Thermal conductivity of dense astrophysical plasma: kinetic approach

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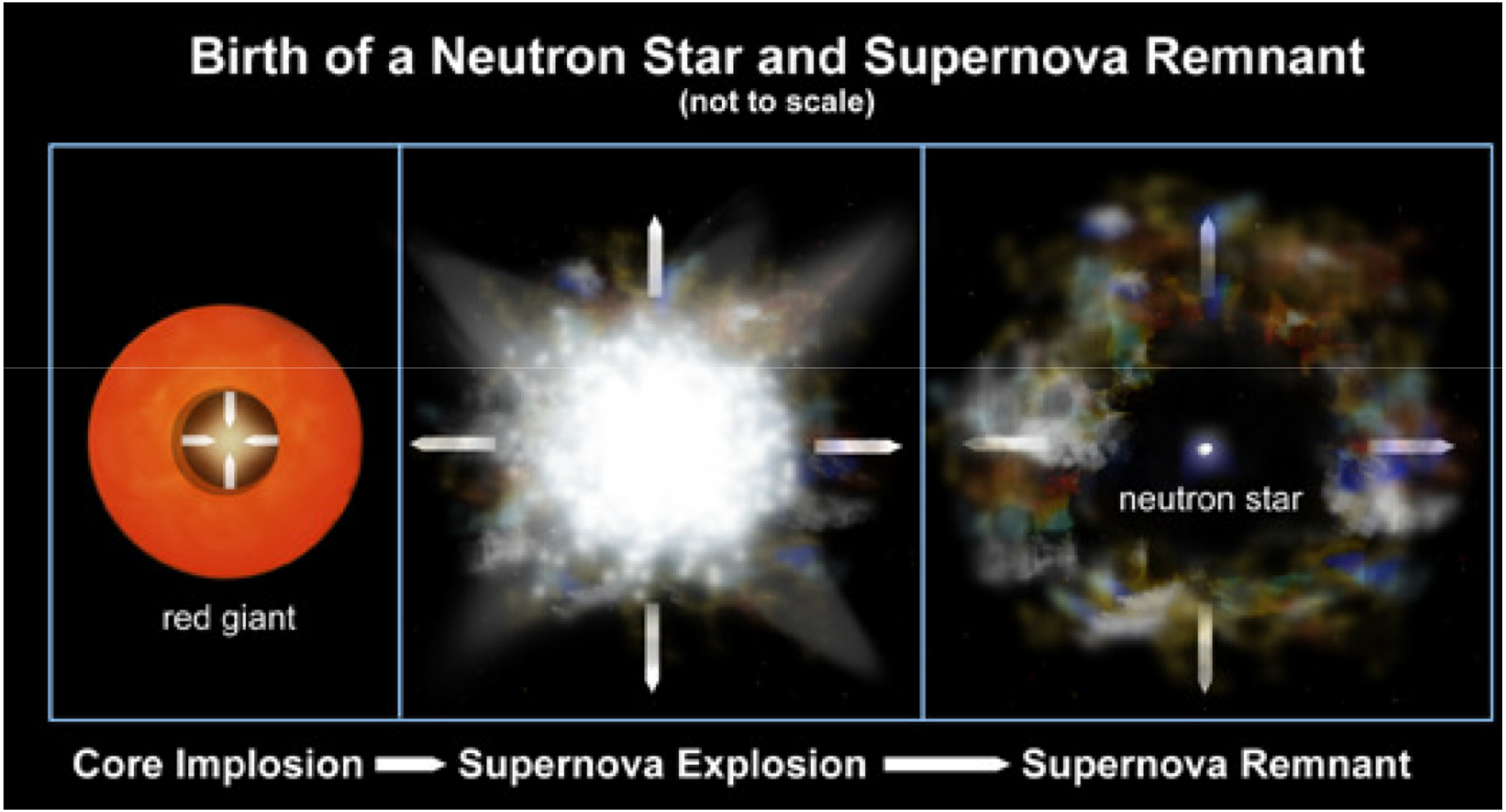
Birth of a Neutron Star and Supernova Remnant
(not to scale)
red giant
neutron star
Core Implosion
Supernova Explosion
Supernova Remnant

Source, RX J	Spin Periods, s	Amplitude/2	Temperature, eV	Absorption line energy, eV
1856.5−3754	7.06	1.5%	60-62	no
0720.4−3125	8.39	11%	85-87	270
1605.3+3249 (RBS 1556)	???	-	93-96	450
1308.6+2127 (RBS 1223)	10.31	18%	102	300
2143.0+0654 (RBS 1774)	9.44	4%	102-104	700
0806.4−4123	11.37	6%	92	460
0420.0−5022	3.45	13%	45	330

Wikipedia: magnificent seven

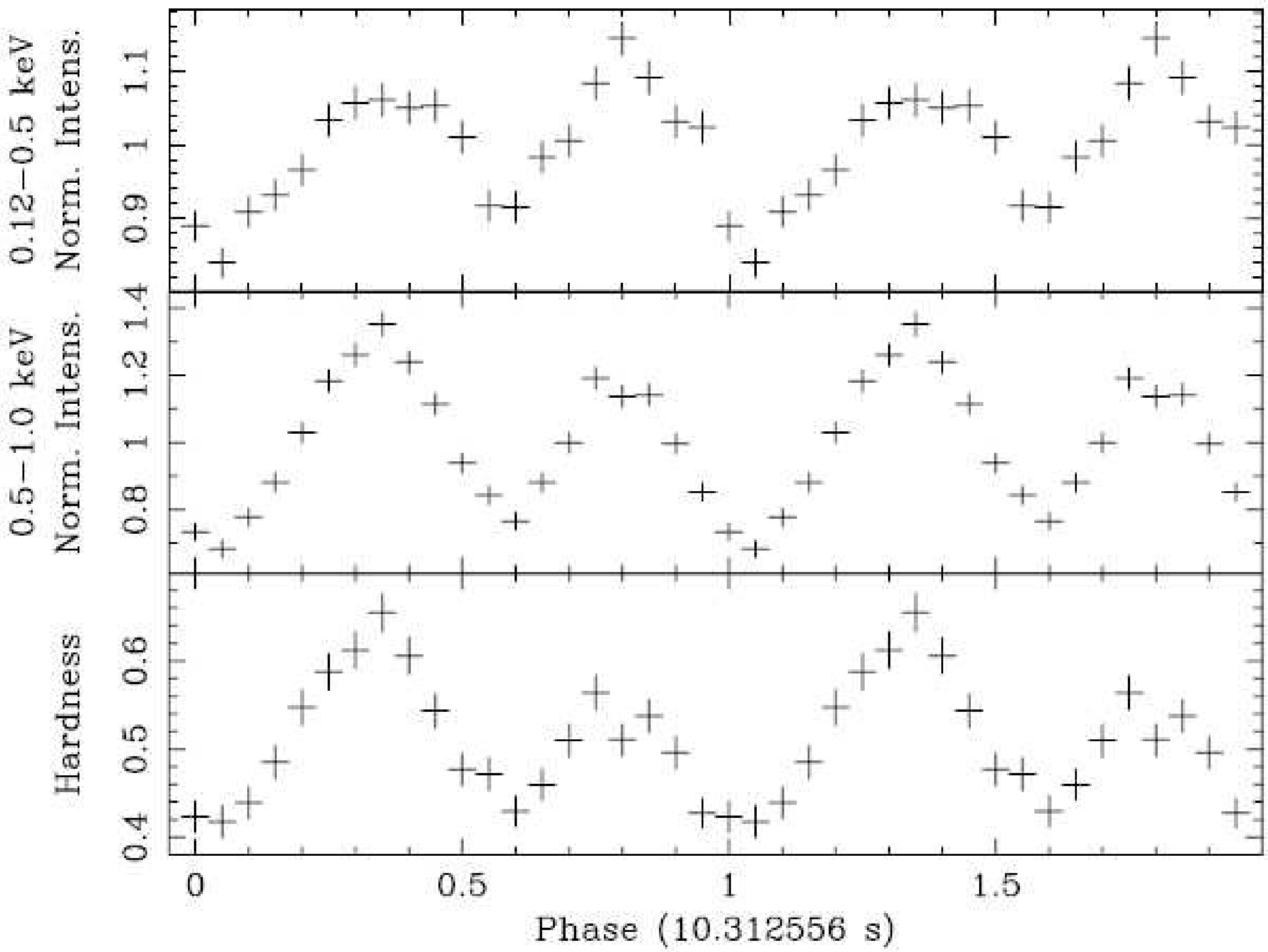

Fig. 3. Pulse profile of RBS1223 in the 0.12–0.5 keV (soft) and 0.5–1.0 keV (hard) energy bands, together with the ratio hard/soft, obtained from the EPIC-pn data of the Jan. 2003 observation.

Nuclear-energy release in neutron-star envelopes, and sources of x-ray emission

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If a neutron star possesses a strong magnetic field, then radiation at frequencies $\nu < eB/2\pi m_e c$ will have an anisotropic directional pattern, and the luminosity of the nonpulsating radiation of the star will become modulated with a period equal to the rotation period. In order to determine the depth of modulation as well as the character of the periodic spectrum variations, model atmospheres of magnetized neutron stars must be calculated. As a neutron star cools, the pole will become hotter than the equator because of anisotropic heat conductivity, and the modulation depth will be greater. In our case the luminosity

TRANSPORT PROPERTIES OF DENSE MATTER

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Except for the outer region of the star $\beta\omega \ll 1$, the electrons are relativistic, and the Landau levels are smeared out thermally. In this case the main effect of the magnetic fields will be to introduce anisotropy into the transport coefficients. To see this in the simplest case, consider a single example of electrons moving in $\boldsymbol{E}$ and $\boldsymbol{B}$ fields; then

$$m^*\left(\ddot{\boldsymbol{x}} + \frac{1}{\tau}\dot{\boldsymbol{x}}\right) = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}/c)\,, \tag{109}$$

where we have assumed the existence of a relaxation time, τ, to describe the collisions with other particles. Now the conductivity is a tensor, σ_{ij}, defined by

$$j_i = \sigma_{ij}E_j\,, \tag{110}$$

where the electric current, j_i, is

$$j_i = n_e v_i\,. \tag{111}$$

Solving the equation of motion for steady-state solutions,

$$\sigma_{ij} = \frac{n_e{}^2\tau}{m^*}\frac{1}{1+(\omega_c\tau)^2}\begin{bmatrix} 1 & \omega_c\tau & 0 \\ -\omega_c\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_c\tau)^2 \end{bmatrix}. \tag{112}$$

Now for large fields and pure materials $\omega_c\tau \gg 1$ and the conductivity is highly anisotropic; it is easy to move along the field lines but difficult to move perpendicular to them. When $\omega_c\tau \ll 1$, then σ is diagonal.

This dependence on the magnetic field is used in most subsequent papers on this subject

$$\frac{\lambda_\perp}{\lambda_\parallel} = \frac{1}{1+(\omega\tau)^2}.$$

Sydney Chapman, T. G. Cowling

The Mathematical Theory of Non-uniform Gases

An Account of the Kinetic Theory of Viscosity, Thermal Conduction and Diffusion in Gases

Cambridge University Press, 1952; 1970.

18.31. *The free-path theory of conduction of heat, and diffusion, in a magnetic field.* We now consider a simple free-path theory of diffusion and heat conduction in an ionized gas at rest in the presence of a magnetic field, using a method somewhat analogous to that of 6.3 and 6.4. As in 18.3, the magnetic field is taken to be parallel to Ox, and in addition it is assumed that the density, temperature, and composition are functions of z alone, and that $X_1 = 0$.

We assume, purely as a convenient rough approximation, that the mean time between successive collisions of a molecule m_s has the same value τ_s, whatever the molecular speed. With this assumption, by an argument similar to that of 5.41, e^{-t/τ_s} is the probability that at any given instant a molecule m_s has travelled without collision for a time at least equal to t.

The number of collisions per unit time experienced by molecules m_s in a volume $\mathbf{r}$, $d\mathbf{r}$ is $n_s d\mathbf{r}/\tau_s$. Let $\chi_s(c_s, z)\, d\mathbf{c}_s d\mathbf{r}/\tau_s$ denote the number of these which result in a molecule m_s entering the velocity-range $\mathbf{c}_s$, $d\mathbf{c}_s$; as the notation implies, it is assumed that χ_s depends only on the magnitude of $\mathbf{c}_s$, and not on its direction. In a gas in the uniform steady state χ_s is identical with Maxwell's function f_s, since the number of molecules entering any velocity-range through collision is equal to the number leaving. In general χ_s will differ only slightly from f_s.

We consider first diffusion. Since free-path methods seem unable to give an adequate theory of thermal diffusion, arising from inequalities of temperature, we assume that the temperature is uniform.

1. Теория теплопроводности и диффузии в магнитном поле, основанная на свободном пробеге

Рассмотрим теперь элементарную, основанную на понятии свободного пробега теорию диффузии и теплопроводности в покоящемся ионизованном газе в присутствии магнитного поля, пользуясь методом, аналогичным примененному в гл. 6, § 3 и 4. Как в § 3 настоящей главы, примем, что магнитное поле параллельно Ox и предположим дополнительно, что плотность, температура и состав зависят только от z и $X_1 = 0$.

Для простоты в качестве грубого приближения допустим, что среднее время между последовательными столкновениями молекулы m_s имеет одно и то же значение τ_s, независимо от ее скорости. При этом в силу рассуждений, подобных приведенным в гл. 5, § 4, п. 1, e^{-t/τ_s} выражает вероятность того, что к данному моменту молекула m_s двигалась без столкновений не менее, чем в течение времени t.

Число столкновений, совершаемых в единицу времени молекулами m_s в объеме $\mathbf{r}$, $d\mathbf{r}$, равно $n_s d\mathbf{r}/\tau_s$. Пусть $\chi(c_s, z)\, d\mathbf{c}_s d\mathbf{r}/\tau_s$ означает часть этих столкновений, в результате которых молекула m_s поступает в интервал скоростей $\mathbf{c}_s$, $d\mathbf{c}_s$. Как показывает обозначение, предполагается, что χ_s зависит только от величины $\mathbf{c}_s$ и не зависит от направления. Если газ находится в однородном стационарном состоянии, χ_s совпадает с максвелловской функцией f_s, так как число молекул, поступающих в любой интервал скоростей за счет столкновений, равно числу молекул, покидающих его. В общем случае χ_s лишь слабо отличается от f_s.

Рассмотрим сначала диффузию. Поскольку методы, основанные на свободном пробеге, по-видимому, неприменимы к

$$\overline{w}_1 = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\int_0^\infty \sin \omega_1 t\, e^{-t/\tau_1}\frac{dt}{\omega_1 \tau_1}$$

$$= \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{\tau_1}{1+\omega_1^2\tau_1^2}. \qquad \ldots\ldots 3$$

The velocity of diffusion of the molecules m_1 when the magnetic field is absent is found by putting $\omega_1 = 0$ in 3. The presence of the magnetic field accordingly results in a reduction in the velocity of diffusion parallel to Oz in the ratio $1 : (1 + \omega_1^2\tau_1^2)$.

In addition, the magnetic field produces a velocity of diffusion in the direction of Oy. By applying an argument similar to that used in proving 2, to the passage of molecules across an element dS of the plane $y = 0$, we find that

$$\bar{v}_1 = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\int_0^\infty (1-\cos\omega_1 t)\,\epsilon^{-t/\tau_1}\frac{dt}{\omega_1\tau_1}$$

$$= \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{\omega_1\tau_1^2}{1+\omega_1^2\tau_1^2}. \qquad \ldots\ldots 4$$

Hence in addition to the ordinary diffusion there is a flow of molecules perpendicular both to $\boldsymbol{H}$ and to the direction of the ordinary diffusion; we may call it *transverse* diffusion.

(Hall current)

Clearly **3** and **4** can only be approximate results; it is actually not possible, with any values of τ_1 and τ_2, for these formulae to be consistent, for all values of H, with the conditions that the gas as a whole should be at rest,* namely

$$n_1 m_1 \bar{v}_1 + n_2 m_2 \bar{v}_2 = 0, \quad n_1 m_1 \bar{w}_1 + n_2 m_2 \bar{w}_2 = 0. \qquad \ldots\ldots 5$$

Consequently results based on **3** and **4** must be treated with reserve; we cannot expect to deduce from them more than the relative order of magnitude of the direct and Hall currents, and the order of magnitude of the reduction in the conductivity. Using **3** and **4**, the direct and Hall currents are found to be

$$j_z = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{n_1 e_1 \tau_1}{1+\omega_1^2\tau_1^2} + \left(Z_2 - \frac{1}{\rho_2}\frac{\partial p_2}{\partial z}\right)\frac{n_2 e_2 \tau_2}{1+\omega_2^2\tau_2^2} \qquad \ldots\ldots 6$$

$$j_y = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{n_1 e_1 \omega_1 \tau_1^2}{1+\omega_1^2\tau_1^2} + \left(Z_2 - \frac{1}{\rho_2}\frac{\partial p_2}{\partial z}\right)\frac{n_2 e_2 \omega_2 \tau_2^2}{1+\omega_2^2\tau_2^2}. \qquad \ldots\ldots 7$$

Ясно, что соотношения (3.7) и (3.8) могут быть только приближенными; при любых значениях τ_1 и τ_2 эти формулы фактически не могут быть согласованы для всех значений H с условиями того, что газ в целом покоится[1)]

$$n_1 m_1 \bar{v}_1 + n_2 m_2 \bar{v}_2 = 0, \qquad n_1 m_1 \bar{w}_1 + n_2 m_2 \bar{w}_2 = 0. \tag{3.9}$$

Таким образом, к результатам, основанным на (3.7) и (3.8), следует относиться с осторожностью; мы не вправе рассчитывать на получение из них чего-либо большего, чем сравнительного порядка величины прямого тока и тока Холла, а также порядка величины снижения электропроводности. Используя (3.7) и (3.8), для прямого тока и тока Холла получаем

$$j_z = \left(Z_1 - \frac{1}{\varrho_1}\frac{\partial p_1}{\partial z}\right)\frac{n_1 e_1 \tau_1}{1+\omega_1^2\tau_1^2} + \left(Z_2 - \frac{1}{\varrho_2}\frac{\partial p_2}{\partial z}\right)\frac{n_2 e_2 \tau_2}{1+\omega_2^2\tau_2^2}, \tag{3.10}$$

$$j_y = \left(Z_1 - \frac{1}{\varrho_1}\frac{\partial p_1}{\partial z}\right)\frac{n_1 e_1 \omega_1 \tau_1^2}{1+\omega_1^2\tau_1^2} + \left(Z_2 - \frac{1}{\varrho_2}\frac{\partial p_2}{\partial z}\right)\frac{n_2 e_2 \omega_2 \tau_2^2}{1+\omega_2^2\tau_2^2}. \tag{3.11}$$

Transport coefficients from kinetic Boltzmann equation

Calculation of thermal conductivity coefficients of electrons in magnetized dense matter

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BOLTZMANN EQUATIONS

$$\frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial r_i} - \frac{e}{m_e}(E_i + \frac{1}{c}\varepsilon_{ikl} c_k B_l)\frac{\partial f}{\partial c_i} + J = 0.$$

$$J = J_{ee} + J_{eN} = R \int [f' f_1'(1-f)(1-f_1) -$$
$$- f f_1 (1 - f')(1 - f_1')] \times g_{ee} b db d\varepsilon dc_{1i} +$$
$$+ \int [f' f_N'(1-f) - f f_N (1 - f')] \times g_{eN} b db d\varepsilon dc_{N i}.$$

$$\frac{df}{dt} + v_i \frac{\partial f}{\partial r_i} - \left[\frac{e}{m_e}(E_i + \frac{1}{c}\varepsilon_{ikl} v_k B_l) + \frac{dc_{0i}}{dt}\right] \frac{\partial f}{\partial v_i}$$
$$- \frac{e}{m_e c}\varepsilon_{ikl} v_k B_l \frac{\partial f}{\partial v_i} - \frac{\partial f}{\partial v_i} v_k \frac{\partial c_{0i}}{\partial r_k} + J = 0,$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + c_{0i}\frac{\partial}{\partial r_i}.$$

TRANSFER EQUATIONS

$$\frac{dn_e}{dt} + n_e \frac{\partial c_{0i}}{\partial r_i} + \frac{\partial}{\partial r_i}(n_e \langle v_i \rangle) = 0,$$

$$\rho \frac{dc_{0i}}{dt} = \frac{1}{c} \varepsilon_{ikl} j_k B_l - \frac{\partial \Pi_{ik}}{\partial r_k},$$

$$\frac{3}{2} k n_e \frac{dT}{dt} - \frac{3}{2} kT \frac{\partial}{\partial r_i}(n_e \langle v_i \rangle) + \frac{\partial q_{ei}}{\partial r_i} + \Pi^e_{ik} \frac{\partial c_{0i}}{\partial r_k} =$$
$$= j_i (E_i + \frac{1}{c} \varepsilon_{ikl} c_{0k} B_l) - \rho_e \langle v_i \rangle \frac{dc_{0i}}{dt},$$

$$\langle v_{\alpha i} \rangle = \frac{R}{n_\alpha} \int f v_{\alpha i} dc_{\alpha i}, \quad n_e = R \int f dc_{ei},$$

$$c_{0i} = \frac{1}{\rho} \sum_\alpha \rho_\alpha \langle c_{ai} \rangle, \quad j_i = -n_e e \langle v_i \rangle,$$

$$q_{\alpha i} = \frac{1}{2} n_\alpha m_\alpha \langle v_\alpha^2 v_{\alpha i} \rangle.$$

EQUATIONS FOR THE FIRST APPROXIMATION FUNCTION

$$f_0 = [1 + \exp\left(\frac{m_e v^2 - 2\mu}{2kT}\right)]^{-1}, \quad R\int f_0 dv_i = n_e.$$

$$n_e = 2\left(\frac{kTm_e}{2\pi\hbar^2}\right)^{3/2} G_{3/2}(x_0),$$

$$P_e = 2kT\left(\frac{kTm_e}{2\pi\hbar^2}\right)^{3/2} G_{5/2}(x_0),$$

$$G_n(x_0) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{x^{n-1}dx}{1+exp(x-x_0)}, \;\; x_0 = \frac{\mu}{kT}:$$

where $G_n(x_0)$ are Fermi integrals.

$$f = f_0[1 + \chi(1 - f_0)]$$

$$\chi = -A_i \frac{\partial \ln T}{\partial r_i} - n_e D_i d_i \frac{G_{5/2}}{G_{3/2}},$$

$$d_i = \frac{\rho_N}{\rho} \frac{\partial \ln P_e}{\partial r_i} - \frac{\rho_e}{P_e} \frac{1}{\rho} \frac{\partial P_N}{\partial r_i} + \frac{e}{kT}(E_i + \frac{1}{c}\varepsilon_{ikl} c_{0k} B_l).$$

$$A_i = A^{(1)} v_i + A^{(2)} \varepsilon_{ijk} v_j B_k + A^{(3)} B_i (v_j B_j),$$

$$\xi = A^{(1)} + iBA^{(2)}$$

Integro-differential equation for \xi

is solved by expansion on orthogonal polinomials

For non-degenerate electrons

$$(1-s)^{-\frac{3}{2}-1}e^{\frac{xs}{1-s}} = \Sigma S_{3/2}^{(p)}(x)s^p.$$

Sonyne polynomials are orthogonal:

$$\int_0^\infty e^{-x}S_{3/2}^{(p)}(x)S_{3/2}^{(q)}(x)x^{3/2}dx = \frac{\Gamma(p+\frac{5}{2})}{p!}\delta_{pq},$$

and

$$S_{3/2}^{(0)}(x) = 1, \quad S_{3/2}^{(1)}(x) = \frac{5}{2} - x,$$

$$S_{3/2}^{(2)}(x) = \frac{35}{8} - \frac{7}{2}x + \frac{1}{2}x^2.$$

For partially degenerate electrons

$$Q_0(x) = 1, \quad Q_1(x) = \frac{5G_{5/2}}{2G_{3/2}} - x,$$

$$Q_2(x) = \frac{35}{8}\frac{G_{7/2}}{G_{3/2}} - \frac{7G_{7/2}}{2G_{5/2}}x + \frac{1}{2}x^2, \quad x = u^2.$$

$$\xi = a_0 Q_0 + a_1 Q_1 + a_2 Q_2,$$

$$A^{(3)} = c_0 Q_0 + c_1 Q_1 + c_2 Q_2.$$

System of equations for a_i contains matrix elements.

electron-nuclei interaction:

$$b_{jk} = R \int f_0 f_{N0} (1 - f_0^{'}) Q_j(u^2) u_i [Q_k(u^2) u_i$$

$$- Q_k(u^{'2}) u_i^{'}] g_{eN} b db d\varepsilon dc_{Ni} dc_i,$$

$$k \geq 0.$$

electron-electron interaction:

$$a_{jk} = R^2 \int f_0 f_{01}(1 - f_0^{'})(1 - f_{01}^{'}) Q_j(u^2) u_i [Q_k(u^2) u_i + Q_k(u_1^2) u_{1i} - Q_k(u^{'2}) u_i^{'} - Q_k(u_1^{'2}) u_{1i}^{'}] g_{ee} b db d\varepsilon dc_{1i} dc_i,$$

The average frequency of electron-ion collisions

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\frac{Z^2 e^4 n_N \Lambda}{(kT)^{3/2} G_{3/2}} \frac{1}{1 + e^{-x_0}}.$$

In the limiting cases it is expressed as

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\frac{Z^2 \mathrm{e}^4 n_N \Lambda}{(kT)^{3/2}} \qquad (ND)$$

$$= \frac{32\pi^2}{3} m_e \frac{Z^2 \mathrm{e}^4 \Lambda n_N}{h^3 n_\mathrm{e}} \qquad (D).$$

The average time τ_{ei} between (ei) collisions is the inverse value of ν_{ei}, and is written as

$$\tau_{nd} = \frac{1}{\nu_{nd}} = \frac{3}{4}\sqrt{\frac{m_e}{2\pi}}\frac{(kT)^{3/2}}{Z^2 \mathrm{e}^4 n_N \Lambda},$$

$$\tau_d = \frac{1}{\nu_d} = \frac{3h^3 n_e}{32\pi^2 m_e Z^2 e^4 \Lambda n_N}.$$

Coulomb logarithm is written in the form

$$\Lambda = \frac{1}{2}\ln(1 + v_{0max}^2) \approx \bar{\Lambda}_v = \ln\left(\frac{b_{\max}\bar{v}_e^2 m_e}{Ze^2}\right), \quad \Lambda \gg 1,$$

The heat flux is expressed via the heat conductivity tensor in the form

$$q_i = -\lambda_{ik}\frac{\partial T}{\partial r_k},$$

$$\lambda_{ik} = \frac{5}{2}\frac{k^2 T n_e}{m_e}\frac{G_{5/2}}{G_{3/2}}\left\{\left[a_0^1 - \left(\frac{7}{2}\frac{G_{7/2}}{G_{5/2}} - \frac{5}{2}\frac{G_{5/2}}{G_{3/2}}\right)a_1^1\right]\delta_{ik}\right.$$

$$- \varepsilon_{ikn}B_n\left[b_0^1 - \left(\frac{7}{2}\frac{G_{7/2}}{G_{5/2}} - \frac{5}{2}\frac{G_{5/2}}{G_{3/2}}\right)b_1^1\right]$$

$$\left.+B_i B_k\left[c_0^1 - \left(\frac{7}{2}\frac{G_{7/2}}{G_{5/2}} - \frac{5}{2}\frac{G_{5/2}}{G_{3/2}}\right)c_1^1\right]\right\}$$

$$a_0 = a_0^1 + iBb_0^1, \qquad a_1 = a_1^1 + iBb_1^1$$

$$B^2\, c_0^1 = (a_0^1)_{B=0} - a_0^1, \quad B^2\, c_1^1 = (a_1^1)_{B=0} - a_1^1$$

For non-degenerate electrons tensor can be written as follows:

$$\lambda_{ik} = \frac{5}{2}\frac{k^2 T n_e}{m_e}\left[(a_0^1 - a_1^1)\delta_{ik} - \varepsilon_{ikn} B_n (b_0^1 - b_1^1) + B_i B_k (c_0^1 - c_1^1)\right]$$

The heat flux

$$q_i = -\frac{5}{2}\frac{k^2 T n_e}{m_e}\left[(a_0^1 - a_1^1)\delta_{ik} - \varepsilon_{ikn} B_n (b_0^1 - b_1^1) + B_i B_k (c_0^1 - c_1^1)\right]\frac{\partial T}{\partial r_k} = q_i^{(1)} + q_I^{(2)} + q_i^{(3)}$$

$$q_i^{(1)} = -\frac{5}{2}\frac{k^2 T n_e}{m_e}(a_0^1 - a_1^1)\frac{\partial T}{\partial r_i} = -\lambda_{nd}^{(1)}\frac{\partial T}{\partial r_i},$$

$$q_i^{(2)} = \frac{5}{2}\frac{k^2 T n_e}{m_e}\varepsilon_{ikn} B_n (b_0^1 - b_1^1)\frac{\partial T}{\partial r_k} = -\varepsilon_{ikn} B_n \lambda_{nd}^{(2)}\frac{\partial T}{\partial r_k},$$

$$q_i^{(3)} = -\frac{5}{2}\frac{k^2 T n_e}{m_e} B_i B_k (c_0^1 - c_1^1)\frac{\partial T}{\partial r_k} = -B_i B_k \lambda_{nd}^{(3)}\frac{\partial T}{\partial r_k}.$$

COMPARISON OF THE EXACT SOLUTION IN LORENTZ APPROXIMATION FOR A HEAT CONDUCTIVITY WITH POLYNOMIAL EXPANSION

Exact solution in Lorentz approximation

$$q_i = -\frac{640k}{\Lambda}\frac{m_e(kT)^4}{n_N Z^2 e^4 h^3}\left(G_5 - \frac{1}{2}\frac{G_{5/2}}{G_{3/2}}G_4\right)\frac{\partial T}{\partial r_\mathrm{i}}.$$

$$\lambda_e^l = \frac{40\sqrt{2}}{\pi^{3/2}\Lambda}k\frac{n_e}{n_N}\frac{(kT)^{5/2}}{\mathrm{e}^4 Z^2\sqrt{m_e}} = \frac{320}{3\pi}\frac{k^2 T n_e}{m_e}\tau_{nd} \quad (ND)$$

$$= \frac{5}{64\Lambda}\frac{k^2 T n_\mathrm{e}^2 h^3}{m_\mathrm{e}^2 n_N Z^2 e^4} = \frac{5\pi^2}{6}\frac{k^2 T n_e}{m_e}\tau_d \quad (D).$$

HEAT CONDUCTIVITY OF STRONGLY DEGENERATE ELECTRONS IN PRESENCE OF MAGNETIC FIELD: LORENTZ APPROXIMATION

$$\xi = \frac{u^2 - \frac{5}{2}\frac{G_{5/2}}{G_{3/2}}}{4\pi n_N \left(\frac{m_e}{2kT}\right)^{3/2} \frac{e^4 Z^2}{m_e^2 u^3}\Lambda - i\omega}.$$

(Chapman, Cowling, 1970)

$$\xi = A^{(1)} + iBA^{(2)},$$

$$A^{(1)} = \frac{(u^2 - \frac{5}{2}\frac{G_{5/2}}{G_{3/2}})4\pi n_N \left(\frac{m_e}{2kT}\right)^{3/2} \frac{e^4 Z^2}{m_e^2 u^3}\Lambda}{\left[4\pi n_N \left(\frac{m_e}{2kT}\right)^{3/2} \frac{e^4 Z^2}{m_e^2 u^3}\Lambda\right]^2 + \omega^2}.$$

$$A^{(2)} = \frac{\omega}{B}\frac{u^2 - \frac{5}{2}\frac{G_{5/2}}{G_{3/2}}}{\left[4\pi n_N \left(\frac{m_e}{2kT}\right)^{3/2} \frac{e^4 Z^2}{m_e^2 u^3}\Lambda\right]^2 + \omega^2}.$$

$$A^{(3)} = A^{(1)}(B = 0) - A^{(1)}.$$

$$q_i = -\frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\left[\delta_{ij}\int_0^\infty f_0(1-f_0)A^{(1)}x^{5/2}dx - \varepsilon_{ijk}B_k\int_0^\infty f_0(1-f_0)A^{(2)}x^{5/2}dx\right.$$

$$\left.+B_iB_j\int_0^\infty f_0(1-f_0)A^{(3)}x^{5/2}dx\right]\frac{\partial T}{\partial x_j} = q_i^{(1)} + q_i^{(2)} + q_i^{(3)}, \qquad x = u^2,$$

$$q_i^{(1)} = -\frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\int_0^\infty f_0(1-f_0)A^{(1)}x^{5/2}dx\frac{\partial T}{\partial x_i} = -\lambda_{sd}^{(1)}\frac{\partial T}{\partial x_i},$$

$$q_i^{(2)} = \varepsilon_{ijk}B_k\frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\int_0^\infty f_0(1-f_0)A^{(2)}x^{5/2}dx\frac{\partial T}{\partial x_j} = -\varepsilon_{ijk}B_k\lambda_{sd}^{(2)}\frac{\partial T}{\partial x_j},$$

$$q_i^{(3)} = -B_iB_j\frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\int_0^\infty f_0(1-f_0)A^{(3)}x^{5/2}dx\frac{\partial T}{\partial x_j} = -B_iB_j\lambda_{sd}^{(3)}\frac{\partial T}{\partial x_j},$$

$$\lambda^{(1)} = \frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\int_0^\infty f_0 \frac{d\left(A^{(1)}x^{5/2}\right)}{dx}dx$$

$$\lambda^{(2)} = -\frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\int_0^\infty f_0 \frac{d\left(A^{(2)}x^{5/2}\right)}{dx}dx$$

$$B^2 A^{(3)} = A^{(1)}(B=0) - A^{(1)},$$
$$B^2 \lambda^{(3)} = \lambda^{(1)}(B=0) - \lambda^{(1)}.$$

$$\int_0^\infty \frac{f(x)dx}{e^{x-x_0}+1} = \int_0^{x_0} f(x)dx + \frac{\pi^2}{6}f'(x_0) + \ldots$$

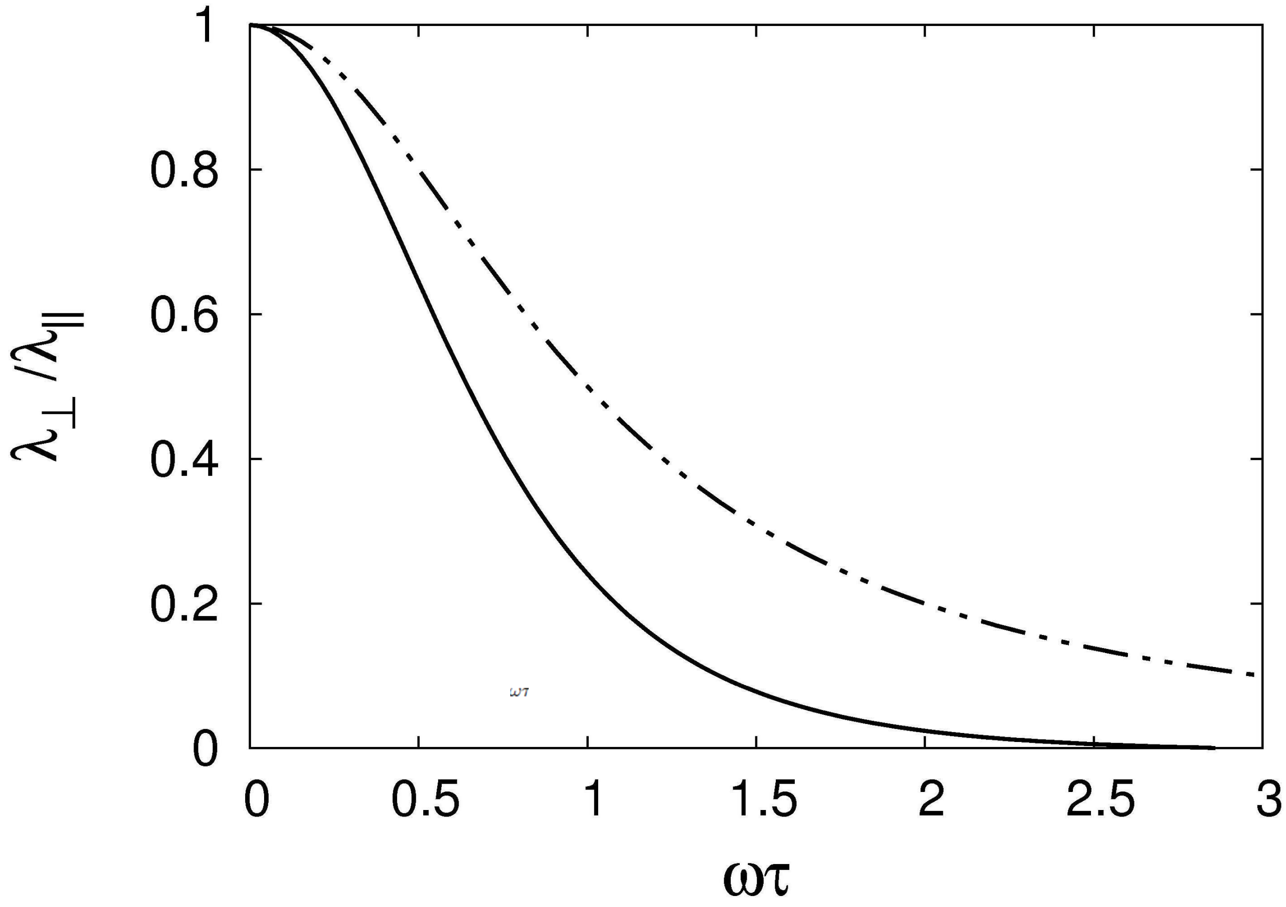
For strongly degenerate electrons at $x_0 \gg 1$

$$\lambda^{(1)} = \frac{5\pi^2}{6}\frac{k^2 T n_e}{m_e}\tau_d \left\{ \frac{1}{1+\omega^2\tau_d^2} - \frac{6}{5}\frac{\omega^2\tau_d^2}{(1+\omega^2\tau_d^2)^2} - \frac{\pi^2}{10}\left[\frac{1}{1+\omega^2\tau_d^2\left(\frac{x^3}{x_0^3}\right)}\right]'' \Big|_{x=x_0} \right\},$$

$$\lambda^{(2)} = -\frac{4\pi^2}{3}\frac{k^2 T n_e}{m_e}\frac{\tau_d^2\omega}{B} \left\{ \frac{1}{1+\omega^2\tau_d^2} - \frac{3}{4}\frac{\omega^2\tau_d^2}{(1+\omega^2\tau_d^2)^2} - \frac{\pi^2}{16}\left[\frac{1}{1+\omega^2\tau_d^2\left(\frac{x^3}{x_0^3}\right)}\right]'' \Big|_{x=x_0} \right\},$$

$$\omega\tau \ll 2\pi. \tag{180}$$

Therefore the consideration in this paper could be safely applied at $\omega\tau \lesssim 1$, and for larger $\omega\tau$ only qualitative estimations could be obtained.



The plots of the ratio $\lambda_{\perp}/\lambda_{\|}$ as a function of $\omega\tau$ are presented for phenomenologically obtained heat conductivity (dash-dot line) for comparison with heat conductivity obtained by the solution of Boltzmann equation in Lorentz approximation (solid line) with $kT = 0.09E_f$.

The transport coefficients calculated here determine a heat flux carried by electrons in the case of zero diffusion vector d_i. In a general case of nonzero diffusion vector d_i and temperature gradient $\partial T/\partial x_i$, the heat and diffusion (electrical current) fluxes are connected with each other, and are defined by 4 kinetic coefficients [10], having a tensor structure in presence of a magnetic field. The general consideration of heat and electrical conductivity of degenerate electrons will be done elsewhere.

Thank you for your attention!