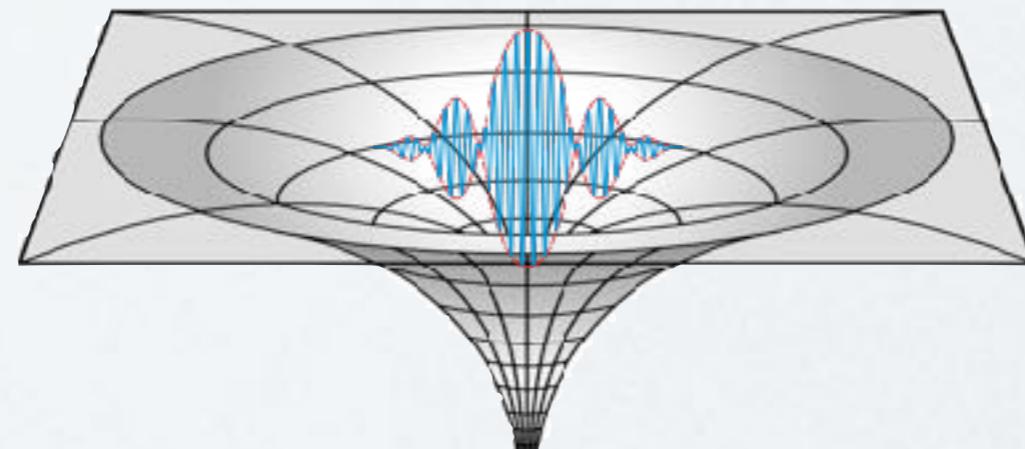


Quantum post-Newtonian corpuscular models of black holes (and cosmology)

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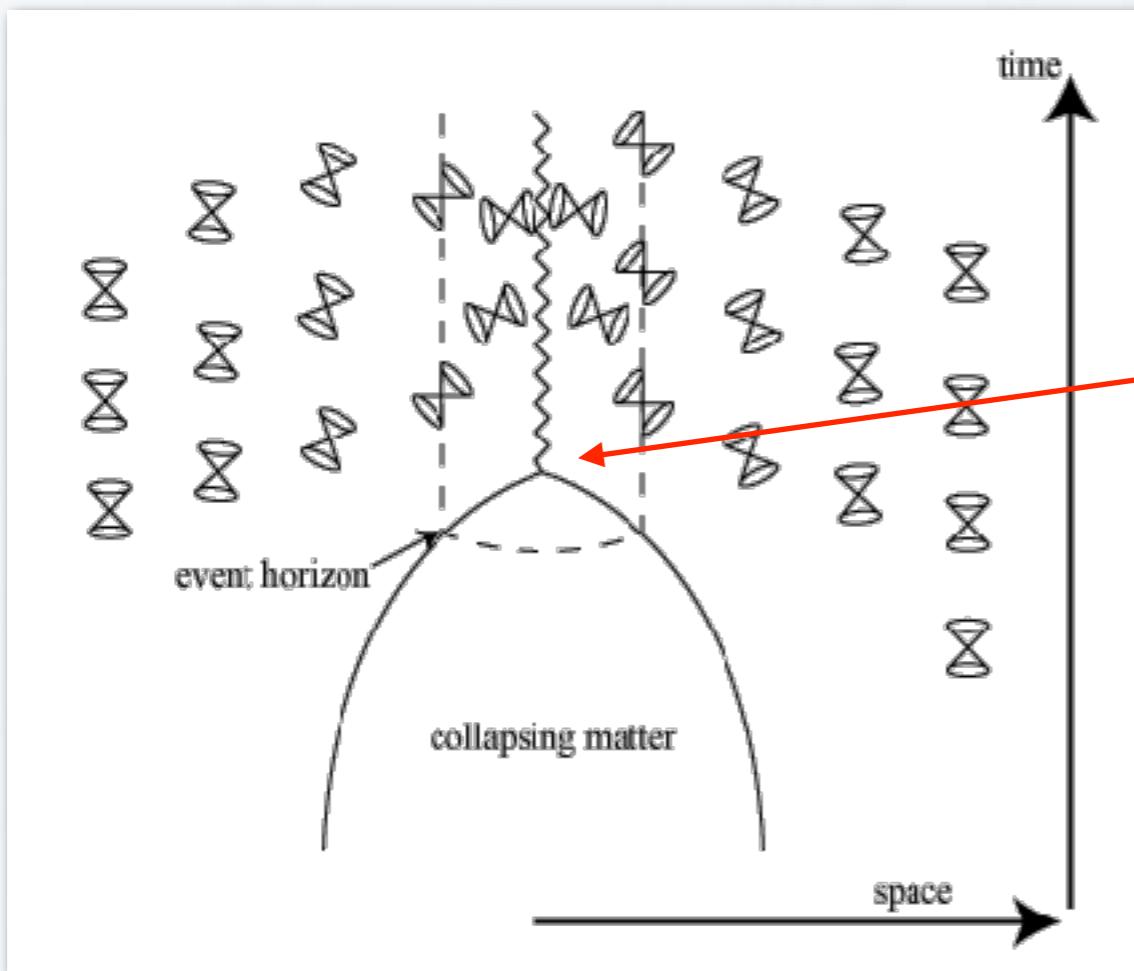
Plan of the talk

1. Motivation: understanding gravitational collapse and black hole formation in quantum theory —> Cosmology
2. Results I: R.C., A. Giugno, A. Giusti, *Matter and gravitons in the gravitational collapse*, PLB 763 (2016)337 [arXiv: 1606.04744]
3. Results II: R.C., A. Giugno, A. Giusti, M. Lenzi, *Quantum corpuscular corrections to the Newtonian potential*, arXiv:1702.05918
4. Outlook: a conjecture for cosmology

1) Gravitational collapse

Standard CLASSICAL picture = General Relativity
CLASSICAL matter and CLASSICAL* space-time

Oppenheimer-Snyder model



Central Singularity
(GR singularity theorems)



But matter is **QUANTUM...!**

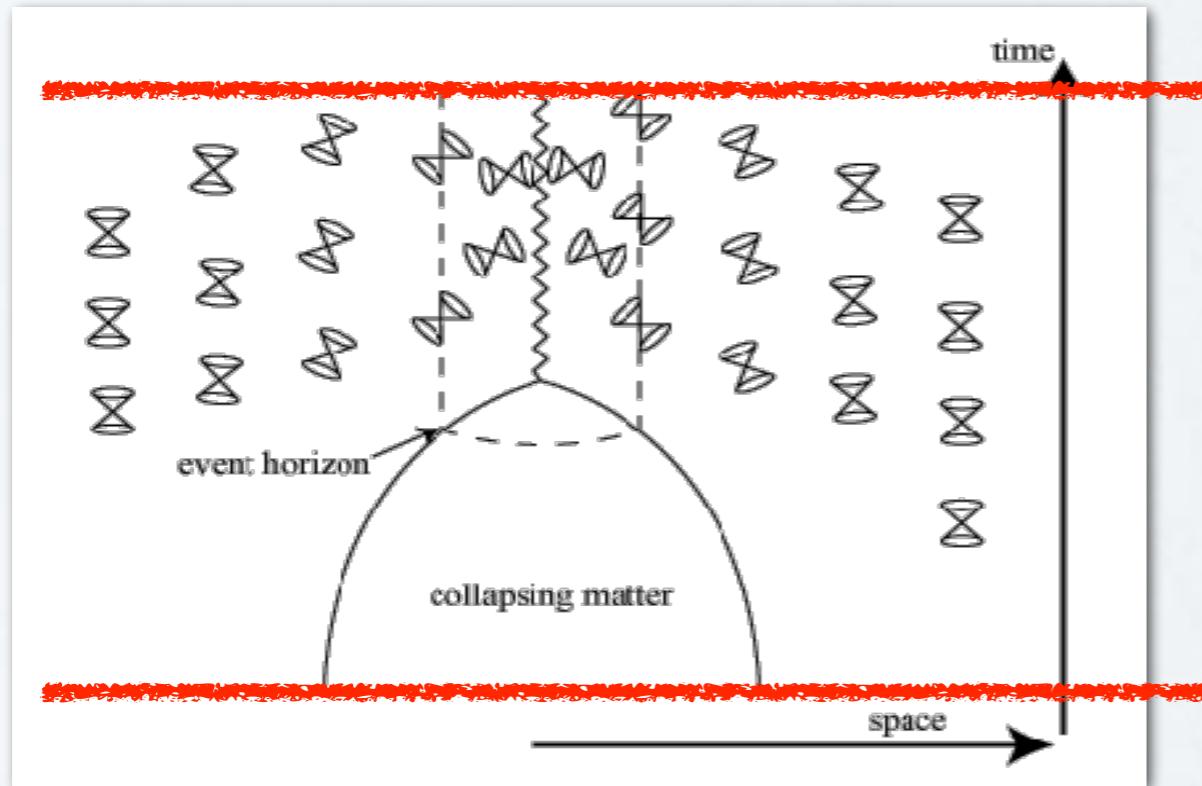
* geometrical

1) Gravitational collapse

Standard SEMICLASSICAL picture = QFT on curved space-time

Background: CLASSICAL matter and CLASSICAL space-time

Foreground: QUANTUM particles



$|0; t = +\infty\rangle$

$$e^{-\frac{i}{\hbar} \int \hat{H} dt}$$

$|0; t = -\infty\rangle$

$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$



Hawking radiation \sim physical paradoxes \sim perhaps wrong background
or Quantum Gravity...?

1) Gravitational collapse

Question: Quantum Gravity = Planck density or larger spatial scale?



1) Quantum Physics = Compton length or larger scale?

- a) atoms and molecules
- b) macroscopic condensates
- c) ...

4) Gravitation = gravitational radius or smaller scale?

- a) asymptotic safety scenarios
- b) non-commutative space-times
- c) ...

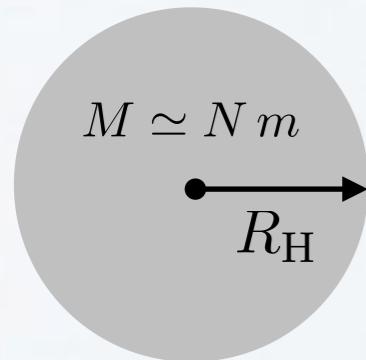


Poor man answer: Compare classical models with their quantum version

2) Baryons and Gravitons

[Dvali & Gomez]

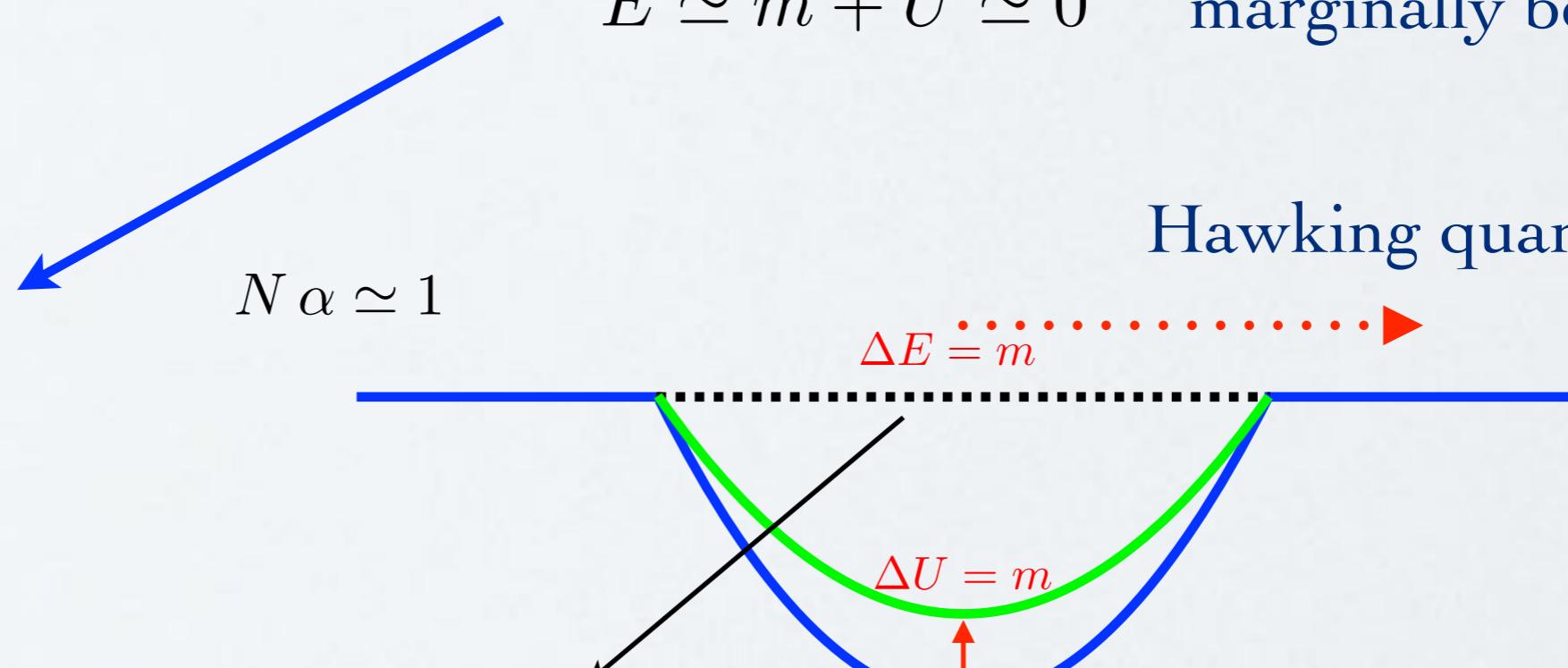
BH = self-sustained BEC of gravitons (effective mass $m = -\epsilon_G$)



$$U \simeq -m \frac{\ell_p}{m_p} \frac{M \sim N m}{r \sim \lambda_m \sim \frac{\ell_p m_p}{m}} \simeq -m N \left(\alpha \sim \frac{m^2}{m_p^2} \right)$$

$E \simeq m + U \simeq 0$ “marginally bound”

$R_H \simeq \sqrt{N} \ell_p$
 $m \simeq \frac{m_p}{\sqrt{N}}$
 $N \gg 1$



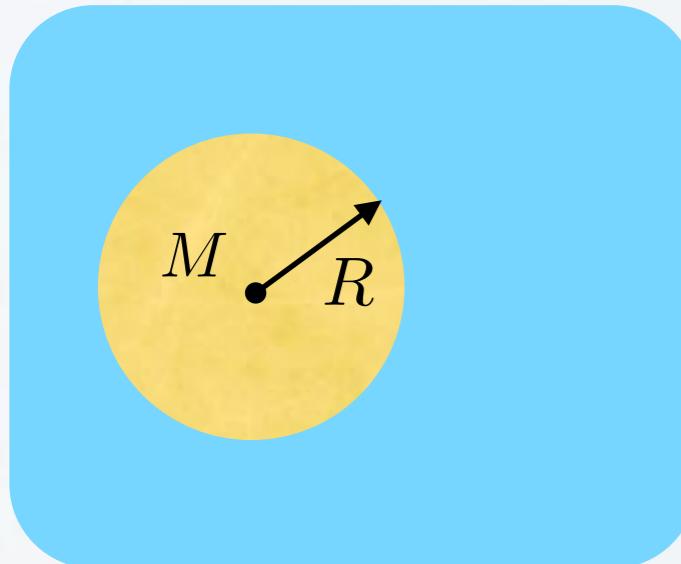
$$\Gamma \sim \frac{1}{N^2} N^2 \frac{1}{\sqrt{N} \ell_p}$$

Hawking evaporation:

$$\dot{M} \simeq m \frac{\dot{N}}{\sqrt{N}} \sim -m \frac{\Gamma}{\sqrt{N}} \sim -\frac{m_p^3 / \ell_p}{M^2}$$

“Energy balance”: $H \equiv H_B + H_G = M$

Star



$$H_B(\infty) = \mu N_B \simeq M$$

$$H = M + K_B(R) + U_{BG}(R) + U_{BB}(R) + U_{GG}(R)$$

kinetic

pressure

$$U_{BG}(R) \simeq N_B \mu \phi_N(R) \simeq -N_B \mu \frac{\ell_p M}{m_p R} = -\frac{M^2 \ell_p}{m_p R}$$

$$\boxed{N_G \simeq \frac{M^2}{m_p^2} \sim \frac{R_H^2}{\ell_p^2}}$$

$$\epsilon_G \simeq -\frac{\ell_p}{R} m_p$$

Newton

$$U_{BG}(R) \simeq N_G \epsilon_G(R)$$

Coherent state

$$U_{GG}(R) \simeq N_G \epsilon_G(R) \phi_N(R) \simeq N_G \frac{M \ell_p^2}{R^2} \quad \leftarrow \text{GR correction!}$$

$$U_{GG}(R_H) \simeq -U_{BG}(R_H) \simeq M$$



“marginally bound”

Black Hole

$$K_B(R_H) + U_{BB}(R_H) \simeq 0$$

$$R_H \simeq \sqrt{N} \ell_p$$

$$m \simeq \frac{m_p}{\sqrt{N}}$$

3) Post-Newtonian effective theory

ArXiv:1702.05918

Einstein-Hilbert action:

$$S_{\text{EH}} = \int d^4x \sqrt{-g} \left(\frac{m_p}{16\pi\ell_p} \mathcal{R} + \mathcal{L}_M \right) \quad \mathcal{L}_M = \rho$$

1) Weak field:

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$$

2) Static non-relativistic motion:

$$h_{\mu\nu} \simeq h_{00} = -2V$$

3) De Donder gauge:

$$0 = 2\partial^\mu h_{\mu\nu} - \partial_\nu h \simeq \partial_t V$$

4) Fierz-Pauli and some guessing ...

Effective scalar action:

$$S[V] = 4\pi \int \epsilon dt \int_0^\infty r^2 dr \left\{ \frac{m_p}{8\pi\ell_p} V \Delta V - \rho V \right\}$$

$$+ \frac{\epsilon}{2} \left[\frac{m_p}{4\pi\ell_p} (V')^2 + V \rho \right] V$$

A) time measure

B) Newtonian

C) Post-Newtonian



3) Post-Newtonian effective theory

ArXiv:1702.05918

A) time measure:

$$ds^2 = - \left(1 - \frac{2M}{\tilde{r}}\right) d\tilde{t}^2 + \left(1 - \frac{2M}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2$$

Radial geodesic fall:

$$\frac{d^2\tilde{r}}{d\tau^2} = -\frac{M}{\tilde{r}^2}$$

From area to distance: $dr = \frac{d\tilde{r}}{\sqrt{1 - \frac{2M}{\tilde{r}}}}$ (... irrelevant)

i) From proper time to Schwarzschild time: $d\tau = \left(1 - \frac{2M}{\tilde{r}}\right) \frac{m}{E} d\tilde{t}$

ii) From Schwarzschild time to static proper time: $dt = \left(1 - \frac{2M}{r}\right)^{1/2} d\tilde{t}$

1) Weak field:

2) Static non-relativistic motion:

$$\frac{d^2r}{d\tilde{t}^2} \simeq -\frac{d}{dr} \left(-\frac{M}{r} + \frac{2M^2}{r^2} \right)$$

Standard post-Newtonian from isotropic metric:

$$\frac{d^2r}{dt^2} \simeq -\frac{d}{dr} \left(-\frac{M}{r} + \frac{M^2}{r^2} \right)$$

B) Newtonian potential:

$$H[V_N] = -L[V_N] = 4\pi \int_0^\infty r^2 dr \left(-\frac{m_p}{8\pi\ell_p} V_N \Delta V_N + \rho V_N \right)$$

Field equation:

$$\Delta V_N = 4\pi \frac{\ell_p}{m_p} \rho$$

Gravitational potential energy:

$$U_N(r) = 2\pi \int_0^r \bar{r}^2 d\bar{r} \rho(\bar{r}) V_N(\bar{r})$$

Gravitational potential energy density :

$$\begin{aligned} J_V(r) &= \frac{1}{4\pi r^2} \frac{d}{dr} U_N(r) \\ &= -\frac{m_p}{8\pi\ell_p} [V'_N(r)]^2 \end{aligned}$$

Gravitational potential self-energy:

$$U_{GG} \sim \int r^2 dr J_V V$$

3) Post-Newtonian effective theory

ArXiv:1702.05918

C) Post-Newtonian

$$L[V] = -4\pi \int_0^\infty r^2 dr \left[\frac{m_p}{8\pi\ell_p} (1 - 4q_\Phi V) (V')^2 + q_B V \rho (1 - 2q_\Phi V) \right]$$

Field equation:

$$(1 - 4q_\Phi V) \Delta V = 4\pi q_B \frac{\ell_p}{m_p} \rho (1 - 4q_\Phi V) + 2q_\Phi (V')^2$$

$$V(r) = V_{(0)}(r) + q_\Phi V_{(1)}(r)$$

0 - order:

$$\Delta V_{(0)} = 4\pi q_B \frac{\ell_p}{m_p} \rho \quad q_B = 1$$

1 - order:

$$\Delta V_{(1)} = 2 \left(V'_{(0)} \right)^2$$

Potential energy:

$$U_{BG} = 2\pi q_B \int_0^\infty r^2 dr \rho \left[V_{(0)} + q_\Phi \left(V_{(1)} - 4V_{(0)}^2 \right) \right]$$

$$U_{GG} = -3q_\Phi \frac{\ell_p}{m_p} \int_0^\infty r^2 dr V_{(0)} \left(V'_{(0)} \right)^2$$

3) Post-Newtonian effective theory

ArXiv:1702.05918

Classical solutions:

$$0\text{-order: } \Delta V_{(0)} = 4\pi q_B \frac{\ell_p}{m_p} \rho \quad \xrightarrow{\Delta j_0(kr) = -k^2 j_0(kr)} \tilde{V}_{(0)}(k) = -4\pi q_B \frac{\ell_p \tilde{\rho}(k)}{m_p k^2}$$

$$1\text{-order: } \Delta V_{(1)}(r) = 2 \left[V'_{(0)}(r) \right]^2$$

Ex1) Point-like source:

$$\rho = M_0 \delta^{(3)}(\mathbf{x}) = \frac{M_0}{4\pi r^2} \delta(r)$$

$$(0+1) \text{ potential: } V \simeq -q_B \frac{\ell_p M}{m_p r} + q_\Phi q_B^2 \frac{\ell_p^2 M^2}{m_p^2 r^2}$$

$$(0+1) \text{ energy: } U_{\text{BG}} \simeq -q_B^2 \frac{\ell_p M_0 M}{2 m_p r_0} - q_B^3 q_\Phi \frac{3 \ell_p^2 M^3}{2 m_p^2 r_0^2}$$

$\delta(r) \rightarrow \delta(r - r_0)$

$$U_{\text{GG}} \simeq q_B^3 q_\Phi \frac{3 \ell_p^2 M^3}{2 m_p^2 r_0^2}$$

$$M = M_0 + q_\Phi M_1$$

Arbitrary!

Post-Newtonian too large!

NO maximal packing for point source!

3) Post-Newtonian effective theory

ArXiv:1702.05918

Ex2) Homogeneous star:

$$\rho(r) = \frac{3M_0}{4\pi R^3} \Theta(R - r)$$

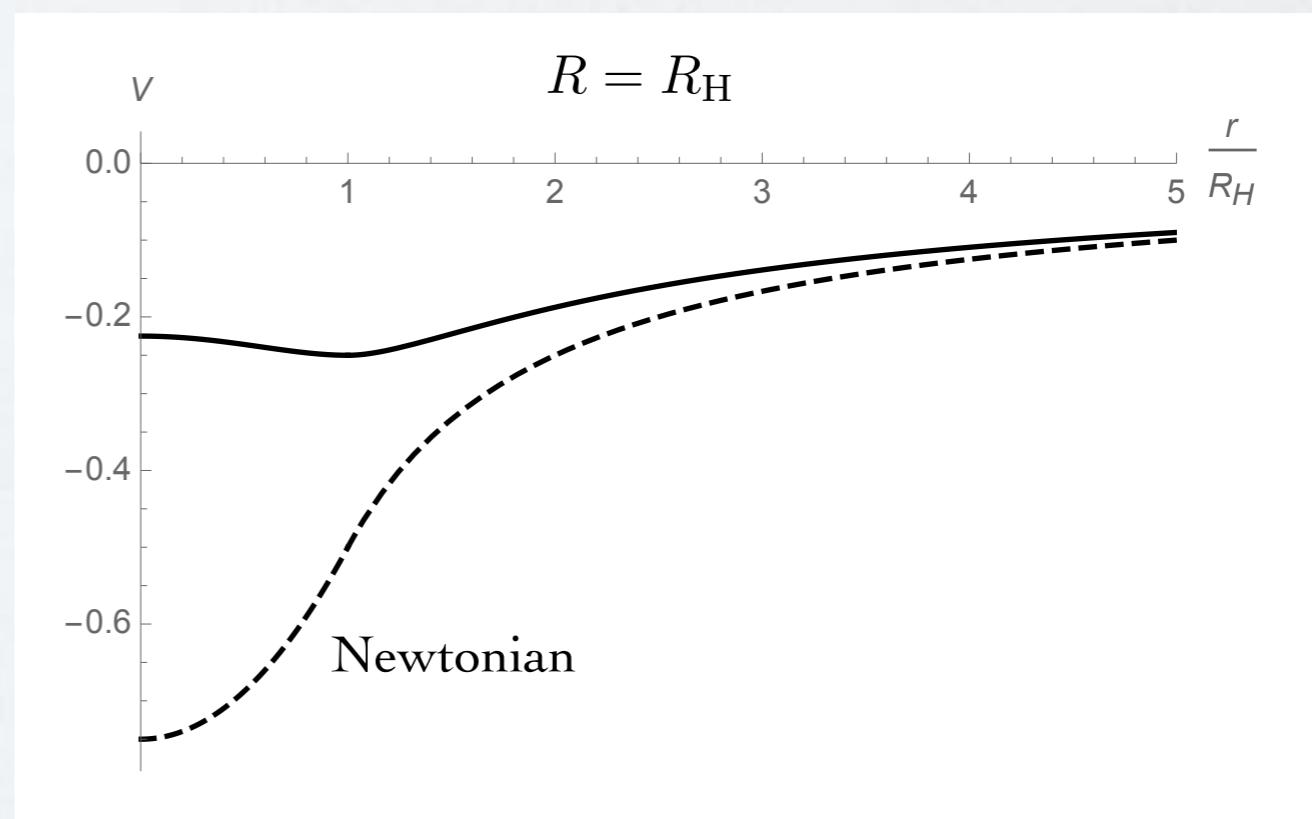
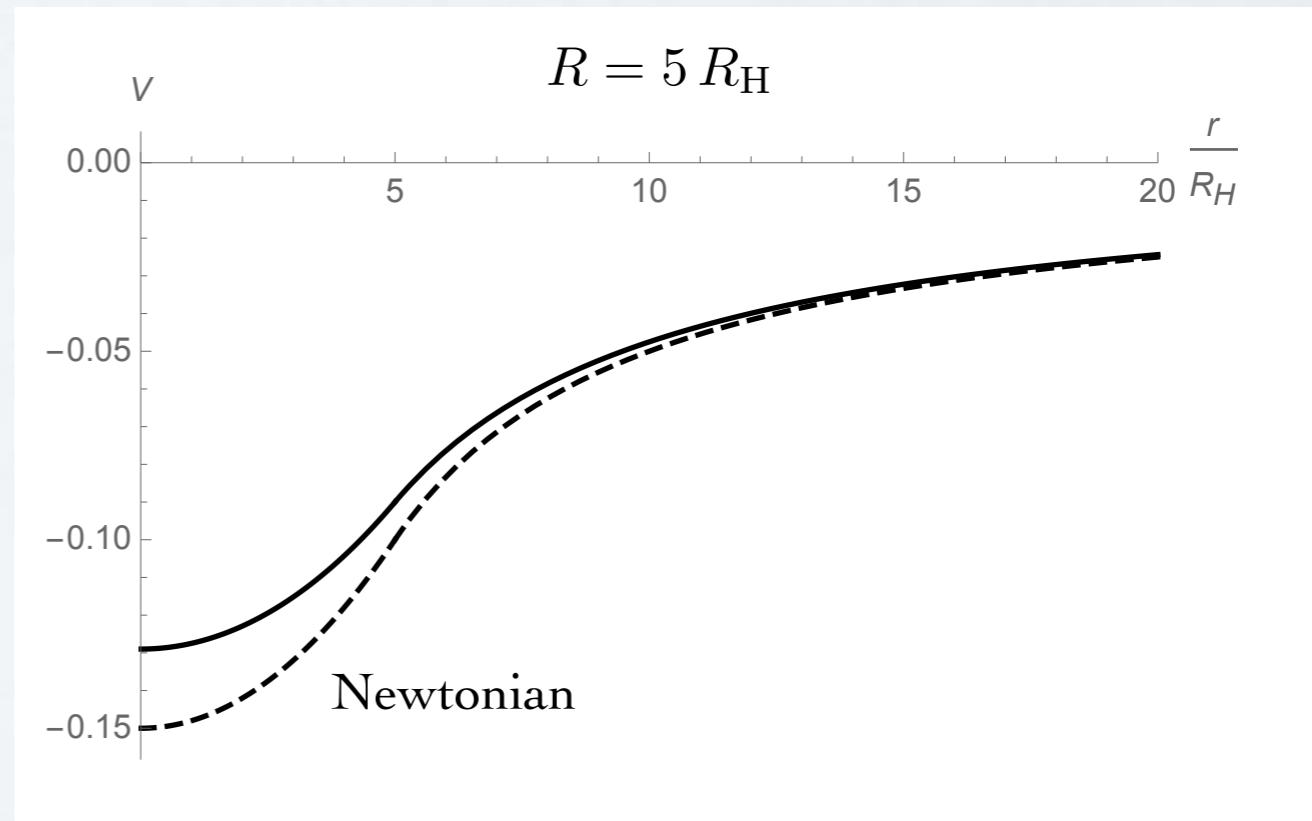
(0 + 1) potential:

(0 + 1) energy:

$$U(R) \simeq -q_B^2 \frac{3\ell_p M^2}{5m_p R} + q_B^3 q_\Phi \frac{249\ell_p^2 M^3}{175m_p^2 R^2}$$

“marginally bound” for

$$R \simeq 1.2 R_H$$



3) Post-Newtonian effective theory

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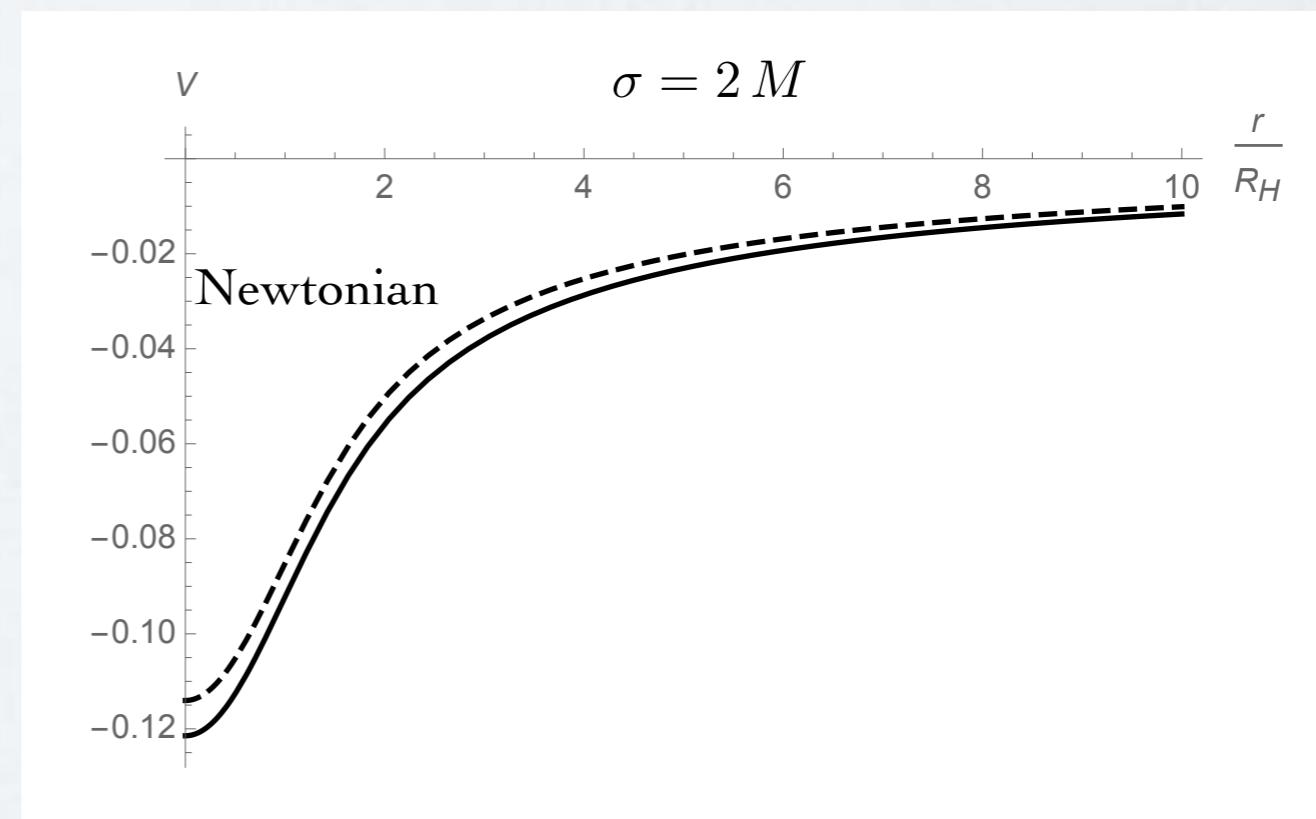
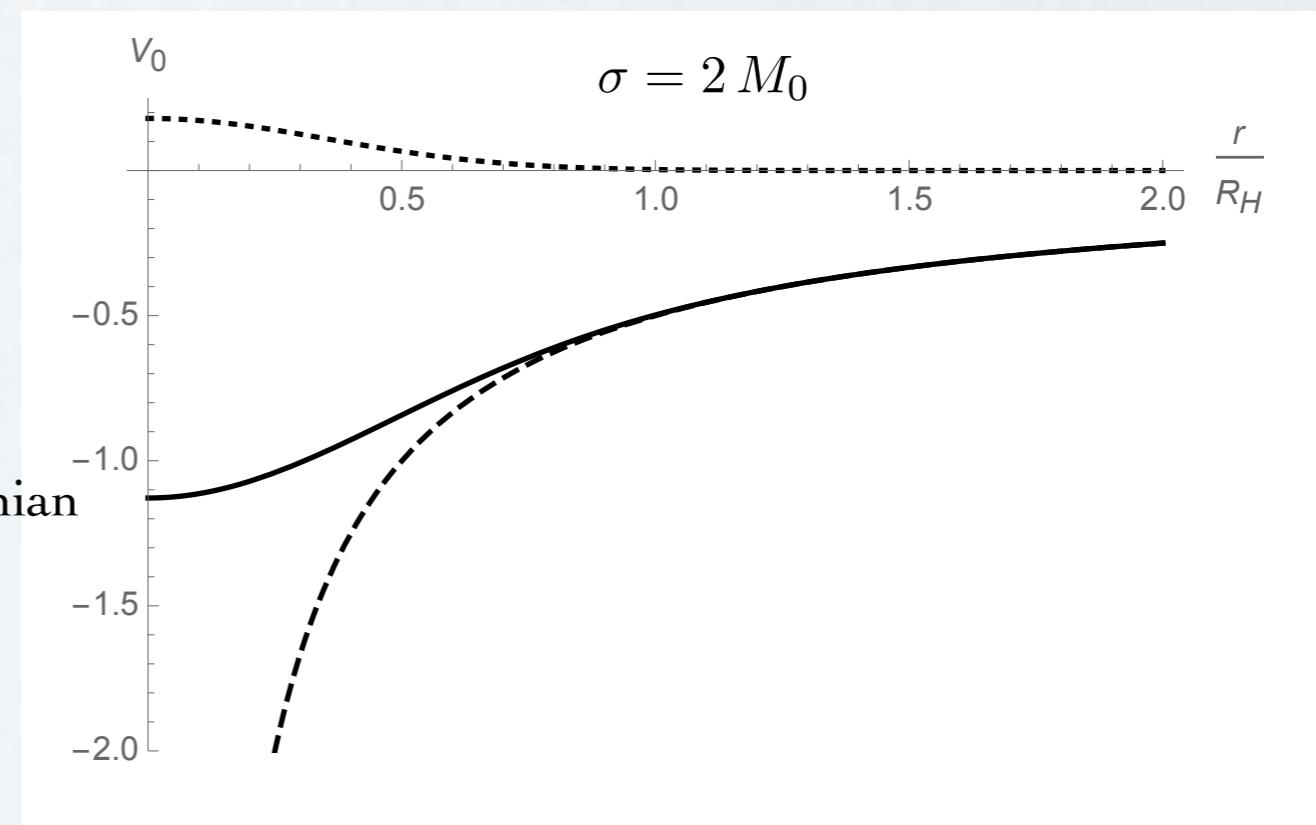
Ex3) Gaussian star:

$$\rho(r) = \frac{M_0 e^{-\frac{r^2}{\sigma^2}}}{\pi^{3/2} \sigma^3}$$

(0 + 1) potential:

Newtonian
 M_0

$$M_0 \rightarrow M$$



Quantum scalar field:

Field equation: $\square\Phi = 0 \longrightarrow \hat{\Phi}(t, r) = \ell_p \int_0^\infty \frac{k^2 dk}{2\pi^2 \sqrt{2k}} j_0(kr) (\hat{a}_k e^{ikt} + \hat{a}_k^\dagger e^{-ikt})$

Commutators:

$$[\hat{a}_p, \hat{a}_k^\dagger] = \frac{2\pi^2}{k^2} \delta(p - k)$$

Vacuum:

$$\hat{a}_k |0\rangle = 0$$

Newtonian coherent state:

$$\hat{a}_k |g\rangle = e^{-ikt} g_k |g\rangle$$

$$g_k = \sqrt{\frac{k}{2}} \frac{\tilde{V}_c(k)}{m_p} = -\frac{q_B \tilde{\rho}(k)}{m_p \sqrt{2k^3}}$$

Normalisation: $\langle g|g\rangle = 1$ 

$$\langle g| \hat{\Phi}(t, r) |g\rangle = \int_0^\infty \frac{k^2 dk}{2\pi^2} j_0(kr) \tilde{V}_c(k) = V_c(r)$$

3) Post-Newtonian effective theory

ArXiv:1702.05918

Normalisation \sim occupation number:

$$|g\rangle = e^{-\frac{N_G}{2}} \exp \left\{ \int_0^\infty \frac{k^2 dk}{2\pi^2} g_k \hat{a}_k^\dagger \right\} |0\rangle \quad \longrightarrow \quad N_G = \int_0^\infty \frac{k^2 dk}{2\pi^2} g_k^2 = \langle g | \int_0^\infty \frac{k^2 dk}{2\pi^2} \hat{a}_k^\dagger \hat{a}_k | g \rangle$$

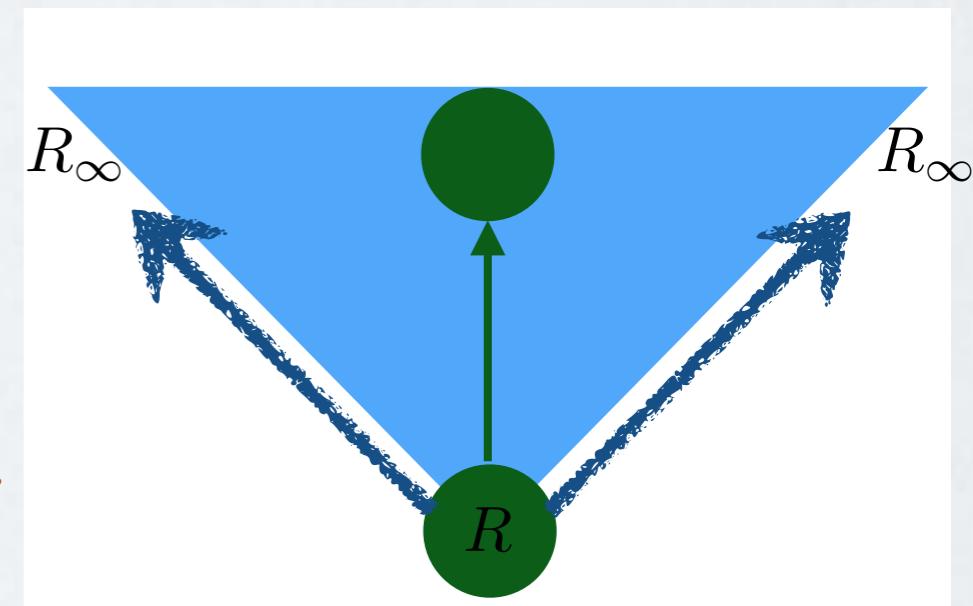
Typically:

$$N_G \sim \frac{M^2}{m_p^2} \ln \left(\frac{\Lambda}{k_0} \right) \sim \frac{M^2}{m_p^2} \ln \left(\frac{R_\infty}{R} \right)$$

UV cut-off Size of static field
 IR cut-off Source size

$\frac{dN_G}{N_G} \sim \frac{dM}{M} - \frac{1}{\ln(R_\infty/R)} \frac{dR}{R}$

Dynamics...



3) Post-Newtonian effective theory

ArXiv:1702.05918

Post-Newtonian corrections:

$$\Delta V_{(1)} = 2 \frac{\ell_p}{m_p} \langle g | \left(\hat{\Phi}' \right)^2 | g \rangle = 2 \left(V'_{(0)} \right)^2 + \cancel{J_0}$$

Vacuum

New coherent state:

$$\hat{a}_k |g'\rangle \simeq g_k |g\rangle + q_\Phi \delta g_k |g\rangle$$

$$\sqrt{\frac{\ell_p}{m_p}} \langle g' | \hat{\Phi} | g' \rangle \simeq V_{(0)} + q_\Phi V_{(1)}$$

One-mode occupation: $\bar{k} \sim R^{-1}$

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$$\delta g_{\bar{k}} \sim -\ell_p \bar{k}^{5/2} g_{\bar{k}}^2$$

Typically:

$$\delta g_{\bar{k}} \sim \frac{R_H}{R} g_{\bar{k}}$$

OK for large source

For black hole formation?

3) Post-Newtonian effective theory

Unpublished

De Sitter:

$$\begin{aligned} G_N \rho_\Lambda = \left(\frac{\dot{a}}{a} \right)^2 &= \frac{1}{L_\Lambda^2} & \xrightarrow{\hspace{2cm}} & \\ G_N \rho_\Lambda L_\Lambda^3 &= G_N M_\Lambda = L_\Lambda & \downarrow & \\ M_\Lambda = N_\Lambda \frac{\ell_p m_p}{L_\Lambda} & \xrightarrow{\hspace{2cm}} & L_\Lambda &= \sqrt{N_\Lambda} \ell_p \end{aligned}$$

Quantum (post-Newtonian) correction in cosmology:

$$\begin{aligned} U_N^{(0)} + U_{\text{PN}}^{(0)} &= 0 \\ U_N^{(0)} = N_\Lambda \varepsilon_\Lambda = -N_\Lambda \frac{\ell_p m_p}{L_\Lambda} &\quad \uparrow \quad \uparrow \\ R & \\ U_{\text{PN}}^{(0)} = N_\Lambda \frac{\sqrt{N_\Lambda} \ell_p^2 m_p}{L_\Lambda^2} &\quad \updownarrow \\ R^2 & \end{aligned}$$

4) Outlook

1) Quantum post-Newtonian corrections:

- a. phenomenological consequences for (neutron) stars
- b. phenomenological consequences for black hole formation

2) Thermal fluctuations:

- a. quantum matter effects
- b. Hawking radiation

3) Inner quantum structure of black holes

4) Quantum cosmology: DE and DM?

Thank you!