Black holes in vector-tensor theories

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based on arXiv:1703.07676

Ginzburg conference on physics, May 29, 2017

Why modify gravity?

Why modify gravity?
- cosmological constant problems,
- non-renormalizability problem,
- benchmarks for testing General Relativity
- theoretical curiosity.

Many ways to modify gravity:
- f(R), scalar-tensor theories,
- Galileons, Horndeski (and beyond) theory, KGB, Fab-four,
- higher-dimensions, DGP,
- Horava, Khronometric
- massive gravity
- Vector-tensor theories (EM, Proca, "Extended" Proca)

Propagating massless graviton and propagating (massive) vector

Galileon/Horndeski theory

Most general galileon shift-symmetric action:

$$\mathcal{L}_2 = K(X)$$

$$\mathcal{L}_3 = G^{(3)}(X)\,\Box\varphi$$

$$\mathcal{L}_4 = G^{(4)}_{,X}(X)\left[(\Box\varphi)^2 - (\nabla\nabla\varphi)^2\right] + R\,G^{(4)}(X),$$

$$\mathcal{L}_5 = G^{(5)}_{,X}(X)\left[(\Box\varphi)^3 - 3\Box\varphi\,(\nabla\nabla\varphi)^2 + 2\,(\nabla\nabla\varphi)^3\right] - 6G_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\varphi\,G^{(5)}(X)$$

Equations of motion are of the second order both in
the metric and the scalar field ->
No extra degrees of freedom

Black holes are bald (?)

- Gravitational collapse...
- Black holes eat or expel surrounding matter
- Their stationary phase is characterised by a limited number of charges
- No details about collapse
- Black holes are bald

No hair theorems/arguments dictate that adding degrees of freedom lead to trivial (General Relativity) or singular solutions.

E.g. in the standard scalar-tensor theories BH solutions are GR black holes with constant scalar.

No hair for galileon

Hui&Nicolis'12

Shift-symmetric galileon, with arbitrary $G_2(X),\ G_2(X),\ G_4(X),\ G_5(X)$
Assume that:

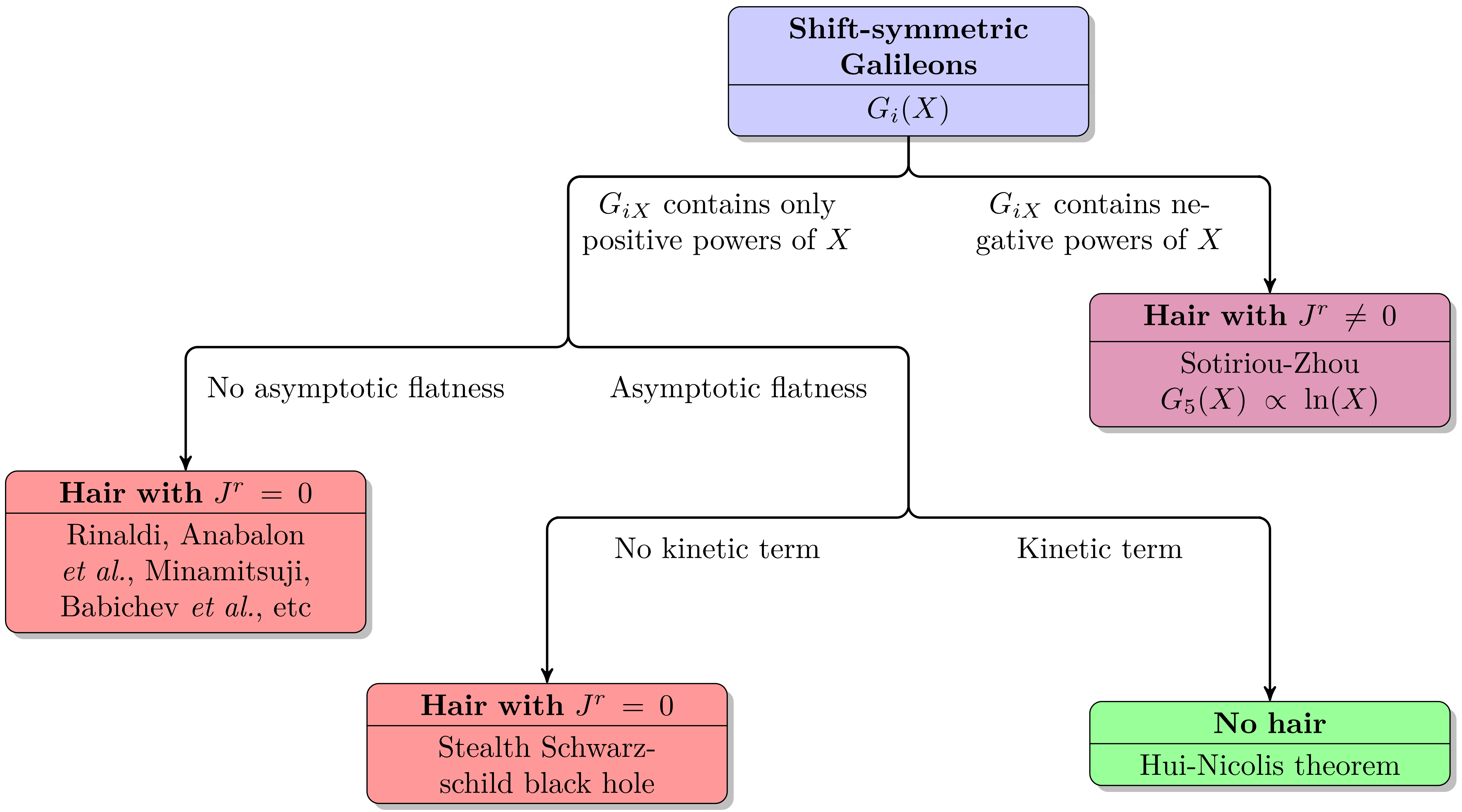
(i) spacetime and scalar field is static spherically symmetric,

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \qquad \phi = \phi(r)$$

(ii) spacetime is asymptotically flat, and $\phi' \to 0 \text{ as } r \to \infty$
and the norm of the current J^2 is finite (at the horizon)
(iii) there is a canonical kinetic term in the action and G_i are such that their derivatives $dG(X)_i/dX$ contain only positive or zero powers of X

A no-hair theorem then follows: the metric is Schwarzschild and the scalar field is constant

Avoiding no-hair theorem



Constructing hairs

Babichev, Charmousis'13

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{A(r)} + \rho(r)^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2).$$

$$\phi = qt + \psi(r)$$

Time-dependent scalar !

The only consistent solution for this ansatz is when $J^r = 0$

$$-qJ^r = \mathcal{E}_{tr} f$$

The norm of the current:

$$J^\mu J_\mu = -A(J^t)^2 + (J^r)^2/A,$$

The physical requirement of no-hair theorem is automatically satisfied by virtue of EOMs.

Explicit example

Babichev, Charmousis'13

$$\mathcal{L}^{\Lambda\mathrm{CGJ}} = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda.$$

follows from general galileon with $G_4 = 1 + \beta X$ and $G_2 = -2\Lambda + 2\eta X.$

The general solution is given by the solution of the algebraic equation:

$$(q\beta)^2\left(\kappa + \frac{r^2}{2\beta}\right)^2 - \left(2\kappa + (1-2\beta\Lambda)\frac{r^2}{2\beta}\right)k(r) + C_0 k^{3/2}(r) = 0,$$

$$h(r) = -\frac{\mu}{r} + \frac{1}{\beta r}\int \frac{k(r)}{\kappa + \frac{r^2}{2\beta}}dr, \qquad f = \frac{(\kappa + \frac{r^2}{2\beta})^2\beta h}{k(r)},$$

$$\psi' = \pm\frac{\sqrt{r}}{h(\kappa + \frac{r^2}{2\beta})}\left(q^2(\kappa + \frac{r^2}{2\beta})h' - \frac{1+2\beta\Lambda}{4\beta^2}(h^2r^2)'\right)^{1/2}.$$

Explicit solutions

Asymptotically dS/AdS:

$$f = h = 1 - \frac{\mu}{r} - \frac{\Lambda_{\rm eff}}{3} r^2, \qquad \psi' = \pm \frac{q}{h}\sqrt{1-h}, \qquad \Lambda_{\rm eff} = -\frac{1}{2\beta}$$

Asymptotically static universe:

$$h = 1 - \frac{\mu}{r}, \quad f = \left(1 - \frac{\mu}{r}\right)\left(1 + \frac{\eta r^2}{\beta}\right) \qquad \psi' = \pm \frac{q}{h}\sqrt{\frac{\mu}{r(1+\frac{\eta}{\beta}r^2)}}$$

Asymptotically flat (no standard kinetic term)

$$f = h = 1 - \frac{\mu}{r} \qquad \psi' = \pm q\sqrt{\mu r}/(r-\mu).$$

Hairy black holes

Motivation

$$S = \int d^4x\sqrt{-g}\left[\zeta R - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda\right]$$

Let us replace the derivative of the scalar by a vector

$$\partial_\mu\phi \to A_\mu$$

$$\eta(\partial\phi)^2 \to \mu^2 A^2$$

$$\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \to \beta G^{\mu\nu}A_\mu A_\nu$$

Vector “John” term

$$-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$$

Also add Maxwell term for vector

The theory

Einstein-Proca theory +Cosmological term+ extra Galileon-like term:

$$S[g,A]=\int\sqrt{-g}\,d^4x\left[R-2\Lambda-\frac{1}{4}\mathcal{F}^2-\frac{\mu^2}{2}A^2+\beta G_{\mu\nu}A^\mu A^\nu\right]$$

$$G_{\mu\nu}+\Lambda g_{\mu\nu}-\frac{1}{2}\left[F_{\mu\sigma}F_{\nu}^{\ \sigma}-\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right]-\frac{\mu^2}{2}\left(A_\mu A_\nu-\frac{1}{2}g_{\mu\nu}A^2\right)-\beta Z_{\mu\nu}=0$$

$$\nabla_\mu(F^{\mu\nu})-\mu^2A^\nu+2\beta A_\mu G^{\mu\nu}=0$$

$$\begin{aligned}Z_{\mu\nu}=\ &\frac{1}{2}A^2R_{\mu\nu}+\frac{1}{2}R\,A_\mu A_\nu-2A^\alpha R_{\alpha(\mu}A_{\nu)}-\frac{1}{2}\nabla_\mu\nabla_\nu A^2+\nabla_\alpha\nabla_{(\mu}\left(A_{\nu)}A^\alpha\right)\\&-\frac{1}{2}\Box(A_\mu A_\nu)+\frac{1}{2}g_{\mu\nu}\left(G_{\alpha\beta}A^\alpha A^\beta+\Box A^2-\nabla_\alpha\nabla_\beta(A^\alpha A^\beta)\right)\end{aligned}$$

Ansatz

Static metric and vector:

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{2,\kappa}$$

$$A_\mu dx^\mu = a(r)dt + \chi(r)dr$$

$$\kappa = 0, \pm 1$$

Curvature of base manifold

Compare with the case of scalar field:
the same ansatz for the metric, and

$$\phi(t,r) = q\,t + \psi(r)$$

$$\partial_\mu \phi \, dx^\mu = q dt + \psi'(r) dr$$

$tt, \; rr$ components of the metric equations

$t, \; r$ components of the vector equations

Equations of motion

The r-component of the vector equation gives the relation of the metric functions,

$$f(r) = \frac{h(r)\left(\mu^2 r^2 + 2\beta\kappa\right)}{2\beta\,(r\,h)'}$$

From rr-component of the Einstein equations one gets,

$$\chi^2(r) = \frac{r\Big[(\frac{\mu^2}{2}r^2 + \beta\kappa)\left(\beta a^2 h' - 2\beta a a' h' - \frac{1}{4} r h (a')^2\right) - \frac{1}{2}(r^2 h^2)'(\frac{\mu^2}{2} + \beta\Lambda)\Big]}{h^2(\frac{\mu^2}{2}r^2 + \beta\kappa)^2}$$

2 equations are solved and there are 2 are to solve

Equations of motion

Substitution: $$h(r) = -\frac{2M}{r} + \frac{1}{r}\int \frac{k(r)}{\mu^2 r^2 + 2\beta\kappa} dr$$

Two master equations:

$$\left[\frac{(\mu^2 r^2 + 2\beta\kappa)(r\ a)'}{\sqrt{k(r)}}\right]' = (1-4\beta)a(r)\left[\frac{(\mu^2 r^2 + 2\beta\kappa)}{\sqrt{k(r)}}\right]'$$

$$C_1 k^{3/2} - k\left[2\beta\kappa + r^2(\frac{\mu^2}{2} - \beta\Lambda)\right] + \frac{1}{8}\left(\mu^2 r^2 + 2\beta\kappa\right)^2\left[[(ra)']^2 - (1-4\beta)(a^2 r)'\right] = 0$$

C_1 is an integration constant

Special case: $\beta = 1/4$

The case $\beta = 1/4$ and spherical symmetry

see also Chagoya et al'16

$$a(r) = \frac{Q}{r} + \frac{Q_2}{2C_1\mu^3}\left[-\frac{\sqrt{2}}{4}\left((Q_2^2-2)\mu^2-\Lambda\right)\frac{\arctan(\mu\sqrt{2}r)}{r} + \mu\left(\mu^2-\frac{\Lambda}{2}\right)\right]$$

$$h(r) = \frac{(2Q_2^2\mu^2-6\mu^2-\Lambda)(\Lambda-2\mu^2)}{2(4C_1\mu^2)^2} - \frac{2M}{r} - \Lambda_{eff}\left[\frac{r^2}{3} + \frac{(Q_2^2\mu^2-2\mu^2-\Lambda)^2}{2\sqrt{2}\mu^3 r(\Lambda-2\mu^2)^2}\arctan\left(\sqrt{2}r\mu\right)\right]$$

C_1 is a gauge choice, reparametrisation of time, fixing it such that asymptotically adS: $\Lambda_{eff} = -2\mu^2$

The case $\beta = 1/4$ and spherical symmetry

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$$h(r) = \frac{2\mu^2}{3}r^2 + \frac{2Q_2^2\mu^2-6\mu^2-\Lambda}{\Lambda-2\mu^2} - \frac{2M}{r} + \frac{(Q_2^2\mu^2-2\mu^2-\Lambda)^2}{\sqrt{2}\mu(\Lambda-2\mu^2)^2}\frac{\arctan\left(\sqrt{2}r\mu\right)}{r}$$

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The case $\beta = 1/4$ and spherical symmetry

$M, \, Q, \, Q_2$ are physical quantities

M	a part of overall mass
Q	Column charge
Q_2	extra charge related to the breaking of gauge symmetry

stealth Schwarzschild soliton

$$Q_2^2\mu^2 - 2\mu^2 - \Lambda = 0$$

$$\Rightarrow \quad h(r) = \frac{2\mu^2}{3}r^2 + 1 - \frac{2M}{r}$$

adS soliton

$$M = 0,\ Q_2 \neq 0$$

✗

- No horizon
- Regular everywhere solution
- The mass of the soliton is not zero

$$r \to \infty \quad \Rightarrow \quad M_{eff} = -\frac{\pi(Q_2^2\mu^2 - 2\mu^2 - \Lambda)^2}{2\sqrt{2}\mu(\Lambda - 2\mu^2)^2}$$

adS soliton

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Solutions with planar horizon

$$\kappa = 0$$

- Asymptotically sdS black holes
- $M,\ Q,\ Q_2$ are physical quantities

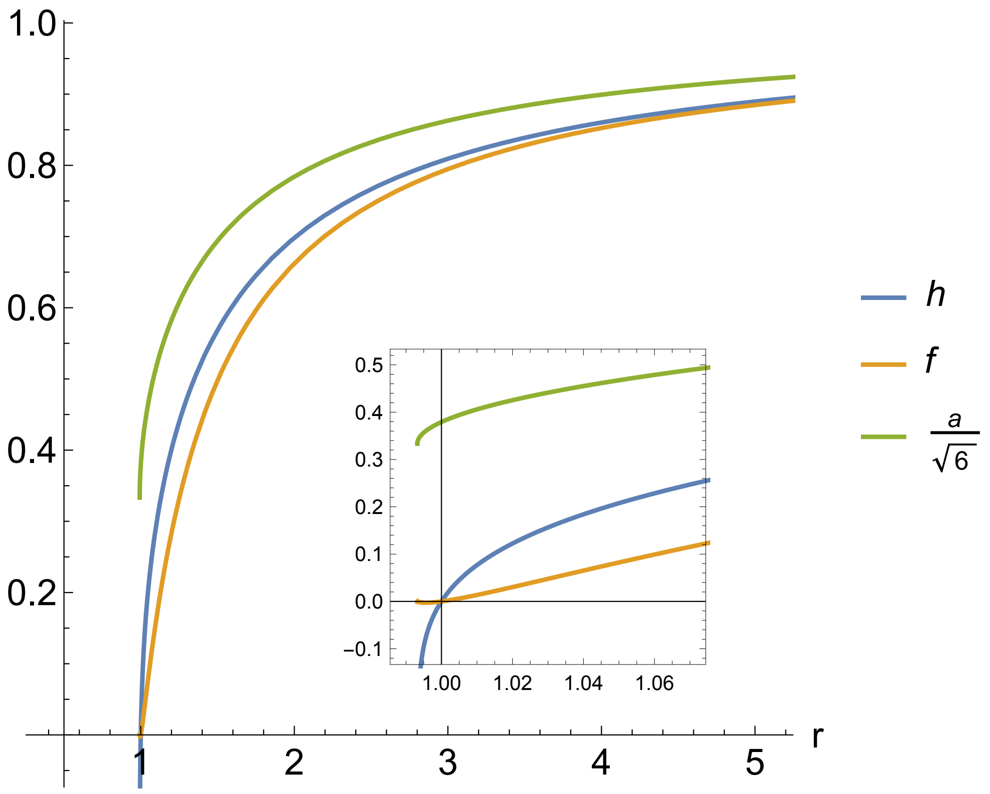
$$a(r) = \frac{Q}{r} - \frac{Q_2}{8C_1\mu^2 r^2}\left[\left(2\Lambda - 4\mu^2\right) r^2 - Q_2^2\right]$$

$$h(r) = \frac{2Q_2^2\mu^2}{\Lambda - 2\mu^2} + r^2\frac{2\mu^2}{3} - \frac{2M}{r} - \frac{(Q_2^2\mu)^2}{2r^2(\Lambda - 2\mu^2)^2}$$

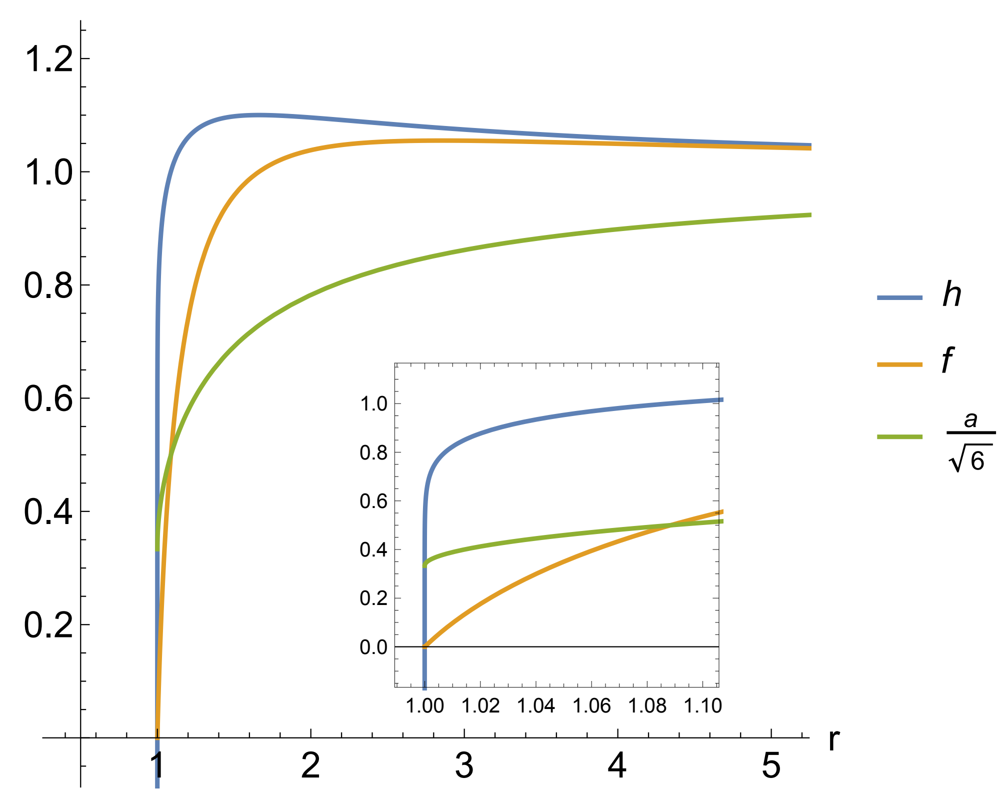
The case $\beta \neq 1/4$ and spherical symmetry

The case $\kappa = 1$, for $\mu = \Lambda = 0$ or $\mu^2 = 2\beta\Lambda$

- For some particular β equations are analytically solvable
- Asymptotically flat black holes



$M = 0$



$M < 0$

The case $\beta \neq 1/4$ and spherical symmetry

Lifshitz black holes: Topological $\kappa = 0$ case

$$ds^2 = -r^{2z}\left(1 - \frac{2M}{r^{2z+1}}\right)dt^2 + \frac{dr^2}{r^2\left(1 - \frac{2M}{r^{2z+1}}\right)} + r^2(dx_1^2 + dx_2^2)$$

$$z = \frac{2\beta}{2\beta - 1}$$

Lifshitz exponent

Conclusions

- We studied extended Einstein-Proca theory
- Black holes with hairs
- Solitons
- Asymptotically flat or adS