On micro-states of 4-d Black Holes and their string origin

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Recent papers

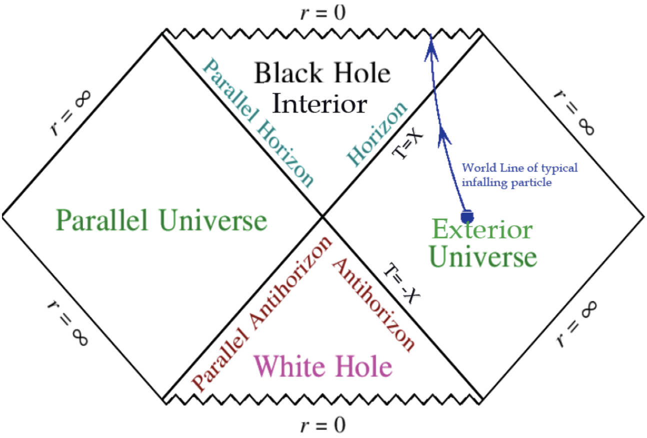
- Black Holes from String Theory
  - M. Bianchi, J.F. Morales, L. Pieri, N. Zinnato "More on microstate geometries of 4d black holes"
  - A. Addazi, M. Bianchi, G. Veneziano "Glimpses of black hole formation/evaporation in highly inelastic, ultra-planckian string collisions"
  - M. Bianchi, J.F. Morales, L. Pieri, "Stringy origin of 4d black hole microstates"
- Soft limits of Scattering Amplitudes
  - M. Bianchi, A. L. Guerrieri, Y-t Huang, C-J Lee, C-K Wen "Exploring soft constraints on effective actions"
  - M. Bianchi, A. L. Guerrieri "On the soft limit of tree-level string amplitudes"
  - M. Bianchi, A. L. Guerrieri "On the soft limit of closed string amplitudes with massive states"

Plan of the Talk

- Black Holes in GR and Information Paradox
- String Theory and the Fuzz-ball Proposal
- 4-d BH micro-state geometries from string amplitudes
- L, K and M solutions from open string condensates at intersecting D3-branes
- Multi-center ansatz, Bubble equations and ‘regularity’
- Summary, conclusions and future directions

Introducing the Black Hole

$$Black\ Hole \equiv [M - J^{-}(\mathcal{I}^{+})]$$



Getting acquainted with the Black Hole

Schwarschild solution ($c = 1,\ G = 1$)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$r = 2GM = r_S$ (coord. singularity): horizon $H = M \cap \dot{J}^-(\mathcal{J}^+)$
In words: boundary of the causal past of null infinity
In practice: light/signal cannot escape to infinity

- Singularity Theorems: Trapped Surface $\Rightarrow$ Singularity
- Cosmic Censorship: Singularity $\Rightarrow$ Horizon
- Area Theorem: $\delta A_H \geq 0$ (... Raychaudhuri equation)
- No Hair theorem: stationary, asymptotically flat BH's fully characterized by mass M, charge Q, angular momentum J (Kerr-Newman solution)

Black Hole Thermodynamics

Black Hole as a black body ($k_B = 1$):

$$dM = \frac{\kappa}{8\pi} dA + \dots$$

where $\kappa =$ surface gravity, constant on (Killing) horizon

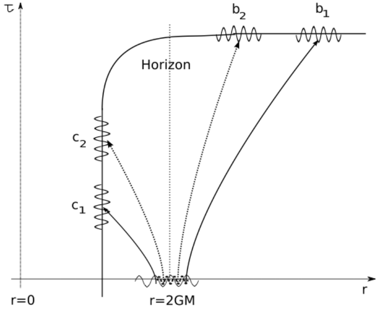
$$T_{BH} = \frac{\kappa}{2\pi} = \frac{1}{8\pi M} \quad , \quad S_{Bek-Hawk} = \frac{1}{4} A$$

Yet negative specific heat ...
Where are the micro-states?

$$S_{GR+QFT}(BH) = log(N_{micro-states}) = log(1) = 0 \;\; (!!??)$$

In GR a BH does not emit.
Semi-classically: Hawking radiation, a BH evaporates!

Information Paradox



- Pure state enters into a BH.
- Emitted radiation is thermal (no information), but entangled with BH.
- Emitted particles do not depend on the state of earlier produced pairs (why? ...).
- BH completely evaporates: there is nothing to be entangled with.
- At the end, only radiation in a mixed state ⇒ lost unitarity.

Information Paradox: Possible Resolutions

The paradox cannot be solved by adding small corrections to the semi-classical computation and information cannot be recovered at the last stages of evaporation.

- Loss of unitarity [Hawking]
- Remnants, Baby Universe [Susskind]
- Non Local BH-radiation interactions [Maldacena-Susskind, Raju-Papadodimas]
- Hairs in the asymptotic structure of space-time [Hawking, Perry, Strominger; Dvali, Gomez, Lüst], ...
- The 'horizon' is no more in an "information free vacuum" [Lunin, Mathur]

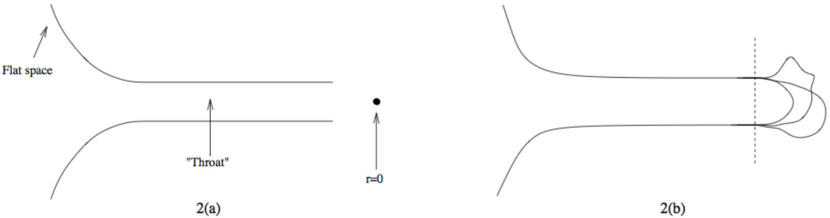
We will explore the last possibility. Rather than only solving an *ad hoc* problem, this resolution emerges naturally from String Theory, fitting into a bigger picture for Quantum Gravity.

Fuzz-ball Proposal [Lunin, Mathur, Bena, Giusto, Russo, Shigemori, Skenderis, Taylor, Turton, Warner]

Every (BPS) Black-Hole micro-state is dual to a smooth, horizon-less (super)gravity solution. NO singularity

Quantum Gravity effects are horizon-sized due to huge phase space.

Would-be horizon carries information ... the paradox is solved.



Far away fuzz-ball resembles a BH: every micro-state has the same asymptotic charges (M, J, Q) as the would-be BH.

The boundary of the region where micro-states differ from BH satisfy $S \approx A/4$. [S. Mathur (2005)]

Classical BH arises as "coarse-grained" description when only the geometry outside the "horizon" is taken into account

BHs in String Theory: The Naive D1-D5

Black Holes in string theory can be constructed as bound states of intersecting (Dp/M)branes. E.g. 'small' BPS BH in D=5

Brane	t	x_1	x_2	x_3	x_4	y_5	y_6	y_7	y_8	y_9
$D1$	$-$	.	.	.	.	$-$	.	.	.	.
$D5$	$-$	.	.	.	.	$-$	$-$	$-$	$-$	$-$

Harmonic function rule: superpose the harmonics with an exponent $-1/2$ for N directions and $1/2$ for D directions

$$ds^2 = (H_1 H_5)^{-1/2}(-dt^2 + dy_5^2) + (H_1 H_5)^{1/2}(dx_1^2 + \ldots dx_4^2)$$

$$+H_1^{1/2} H_5^{-1/2}(dy_6^2 + \ldots dy_9^2)$$

$$F_{01m} = \partial_m H_1^{-1} \quad F_{0\ldots 5m} = \partial_m H_5^{-1} \quad e^{\phi} = H_1^{1/2} H_5^{-1/2}$$

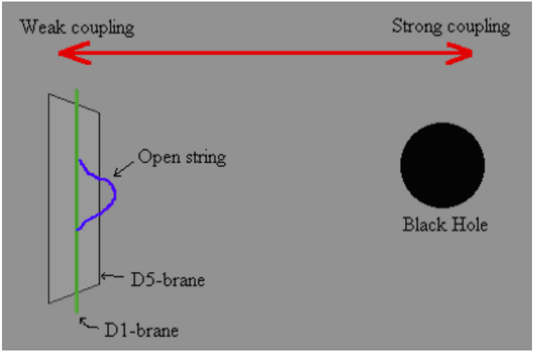
The D1-D5 system is U-dual to F1-P or D3-D3'

Naive D1-D5-P: Strong Coupling

Strong Coupling: Supergravity and "real" Black Hole in $D = 5$.
Small curvature at the horizon: $g_s Q >> 1$.
Macroscopic (geometric) entropy $S_{BH} = 2\pi\sqrt{Q_1 Q_5 Q_P}$

$$ds^2 = (H_1 H_5)^{-1/2}[-dt^2 + dy_5^2 + (H_P - 1)(dt + dy_5)^2]$$

$$+(H_1 H_5)^{1/2}(dx_1^2 + \ldots dx_4^2) + H_1^{1/2} H_5^{-1/2}(dy_6^2 + \ldots dy_9^2)$$



D1-D5 Fuzz-ball

$$ds^2 = -(H_1 H_5)^{-1/2}[(dt + A_i dx^i)^2 - (dy_5 + B_i dx^i)^2]$$
$$+(H_1 H_5)^{1/2} \sum_{i=1}^{4} dx_i^2 + (H_1/H_5)^{1/2} \sum_{a=1}^{4} dy_a^2$$
$$H_1 = 1 + \frac{Q_1}{\ell} \int_0^\ell \frac{dv}{|\vec{x} - \vec{F}(v)|^2} \quad H_5 = 1 + \frac{Q_1}{\ell} \int_0^\ell \frac{dv |\dot{F}(v)|^2}{|\vec{x} - \vec{F}(v)|^2}$$
$$A_i = \frac{Q_1}{\ell} \int_0^\ell \frac{dv \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2} \quad dB = \star_4 dA \quad v = t - y_5$$

E.g. circle: $F_1 = cos(2\pi v/\ell)$, $F_2 = sin(2\pi v/\ell)$, $F_3 = F_4 = 0$
Regular geometry! Coordinate singularity on the curve $x^i = F^i(v)$ resolved into K-K monopole: D1D5 fuzz-ball horizon-less and regular! Throat of the hole ends in a smooth "cap", whose shape, determined by $F(v)$ profile, discriminates different micro-states ('hairs'). Entropy $S_{micro} = 2\sqrt{2}\pi\sqrt{Q_1 Q_5}$ from CFT or from 'geometric quantization' of the profile $F(v) \sim$ transverse oscillations of the string in $\mathbb{R}^4$ in the F1-P 'frame'

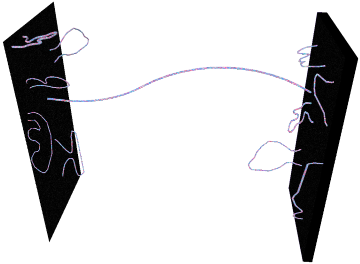
Naive D1-D5-P: Weak Coupling

Fuzz-ball proposal proven in the 2 charge case, yet 'small' BPS BH with zero horizon area in the supergravity limit

Large BH's require 3 charges in $D = 5$ or 4 charges in $D = 4$.

Weak Coupling: D-branes and open strings with $g_s Q << 1$.

For BPS BH's in $D = 5$: $S_{micro} = S_{MACRO}$. [Strominger, Vafa (1996)]



For $V_{T_4} << R^4_{S_1}$, $d = 1 + 1$, $\mathcal{N} = (4, 4)$, gauge group $U(Q_1) \times U(Q_5)$.

Central charge $c = n_{bose} + \frac{1}{2} n_{fermion} = 6 N_1 N_5$, from $(1, 5)$ strings.

For large charges, degeneracy given by C(H)ardy-Ramanujan formula:

$d(Q_P) \sim e^{2\pi \sqrt{c Q_P / 6}} \Rightarrow S_{micro} = log(d(Q_P)) = S_{MACRO}$

But what are the micro-states in the gravity picture?

Part II

4-d BH micro-state geometries from string amplitudes

Stringy Origin of 4d BPS Black Holes Micro-states

Enormous progress in 5-d [Bena, Giusto, Gibbons, Martinec, Russo, Shigemori, Warner, ...]

Much less known in 4-d !

Our goal: recover micro-state geometries in supergravity from the underlying fundamental string theory description

In particular we consider bound-states of 4 stacks of (orthogonally) intersecting D3-branes on T^6 in Type IIB

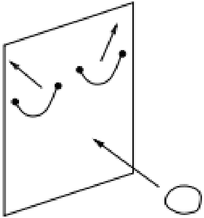
dual to D2-D2-D2-D6 in Type IIA or M2-M5-P-KK6 in M-theory

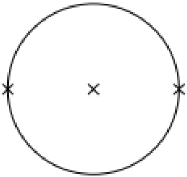
Brane	t	x_1	x_2	x_3	y_1	$\tilde{y}_1$	y_2	$\tilde{y}_2$	y_3	$\tilde{y}_3$
$D3_0$	—	.	.	.	—	.	—	.	—	.
$D3_1$	—	.	.	.	—	.	.	—	.	—
$D3_2$	—	.	.	.	.	—	—	.	.	—
$D3_3$	—	.	.	.	.	—	.	—	—	.

We will derive a 1:1 relation between open string condensates and fields in the bulk for a large class of 4d BPS BH's

Mixed Open-Closed Scattering Amplitudes

The micro-state geometries will be derived from mixed open-closed disk amplitudes, computing the emission rate of massless closed strings from open string condensates binding different stacks of branes.





Closed String Fields

$$g_{MN},\ b_{MN},\ C^{(4)}_{MNPQ}$$

Open String Fields

$$\mu^A,\ \phi^i$$

From Amplitudes to Supergravity Fields

We will work at leading order in g_s (disk), take all open string momenta equal (or tending) to zero and the closed string momentum k only in non compact space directions.

$$\mathcal{A}(h,k) \propto \int \frac{d^{2+n}z}{V_{CKV}} \langle W_{closed}(h,k); z, \bar{z}) V_{open}(z_1) \dots V_{open}(z_n) \rangle$$

The relation is well defined only if the disk diagram cannot factorize via the exchange of open-string states
Choose 'polarizations' of open strings in such a way that no factorization diagram be allowed
The deviation from flat space of a closed-string field is extracted from the string amplitude:

$$\delta\tilde{\phi}(k) = -\frac{i}{k^2}\frac{\delta\mathcal{A}(h,k)}{\delta h} \quad \rightarrow \quad \delta\phi(x) = \int \frac{d^3k}{(2\pi)^2}\tilde{\phi}(k)e^{ikx}$$

Supergravity Solution: the Love-ful Eight

Type IIB supergravity equations (with $\phi = g_s$, $C_0 = C_2 = B_2 = 0$)

$$R_{MN} = \frac{1}{4\cdot 4!} F_{MP_1P_2P_3P_4}\, F_N^{P_1P_2P_3P_4} \qquad F_5 = *_{10} F_5 \qquad F_5 = dC_4$$

8 harmonic functions $H_a = \{V, L_I, K^I, M\}, I = 1, 2, 3$ (STU model)

$$ds^2 = -e^{2U}(dt + w)^2 + e^{-2U}\,|d\vec{x}|^2 + \sum_{I=1}^{3}\left[\frac{dy_I^2}{Ve^{2U} Z_I} + Ve^{2U} Z_I\, \tilde{e}_I^2\right]$$

$$C_4 = \alpha_0 \cdot \tilde{e}_1 \cdot \tilde{e}_2 \cdot \tilde{e}_3 + \beta_0 \cdot dy_1 \cdot dy_2 \cdot dy_3 + \frac{\epsilon_{IJK}}{2}\left(\alpha_I \cdot dy_I \cdot \tilde{e}_J \cdot \tilde{e}_K + \beta_I \cdot \tilde{e}_I \cdot dy_J \cdot dy_K\right)$$

where $\cdot = \wedge$, ϵ_{IJK} (reduced) intersection form for 3-cycles in T^6,

$$Z_I = L_I + \frac{|\epsilon_{IJK}|}{2}\frac{K^J K^K}{V} \quad , \quad \mu = \frac{M}{2} + \frac{L_I K^I}{2\,V} + \frac{|\epsilon_{IJK}|}{6}\frac{K^I K^J K^K}{V^2}$$

$$e^{-4U} = Z_1 Z_2 Z_3 V - \mu^2 V^2$$

$$*_3 dw = V d\mu - \mu dV - V Z_I d\frac{K^I}{V} \quad , \quad \tilde{e}_I = d\tilde{y}_I - \left(\frac{K^I}{V} - \frac{\mu}{Z_I}\right) dy_I$$

Harmonic Multipole Expansion

Setting $\ell_{D3} = 4\,\pi\, g_s\, (\alpha')^2 / V_{D3} = 1$

$$L_I \approx 1 + \frac{N_I}{|x|} \quad V \approx 1 + \frac{N_0}{|x|} \quad but \quad K^I \approx c_i^{K^I} \frac{x^i}{|x|^3} \quad M \approx c_i^M \frac{x^i}{|x|^3}$$

Multi-pole expansion $H_a(x) = h_a + \sum_{n=0}^{\infty} c^a_{i_1 \ldots i_n} P_{i_1 \ldots i_n}(x)$ with

$$P_{i_1 \ldots i_n}(x) = \int \frac{d^3 k}{(2\pi)^3} e^{ikx} \tilde{P}_{i_1 \ldots i_n}(k) \qquad \tilde{P}_{i_1 \ldots i_n}(k) = \frac{4\pi\, i^n}{n!\, k^2} k_{i_1} \ldots k_{i_n}$$

$P(x)$'s singular at $x = 0$, but for appropriate choice of $c_{i_1 \ldots i_n}$ infinite sum may produce a fuzzy and smooth geometry.

Three classes of solutions: L, K and M solutions related to 1, 2 and 4 open-string insertions on the boundary of the disk.

The "superposition" of L, K and M solutions produces SUGRA micro-state geometries.

L Solution

L solutions are geometries that fall-off at infinity as Q_i/r, corresponding to a single stack of branes.

$$V = L(x) \qquad M = K^I = 0 \qquad L_I = 1$$

At linear order in ℓ_{D3} one finds:

$$\delta g_{MN} dx^M dx^N = \frac{\delta L}{2}\left[dt^2 - \sum_{i=1}^{3}(dy_i^2 - dx_i^2 - d\tilde{y}_i^2)\right] + \ldots$$

$$\delta C_4 = -\delta L \wedge dt \wedge dy_1 \wedge dy_2 \wedge dy_3 + A \wedge d\tilde{y}_1 \wedge d\tilde{y}_2 \wedge d\tilde{y}_3 + \ldots$$

with $\delta L = L - 1$ and A both of order ℓ_{D3}. One can take:

$$L = 1 + \frac{\ell_{D3} N_0}{|x|} + \ldots \qquad *_3 dL = dA$$

One-boundary Amplitude

Very well known result, modulo 'untwisted' open-string insertions

$$\mathcal{A}_{NS-NS,\xi(\phi)} = \left\langle c\bar{c} W^{(-1,-1)}_{NS-NS}(z,\bar{z}) c V^{(0)}_{\xi(\phi)}(z_1) \right\rangle = i\, c_{\rm NS}\ {\rm tr}(ER)\xi(k)$$

where $E = h + b$, R reflection matrix ($+1$ Neumann, -1 Dirichlet)

$$W^{(-1,-1)}_{NSNS}(z,\bar{z}) = c_{\rm NS}\, (ER)_{MN}\, e^{-\varphi}\psi^M e^{ikX}(z)\, e^{-\varphi}\psi^N e^{ikRX}(\bar{z})$$

$$V^{(0)}_{\xi(\phi)}(z_1) = \sum_{n=0}^{\infty} \xi_{i_1\dots i_n}\, \partial X^{i_1}(z_1) \prod_{a=2}^{n} \int_{-\infty}^{\infty} \frac{dz_a}{2\pi}\, \partial X^{i_a}(z_a)$$

with $\xi(\phi) = \sum_{n=0}^{\infty} \xi_{i_1\dots i_n} \phi^{i_1} \dots \phi^{i_n}$ and $z_a = \bar{z}_a$ (open strings)

The asymptotic deviation from the flat metric can be extracted:

$$\delta\tilde{g}_{MN}(k) = \left(-\frac{i}{k^2}\right) \sum_{n=0}^{\infty} \frac{\delta\mathcal{A}_{NS-NS,\phi^n}}{\delta h_{MN}} = c_{\rm NS} \frac{\xi(k)}{k^2}\, (\eta R)_{MN}$$

After Fourier transform one finds agreement with SUGRA:

$$\delta g_{MN} = \int \frac{d^3k}{(2\pi)^3} \delta\tilde{g}_{MN} = -\frac{1}{2}(\eta R)_{MN}\, \delta L(x) \quad \textit{and} \quad \delta b_{MN} = 0\,!$$

In particular, for a single D3 brane at position $x = a$: $\xi(\phi) \sim e^{i\,a\,\phi}$

K Solution

K solutions are geometries that fall-off at infinity as $Q_i Q_j / r^2$

$$K^3 = -M = K(x) \qquad \mu = 0 \qquad L_I = V = 1 \qquad K^1 = K^2 = 0$$

They are associated to fermionic bilinears localized at the intersection of two branes and in general they carry angular momentum.

At linear order in ℓ_{D3} one finds ($*_3 dw = -dK$):

$$\delta g_{MN} dx^M dx^N = -2\, w\, dt - 2\, K\, dy_3 d\tilde{y}_3 + \dots$$

$$\delta C_4 = (K\, dt \wedge dy_3 - w \wedge d\tilde{y}_3) \wedge (dy_1 \wedge d\tilde{y}_2 + d\tilde{y}_1 \wedge dy_2)$$

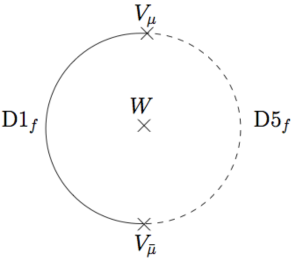
For example one can take K to be

$$K \approx \frac{v_i x_i}{|x|^3} \qquad w \approx \epsilon_{ijk}\, v_i \frac{x_j\, dx_k}{|x|^3}$$

Two-boundary Amplitude

$$\mathcal{A}^{NS-NS}_{\mu^2,\xi(\phi)} = \int dz_4 \left\langle c(z_1) V_{\bar\mu}(z_1)\, c(z_2) V_\mu(z_2)\, c(z_3) W(z_3,z_4)\, V_{\xi(\phi)} \right\rangle$$

where $V_{\bar\mu}(z_1) = \bar\mu^A\, e^{-\varphi/2}\, S_A \sigma_2\sigma_3 \qquad V_\mu(z_2) = \mu^B\, e^{-\varphi/2}\, S_B\sigma_2\sigma_3$



$$\left\langle \mathrm{tr}\, \bar\mu^{(A}\mu^{B)} \right\rangle = \frac{1}{3!}\, v^{MNP}\, \Gamma^{AB}_{MNP} \qquad \mathcal{A}^{NS-NS}_{\mu^2,\xi(\phi)} = \frac{\xi(k)}{3!}\, (ER)_{MN}\, k_P\, v^{MNP}$$

with $v^{MNP} \in$ **10** of SO(6) (NO **6**!!) e.g. for $v_{3 y_3 \tilde y_3} = -v_{12t} = 4\pi v$

$$\delta g_{2t} = -v\,\frac{x_1}{|x|^3} \qquad \delta g_{1t} = v\,\frac{x_2}{|x|^3} \qquad \delta g_{y_3\tilde y_3} = -v\,\frac{x_3}{|x|^3}$$

M Solution

M solutions are geometries that fall-off at infinity as $Q_1 Q_2 Q_3 Q_4/r^3$, associated to choice $c_i^M + \sum_{i=1}^3 c_i^K = 0$, e.g.

$$K^2 = M = M(x) \qquad \mu = M \qquad L_I = V = 1 \qquad K^1 = K^3 = 0$$

$$\delta g_{MN} dx^M dx^N = 2M\,(dy_1\, d\tilde{y}_1 + dy_3\, d\tilde{y}_3) + \dots$$

$$\delta C_4 = -M\, dt \wedge (dy_1 \wedge d\tilde{y}_2 \wedge dy_3 + d\tilde{y}_1 \wedge d\tilde{y}_2 \wedge d\tilde{y}_3)$$

$$+ w_2 \wedge (dy_1 \wedge dy_2 \wedge dy_3 + d\tilde{y}_1 \wedge dy_2 \wedge d\tilde{y}_3) + \dots$$

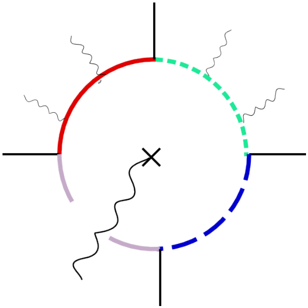
with $w_2 = *_3 dM$

In particular one can take the harmonic M to be 'quadruple'

$$M \approx v_{ij} \frac{3\, x_i\, x_j - \delta_{ij} |x|^2}{|x|^5}$$

Four-Boundary Amplitude

Insertion of four fermions $\mu_{a,a+1}$ starting on D3-branes of type a and ending on D3-branes of type $a+1$ with $a = 0, 1, 2, 3 \pmod 4$)
Even if each intersection preserves $\mathcal{N} = 2$ SUSY (1/4 BPS), so that each fermion $\mu_{a,a+1}$ paired with its conjugate $\bar{\mu}_{a,a+1}$, whole configuration preserves only $\mathcal{N} = 1$ SUSY (1/8 BPS).
The condensate is complex e.g. $\mu_1\mu_2\bar{\mu}_3\bar{\mu}_4 \neq \bar{\mu}_1\bar{\mu}_2\mu_3\mu_4$



$$\mathcal{A}^{NS-NS}_{\mu^4,\xi(\phi)} = \int dzd^2w \left\langle cV_{\mu_1}(z_1)cV_{\mu_2}(z_2)V_{\bar{\mu}_3}(z{=}\bar{z})cV_{\bar{\mu}_4}(z_4)W_{NSNS}(w,\bar{w})V_{\xi(\phi)}\right\rangle$$

Four-Boundary Amplitude

$$\left\langle \mathrm{tr}\, \mu_1^{(\alpha}\, \mu_2^{\beta)}\, \bar{\mu}_3^{(\dot\alpha}\, \bar{\mu}_4^{\dot\beta)} \right\rangle = \frac{2\pi v^{ij}}{c_{\mathrm{NS}}\, \mathcal{I}_1}\, \sigma_i^{\alpha\dot\alpha} \bar{\sigma}_j^{\beta\dot\beta} \quad v^{ij} \in (\mathbf{3},\mathbf{3}) \textit{ of } SU_L(2) \times SU_R(2)$$

Need Z_2 twist field correlator on the boundary of the disk

$$\langle \sigma_2(z_1)\sigma_2(z_2)\sigma_2(z_3)\sigma_2(z_4)\rangle = f\left(\frac{z_{14}z_{23}}{z_{13}z_{24}}\right)\left(\frac{z_{13}z_{24}}{z_{12}z_{23}z_{34}z_{41}}\right)^{1/4}$$

where $f(x) = \frac{\Lambda(x)}{\sqrt{F(x)F(1-x)}}$ with $F(x) = {}_2F_1(1/2, 1/2; 1; x)$ and

$\Lambda(x) = \sum_{n_1,n_2} \exp\left\{-\frac{2\pi}{\alpha'}\left[\frac{F(1-x)}{F(x)}\, n_1^2 R_1^2 + \frac{F(x)}{F(1-x)}\, n_2^2 R_2^2\right]\right\} \approx 1$

$$\mathcal{A}^{NS-NS}_{\mu^4,\xi(\phi)} = \left[(ER)_{[1\bar{1}]} + (ER)_{[3\bar{3}]}\right] k_i k_j\, v^{ij}\, \xi(k)$$

so that $\delta\tilde{g}_{1\bar{1}} = \delta\tilde{g}_{3\bar{3}} = -2\pi i \xi(k) v^{ij} k_i k_j / k^2$
Agreement with SUGRA solution to leading order in ℓ_{D3}.
One can even turn on different condensates to get new solutions

$$\widetilde{\mathcal{O}}^{\alpha\dot\alpha\beta\dot\beta} = \mathrm{tr}\, \mu_1^{(\alpha}\, \bar{\mu}_2^{(\dot\alpha}\, \mu_3^{\beta)}\, \bar{\mu}_4^{\dot\beta)} \quad \textit{or} \quad \hat{\mathcal{O}}^{(\alpha\beta\gamma)\dot\beta} = \mathrm{tr}\, \mu_1^{(\alpha}\, \mu_2^{\beta}\, \mu_3^{\gamma)}\, \bar{\mu}_4^{\dot\beta}$$

Four Boundary: Some speculations on the entropy

- In some sense, thanks to the presence of $\mathcal{N}=2$ SUSY preserving $D3_aD3_b$ pairs, the $D3^4$ more closely related than $D1D5P$ to $D1D5$ system. 'Realistic' four-charge case may turn out to be simpler than three-charge case!
- The number of disks with four different boundaries grows as $Q_1Q_2Q_3Q_4 = \mathcal{I}_4$. One can attempt the calculation of the entropy via geometric quantization by introducing suitable profile-dependent harmonic functions, as in the D1-D5 case.
- A family of asymptotically $AdS_2 \times S^2 \times T^6$ geometries has been found and shown to be regular. Harmonic functions written in terms of an arbitrary profile [Lunin (2015)]

$$H(\vec{x}) = h_{reg}(\vec{x}) + \int_0^{2\pi} \frac{dv}{2\pi} \frac{1}{|\vec{x}-\vec{F}(v)|} \sqrt{1 + \frac{(\vec{x}-\vec{F})\vec{A}(v)}{|\vec{x}-\vec{F}|^2}}$$

- For asymptotically flat solutions in 4d, no-go theorem: NO non-singular solutions in GR. Either include higher-derivative terms or get 'generalised' regularity in five or higher dimension

Part III.
Multi-center ansatz, Bubble Equations boundary conditions and ‘regularity’

From 4 to 10 (or 11) dimensions and back: STU et cetera

4-dim $\mathcal{M}_{STU} = [SL(2,R)/U(1)]^3 \subset E_{7(+7)}/SU(8) = \mathcal{M}_{\mathcal{N}=8}$

$$\mathcal{L}_{STU\sim U_1U_2U_3} = \frac{1}{16\pi G}\left(R_4 - \sum_{I=1}^{3}\frac{\partial_\mu U_I \partial^\mu \bar{U}_I}{2ImU_I{}^2} - \frac{1}{4}F_a\mathcal{I}^{ab}F_b - \frac{1}{4}F_a\mathcal{R}^{ab}\widetilde{F}_b\right)$$

10-dim uplift

$$ds^2_{10} = -e^{2U}(dt+w)^2 + e^{-2U}\,|d\vec{x}|^2 + \sum_{I=1}^{3}\left[\frac{dy_I^2}{Ve^{2U}Z_I} + Ve^{2U}Z_I\,\tilde{e}_I^2\right]$$

where $Z_I = L_I + \frac{|\epsilon_{IJK}|}{2}\frac{K^JK^K}{V}$, $\mu = \frac{M}{2} + \frac{L_IK^I}{2\,V} + \frac{|\epsilon_{IJK}|}{6}\,\frac{K^IK^JK^K}{V^2}$ and

$$e^{-4U} = \mathcal{I}_4(L_I, V, K^I, M) = Z_1Z_2Z_3V - \mu^2V^2 = L_1\,L_2\,L_3\,V - K^1\,K^2\,K^3\,M$$

$$+\frac{1}{2}\sum_{I>J}^{3}K^IK^JL_IL_J - \frac{1}{2}MV\sum_{I=1}^{3}K^IL_I - \frac{1}{4}M^2V^2 - \frac{1}{4}\sum_{I=1}^{3}(K^I)^2L_I^2$$

11-dim uplift $ds_{T^6} = \sum_{I=1}^{3} Z_I^{-1}\,(Z_1\,Z_2\,Z_3)^{\frac{1}{3}}\,(dy_I^2 + d\tilde{y}_I^2)$ and

$$ds^2_5 = -\frac{[dt+\mu(d\Psi+w_0)+w]^2}{(Z_1\,Z_2\,Z_3)^{\frac{2}{3}}} + (Z_1\,Z_2\,Z_3)^{\frac{1}{3}}\left[V^{-1}(d\Psi+w_0)^2 + V|d\vec{x}|^2\right]$$

Asymptotic geometry and charges

(Later on $16\pi G = 1$)

$$\mathfrak{M} = \frac{1}{8\pi G}\int_{S^2_\infty} \star_4 \, d\xi^{(t)} \quad , \quad J = -\frac{1}{16\pi G}\int_{S^2_\infty} \star_4 \, d\xi^{(\phi)} \quad ,$$

$$Q^a = \frac{1}{4\pi}\int_{S^2_\infty} (\mathcal{I}^{ab} \star_4 F_b - \mathcal{R}^{ab} F_b) \quad , \quad P_a = \frac{1}{4\pi}\int_{S^2_\infty} F_a$$

Boundary conditions and charges for orthogonal branes

$$V \approx 1 + \frac{v}{r} \qquad L_I \approx 1 + \frac{\ell_I}{r} \qquad K^I = M \approx 0$$

$$\mathfrak{M} = v + \ell_1 + \ell_2 + \ell_3 \, , \; P = (v, 0, 0, 0) \, , \; Q = (0, \ell_1, \ell_2, \ell_3) \, , \; J = 0$$

Boundary conditions and charges for branes at angle

$$V \approx 1 + \frac{v}{r} \quad L_I \approx 1 + \frac{\ell_I}{r} \quad K^1 \approx g + \frac{k^1}{r} \qquad K^2 \approx g \quad K^3 = M = 0$$

$$\mathfrak{M} = v + \ell_1 + \ell_2 + \ell_3, P = (v, -g(\ell_1 + \ell_2), 0, 0), Q = (0, \ell_1, \ell_2, \ell_3), J = 0$$

Micro-state geometries

Multi-center Taub-NUT ansatz ($r_i = |\vec{x} - \vec{x}_i|$, $i = 1, \dots N$)

[Bena, Warner, Gibbons, Cvetic, Lu, Pope, ...]

$$V = v_0 + \sum_{i=1}^{N} \frac{q_i}{r_i}, L_I = \ell_{0I} + \sum_{i=1}^{N} \frac{\ell_{I,i}}{r_i}, K^I = k_0^I + \sum_{i=1}^{N} \frac{k_i^I}{r_i}, M = m_0 + \sum_{i=1}^{N} \frac{m_i}{r_i}$$

Near each center, $R^4/Z_{|q_i|}$, asymptotically $R^3 \times S^1_\Psi$
Geometry factorises, i.e. regular in 5-d (!), if near the centers

$$Z_I\Big|_{r_i \approx 0} \approx \zeta^i_I \,(finite) \quad and \quad \mu\Big|_{r_i \approx 0} \approx 0 \,(zero)$$

Absence of horizons and closed time-like curves requires

$$Z_I V > 0 \qquad and \qquad e^{2U} > 0$$

w closed exact form near the centres

Bubble equations

Z_I finite near the centers if

$$\ell_{I,i} = -\frac{|\epsilon_{IJK}|}{2} \frac{k_i^J \, k_i^K}{q_i} \quad , \quad m_i = \frac{k_i^1 \, k_i^2 \, k_i^3}{q_i^2}$$

μ vanishes near the centers if Bubble Equations are satisfied

$$\sum_{j=1}^{N} \frac{\Pi_{ij}}{r_{ij}} + v_0 \, \frac{k_i^1 \, k_i^2 \, k_i^3}{q_i^2} - \sum_{I=1}^{3} \ell_{0I} \, k_i^I - |\epsilon_{IJK}| \, \frac{k_0^I \, k_i^J \, k_i^K}{2 \, q_i} - m_0 q_i = 0$$

with $\Pi_{ij} = (q_i q_j)^{-2} \prod_{I=1}^{3} \left(k_i^I q_j - k_j^I q_i \right)$ and $r_{ij} = |\vec{x}_i - \vec{x}_j|$
Bubble equations imply absence of pernicious Dirac-Misner strings

$$*_3 dw = \frac{1}{2} \sum_{i,j=1}^{N} \Pi_{ij} \, \left(\frac{1}{r_j} - \frac{1}{r_{ij}} \right) d \frac{1}{r_i} = \frac{1}{4} \sum_{i,j=1}^{N} \Pi_{ij} \, \omega_{ij}$$

with $\omega_{ij} = (\vec{n}_i + \vec{n}_{ij}) \cdot (\vec{n}_j - \vec{n}_{ij}) d\phi_{ij} / r_{ij}$ free of D-M strings along lines between two centers, since numerator vanishes there

Scaling solutions

If the coefficients k_i^I satisfy

$$v_0\, m_i - \sum_{I=1}^{3} \ell_{0I}\, k_i^I + k_0^I\, \ell_{Ii} - m_0 q_i = 0$$

invariance under rigid rescaling of the positions of the centres

$$\vec{x}_i \longrightarrow \lambda \vec{x}_i$$

Multiplying (...) by the positions of the centers $\vec{x}_i$, the solution can be shown to carry zero angular momentum

$$\vec{J} = m_0\, \vec{v}_2 - v_0\, \vec{m}_2 + \ell_{0I}\, \vec{k}_2^I - k_0^I\, \vec{\ell}_{2I} = 0$$

in agreement with (Sen's) expectations for micro-states

Fuzz-balls of orthogonal branes

Boundary conditions

$$\ell_{0I} = v_0 = 1 \qquad m_0 = m = k_0^I = k^I = 0$$

For $q_i = 1$ (to avoid orbifold singularities, for simplicity)

$$P_0 = N \quad , \quad Q_I = -\sum_{i=1}^{N} \frac{|\epsilon_{IJK}| k_i^J \, k_i^K}{2}$$

Bubble Equations ($q_i = 1!$)

$$\sum_{j \neq i}^{N} \frac{\prod_{I=1}^{3}(k_i^I - k_j^I)}{r_{ij}} + {k^1}_i {k^2}_i {k^3}_i - \sum_{I=1}^{3} {k^I}_i = 0$$

absence of horizons and of closed time-like curves requires

$$Z_I \, V > 0 \qquad and \qquad e^{2U} > 0$$

Configurations with one or two centers fail to meet the BPS requirement $Q_I > 0$. Let us start (and end) with three centers

3-center case $N = 3 = P_0$

$$k^I{}_i = \begin{pmatrix} -n_1\, n_2 & -n_1\, n_3 & n_1\,(n_2 + n_3) \\ n_3 & n_2 & -n_2 - n_3 \\ -n_4 & n_4 & 0 \end{pmatrix}$$

scaling solutions: $n_2 = 0, n_1 = 1, n_3 = n_4 = n$

$$Q_1 = Q_2 = Q_3 = n^2 \quad , \quad any \quad r_{12} = r_{23} = r_{13} = R$$

non-scaling solutions:

- $n_2 = 0,\ n_1 = n_3 = 1,\ n_4 = n$: $r_{13} = r_{23} = R$ undetermined

$$Q_1 = Q_2 = n \qquad Q_3 = 1 \quad r_{12} = \frac{2\,n\,r_{23}}{2\,n + (n-1)\,r_{23}}$$

- $n_2 = n_4 = n,\ n_1 = 1,\ n_3 = 2\,n$: $r_{13} = r_{23} = R$ undetermined

$$Q_1 = Q_2 = n^2 \quad Q_3 = 13\,n^2 \quad r_{12} = \frac{r_{23}}{10 + r_{23}}$$

- $n_2 = 0,\ n_1 = 3\,n,\ n_3 = 2\,n,\ n_4 = n$: $r_{23} < 6\,(2-\sqrt{2})\,n^2$

$$Q_1 = 2\,n^2, Q_2 = 6\,n^2, Q_3 = 3\,n^2, r_{12} = \frac{12\,n^2\,r_{23}}{12\,n^2 - r_{23}}, r_{13} = \frac{6\,n^2\,r_{23}}{6\,n^2 - r_{23}}$$

Fuzz-balls of branes at angle

New boundary conditions

$$\ell_{0I} = v_0 = 1, m_0 = m = k_0^3 = k^3 = k^2 = 0, k_0^1 = k_0^2 = g, k^1 = g(\ell_1+\ell_2)$$

Generalized bubble equations

$$\sum_{j\neq i}^{N} \frac{k_{ij}^{(1)}\, k_{ij}^{(2)}\, k_{ij}^{(3)}}{r_{ij}} + k_i^1\, k_i^2\, k_i^3 - \sum_{I=1}^{3} k_i^I - g\, k_i^2\, k_i^3 - g\, k_i^1\, k_i^3 = 0$$

3-center case, $P_0 = 3$, n_1, n_2, n_3 positive integers, g rational

$$k^I{}_i = \begin{pmatrix} 0 & -n_1 & n_1 + g\, n_3(n_1+n_2) \\ n_2 & 0 & -n_2 \\ -n_3 & n_3 & 0 \end{pmatrix}$$

$$Q_1 = n_2\, n_3, Q_2 = n_1\, n_3, Q_3 = n_1\, n_2 + g\, n_2\, n_3(n_1 + n_2)$$

Some non-scaling solutions

- $n_1 = n_2 = n_3 = n = (2g)^{-1}$

$$k^I{}_i = \begin{pmatrix} 0 & -n & 2n \\ n & 0 & -n \\ -n & n & 0 \end{pmatrix}, \; r_{12} = \frac{4\,n^2\,r_{23}}{6\,n^2 - r_{23}}, \; r_{13} = \frac{4\,n^2\,r_{23}}{3\,n^2 - r_{23}}$$

$$Q_0 = 3 \qquad Q_1 = Q_2 = n^2 \qquad Q_3 = 2\,n^2, \; r_{23} < \frac{9-\sqrt{57}}{2} n^2.$$

- $n_1 = n_2 = n,\; n_3 = 2\,n,\; g = (4\,n)^{-1}$

$$k^I{}_i = \begin{pmatrix} 0 & -n & 2\,n \\ n & 0 & -n \\ -2\,n & 2\,n & 0 \end{pmatrix}, r_{12} = \frac{8\,n^2\,r_{23}}{12\,n^2 + r_{23}}, r_{13} = \frac{8\,n^2\,r_{23}}{6\,n^2 - r_{23}}$$

$$Q_0 = 3 \qquad Q_1 = Q_2 = Q_3 = 2\,n^2 \qquad r_{23} < n^2\left(-11 + \sqrt{145}\right)$$

Moduli space ... very preliminary

Non-compact in general. Yet, expect quantum effects to put a lower bound on R (separation between two centres $\sim$ depth of AdS throat) as well as an upper bound on R (energy gap of typical excitations in CFT dual description)
Compact in the last cases. For the minimal case $Q_0 = 3$, $Q_1 = Q_2 = Q_3 = 2$ get 12 choices of k_i^I ... matches with degeneracy of 'small' BH's

Summary and conclusions

Summary

- Precise dictionary between open string condensates and a large class of 4-d BPS BHs, computing amplitudes of NS-NS (R-R) closed strings in the presence of open string condensates living on D3-branes and/or at their intersections.
  - L solutions fall as Q_i/r and are associated to one boundary amplitudes.
  - K solutions fall as $Q_i Q_j/r^2$ and are associated to two boundary amplitudes (two twisted fermions)
  - M solutions fall as $Q_1 Q_2 Q_3 Q_4/r^3$ and are associated to four boundary amplitudes.

    One would like to identify this contribution as the micro-states of the four charge black hole.
- Multi-center ansatz: bubble equations (generalised), boundary conditions (M, Q, P, J) and "regularity" ... NO horizon, NO singularity (in $D = 5$), NO CTC's
- Scaling vs non-scaling solutions for $N = 3 = P_0$ and their (non-)compact moduli spaces

Comments

- Information paradox: deep conflict between General Relativity and Quantum Mechanics. Large BH entropy $S_{BH} = A_H/4$ vs uniquess of BH's for given M, Q and J.
- Unitarity violated: information neither visible at Horizon (null surface: particles/waves fall in or dilute) nor coded in Hawking radiation ... Need 'new' physics at putative horizon
- Success of String Theory in explaining microscopic origin of BPS BH entropy, yet in a regime where classical BH description not valid ... Need 'horizon-sized' and 'horizon-less' bound-states with same M, Q and J as classical BH: 'fuzz-ball s' or 'micro-state geometries'.
- Only small fraction of expected 'fuzz-balls' known in 5-d and even less in 4-d. Moreover, a-typical / not generic micro-states: carry angular momentum ($J_L J_R \neq 0$ in $D = 5$, $J \neq 0$ in $D = 4$), role in the 'BH ensemble' unclear. CFT description only known in very few case.
- Smooth (regular) geometries in $D = 5$ but NOT in $D = 4$. Need higher dim's and/or higher derivatives i.e. String Theory

Future Directions

- Generalize to D3-brane configurations with generic tilting on orbifolds (e.g. T^6/Z_3)
- Compute the contribution to the entropy of the known configurations (scaling vs non-scaling) and understand their CFT (AdS) and/or Quiver Quantum Mechanics description [Denef, Pioline, Manschoot, Sen, Garavuso, ...]
- Apply similar techniques to scattering of more than one (= two, at most) closed string (massive) states [Garousi, Myers, Klebanov, Hashimoto, D'Appollonio, Di Vecchia, Russo, Veneziano, Turton, MB, Teresi, ...]
- Construct new micro-state SUGRA solutions corresponding to diverse choices of the open string condensates
- Find regular non extremal and realistic (four-charge) geometries
- Study fuzz-ball mergers and GW production ... experimental test of String Theory ?