Cosmological particle creation, conformal invariance and A. D. Sakharov's induced gravity

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Outline

I. Introduction
II. Particle creation — phenomenology
III. Conformal symmetry
IV. Scalar field
V. The “good” hydrodynamical variables and equations of motion
VI. Vacuum equations
“Empty” vacuum and
“Dynamical” vacuum $=$ Dirac sea

I. Introduction

- Particle creation — quantum phenomenon
  Renormalization. Counter-terms.
  A. D. Sakharov — induced gravity
- Ya. B. Zel'dovich, A. A. Starobinsky, L. Parker, S. Fulling, V. N. Lukash, A. A. Grib, S. G. Mamaev, V. A. Mostepanenko
- Back reaction — problems
  Process of creation
  Global geometry: horizons, energy dominance violation
- Phenomenological description of particle creation:
  Eulerian coordinates, "creation law",
  Weyl tensor — renormalizability
- Conformal gravity. History
  Nowadays — Higgs Mechanism
  Creatioon of universe from "nothing" — A. Vilenkin
  Initial states is conformally invariant — R. Penrose, G. 't Hooft

- Sign convention

  signature $(+,-,-,-)$

  $R^{\mu}{}_{\nu\lambda\sigma} = \frac{\partial\Gamma^{\mu}_{\nu\sigma}}{\partial x^{\lambda}} - \frac{\partial\Gamma^{\mu}_{\nu\lambda}}{\partial x^{\sigma}} + \Gamma^{\mu}_{\varkappa\lambda}\Gamma^{\varkappa}_{\nu\sigma} - \Gamma^{\mu}_{\varkappa\sigma}\Gamma^{\varkappa}_{\nu\lambda}$

  $R_{\nu\sigma} = R^{\mu}{}_{\nu\mu\sigma}$

  $R = g^{\nu\sigma}R_{\nu\sigma}$

  $\Gamma^{\lambda}_{\mu\nu}$ — metric connections

- Units

  $\hbar = c = 1$

  $[G] = [\ell]^2 = \frac{1}{[m]^2} = [t]^2$

II. Particle creation — phenomenology

Hydrodynamics of perfect fluid with varying number of particles

Lagrange description is not adequate $\Longrightarrow$ worldline is varied without taking into account the process of its creation $\Longrightarrow$ Euler description

Standard action: (J. R. Ray 1972)

$$
\begin{aligned}
S_{\text{hydro}} &= -\int \varepsilon(X, n)\sqrt{-g}\,dx + \int \lambda_0(u^\mu u_\mu - 1)\sqrt{-g}\,dx \\
&\quad + \int \lambda_2 X_{,\mu} u^\mu \sqrt{-g}\,dx + \int \lambda_1 (n u^\mu)_{;\mu}\sqrt{-g}\,dx
\end{aligned}
$$

$\varepsilon(X, n)$ — invariant energy density

n — invariant particle number density

u^μ — four-velocity of particle flux

X — auxiliary variable (numeration of worldlines)

$\lambda_0,\ \lambda_1,\ \lambda_2$ — Lagrange multipliers

$(n u^\mu)_{;\mu} = 0$ — particle number conservation law

Dynamical variables: $n,\ u^\mu,\ X$

Our proposal: to include into the formalism the "creation law"

$(nu^\mu)_{;\mu} = \Phi(inv) \neq 0$ (V. A. Berezin 1987)

Fundamental result for (cosmological) particle creation

$\Phi = \beta C^2$ (Ya. B. Zel'dovich, A. A. Starobinsky 1977)

$C^\mu{}_{\nu\lambda\sigma}$ — Weyl tensor

$$S_{\rm hydro} = -\int \varepsilon(X,n)\sqrt{-g}\,dx + \int \lambda_0(u^\mu u_\mu - 1)\sqrt{-g}\,dx + \int \lambda_2 X_{,\mu} u^\mu \sqrt{-g}\,dx + \int \lambda_1((nu^\mu)_{;\mu} - \beta C^2)\sqrt{-g}\,dx$$

λ_1 is defined up to an additive constant

$\lambda_1 \to \lambda_1 + \gamma_0, \quad \gamma_0 = const$

$$\gamma_0 \int ((nu^\mu)_{;\mu} - \beta C^2)\sqrt{-g}\,dx = \gamma_0 \int ((n\sqrt{-g}\,u^\mu)_{,\mu} - \beta C^2\sqrt{-g})\,dx$$

$$\to \quad -\gamma_0\beta \int C^2\sqrt{-g}\,dx \quad \to \quad \text{conformal Weyl gravity}$$

III. Conformal invariance — the postulate

- "+" Additional symmetry
  1. Creation of the universe from "nothing"
     A.Vilenkin, Ya. B. Zel'dovich & L.P.Grishchuk, S.W.Hawking...
  2. R. Penrose, G 't Hooft
- "−"
  1. Theory was rejected — absence of massive particles
     H. Weyl, A. Einstein 1921
  2. However, the Higgs mechanism was unknown in 1921!
     G 't Hooft 2014

$$g_{\mu\nu} = \Omega^2 \hat{g}_{\mu\nu}, \qquad \frac{\delta S_{\rm tot}}{\delta \Omega} = 0 \quad \Rightarrow$$

- Conformal multiplier Ω may be considered as an independent variable

$S_{\rm tot} = S_{\rm grav} + S_{\rm matter}$

By definition:

$$\delta S_{\rm matter} = \frac{1}{2}\int T_{\mu\nu}\sqrt{-g}\,\delta g^{\mu\nu}\,dx$$

$$\delta S_{\rm matter} = \frac{1}{2}\int \hat{T}_{\mu\nu}\sqrt{-\hat{g}}\,\delta \hat{g}^{\mu\nu}\,dx$$

Let

$$\delta g^{\mu\nu} = -\frac{2}{\Omega^3}\hat{g}^{\mu\nu}\delta\Omega = -\frac{2}{\Omega}g^{\mu\nu}\delta\Omega$$

Then, if

$$\frac{\delta S_{\rm grav}}{\delta\Omega} = 0 \quad \Rightarrow$$

$$0 = \delta S_{\rm matter} = -\int T_{\mu\nu}g^{\mu\nu}\frac{\delta\Omega}{\Omega}\sqrt{-g}\,dx \quad \Rightarrow \quad {\rm Tr}\,(T_{\mu\nu}) = 0$$

Let

$$\delta g^{\mu\nu} = \Omega^2\delta\hat{g}^{\mu\nu} \quad \Rightarrow \quad \hat{T}_{\mu\nu} = \Omega^2 T_{\mu\nu} \quad (\hat{T}^{\mu}_{\nu} = \Omega^4 T^{\mu}_{\nu},\ \hat{T}^{\mu\nu} = \Omega^6 T^{\mu\nu})$$

IV. What are those fields responsible for particle crreation?

- The simplest case — scalar field

$$S_{\rm scalar} = \int \left(\frac{1}{2}\chi^{\mu}\chi_{\mu} - \frac{1}{2} m^2 \chi^2 \right) \sqrt{-g}\, dx$$

- Conformal transformation (standard):

$$\delta g^{\mu\nu} = \Omega^2 \delta \hat{g}^{\mu\nu}, \quad \chi = \frac{1}{\Omega}\hat{\chi} \quad \Rightarrow$$

$$S_{\rm scalar} = \int \left(\frac{1}{2}\hat{\chi}^{\mu}\hat{\chi}_{\mu} - \frac{1}{\Omega}\hat{\chi}_{\mu}\Omega^{\mu} + \frac{1}{2}\frac{\hat{\chi}^2}{\Omega^2}\Omega_{\mu}\Omega^{\mu} - \frac{1}{2} m^2 \Omega^2 \chi^2 \right) \sqrt{-g}\, dx$$

$$\chi_{\mu} = \chi_{,\mu} \quad \chi^{\mu} = g^{\mu\nu}\chi_{\nu} \quad \hat{\chi}^{\mu} = \hat{g}^{\mu\nu}\hat{\chi}_{\nu} \quad \Omega^{\mu} = g^{\mu\nu}\Omega_{\nu}$$

- How to make $S_{\rm scalar}$ to be conformally *covariant?*

The known prescription: to add to Lagrangian $\frac{R}{12}\chi^2$
R — scalar curvature, constructed on metric $g_{\mu\nu}$ $\Rightarrow$

$$
\begin{aligned}
S_{\text{scalar}} &= \int \left(\frac{1}{2}\chi^{\mu}\chi_{\mu} + \frac{R}{12}\chi^2 - \frac{1}{2}m^2\chi^2 \right) \sqrt{-g}\, dx \\
&= \int \left(\frac{1}{2}\hat{\chi}^{\mu}\hat{\chi}_{\mu} + \frac{\hat{R}}{12}\chi^2 - \frac{1}{2}m^2\Omega^2\hat{\chi}^2 \right) \sqrt{-\hat{g}}\, dx \\
& \quad -\frac{1}{2}\int \left(\hat{\chi}^2 \frac{\Omega^{\lambda}}{\Omega} \right)_{|\lambda} \sqrt{-\hat{g}}\, dx
\end{aligned}
$$

The last term is the surface integral

- $m^2 = 0$ — all is OK?
- But! The correct sign $\Longleftrightarrow$ the incorrect sign
- Our choice:
  1. The "correct" sign for $\hat{R}$: $-\frac{\hat{R}}{12}\hat{\chi}^2$
  2. The "incorrect" sign for kinetic term: $-\frac{1}{2}\hat{\chi}^{\mu}\hat{\chi}_{\mu}$

“Explanatory note”

- Our scalar field χ is not “real” (fundamental). The already created particles are quanta of this field (phenomenology!)
- What remains — is a “vacuum” part of the scalar field, containing conformal anomaly, responsible for particle creation
- The wrong sign sign of kinetic term $\Longrightarrow$ absence of the law energy bound (C-field by Hoyle and Narlikar)
- Scalar filed χ is not a dynamical variable $\Longrightarrow$ there is no variation

How could the field χ know about the conformal transformation?

- It is always possible to choose $\hat{\chi} = \frac{1}{\ell}\,\varphi, \quad \varphi = \Omega$
  $(\chi = \frac{1}{\Omega}\hat{\chi},\ [\ell] = \frac{1}{[m]},\ \hbar = c = 1) \quad \Longrightarrow$
  $$S_{\text{scalar}} = -\frac{1}{\ell^2}\int\left(\frac{1}{2}\varphi^{\mu}\varphi_{\mu} + \frac{\hat{R}}{12}\varphi^2 - \frac{1}{2}m^2\varphi^4\right)\sqrt{-\hat{g}}\,dx$$
- It appears φ^4! $\quad (d = dim\,\mathcal{M} = 4)$!
- Conformal *covariance*
  $\varphi(new) = \tilde{\Omega}\varphi(old), \quad \sqrt{-g(old)} = \tilde{\Omega}^4\sqrt{-g(new)}$
- Cosmological term or quintessence: $\quad 3m^2 = \Lambda$
- Independent variations of φ and $\hat{g}^{\mu\nu}$
- Energy-momentum tensor

$$\hat{T}^{\text{scalar}}_{\mu\nu} = -\frac{1}{\ell^2}\varphi_{\mu}\varphi_{\mu} + \frac{1}{2\ell^2}\varphi^{\sigma}\varphi_{\sigma}\hat{g}_{\mu\nu} - \frac{1}{2\ell^2}m^2\varphi^4\hat{g}_{\mu\nu}$$
$$-\frac{1}{6\ell^2}\left(\varphi^2(\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R}) - 2\left((\varphi\varphi_{\nu})_{|\mu} - (\varphi\varphi^{\sigma})_{|\sigma}\,\hat{g}_{\mu\nu}\right)\right)$$
$$\mathrm{Tr}\ T^{\text{scalar}}_{\mu\nu} = -\frac{1}{\ell^2}\varphi\varphi^{\sigma}_{|\sigma} + \frac{1}{6\ell^2}\hat{R}\varphi^2 - \frac{2}{\ell^2}m^2\varphi^4$$

V. The “good” hydrodynamical variables for hydrodynamical action

$$(nu^\mu)_{;\mu} = \beta C^2 \quad \Rightarrow \quad C^2\sqrt{-g} = \hat{C}^2\sqrt{-\hat{g}}, \quad \sqrt{-g} = \varphi^4\sqrt{-\hat{g}}$$

$$ds^2 = \varphi^2 d\hat{s}^2 \quad \Rightarrow \quad u^\mu = \frac{1}{\varphi}\hat{u}^\mu, \quad \hat{u}^\mu\hat{u}_\mu = 1, \quad u_\mu = \varphi\hat{u}_\mu$$

$$(nu^\mu)_{;\mu}\sqrt{-g} = (nu^\mu\sqrt{-g})_{,\mu} = (\varphi^3 nu^\mu\sqrt{-\hat{g}})_{,\mu} \quad \Rightarrow$$

$$\hat{n} = n\varphi^3\sqrt{-\hat{g}} \quad \Rightarrow \quad N = \int \hat{n}\, d^3x - inv$$

$$\begin{aligned} S_{\rm hydro} \;&=\; -\int \varepsilon(X,n)\sqrt{-g}\,dx + \int \lambda_0(u^\mu u_\mu - 1)\sqrt{-g}\,dx \\ &+ \int \lambda_2 X_{,\mu}u^\mu\sqrt{-g}\,dx + \int \lambda_1 (nu^\mu)_{;\mu}\sqrt{-g}\,dx \quad \Longrightarrow \end{aligned}$$

$$\begin{aligned} S_{\rm hydro} \;&=\; -\int \varepsilon\left(X, \frac{\hat{n}}{\varphi^3\sqrt{-\hat{g}}}\right)\varphi^4\sqrt{-\hat{g}}\,dx + \int \lambda_0(\hat{u}^\mu\hat{u}_\mu - 1)\varphi^4\sqrt{-\hat{g}}\,dx \\ &+ \int \lambda_2 X_{,\mu}\hat{u}^\mu\varphi^3\sqrt{-\hat{g}}\,dx + \int \lambda_1 \left((\hat{n}\hat{u}^\mu)_{,\mu} - \beta\hat{C}^2\sqrt{-\hat{g}}\right) dx \end{aligned}$$

$$\varepsilon + p = n\,\frac{\partial \varepsilon}{\partial n} \qquad \Longrightarrow$$

$$\hat{T}^{\rm hydro}_{\mu\nu} = (\varepsilon + p)\varphi^4 \hat{u}_\mu \hat{u}_\nu - p\varphi^4 \hat{g}_{\mu\nu} - 4\beta\varphi^2 \left((\lambda_1 \hat{C}_{\mu\sigma\nu\lambda})^{|\lambda|\sigma} + \frac{1}{2}\lambda_1 \hat{C}_{\mu\lambda\nu\sigma} \hat{R}^{\lambda\sigma} \right)$$

$$\mathrm{Tr}\; T^{\rm hydro}_{\mu\nu} = (\varepsilon - 3p)\varphi^4$$

$$\mathrm{Tr}\; T^{\rm total}_{\mu\nu} = -\frac{1}{\ell^2}\varphi\varphi^{\sigma}_{|\sigma} + \frac{1}{6\ell^2}\hat{R}\varphi^2 - \frac{2}{\ell^2}m^2\varphi^4 + (\varepsilon - 3p)\varphi^4$$

- Variation of φ: $\frac{\delta S_{\rm total}}{\delta \varphi} = 0 \qquad \Longrightarrow$

$$\frac{1}{\ell^2}({\varphi^{\sigma}}_{|\sigma} - \frac{1}{6}\hat{R}\varphi + 2m^2\varphi^3) + (\varepsilon - 3p)\varphi^3 = 0$$

As it should be!

VI. Vacuum equations

- Without "hats" (!):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{\mathrm{hydro}}$$

$$T_{\mu\nu}^{\mathrm{hydro}} = (\epsilon + p) u_\mu u_\nu - p g_{\mu\nu} - 4\beta B_{\mu\nu}[\lambda_1]$$

$$B_{\mu\nu}[\lambda_1] = (\lambda_1 C_{\mu\sigma\nu\lambda})^{;\lambda;\sigma} + \frac{1}{2} \lambda_1 C_{\mu\lambda\nu\sigma} R^{\lambda\sigma}$$

- "Empty" vacuum:

$$\epsilon = p = 0$$

$$4\alpha_0 B_{\mu\nu}[\lambda_1] + \frac{1}{16\pi G} G_{\mu\nu} - \frac{\Lambda}{16\pi G} g_{\mu\nu} = 0$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

- In terms of φ and $\hat{g}_{\mu\nu}$ $(g_{\mu\nu} = \varphi^2 \hat{g}_{\mu\nu})$:

$$
\begin{aligned}
0 \;=\; & -4\beta\left((\lambda_1 \hat{C}_{\mu\sigma\nu\lambda})^{|\lambda|\sigma} + \frac{1}{2}\lambda_1 \hat{C}_{\mu\lambda\nu\sigma}\hat{R}^{\lambda\sigma}\right) \\
& - \frac{1}{6\ell^2}\left(\varphi^2(\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R}) + 4\varphi_\mu\varphi_\nu - \varphi^\sigma\varphi_\sigma\hat{g}_{\mu\nu}\right. \\
& \left. -2\varphi\varphi_{\nu|\mu} + 2\varphi\varphi^{|\sigma}_{|\sigma}\hat{g}_{\mu\nu} - \Lambda\varphi^4\hat{g}_{\mu\nu}\right)
\end{aligned}
$$

- Trace:

$$
\varphi^{|\sigma}_{|\sigma} - \frac{1}{6}\hat{R}\varphi - \frac{2}{3}\Lambda\varphi^3 = 0
$$

Dirac sea

- Weyl tensor $C_{\mu\sigma\nu\lambda} \neq 0$
- Ingredients: particles of both positive and negative energies

$$\epsilon_{(\pm)},\ p_{(\pm)},\ \vec{v}_{(\pm)},\ u^{\mu}_{(\pm)}$$

- Hydrodynamical equations

$$\begin{aligned} \left(\epsilon_{(\pm)} + p_{(\pm)}\right) u^{(\pm)}_{\mu} + \lambda^{(\pm)}_{2} X^{(\pm)}_{,\mu} - n_{(\pm)} \lambda^{(\pm)}_{1,\mu} &= 0, \\ \frac{\partial \epsilon_{(\pm)}}{\partial X^{(\pm)}} - \left(\lambda^{(\pm)}_{2} u^{(\pm)\sigma}\right)_{;\sigma} &= 0 \end{aligned}$$

- Complete compensations

$$\begin{array}{ccccc} \epsilon_{+} = -\epsilon_{-}, \quad n_{+} = n_{-} & \Rightarrow & p_{+} = -p_{-} & & \\ u^{(+)\mu} = u^{(-)\mu} & \Rightarrow & X^{(+)} = X^{(-)} & \Rightarrow & \\ \lambda^{(+)}_{2} = -\lambda^{(-)}_{2} & \Rightarrow & \lambda^{(+)}_{1,\mu} = -\lambda^{(-)}_{1,\mu} & \Rightarrow & \end{array}$$

$$\boldsymbol{\lambda^{(+)}_{1} + \lambda^{(-)}_{1} = \mathrm{const}}$$

“Dynamical vacuum” equations

$$\mathbf{4\alpha_0 B_{\mu\nu} + \frac{1}{16\pi G} G_{\mu\nu} - \frac{\Lambda}{16\pi G} g_{\mu\nu} = 0}$$

$$B_{\mu\nu} = C_{\mu\sigma\nu\lambda}{}^{;\lambda;\sigma} + \frac{1}{2} C_{\mu\lambda\nu\sigma} R^{\lambda\sigma} \quad \Rightarrow \quad \text{Bach tensor}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad \Rightarrow \quad \text{Einstein tensor}$$

$$\Lambda \quad \Rightarrow \quad \text{Cosmological constant}$$

Thanks to everybody!

The end

Abstract

We constructed a phenomenological model for cosmological particle creation. It allows to take into account the back reaction on the space-time metric not only of the energy-momentum tensor of the already created particles, but also the very process of the creation. Adopting the well-known fact that the particle production rate is proportional to the square of the Weyl tensor, we found that there is no more need to include, ad hoc, the conformal gravity term into the action integral, it is already there. Such a feature is in accordance with the idea of the induced gravity, first proposed by A. D. Sakharov. Assuming, then, the simplest possible (without self-interaction) action for the scalar field responsible for the particle creation, we found that the requirement for the local conformal transformation to be the fundamental physical law, leads not only to the induced scalar term in the action (Einstein-dilaton gravity), but also to the appearance of the self-interaction for the scalar field absent before. Moreover, the exponent of this self-interaction term $(= 4)$ is due to the dimensionality of our space-time.