0.08, σ = 0.25



Phase chimera Amplitude chimera

The intermittency process is random in time.

T.E. Vadivasova, G.I. Strelkova, **S.A. Bogomolov**, and V.S. Anishchenko. CHAOS, **26** (2016).

4. Global transition "coherence-incoherence"

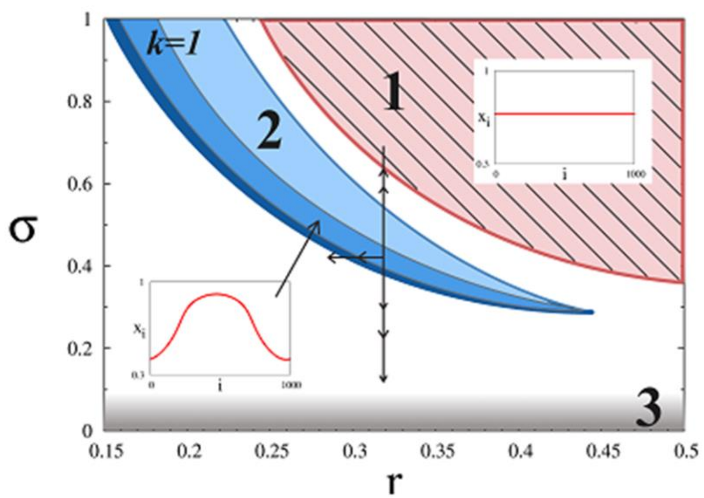
System equation $x_i^{t+1} = f(x_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(x_j^t) - f(x_i^t)]$ with $f(x_i) = ax_i(1 - x_i)$

can be rewritten as follows:

$$x_i^{t+1} = a(1-\sigma)x_i^t(1-x_i^t) + \frac{\sigma}{2P} \sum_{j=i-P,\, j\neq i}^{i+P} ax_j^t(1-x_j^t) \qquad (2)$$

Two limit cases for σ:

1. **σ → 1: the first term in (2) vanishes. The second term describes synchronous chaotic oscillations of the network (region 1).**
2. **σ → 0: the second term in (2) vanishes. The system demonstrates spatio-temporal chaotic regime (region 3).**



When 0 < σ < 1 (in our simulation 0.2 < σ < 0.43), phase, amplitude and amplitude-phase chimera states are realized.

Conclusion

- We have shown that the chimera states can be obtained only in networks of chaotic non-hyperbolic systems and cannot be found in networks of hyperbolic (quasi-hyperbolic) systems.
- The appearance and existence of chimera states in a ring of non-locally coupled chaotic oscillators can be described by using two basic models, namely, the Henon map and the Lozi map as partial elements.
- The global transition “coherence – incoherence” has been explored in detail in the ring of non-locally coupled logistic maps operating in the chaotic regime.
- The peculiarities of stability loss of complete chaotic synchronization and transition to the regime of partial chaotic synchronization have been established.
- Conditions for the appearance of phase and amplitude chimera states have been studied.
- The effect of time intermittency between the phase and amplitude chimeras has been revealed.
- Cross-correlations have been analyzed for all types of chimera states.

Our papers

1. N. Semenova, A. Zakharova, E. Schöll, V. S. Anishchenko, Europhys. Lett. **112** (2015) 40002.
2. N. Semenova, A. Zakharova, E. Schöll, and V. Anishchenko, AIP Conference Proceedings 1738, 210014 (2016); doi: 10.1063/1.4951997.
3. S.A. Bogomolov, G.I. Strelkova, E. Schöll, and V.S. Anishchenko, Amplitude and phase chimeras in an ensemble of chaotic oscillators. Tech. Phys. Lett. **42**(7), 763-766 (2016).
4. S.A. Bogomolov, A.V. Slepnev, G.I. Strelkova, E. Schöll, and V.S. Anishchenko, Mechanisms of Appearance of Amplitude and Phase Chimera States in Ensembles of Nonlocally Coupled Chaotic Systems. Commun. Nonlinear Sci. Numer. Simulat. **43**, 25-36 (2017), http://dx.doi.org/10.2016/j.cnsns.2016.06.024
5. T.E. Vadivasova, G.I. Strelkova, S.A. Bogomolov, and V.S. Anishchenko, Correlation analysis of the coherence-incoherence transition in a ring of nonlocally coupled logistic maps. CHAOS, **26** (2016).

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Thank you very much for your attention!