Holographic approach to heavy-ion collisions

Irina Aref'eva

Steklov Mathematical Institute, RAS

Based on I.A, ``Holography for Heavy Ions Collisions at LHC and NICA, arXiv:1612.08928

May 29 - June 3, 2017
Lebedev Institute,
Moscow, Russia

Outlook

- **Physical picture of Quark-Gluon Plasma in heavy-ion collisions**

- **Why holography?**

- **Results from holography:**

  Fit experimental data via holography:

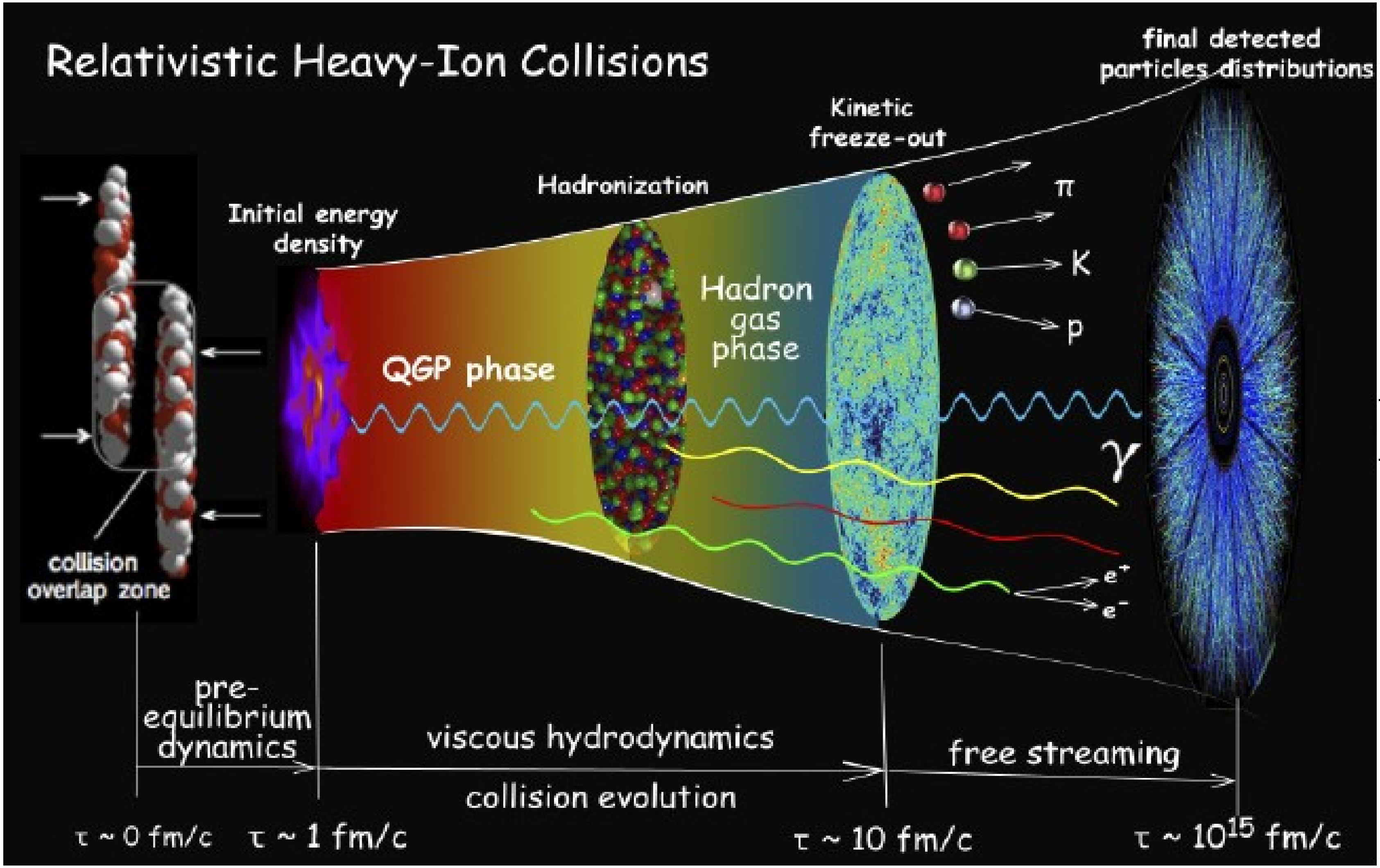
  top-down (top=string theory)
  bottom-up (bottom=5-dim GR+matter)

  Transport coefficients, eta/s, thermalization time, multiplicity,….

  Predict new data (form of QCD Phase Diagram)

- **What is special for NICA (Nuclotron-based Ion Collider fAcility)**

Heavy-Ion Collisions



Picture from: P.Sorensen, C.Shen

Experiments: Heavy Ions collisions produced a medium

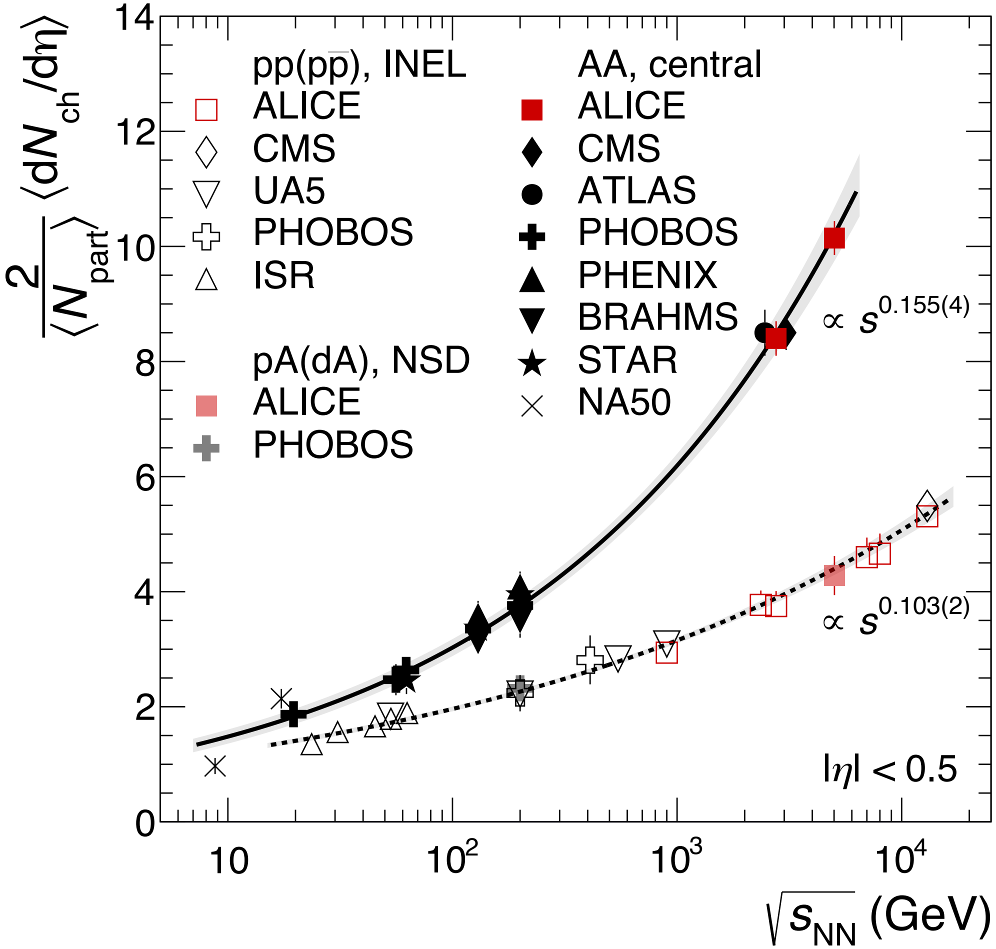
There are strong experimental evidences that RHIC or LHC have created some medium which behaves collectively:

- **modification of particle spectra (compared to p+p)**
- **jet quenching**
- **high p_T-suppression of hadrons**
- **elliptic flow**
- **suppression of quarkonium production**

From observations in HIC

- QGP strong interacting fluid
- Measurement of energy lost (jet quenching, R_{AA}-factor, J/Psi suppressions
- Transport coefficients, extremely small eta/s
- Phase transition (still near small mu)
- Energy dependence of the total multiplicity $s^{0.155}$
- Thermalization time
- Direct photons (electric conductivity)

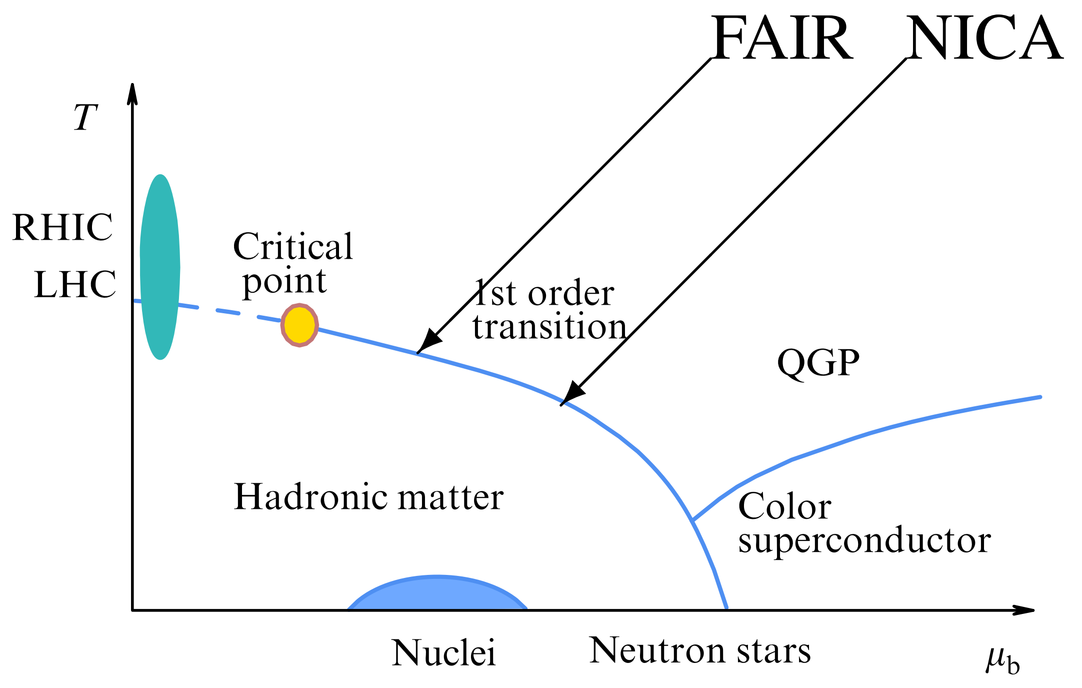
Multiplicity



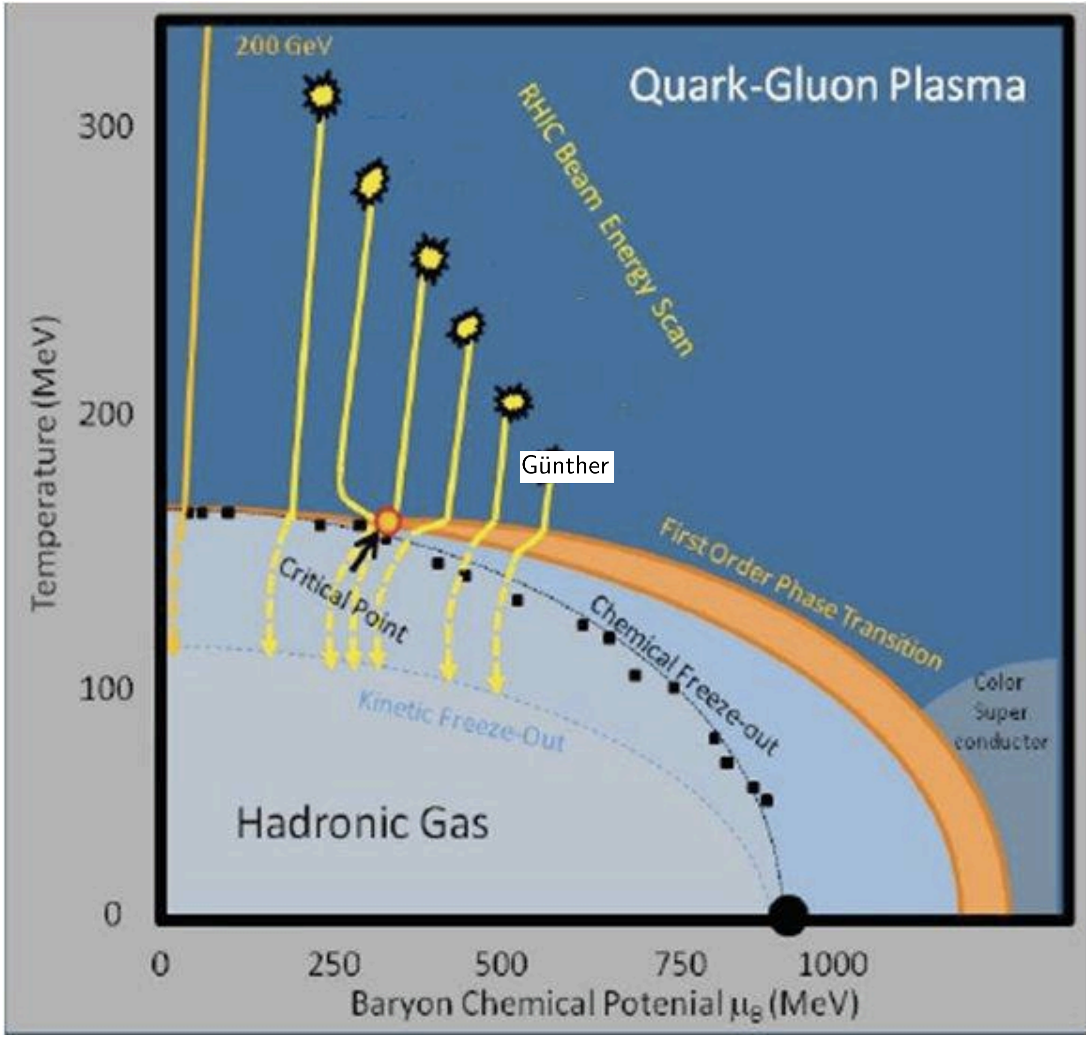
Plot from1512.06104 (ALICE).

$$\mathcal{M}_{LHC} \sim s^{0.155}$$

QCD Phase Diagram



The (T, μ_B)-phase diagram of QCD



Smeared phase transition

From Jana Gu¨nther, WB collaboration, 2016

QGP as a strongly coupled fluid

- **Conclusion from the RHIC and LHC experiments: appearance of QGP (not a weakly coupled gas of quarks and gluons, but a strongly coupled fluid).**

- This makes perturbative methods inapplicable

- The lattice formulation of QCD does not work for description of QGP formation, since we have to study real-time phenomena.

- This has provided a motivation to try to understand the dynamics of QGP through the **gauge/string duality**

Holography:

Connection between

a strongly coupled quantum field theory in a 4-dimensional spacetime

and

a 5-dimensional classical gravity in a special background

In this talk -- bottom-up approach

Starting point - 5-dim background

Background

$$ds^2 = \frac{b(z)}{z^2}\left((-f(z)dt^2 + dx^2) + z^{2-2/\nu}(dy_1^2 + dy_2^2) + \frac{dz^2}{f(z)}\right)$$

$$b(z) = \exp(\frac{cz^2}{2}) \quad f = 1 - (\frac{1}{z_h^{2/\nu+2}} + q^2 z_h^2) z^{2/\nu+2} + q^2 z^{2/\nu+4}$$

Generalization of

1) b(z): O. Andreev, V. Zakharov, Phys.Rev, D749 (2006); JHEP 0704(2007)

Alternative: Gubser et al 0804.0434, 1108.229…

Kiritsis et al, 0903.2859, Evans et al 1002.1885

2) Charge: P.Colangelo, F.Giannuzzi, S.Nicotri, 1008.3116

Y.Yang, P.H.Yuan, 1506.05930

3) Anizotropy: I.A, A. Golubtsova, JHEP 1504 (2015) 011

(this anisotropy reproduces the energy dependence of multiplicity),

Multiplicity with anisotropic (Lifshitz-like) background

$$S = \int d^5x \sqrt{|g|} \left(R[g] + \Lambda - \frac{1}{2}\partial_M \phi \partial^M \phi - \frac{1}{4} e^{\lambda\phi} F_{(2)}^2 \right)$$

$$ds^2 = \frac{1}{z^2} \left(-dt^2 + dx^2 + z^{2-2/\nu} (dy_1^2 + dy_2^2) + dz^2 \right)$$

Shock domain walls collision: ENTROPY

$$\mathcal{M} = \frac{\nu}{2G_5} (8\pi G_5)^{1/(1+\nu)} s^{\frac{1}{2+\nu}}$$

To get $\mathcal{M}_{LHC} \sim s^{0.155(4)}$ $\nu = 4.45$

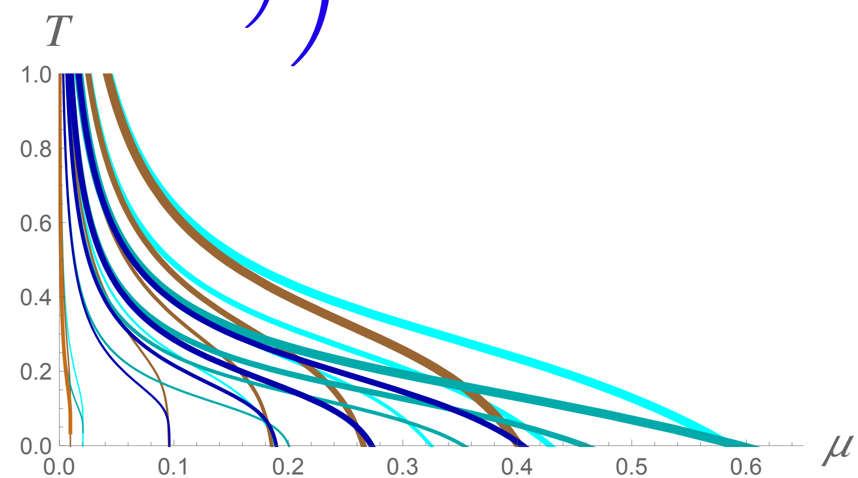
Thermodynamics (in 5 -dim)

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z=z_h} = \frac{1}{2\pi}\,\frac{\frac{1}{\nu}+1}{z_h}\left(1-\frac{1}{\frac{1}{\nu}+1}q^2 z_h^{\frac{2}{\nu}+4}\right)$$

The quark chemical potential $A_0(0) = i\mu$

$$A_0(z) = i\left(\mu - \frac{4\sqrt{3}\,q}{c}\left(1-e^{-cz^2/4}\right)\right)$$

$$T = \frac{\left(\frac{1}{\nu}+1\right)}{2\pi z_h}\left(1-\frac{c^2\mu^2 z_h^{\frac{2}{\nu}+4}}{3\left(\frac{1}{\nu}+1\right)\left(1-e^{-\frac{cz_h^2}{2}}\right)^2}\right)$$

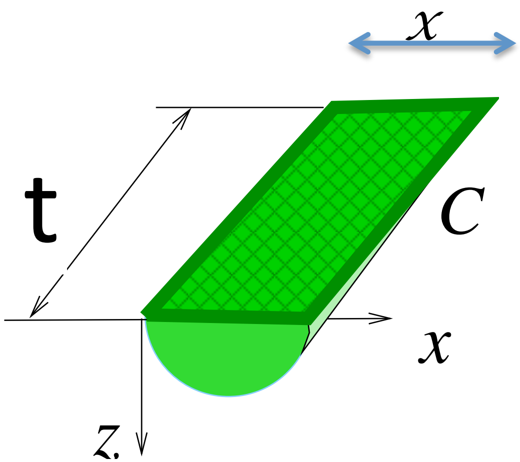


Isotropic (cyan for c=0.42 and brown for c=0.9$ lines)
Anisotropic (darker cyan for c=0.42 and darker blue for c=0.9$ lines with \nu=4
and different values of the charge q=0.01, 0.1, 0.3, 0.5.

Non-sharp confinement/deconfinement phase transition in holography for the anizotropic model

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along x-direction



t>>x

$$W(T,X) = \langle \mathrm{Tr}_F\, e^{i\oint_{T\times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T},$$

Holography for a probe

$$S_{xt} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}.$$

$$S_{xt} = \frac{T}{2\pi\alpha'} \int \frac{b(z)}{z^2} \sqrt{f(z) + z'^2}\, dx.$$

The recipe by Maldacena ('98), Rey et al ('98), Sonnenschein et al ('98)

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along x-direction

$$S_{xt} = \frac{T}{2\pi\alpha'} \int \frac{b(z)}{z^2}\sqrt{f(z) + z'^2}\, dx.$$

$$V_x(z) = \frac{b(z)}{z^2}\sqrt{f(z)}$$

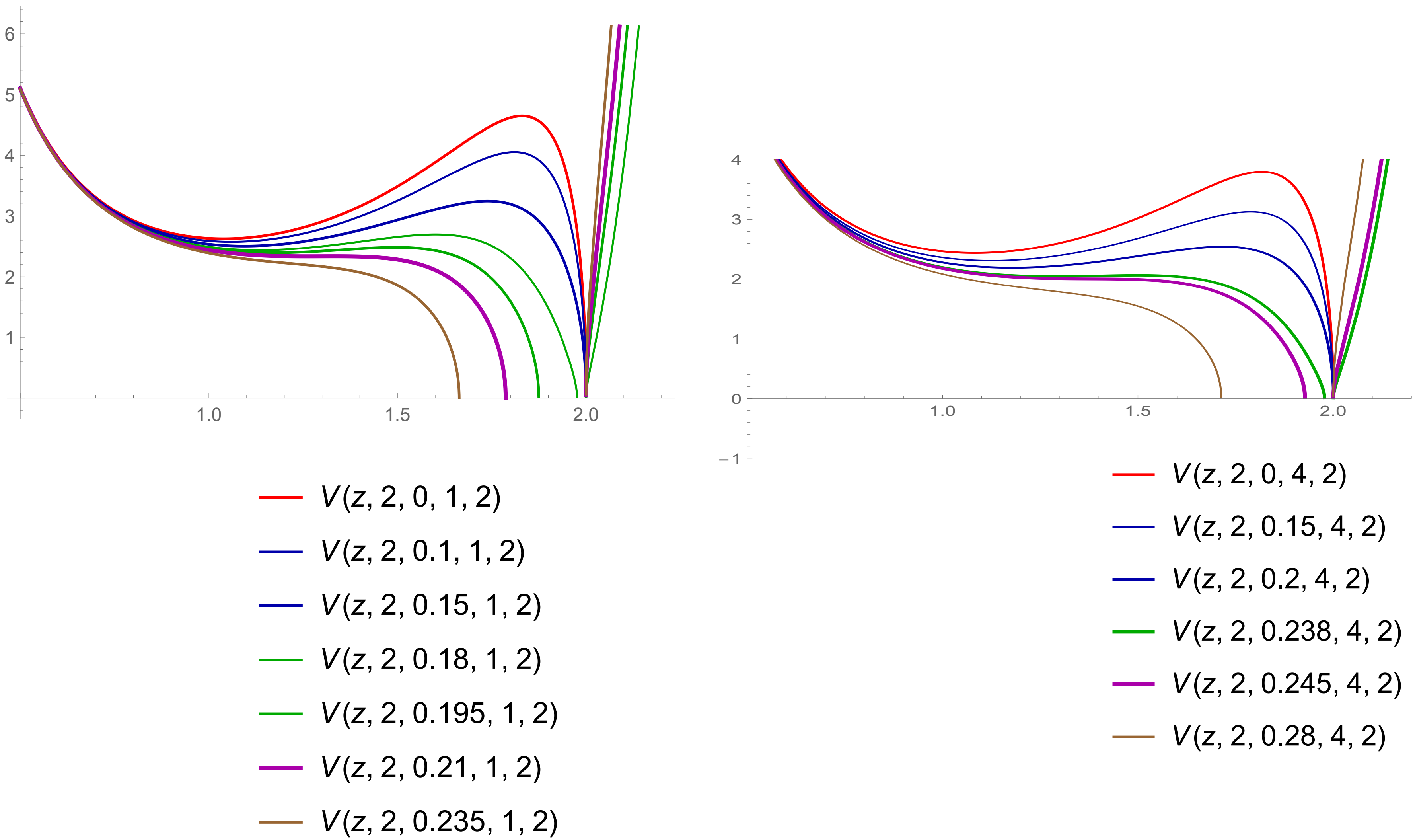
Symmetric parameterization $z(\pm\ell) = 0 \quad z(0) = z_* \quad z'(0) = 0$

Distance between endpoints of the string

$$L_x = 2\int_{\infty}^{z_*} \frac{dz}{z'} = 2\int_0^{z_*} \frac{dz}{\sqrt{f(z)}\sqrt{\left(\frac{V_x^2(z)}{V_x^2(z_*)} - 1\right)}}$$

$z_* < z_{h_{i_0}}$ the smallest of the horizons $f(z) > 0 \;\; \text{for} \;\; 0 < z < z_*$

$$V[z, c, q, \nu, z_h]$$



6
5
4
3
2
1
1.0
1.5
2.0
V(z, 2, 0, 1, 2)
V(z, 2, 0.1, 1, 2)
V(z, 2, 0.15, 1, 2)
V(z, 2, 0.18, 1, 2)
V(z, 2, 0.195, 1, 2)
V(z, 2, 0.21, 1, 2)
V(z, 2, 0.235, 1, 2)
4
3
2
1
0
−1
1.0
1.5
2.0
V(z, 2, 0, 4, 2)
V(z, 2, 0.15, 4, 2)
V(z, 2, 0.2, 4, 2)
V(z, 2, 0.238, 4, 2)
V(z, 2, 0.245, 4, 2)
V(z, 2, 0.28, 4, 2)

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along x-direction

$$E_x = \int_0^{z_*} \frac{dz}{z^2}\left[\frac{b(z)V(z)}{\sqrt{V^2(z)-V^2(z_*)}} - 1\right] - \frac{1}{z_*} + m^{\frac{\nu}{2\nu+2}} e^{c\, m^{-\frac{2\nu}{2\nu+2}}} - \sqrt{\pi c}\,\mathrm{erfi}\left(\sqrt{c}\, m^{-\frac{\nu}{2\nu+2}}\right)$$

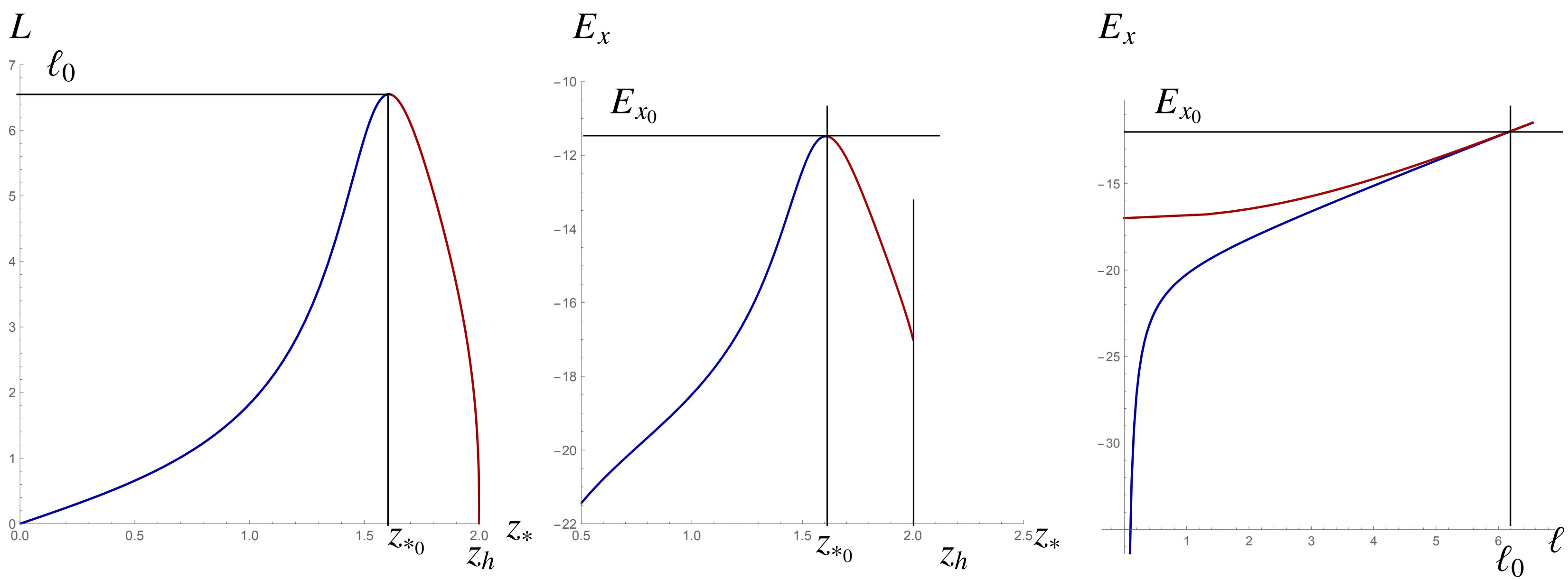
$$z_h = \left(\frac{1}{m}\right)^{\frac{\nu}{2\nu+2}}$$

$$L_x = 2\int_0^{z_*} \frac{dz}{\sqrt{f(z)}\sqrt{\left(\frac{V_x^2(z)}{V_x^2(z_*)} - 1\right)}}$$

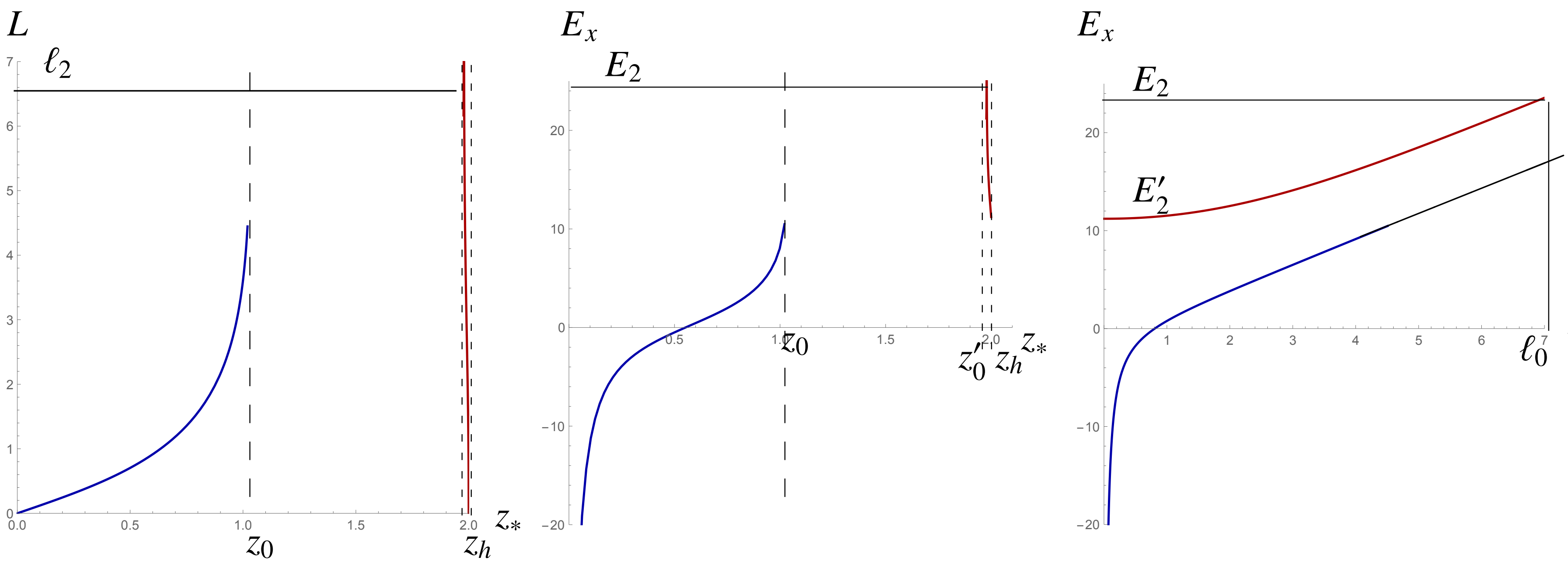
Above the critical point (deconfinment)

There is no extremal point for the “potential” in

the interval $0 < z < z_{h_{i_0}}$



Below the critical point (confinement)



The "potential" is a decreasing function only on the intervals

$0 < z < z_{min}$ and $z_{max} < z < z_{h_{i_0}}$

$$V'(z)|_{z=z_0} = 0, \quad V''(z_0) > 0$$

Similar to analysis performed for the isotropic case:

O. Andreev, V. Zakharov, JHEP 0704(2007)

M.Mia et al, Phys.Lett. B694 (2011)460 (2011)

P.Colangelo, F.Giannuzzi, S.Nicotri, Phys.Rev. D83 (2011) 035015

Few calculations

Backup slide

$$\sqrt{\frac{V(z)}{V(z_0)}^{\,2} - 1} = \sqrt{\frac{V''(z_0)}{V(z_0)}}\,(z - z_0) + \mathcal{O}(z - z_0)^2$$

$$L_1 \underset{z_* \sim z_0}{\sim} -2\sqrt{\frac{V(z_0)}{f(z_0)V''(z_0)}}\ln(z_0 - z_*)$$

$$S_{xt} \underset{z_* \sim z_0}{\sim} -\frac{\mathcal{T}}{2\pi\alpha'}\frac{b(z_0)}{z_0^2}\frac{1}{\sqrt{\frac{V''(z_0)}{V(z_0)}}}\ln(z_0 - z_*)$$

$$S_{xt} \underset{L \to \infty}{\sim} \frac{\mathcal{T}}{\pi\alpha'}\frac{L_1}{2}\frac{b(z_0)\sqrt{f(z_0)}}{z_0^2}$$

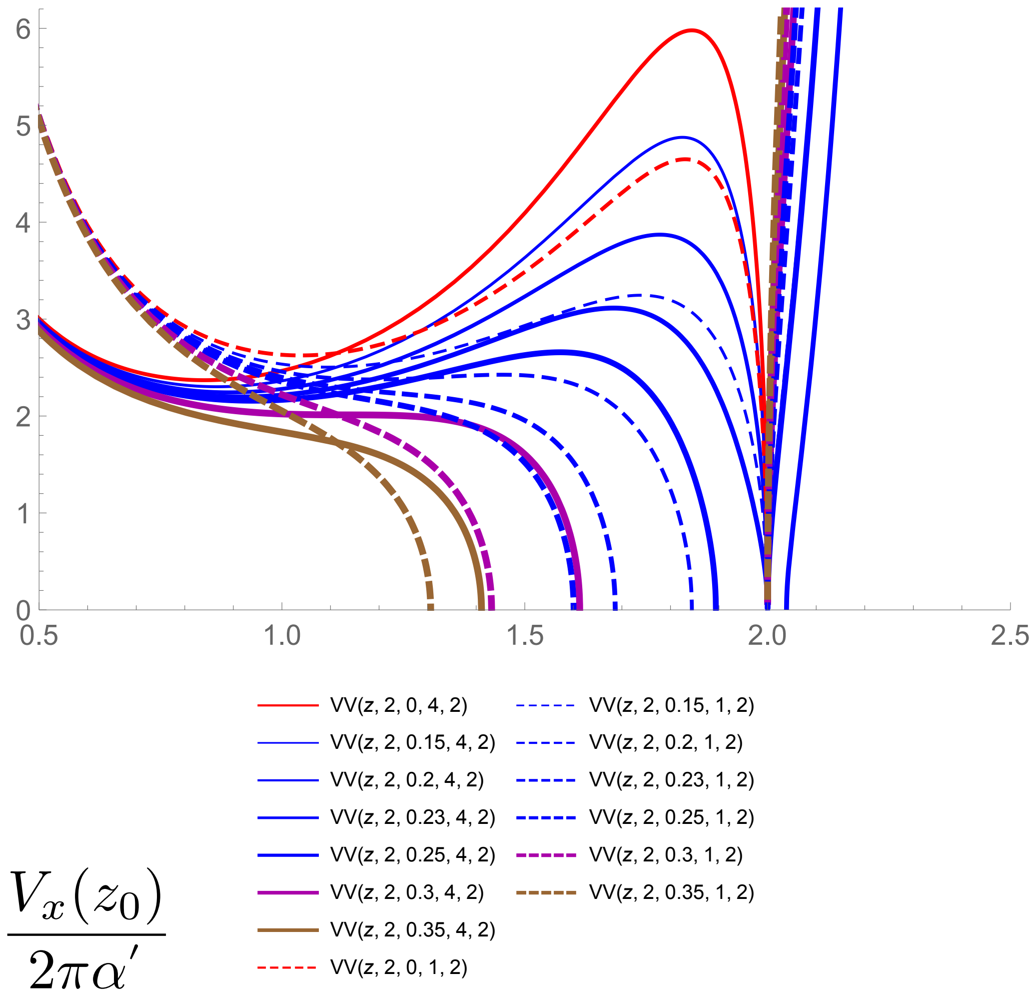
$$\sigma_x = \frac{V_x(z_0)}{2\pi\alpha'}$$

Temporal Wilson loop in the charged quark confinement background

Energy between quarks located along transversal y-direction

$$S_{yt} = \frac{\mathcal{T}}{2\pi\alpha'}\int \frac{b(z)}{z^2}\sqrt{z^{2-2/\nu}f(z)+z'^2}\,dx.$$

The "potential"



$$V_y(z) = \frac{b(z)\sqrt{f(z)}}{z^{1/\nu+1}}$$

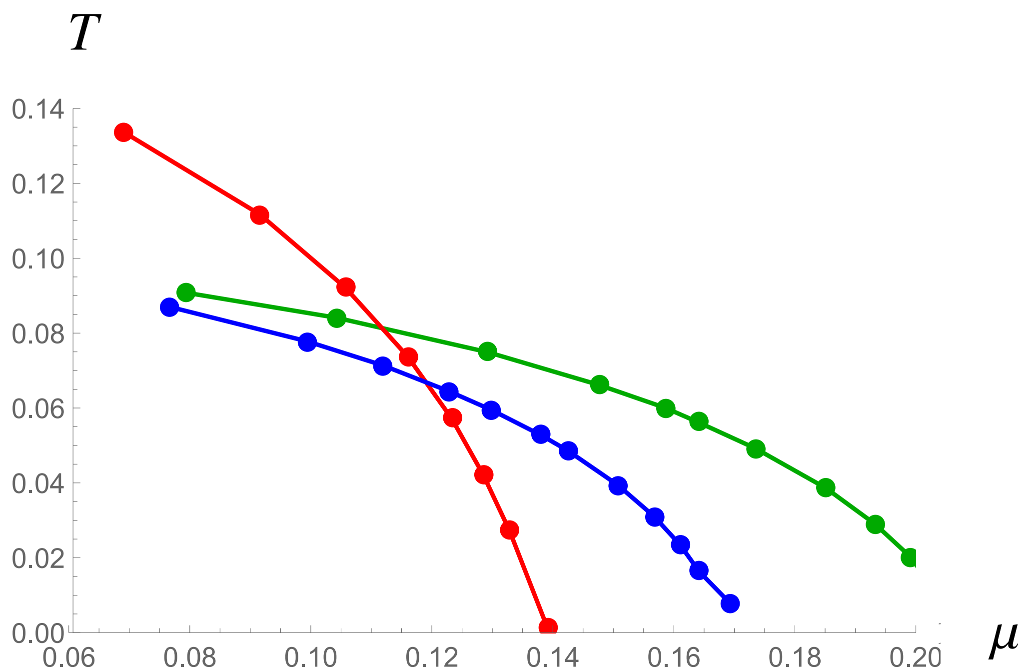
$$\sigma_y = \frac{V_y(z_0)}{2\pi\alpha'}$$

$$V_x(z) = \frac{b(z)}{z^2}\sqrt{f(z)}$$

(dashed lines)

$$\sigma_x = \frac{V_x(z_0)}{2\pi\alpha'}$$

Holographic anisotropic QCD phase diagram



Phase transitions lines divide the plane in two regions:
a hadron phase near the origin,
and a deconfined phase beyond the curve.
The red line corresponds to the isotropic case.

Anisotropic case nu=4:

the blue line corresponds to **quarks located along the longitudinal x-direction** ,
the green line corresponds to **quarks located along the transversal y-direction**

Background

$$ds^2 = \frac{b(z)}{z^2}\left(\left(-f(z)dt^2 + dx^2\right) + z^{2-2/\nu}(dy_1^2 + dy_2^2) + \frac{dz^2}{f(z)} \right)$$

Question: **can we get this metric as solution to E.O.M. that follows from the action**

I.A., Kristina Rannu, work in progress

$$S = \int \frac{d^5x\sqrt{-g}}{16\pi G_5}\left[R - \frac{f_1(\phi)}{4}\,F_{(1)}^2 - \frac{f_2(\phi)}{4}\,F_{(2)}^2 - \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - V(\phi)\right],$$

Ansatz:

$$\phi = \phi(z), \qquad A_\mu^{(1)} = A_0(z), \qquad F^{(2)} = q\ dx^2 \wedge dx^3$$

Special case
(preliminary results):

$$f_1 = z^{-2+\frac{2}{\nu}} \qquad b(z) = e^{\frac{cz^2}{2}}$$

$$A_0(z) = \mu\, \frac{e^{-\frac{cz^2}{4}} - e^{-\frac{cz_H^2}{4}}}{1 - e^{-\frac{cz_H^2}{4}}} \qquad q = \frac{\mu\, e^{\frac{cz_H^2}{4}}}{2\, z_H^{1/\nu}\left(1 - e^{\frac{cz_H^2}{4}}\right)}\sqrt{\frac{\nu c}{1+\nu}}$$

Blackening function g(z)

$$g(z) = 1 - \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\, \frac{3cz^2}{4}\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\, \frac{3cz_H^2}{4}\right)} - \mu^2\, e^{\frac{cz_H^2}{2}}\, \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\, cz^2\right)}{4\, c^{1/\nu}\left(1 - e^{\frac{cz_H^2}{4}}\right)^2} +$$

$$+\mu^2\, e^{\frac{cz_H^2}{2}}\, \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\, cz_H^2\right)}{4\, c^{1/\nu}\left(1 - e^{\frac{cz_H^2}{4}}\right)^2}\, \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\, \frac{3cz^2}{4}\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\, \frac{3cz_H^2}{4}\right)}$$

$$g(z) \approx 1 - z^{2+\frac{2}{\nu}}\left(\frac{1}{z_H^{2+\frac{2}{\nu}}} + q^2 z_H^{2/\nu}\right) + z^{2+\frac{4}{\nu}} q^2,$$

Backup slide

Others functions

$$f_2 = \frac{(\nu-1)\ z^{-4/\nu} e^{\frac{cz^2}{2}}}{4c^{1/\nu} q^2 \nu^2 \left(1 - e^{\frac{cz_H^2}{4}}\right)^2 \left[\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu},\ \frac{3cz_H^2}{4}\right)\right]} \times$$

$$\times \left(\frac{\nu}{4^{1/\nu}} (cz^2)^{1+\frac{1}{\nu}}\ e^{-cz^2} \left\{ 3^{1+\frac{1}{\nu}}\ e^{\frac{cz^2}{4}} \left[4c^{\frac{1}{\nu}} \left(e^{\frac{cz_H^2}{4}} - 1\right)^2 + \mu^2 e^{\frac{cz_H^2}{2}}\ \Gamma\left(1+\frac{1}{\nu},\ cz_H^2\right)\right] + \right.\right.$$

$$+ \mu^2 e^{\frac{cz_H^2}{2}}\ \Gamma\left(1+\frac{1}{\nu}\right) \left(4^{1+\frac{1}{\nu}} - 3^{1+\frac{1}{\nu}}\ e^{\frac{cz^2}{4}}\right) - 4^{1+\frac{1}{\nu}} \mu^2\ e^{\frac{cz_H^2}{2}}\ \Gamma\left(1+\frac{1}{\nu},\ \frac{3cz_H^2}{4}\right) \Bigg\} +$$

$$+ \left((3cz^2-4)\nu - 4\right) \left\{ \Gamma\left(1+\frac{1}{\nu},\ \frac{3cz_H^2}{4}\right) \left[4c^{\frac{1}{\nu}} \left(e^{\frac{cz_H^2}{4}} - 1\right)^2 + \mu^2 e^{\frac{cz_H^2}{2}}\ \Gamma\left(1+\frac{1}{\nu},\ cz^2\right)\right] + \right.$$

$$+ \Gamma\left(1+\frac{1}{\nu},\ \frac{3cz^2}{4}\right) \left[-\ 4c^{\frac{1}{\nu}} \left(e^{\frac{cz_H^2}{4}} - 1\right)^2 + \mu^2 e^{\frac{cz_H^2}{2}}\ \Gamma\left(1+\frac{1}{\nu}\right) - \mu^2 e^{\frac{czH^2}{2}}\ \Gamma\left(1+\frac{1}{\nu},\ cz_H^2\right)\right] +$$

$$\left.\left. + \mu^2 e^{\frac{cz_H^2}{2}}\ \Gamma\left(1+\frac{1}{\nu}\right) \left[\Gamma\left(1+\frac{1}{\nu},\ cz^2\right) + \Gamma\left(1+\frac{1}{\nu},\ \frac{3cz_H^2}{4}\right) - \Gamma\left(1+\frac{1}{\nu},\ cz_H^2\right)\right] \right\}\right).$$

$$\phi = \int_0^z \frac{d\xi}{\nu\xi} \sqrt{\frac{3}{2}\ \nu^2\ c^2\ \xi^4 - 9\nu^2\ c\xi^2 + 4\nu - 4 + C_4} = C + C_1 \log z + \frac{9c\nu}{8\sqrt{\nu-1}} + (\ldots) z^4 + \ldots$$

Direct photons and electric conductivity

The thermal-photon production from the QGP plays an essential role, since photons after they are produced in HIC almost do not interact with the QGP and, therefore, they give us the local information on heavy ion collisions.

The photon-emission rate is related to the retarded correlator of currents in momentum space

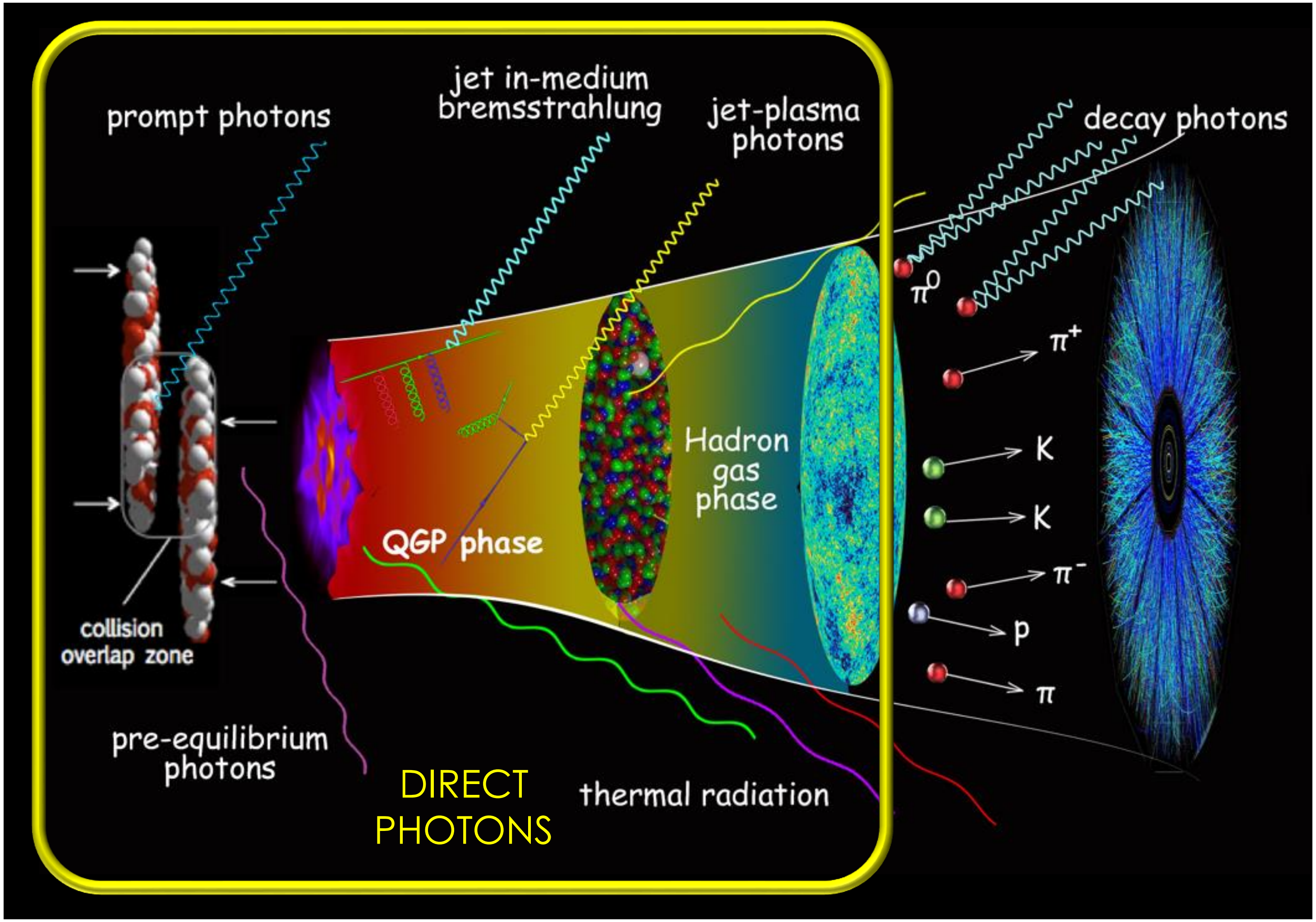
$$G^R_{\mu\nu}(k) = i\int d^4(x-y)e^{ik\cdot(x-y)}\theta(x^0)\langle[J^a_\mu(x), J^b_\nu(0)]\rangle,$$

$$d\Gamma = -\frac{d^3k}{(2\pi)^3}\frac{e^2 n_b(|\mathbf{k}|)}{|\mathbf{k}|}\mathrm{Im}\left[\mathrm{tr}\left(\eta^{\mu\nu}G^{ab\,R}_{\mu\nu}\right)\right]_{k^0=|\mathbf{k}|},$$

S.I.Finazzo and R.Rougemont,
Phys.Rev.D 93, (2016) 034017
I.Iatrakis, E.Kiritsis, C.Shen and D.L.Yang,
arXiv:1609.07208 [hep-ph]

DIRECT PHOTONS

- emerge directly from a particle collison
- represent less than 10% of all detected photons



Experiments can not distinguish between the different sources

[Source: C. Shen, talk at ECT*, Trento 12/2015]

Theoretical models can be used to identify these sources and their relative importance in the spectrum

Direct photons and electric conductivity

$$S_M = -\mathcal{N}\int d^5x\sqrt{-g}\,\frac{V_{dil}(\phi)}{4}F^{MN}F_{MN}$$

$$ds^2 = \frac{b^2(z)}{z^2}\left(-f(z)dt^2 + dx^2 + p(z)(dy_1^2 + dy_2^2) + \frac{dz^2}{f(z)}\right)$$

$$\int d^4x dz\mathcal{V}(z)\left(-\frac{F_{01}^2+F_{02}^2}{fp} - \frac{F_{03}^2}{f} + f\left(\frac{F_{1z}^2+F_{2z}^2}{p} + F_{3r}^2\right) - F_{0z}^2 + \frac{F_{12}^2}{p^2} + \frac{F_{13}^2+F_{23}^2}{p}\right)$$

$$\mathcal{V}(z) = V_{dil}(\phi(z))\frac{b(z)}{z}$$

Boundary conditions

$$\lim_{z\to 0} A_\mu(z,\omega,k) = 1, \qquad \lim_{z\to z_h} A_\mu(z,\omega,k) = 0$$

$$A_\mu(t,\mathbf{x},z) = \int\frac{d^4k}{(2\pi)^4}e^{-i\omega t+i\mathbf{kx}}\mathcal{A}_\mu(z,\omega,k), \quad \mathcal{A}_\mu(z,\omega,k) = A_\mu(z,\omega,k)\,a_\mu(\omega,k)$$

$$k = k_x, \qquad k_{y_1} = k_{y_2} = 0$$

Backup slide

Eqs to solve:

$$E_L'' + \left(\frac{f'}{f}\frac{w^2}{w^2 - fk^2} + \frac{V'}{V}\right) E_L' + \frac{w^2 - fk^2}{f^2} E_L = 0$$

$$E_{\perp,i}'' + \left(-\frac{p'}{p} + \frac{V'}{V} + \frac{f'}{f}\right) E_{\perp,i}' + \frac{w^2 - k^2}{f} E_{\perp,i} = 0, \quad i = 1,2$$

$$E_L = kA_0 + wA_3, \quad E_{\perp,i} = wA_{\perp,i}, \quad i = 1,2.$$

Substitute to

$$S_{boundary} = \int d^4x \left[\frac{f}{w^2 - k^2 f} E_L E_L' + \frac{f}{p\, w^2} E_\perp E_\perp' \right],$$

$$E_\perp E_\perp' \equiv \sum_{i=1,2} E_{\perp,i} E_{\perp,i}'$$

As in isotropic case

Backup slide

$$\zeta_\perp = -\frac{V\,f}{pw}\frac{\partial_r E_i^\perp}{E_i^\perp} \qquad \zeta_L = -\frac{V\,f}{\omega}\frac{\partial_r E_L}{E_L},$$

$$\zeta_\perp' - \frac{w}{f}\left[\frac{p}{V}\,\zeta_\perp^2 + \frac{V}{p}\left(1 - f\frac{k^2}{w^2}\right)\right] = 0.$$

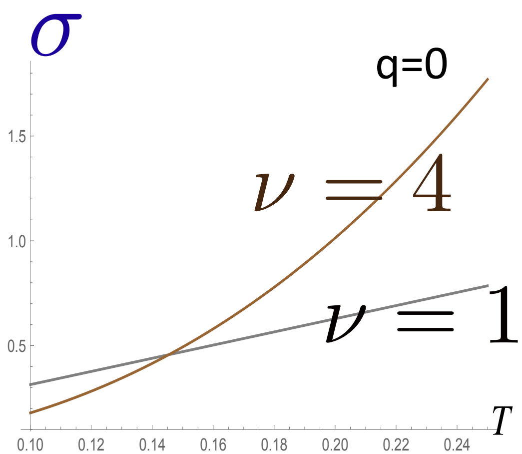
$$\zeta_L' + \frac{k^2 f'}{w^2 - f k^2}\zeta_L - \frac{w}{f}\left[\frac{\zeta_L^2}{V} + V\left(1 - f\frac{k^2}{w^2}\right)\right] = 0$$

$$\zeta_\perp|_{z=0} = \zeta_\perp|_{z=z_h} = i\frac{V}{p}|_{z=z_h}$$

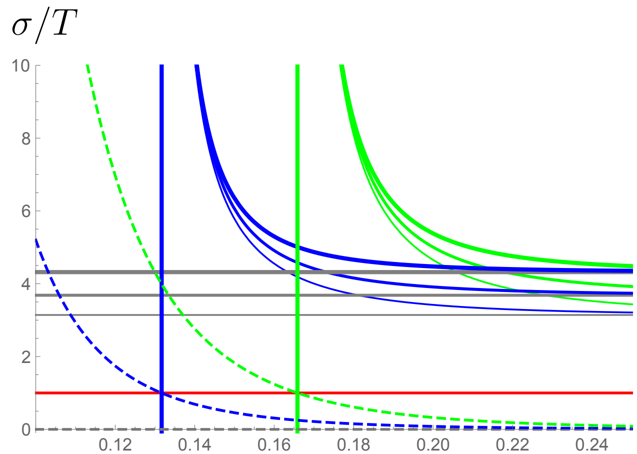
A la membrane paradigm N.Iqbal, H.Liu, Phys. Rev.D 79 (2009) 025023

Electric conductivity

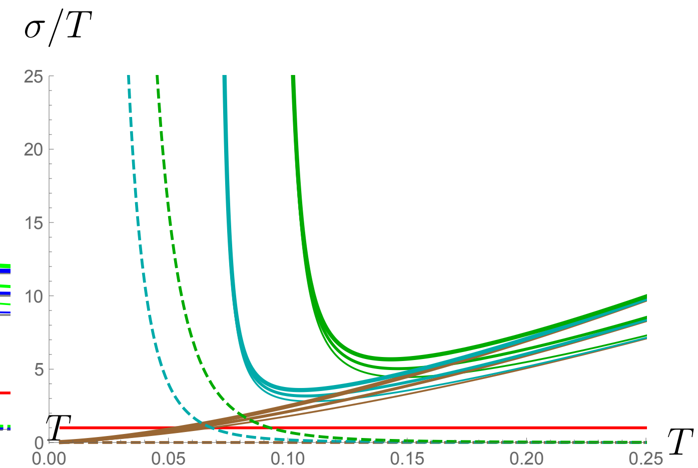
$$ {}_{\perp}(T,\nu,q)| \approx \left(\frac{2\pi\nu}{1+\nu}\right)^{3-2/\nu} \frac{T^{3-2/\nu}}{\left(1-\left(\frac{\nu+1}{\nu}\right)^{\frac{2+3\nu}{\nu}} q^2 \left(\frac{1}{2\pi T}\right)^{\frac{2+4\nu}{\nu}}\right)^{3-2/\nu}} $$



$\nu = 1$



$\nu = 4$



$\nu = 1$ and $\nu = 4$

Isotropic: q=0, 0.1, 0.2 gray, blue and green lines
Anisotropic: q=0, 0.1, 0.2 are shown by brown,
darker cyan and darker green.
Dashed: validity of the approximation (below the red line)

Conclusion:

Holographic models are some kind of phenomenological models with few number of parameters

We have considered the anizotropic model that describes: multiplicity, quark confinement;

predicts:
smeared phase transition,
anizotropy in hadron spectrum (for a short time after collisions)

Anisotropy drastically change standard holographic calculations, in particular,

- Wilson loops, and quark potential
- Jet quenching
- Drag forces
- sheet viscosity and therefore elliptic flows
- susceptibility
- thermalization time