Beyond Newton-Cartan Gravity

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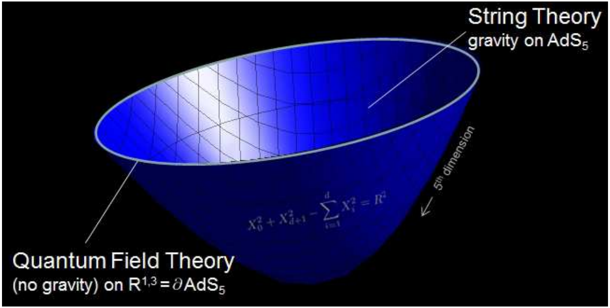
rijksuniversiteit groningen

why non-relativistic gravity ?

Holography



Gravity is not only used to describe the gravitational force!

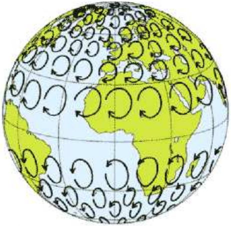
Bagchi, Gopakumar (2009); Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

Condensed Matter

Effective Field Theory (EFT) coupled to NC background fields

serve as response functions and leads to restrictions on EFT

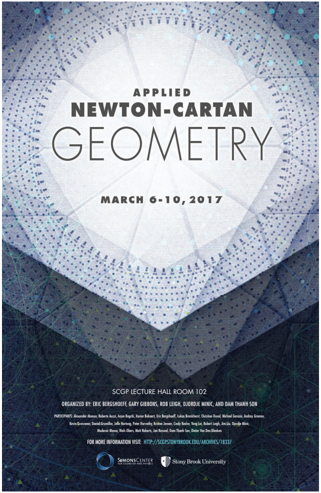
compare to



Coriolis force

Luttinger (1964), Greiter, Wilczek, Witten (1989), Son (2005, 2012), Can, Laskin, Wiegmann (2014)

Jensen (2014), Gromov, Abanov (2015), Gromov, Bradlyn (2017)



Simons workshop 'Applied Newton-Cartan Geometry', March 6-10

Outline

Newton-Cartan Gravity

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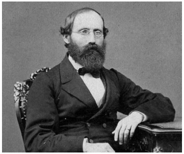
Non-relativistic Gravity

- Free-falling frames: Galilean symmetries
- Constant acceleration: Newtonian gravity/Newton potential $\Phi(x)$
- no frame-independent formulation

  (needs geometry!)



Riemann (1867)

Galilei Symmetries

- time translations: $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!
- space translations: $\delta x^i = \xi^i$ $\quad i = 1, 2, 3$
- spatial rotations: $\delta x^i = \lambda^i{}_j \, x^j$
- Galilean boosts: $\delta x^i = \lambda^i t$

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]} \,, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]} \,,$$

$$[G_a, H] = -P_a \,, \qquad [J_{ab}, J_{cd}] = \delta_{c[a}J_{b]d} - \delta_{a[c}J_{d]b} \,, \qquad a = 1, 2, 3$$

'Gauging' Galilei

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(P)$
Galilean boosts	G^a	$\omega_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\mathcal{R}_{\mu\nu}{}^{ab}(J)$

Imposing Constraints

$\mathcal{R}_{\mu\nu}{}^a(P) = 0$: does only solve for part of $\omega_\mu{}^{ab}$

$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \ \rightarrow \ \tau_\mu = \partial_\mu \tau$: absolute time

‘Gauging’ Bargmann

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	${e_\mu}^a$	${\mathcal{R}_{\mu\nu}}^a(P)$
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spatial rotations	J^{ab}	${\omega_\mu}^{ab}$	${\mathcal{R}_{\mu\nu}}^{ab}(J)$
central charge transf.	Z	m_μ	$\mathcal{R}_{\mu\nu}(Z)$

Imposing Constraints

$${\mathcal{R}_{\mu\nu}}^a(P) = 0\,, \qquad \mathcal{R}_{\mu\nu}(Z) = 0\ :$$ solve for spin-connection fields

$$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \ \rightarrow\ \tau_\mu = \partial_\mu \tau\ :$$ absolute time (‘zero torsion’)

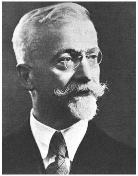
The NC Transformation Rules

The independent NC fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\begin{aligned} \delta\tau_\mu &= 0\,, \\ \delta e_\mu{}^a &= \lambda^a{}_b\, e_\mu{}^b + \lambda^a\tau_\mu\,, \\ \delta m_\mu &= \partial_\mu\sigma + \lambda_a\, e_\mu{}^a \end{aligned}$$

The spin-connection fields $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$ are functions of e, τ and m

The NC Equations of Motion



Élie Cartan 1923

The NC equations of motion are given by

$$\tau^\mu e^\nu{}_a \mathcal{R}_{\mu\nu}{}^a(G) = 0 \qquad \mathbf{1}$$

$$e^\nu{}_a \mathcal{R}_{\mu\nu}{}^{ab}(J) = 0 \qquad \mathbf{a} + (\mathbf{ab})$$

- after gauge-fixing and assuming flat space the first NC e.o.m. becomes $\triangle\Phi = 0$
- there is no known action that gives rise to these equations of motion

Outline

Torsion

Torsion occurs both in Holography and in Condensed Matter

- Lifshitz Holography

  zero torsion, i.e. $\partial_\mu \tau_\nu - \partial_\nu \tau_\mu = 0$, is not conformal invariant

  Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Condensed Matter

  non-relativistic energy-momentum tensor without restrictions requires arbitrary torsion

  Luttinger (1964); Gromov, Abanov (2014); Geracie, Golkar, Roberts (2014); Jensen (2014)

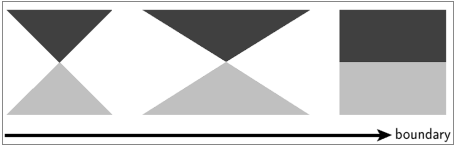
Twistless Torsional

- zero torsion: $\tau_{\mu\nu} \equiv 2\partial_{[\mu}\tau_{\nu]} = 0 \;\rightarrow\; \tau_\mu = \partial_\mu \tau$: absolute time

- twistless torsional or hypersurface orthogonal:

$$\tau_{ab} \equiv e_a^\mu e_b^\nu \tau_{\mu\nu} = 0$$

  is conformal invariant due to identity $e_a{}^\mu \tau_\mu = 0$



- arbitrary torsion

Luttinger (1964); Chatzistavrakidis, Romano, Rosseel + E.B. (2017)

3D Non-relativistic Gravity

Extended Bargmann Symmetries

Papageorgiou, Schroers (2009); Rosseel + E.B., work in progress

Galilei $\overset{\text{'Mass'}}{\Longrightarrow}$ Bargmann $\overset{\text{'Spin'}}{\Longrightarrow}$ Extended Bargmann

Lévy-Leblond (1972), Jackiw, Nair (2000)

$$[H, G_a] = -\epsilon_{ab}P_b\,, \qquad [J, G_a] = -\epsilon_{ab}G_b\,, \qquad [J, P_a] = -\epsilon_{ab}P_b\,,$$

$$[G_a, P_b] = \epsilon_{ab}M\,, \qquad [G_a, G_b] = \epsilon_{ab}S$$

3D: Fractional Quantum Hall Effect, Anyons, Emergent U(1)

3D Extended Bargmann Gravity

Rosseel + E.B. (2016); Hartong, Obers (2016)

3D extended Bargmann has invariant, non-degenerate bilinear form:

$$S = \frac{k}{4\pi}\int d^3x \left(\epsilon^{\mu\nu\rho} e_\mu{}^a R_{\nu\rho}{}^a(G) - \epsilon^{\mu\nu\rho} m_\mu R_{\nu\rho}(J) - \epsilon^{\mu\nu\rho}\tau_\mu R_{\nu\rho}(S)\right)$$

Non-Relativistic Limit

$$S = \frac{k}{4\pi}\int d^3x \left(\epsilon^{\mu\nu\rho} E_\mu{}^A R^A_{\nu\rho}(J) + 2\epsilon^{\mu\nu\rho} Z_{1\mu}\partial_\nu Z_{2\rho}\right)$$

Fractional Quantum Hole Effect: massive spin-2 GMP mode, bi-metric gravity, higher spins, W_∞-symmetry

Gromov, Son (2017)

Outline

Take Home Message

Newton-Cartan gravity is a rich field

with new links and cross-fertilization between

String Theory, Gravity and Condensed Matter !