ONE-LOOP DIVERGENCES IN SIX DIMENSIONAL $\mathcal{N} = (1,0)$ AND $\mathcal{N} = (1,1)$ SUPERSYMMETRIC GAUGE THEORIES

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Tomsk

GINZBURG CENTENNIAL CONFERENCE ON PHYSICS,
Lebedev Physical Institute, Moscow, May 29-July 3, 2017

Goals

- Construction of the background field methods and getting the manifestly supersymmetric and gauge invariant effective action for $6D, \mathcal{N} = (1,0)$ non-Abelian vector multiplet coupled to hypermultiplet
- Calculation of the one-loop off-shell divergences in vector multiplet and hypermultiplet sectors
- Analysis of divergences in the $\mathcal{N} = (1,1)$ SYM theory and prove that the one-loop off-shell divergence are completely absent

Based mainly on:
I.L.B, E.A. Ivanov, K.V. Stepanayantz, B.M. Merzlikin, Phys.Lett, B763, 375, 2016, arXiv:1609.00975[hep-th];
I.L.B, E.A. Ivanov, K.V. Stepanayantz, B.M. Merzlikin, JHEP, 01 (2017) 128, arXiv:1612.03190[hep-th].

The modern interest to $6D$ supersymmetric gauge theories is stipulated by the following problems:

1. Problem of describing the quantum structure of six-dimensional supersymmetric gauge theories dimensionally reduced from superstrings (N. Seiberg, E. Witten, 1996; N. Seiberg, 1997).

2. Problem of field description of the interacting multiple $M5$-branes (see e.g. review J. Bagger, N. Lambert, S. Mikhu, C. Papageorgakis, Phys.Repts. 527 (2013) 1).

- Hypothetic M-theory is characterized by two extended objects: $M2$-brane and $M5$-brane.
- The field description of interacting multiple $M2$-branes is given by Bagger-Lambert-Gustavsson (BGL) theory which is $3D$, $\mathcal{N}=8$ supersymmetric gauge theory.
- Lagrangian description of the interacting multiple $M5$-branes is not constructed.

3. Problem of miraculous cancellation of some on-shell divergences in higher dimensional maximally supersymmetric gauge theories (theories with 16 supercharges). All these theories are non-renormalizable by power counting.

- Field limit of superstring amplitude shows that $6D, \mathcal{N} = (1,1)$ SYM theory is on-shell finite at one-loop (M.B. Green, J.H. Schwarz, L. Brink, 1982).
- Analysis based on on-shell supersymmetries, gauge invariance and field redefinitions (P.S. Howe, K.S. Stelle, 1984, 2003; G. Bossard, P.S. Howe, K.S. Stelle, 2009).
- Direct one-loop and two-loop on-shell calculations (E.S. Fradkin, A.A. Tseytlin, 1983; N. Marcus, A. Sagnotti, 1984, 1985.)
- Direct calculations of scattering amplitudes in $6D$ theory up to five loops and in $D8, 10$ theories up to four loops (L.V. Bork, D.I. Kazakov, M.V. Kompaniets, D.M. Tolkachev, D.E. Vlasenko, 2015).

Results: On-shell divergences in $6D$ theory start at three loops. One-shell divergences in $8D$ and $10D$ theories start at one loop.

Some properties of $6D, \mathcal{N} = (1,1)$ SYM theory

Purpose: to show that the $\mathcal{N} = (1,1)$ SYM theory is off-shell finite at one-loop in spite of it is non-remormalizable on power counting

$6D, \mathcal{N} = (1,1)$ SYM theory possesses some properties close or analogous to $4D, \mathcal{N} = 4$ SYM theory.

- The $6D, \mathcal{N} = (1,1)$ SYM theory can be formulated in harmonic superspace as well as the $4D, \mathcal{N} = 4$ SYM theory
- The $6D, \mathcal{N} = (1,1)$ SYM theory possesses the manifest $\mathcal{N} = (1,0)$ supersymmetry and additional hidden $\mathcal{N} = (0,1)$ supersymmetry analogous to $4D, \mathcal{N} = 4$ SYM theory where there is the manifest $\mathcal{N} = 2$ supersymmetry and additional hidden $\mathcal{N} = 2$ supersymmetry
- The $6D, \mathcal{N} = (1,1)$ SYM theory as well as the $4D, \mathcal{N} = 4$ SYM theory is characterized by the non-trivial moduli space
- The $6D, \mathcal{N} = (1,1)$ SYM theory is anomaly free as well as the $4D, \mathcal{N} = 4$ SYM theory and satisfies some non-renormaization theorems

Reminder of the notions

- Supersymmetry is extension of special relativity symmetry by fermionic generators.
- Supersymmetry unifies the bosonic and fermionic fields into one supermultiplet.
- Supersymmetric field models can be formulated in terms of conventional bosonic and fermionic fields. Component approach.
- In some cases the supersymmetric field models can be formulated in terms of superfields. A superfield depends on space-time coordinates x and some number of anicommuting (Grassmann) coordinates θ. The coefficients of expansion of superfield in anticommuting coordinates are the conventional bosonic and fermionic fields of supermultiplet.
- Advantage of component formulation: close relation with conventional field theory, convenient in classical field theory and to calculate the scattering amplitudes. Disadvantage: supersymmetry is not manifest.
- Advantage of superfield formulation: manifest supersymmetry, convenient in quantum field theory to study off-shell effects, simple proof of the non-renormalization theorems.
- Problem of unconstrained formulation. Harmonic superspace approach.

Plan

- Review part
  1. $6D$ supersymmetry
  2. $6D, \mathcal{N} = (1,0)$ harmonic superspace
  3. $6D, \mathcal{N} = (1,0)$ hypermultiplet
  4. $6D, \mathcal{N} = (1,0)$ vector multiplet
  5. Action for vector multiplet coupled to hypermultiplet
  6. $\mathcal{N} = (1,1)$ SYM theory in terms of $\mathcal{N} = (1,0)$ harmonic supersfields
- Background field method
- Structure of one-loop counterterms
- Divergent part of one-loop effective action
- Summary

P.S. Howe, G. Sierra, P.K. Townsend, 1983.

$6D$ Minkowski space

- Coordinates x^M, $M = 0,1,2,3,4,5$
- Metric $\eta_{MN} = diag(1,-1,-1,-1,-1,-1)$
- Proper Lorentz group $SO(1,5)$

Two types of $6D$ Spinors

- Left $(1,0)$ spinors ψ_a, $a = 1,2,3,4$
- Right $(0,1)$ spinors ϕ^a, $a = 1,2,3,4$

Dirac matrices

- 8×8 Dirac matrices Γ_M,

$$\Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2\eta_{MN}$$

- Representation of the Dirac matrices

$$\Gamma_M = \begin{pmatrix} 0 & \tilde{\gamma}_M \\ -\gamma_M & 0 \end{pmatrix},$$

- Antisymmetric Pauli-type matrices γ_M and $\tilde{\gamma}_M$,

$$\gamma_M \tilde{\gamma}_N + \gamma_N \tilde{\gamma}_M = -2\eta_{MN}$$

$$(\tilde{\gamma}_M)^{ab} = \frac{1}{2}\epsilon^{abcd}(\gamma_M)_{cd}$$

- Spinor representation of the vectors, $V_{ab} = \frac{1}{2}(\gamma^M)_{ab} V_M$

$6D$ superalgebra

- Two types of independent supercharges
  Q^I_a, Q^a_J, $I = 1, ..., 2m; J = 1, ..., 2n$
- Anticommutational relations for supercharges

$$\{Q^I_a, Q^K_b\} = 2\Omega^{IK} P_{ab}$$

$$\{Q^a_J, Q^b_L\} = 2\Omega_{JL} P^{ab}$$

  Matrix Ω_{IK} belongs to $USp(2n)$ group (R-symmetry group), $\Omega_{IK}\Omega^{KJ} = \delta^J_I$
- $\mathcal{N} = (m, n)$ supersymmetry
- $\mathcal{N} = (1, 0)$ superspace
  Coordinates $z = (x^M, \theta^a_i), i = 1, 2$
- Basic spinor derivatives

$$D^i_a = \frac{\partial}{\partial \theta^a_i} - i\theta^{ib}\partial_{ab}, \quad \{D^i_a, D^j_b\} = -2i\Omega^{ij}\partial_{ab}$$

Harmonic superspace

Basic references:

$4D$

A.Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, E. Sokatchev, 1985.
A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001.

General purpose: to formulate $\mathcal{N}=2$ models in terms of unconstrained $\mathcal{N}=2$ superfields. General idea: to use the parameters $u^{\pm i}(i=1,2)$ (harmonics) related to $SU(2)$ automorphism group of the $\mathcal{N}=2$ superalgebra and parameterizing the 2-sphere,
$u^{+i}u^{-}_{i}=1$
It allows to introduce the $\mathcal{N}=2$ superfields with the same number of anticommuting coordinates as in case of the $\mathcal{N}=1$ supersymmetry. Prices for this are the extra bosonic variables, harmonics $u^{\pm i}$.

$6D$

P.S. Howe, K.S. Stelle, P.C. West, 1985.
B.M. Zupnik, 1986; 1999.
G. Bossard, E. Ivanov, A. Smilga, JHEP, 2015.

$6D, (1,0)$ harmonic superspace

$(1,0)$ harmonic superspace

- $USp(2) \sim SU(2), I = i$ The same harmonics $u^{\pm i}$ as in $4D, \mathcal{N}=2$ supersymmetry
- Harmonic $6D, (1,0)$ superspace with coordinates $Z = (x^M, \theta^a_i, u^{\pm i})$
- Analytic basis $Z_{(an)} = (x^M_{(an)}, \theta^{\pm a}, u^{\pm}_i)$,
  $x^M_{(an)} = x^M + \frac{i}{2}\theta^{-a}(\gamma^M)_{ab}\theta^{+b}, \quad \theta^{\pm a} = u^{\pm}_i \theta^{ai}$
  The coordinates $\zeta = (x^M_{(an)}, \theta^{+a}, u^{\pm}_i)$ form a subspace closed under $(1,0)$ supersymmetry
- The harmonic derivatives

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + i\theta^{+} \not{\partial}\theta^{+} + \theta^{+a}\frac{\partial}{\partial\theta^{-a}},$$

$$D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} + i\theta^{-} \not{\partial}\theta^{-} + \theta^{-a}\frac{\partial}{\partial\theta^{+a}},$$

$$D^{0} = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{+a}\frac{\partial}{\partial\theta^{+a}} - \theta^{-a}\frac{\partial}{\partial\theta^{-a}}$$

- Spinor derivatives in the analytic basis

$$D^+_a = \frac{\partial}{\partial\theta^{-a}}, \quad D^-_a = -\frac{\partial}{\partial\theta^{+a}} - 2i\partial_{ab}\theta^{-b}, \quad \{D^+_a, D^-_b\} = 2i\partial_{ab}$$

Hypermultiplet in conventional $6D$ superspace

- The $\mathcal{N}=(1,0)$ hypermultiplet is described in conventional $6D, \mathcal{N}=(1,0)$ superspace by the superfields $q^i(x,\theta)$ and their conjugate $\bar{q}_i(x,\theta)$, where $\bar{q}_i=(q^i)^+$ under the constraint

$$D_a^{(i}q^{j)}(x,\theta)=0$$

- On-shell component form of the hypermultiplet

$$q^i(z)=f^i(x)+\theta^{ai}\psi_a(x)$$

where the scalar field $f^i(x)$ and the spinor field $\psi_a(x)$ satisfy the equations $\Box f^i=0, \partial^{ab}\psi_b=0$

- The on-shell $\mathcal{N}=(1,0)$ hypermultiplet in six dimensions has 2 bosonic+2 fermionic complex degrees of freedom.

Hypermultiplet in harmonic superspace: off-shell Lagrangian formulation

- Off-shell hypermultiplet is described by the analytic superfield $q^+_A(\zeta, u)$, $D^+_a q^+_A(\zeta, u) = 0$, satisfying the reality condition $\widetilde{(q^{+A})} \equiv q^+_A = \varepsilon_{AB} q^{+B}$. Pauli-Gürsey indices $A, B = 1, 2$
- Off-shell hypermultiplet harmonic superfield contains infinite set of auxiliary fields which vanish on-shell due to the equations of motion

$$D^{++} q^+(\zeta, u) = 0$$

- The equations of motion follow from the action

$$S_{HYPER} = -\frac{1}{2} \int d\zeta^{(-4)} du \; q^{+A} D^{++} q^+_A$$

Here $d\zeta^{(-4)} = d^6x d^4\theta^+$.

The $\mathcal{N}=(1,0)$ non-Abelian vector multiplet in $6D$ conventional superspace

- Gauge covariant derivatives

$$\nabla_{\mathcal{M}} = D_{\mathcal{M}} + \mathcal{A}_{\mathcal{M}}, \quad [\nabla_{\mathcal{M}}, \nabla_{\mathcal{N}}] = T_{\mathcal{MN}}{}^{\mathcal{L}}\nabla_{\mathcal{L}} + F_{\mathcal{MN}}$$

  with $D_{\mathcal{M}} = \{\partial_M, D^i_a\}$ being the flat covariant derivatives and $\mathcal{A}_M$ being the gauge connection taking the values in the Lie algebra of the gauge group.

- The constraints

$$F^{ij}_{ab} = 0, \quad \{\nabla^i_a, \nabla^j_b\} = -2i\varepsilon^{ij}\nabla_{ab}, \quad [\nabla^i_c, \nabla_{ab}] = -2i\varepsilon_{abcd}W^{id}$$

  Here W^{ia} is the superfield strength obeying the Bianchi identities.

The constraints are solved in the framework of the harmonic superspace

The $\mathcal{N} = (1,0)$ non-Abelian vector multiplet in $6D$, $\mathcal{N} = (1,0)$ harmonic superspace

- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + V^{++}$$

  Connection V^{++}, takes the values in the Lie algebra of the gauge group, this is an unconstrained analytic potential of the $6D, \mathcal{N} = (1,0)$ SYM theory.
- On-shell contents: $V^{++} = \theta^{+a}\theta^{+b}A_{ab} + 2(\theta^{+})_a\lambda^{-a}$, A_{ab} is a vector field, $\lambda^{-a} = \lambda^{ai}u_i, \lambda^{ai}$ is a spinor field.
- The superfield action of $6D, \mathcal{N} = (1,0)$ SYM theory is written in the form

$$S_{SYM} = \frac{1}{f^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{tr}\int d^{14}z du_1 \ldots du_n \frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)}$$

  Here f is the dimensional coupling constant ($[f] = -1$)
- Gauge transformations

$$V^{++\prime} = -ie^{i\lambda}D^{++}e^{-i\lambda} + e^{i\lambda}V^{++}e^{-i\lambda}, \qquad q^{+\prime} = e^{i\lambda}q^{+}$$

Theory of $\mathcal{N}=(1,0)$ non-Abelian vector multiplet coupled to hypermultiplet

- Action

$$S[V^{++},q^{+}]=\frac{1}{f^2}\sum_{n=2}^{\infty}\frac{(-i)^n}{n}\mathrm{tr}\int d^{14}z\,du_1\ldots du_n\,\frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)}$$

$$-\int d\zeta^{-4}du\tilde{q}^{+}\nabla^{++}q^{+}$$

- Harmonic covariant derivative

$$\nabla^{++}=D^{++}+iV^{++}$$

- Equations of motion

$$\frac{1}{2f^2}F^{++}-i\tilde{q}^{+}q^{+}=0\,,\qquad \nabla^{++}q^{+}=0\,.$$

$$F^{++}=(D^{+})^4V^{--},\ D^{++}V^{--}-D^{--}V^{++}+i[V^{++},V^{--}]=0$$

$\mathcal{N} = (1,1)$ SYM theory

$\mathcal{N} = (1,1)$ SYM theory can be formulated in terms of $\mathcal{N} = (1,0)$ harmonic superfields as the $\mathcal{N} = (1,0)$ vector multiplet coupled to hypermultiplet in adjoint representation. The theory is manifestly $\mathcal{N} = (1,0)$ supersymmetric and possesses the extra hidden $\mathcal{N} = (0,1)$ supersymmetry.

- Action

$$S[V^{++}, q^{+}] = S_{SYM}[V^{++}] + S_{HYPER}[q^{+}, V^{++}]$$

- The action is manifestly $\mathcal{N} = (1,0)$ supersymmetric.
- The action is invariant under the transformations of extra hidden $\mathcal{N} = (0,1)$ supersymmetry

$$\delta V^{++} = \epsilon^{+} q^{+}, \qquad \delta q^{+} = -(D^{+})^{4}(\epsilon^{-} V^{--})$$

- We start with harmonic superfield formulations of vector multiplet coupled to hypermultiplet.
- Effective action is formulated in the framework of the harmonic superfield background field method. It provides manifest $\mathcal{N} = (1,0)$ supersymmetry and gauge invariance of effective action under the classical gauge transformations.
- Effective action is calculated on the base of superfield proper-time technique. It provides preservation of manifest $\mathcal{N} = (1,0)$ supersymmetry and manifest gauge invariance at all steps of calculations.
- We study the model where the $\mathcal{N} = (1,0)$ vector multiplet interacts with hypermultiplet in the arbitrary representation of the gauge group. Then, we assume in the final result for one-loop divergences, that this representation is adjoint what corresponds to $\mathcal{N} = (1,1)$ SYM theory.

Background field method

Aim: construction of gauge invariant effective action

Realization

- The superfields V^{++}, q^+ are splitting into the sum of the background superfields V^{++}, Q^+ and the quantum superfields v^{++}, q^+

$$V^{++} \to V^{++} + f v^{++}, \qquad q^+ \to Q^+ + q^+$$

- The action is expending in a power series in quantum fields. As a result, we obtain the initial action $S[V^{++}, q^+]$ as a functional $\tilde{S}[v^{++}, q^+; V^{++}, Q^+]$ of background superfields and quantum superfields.
- The gauge-fixing function are imposed only on quantum superfiled

$$\mathcal{F}^{(+4)}_\tau = D^{++} v^{++}_\tau = e^{-ib} (\nabla^{++} v^{++}) e^{ib} = e^{-ib} \mathcal{F}^{(+4)} e^{ib} \ ,$$

  where $b(z)$ is a background-dependent gauge bridge superfield and τ means τ-frame. In the non-Abelian gauge theory, the gauge-fixing function is background-dependent.
- Faddev-Popov procedure is used. Ones obtain the effective action $\Gamma[V^{++}, Q^+]$ which is gauge invariant under the classical gauge transformations. Background field construction in the case under consideration is analogous to one in $4D, \mathcal{N} = 2$ SYM theory (I.L.B, B.A. Ovrut, S.M. Kuzenko, 1998).

- The effective action $\Gamma[V^{++}, Q^+]$ is written in terms of path integral

$$e^{i\Gamma[V^{++},Q^+]} = Det^{1/2}\,\widehat{\Box} \int \mathcal{D}v^{++}\,\mathcal{D}q^+\,\mathcal{D}\mathbf{b}\,\mathcal{D}\mathbf{c}\,\mathcal{D}\varphi\; e^{iS_{quant}[v^{++},q^+,\mathbf{b},\mathbf{c},\varphi,V^{++},Q^+]}$$

- The quantum action S_{quant} has the structure

$$S_{quant} = S[V^{++} + fv^{++}, Q^+ + q^+] +$$

$$+S_{GF}[v^{++}, V^{++}] + S_{FP}[\mathbf{b}, \mathbf{c}, v^{++}, V^{++}] + S_{NK}[\varphi, V^{++}].$$

- Gauge fixing term $S_{GF}[v^{++}, V^{++}]$, Faddeed-Popov ghost action $S_{FP}[\mathbf{b}, \mathbf{c}, v^{++}, V^{++}]$, Nelson-Kalosh ghost action $S_{NK}[\varphi, V^{++}]$
- Operator $\widehat{\Box}$

$$\widehat{\Box} = \eta^{MN}\nabla_M\nabla_N + W^{+a}\nabla^-_a + F^{++}\nabla^{--} - \frac{1}{2}(\nabla^{--}F^{++})$$

- All ghosts are the analytic superfields

One-loop approximation. Only quadratic in quantum fields and ghosts terms are taken into account in the path integral for effective action. It gives after some transformation the one-loop contribution $\Gamma^{(1)}[V^{++}, Q^{+}]$ to effective action in terms of formal functional determinants in analytic subspace of harmonic superspace

$$\Gamma^{(1)}[V^{++}, Q] = \frac{i}{2} Tr \ln[\widehat{\Box}^{AB} - 2f^2 Q^{+m} (T^A G_{(1,1)} T^B)_m{}^n Q^+_n] -$$

$$-\frac{i}{2} Tr \ln \widehat{\Box} - iTr \ln(\nabla^{++})^2_{\text{Adj}} + \frac{i}{2} Tr \ln(\nabla^{++})^2_{\text{Adj}} + iTr \ln \nabla^{++}_{\text{R}}$$

As usual, $TrlnO \sim DetO$, Tr means the functional trace in analytic subspace and matrix trace.
$(T^A)_m{}^n$ are are generators of the representation for the hypermultiplet.
The $G_{(1,1)}$ is the Green function for the operator ∇^{++}.
Index A numerates the generators, $V^{++} = V^{++A} T^A$. Operator $\widehat{\Box}$ acts on the components V^{++A} as $(\widehat{\Box} V^{++})^A = \widehat{\Box}^{AB} V^{++B}$
Adj and R mean that the corresponding operators are taken in the adjoint representation and in the representation for hypermultiplet.

Superfield Feynman diagrams (supergraphs)

- Perturbation theory can be given in terms of Feynman diagrams formulated in superspace
- Vector multiplet propagator

$$G^{(2,2)}(1|2) = -2\frac{(D_1^+)^4}{\Box_1}\delta^{14}(z_1 - z_2)\delta^{(-2,2)}(u_1, u_2)$$

- Hypermultiplet propagator

$$G^{(1,1)}(1|2) = \frac{(D_1^+)^4(D_2^+)^4}{\Box_1}\frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}$$

- Ghost propagators have the analogous structure
- Superspace delta-function

$$\delta^{14}(z_1 - z_2) = \delta^6(x_1 - x_2)\delta^8(\theta_1 - \theta_2)$$

- The vertices are taken from the superfield action as usual

Power counting

Superficial degree of divergence ω-total degree in momenta in loop integral.

- Consider the L loop supergraph G with P propagators, V vertices, N_Q external hypermultiplet legs, and an arbitrary vector multiplet external legs.
- One can prove that due to the Grassmann delta-functions in the propagators, any supergraph for effective action contains only a single integral over $d^8\theta$ (non-renormalization theorem).
- Mass dimensions: $[x] = -1$, $[p] = 1$, $[\int d^6p] = 6$, $[\theta] = -\frac{1}{2}$, $[\int d^8\theta] = 4$, $[q^+] = 1$, $[V^{++}] = 0$.
- After summing all dimensions and using some identities, power counting gives $\omega(G) = 2L - N_Q - \frac{1}{2}N_D$
- N_D is a number of spinor derivarives acting on external lines
- A number of space-time derivatives in the counterterms increases with L. The theory is multiplicatively non-renormalizable.
- One loop approximation $\omega_{1-loop}(G) = 2 - N_Q$
- The possible divergences correspond to $\omega_{1-loop} = 2$ and $\omega_{1-loop} = 0$

Calculations of ω are analogous to ones in $4D, \mathcal{N} = 2$ gauge theory (ILB, S.M. Kuzenko, B.A. Ovrut, 1998).

Possible candidate for one-loop divergences can be constructed on the based of dimensions, gauge invariance and $\mathcal{N}=(1,0)$ supersymmetry in the form (G. Bossard, E. Ivanov, A. Smilga, 2015)

$$\Gamma^{(1)}_{div} = \int d\zeta^{(-4)}\,du\,\Big[c_1(F^{++A})^2 + ic_2F^{++A}(\widetilde{q}^{+})^m(T^A)_m{}^n(q^+)_n + c_3\Big((\widetilde{q}^{+})^m(q^+)_m\Big)^2$$

Where c_1, c_2, c_3 are arbitrary dimensionless real numbers.

- Let $N_Q=0$, $N_D=0$, so that ω=2, and we use the dimensional regularization. Then the only admissible counterterm in the gauge multiplet sector is given by the first term in $\Gamma^{(1)}_{div}$, with dimensionless divergent coefficient c_1. Being dimensionless, this coefficient must be proportional to $1/\varepsilon$, where $\varepsilon = d-6$ is a regularization parameter.
- Let $N_Q=2$, $N_D=0$ so that $\omega=0$ and we use the dimensional regularization. The admissible counterterm is given by the second term in $\Gamma^{(1)}_{div}$ with the dimensionless divergent coefficient c_2, which must be proportional to $1/\varepsilon$.
- Let $N_Q=4$, $N_D=0$ so that $\omega=-2$. It means that $c_3=0$ in $\Gamma^{(1)}_{div}$ because the relevant structure corresponds to the convergent graphs.

Superfield proper-time technique

Calculating the one-loop divergences of superfield functional determinants is carried out in the framework of proper-time technique. Such technique allows us to preserve the manifest gauge invariance and manifest $\mathcal{N} = (1,0)$ supersymmetry at all steps of calculations.

General scheme of calculations

- Proper-time representation

$$TrlnO \sim Tr \int_0^\infty \frac{d(is)}{(is)^{1+\varepsilon}} e^{isO_1} \delta(1,2)|_{2=1}$$

- Here s is the proper-time parameter and ε is a parameter of dimensional regularization.
- Typically the $\delta(1,2)$ contains $\delta^8(\theta_1 - \theta_2)$, which vanishes at $\theta_1 = \theta_2$
- Typically the operator O contains some number of spinor derivatives D^+_a, D^-_a which act on the Grassmann delta-functions $\delta^8(\theta_1 - \theta_2)$ and can kill them. Non-zero result will be only if all these δ-functions are killed.
- Only these terms are taking into account which have the pole $\frac{1}{\varepsilon}$ after integration over proper-time.

Results of calculations

$$\Gamma^{(1)}_{div}[V^{++}, Q^{+}] = \frac{C_2 - T(R)}{3(4\pi)^3\varepsilon} tr \int d\zeta^{(-4)} du (F^{++})^2 -$$

$$-\frac{2if^2}{(4\pi)^3\varepsilon} \int d\zeta^{(-4)} du \widetilde{Q}^{+m} (C_2 \delta_m{}^n - C(R)_m{}^n) F^{++} Q^{+}{}_n.$$

- The quantities $C_2, T(R), C(R)$ are defined as follows

$$tr(T^A T^B) = T(R)\delta^{AB}$$

$$tr(T^A_{Adj} T^B_{Adj}) = f^{ACD} f^{BCD} = C_2 \delta^{AB}$$

$$(T^A T^A)_m{}^n = C(R)_m{}^n.$$

- Results of calculations correspond to analysis done on the base of power counting. The coefficients c_1, c_2 are found. The coefficient $c_3 = 0$ as we expected.
- In $\mathcal{N} = (1,1)$ SYM theory, the hypermultiplet is in the same representation as the vector multiplet. Then $C_2 = T(R) = C(R)$. Then $\Gamma^{(1)}_{div}[V^{++}, Q^{+}] = 0!$

- The six-dimensional $\mathcal{N} = (1,0)$ supersymmetric theory of the non-Abelian vector multiplet coupled to hypermultiplet in the $6D$, $\mathcal{N} = (1,0)$ harmonic superspace was considered.
- Background field method in harmonic superspace was constructed .
- Manifestly supersymmetric and gauge invariant effective action, depending both on vector multiplet anf hypermultiplet superfields, was defined.
- Superficial degree of divergence is evaluated and structure of one-loop counterterms was studied.
- An efficient manifestly gauge invariant and $\mathcal{N} = (1,0)$ supersymmetric technique to calculate the one-loop effective action was developed. As an application of this technique, we found the one-loop divergences of the theory under consideration.
- It is proved that $\mathcal{N} = (1,1)$ SYM theory is one-loop off-shell finite. There is no need to use the equations of motion to prove this property.

THANK YOU VERY MUCH!