Mechanism of the explosion of the collapsing supernovae

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Study the gravitational collapse

1. *The SN mechanism is one of the Ginzburg problems of the modern physics. Also GRB (some connected with SNe) is an unsolved problem.*
2. We are developing the numerical methods for the carry out the radiative hydrodynamic with taking into account the neutrino transport. In the spherically symmetric case we can solve the kinetic Bolzmann equations for the neutrino of the di¤erent kinds with taking into account all reactions of the weak interactions. 1D approach is not enough to explain the Supernova explosion, because 3D instabilities.
3. The kinetic Boltzmann equations in 3D become very complicated for the numerical simulations due to large dimensions of the phase space and di¤erent timescales of processes. The interesting result of simulations with simpli…ed 3D models is the prediction of the high-energy neutrino have to be registered $\gtrsim$ 10 MeV.

Physics and energy scales

1. Type II Supernova is collapsing of the center mass $\lesssim 2M_\odot$ of a massive star. At the compression of $M_\odot = 2 \cdot 10^{33}$ g to the radius of NS 10 km we obtain the energy $\sim GM_\odot / R_{\rm NS} \sim 10^{54}$ erg. It is 100 MeV per a nucleon with a mass $m_n = 1.67 \cdot 10^{-24}$ g, while the thermonuclear reactions give 10 MeV/nucleon. The neutrino di¤erent type carry the energy.
2. Due to the di¤erent timescale we can consider 2 di¤erent processes: gravitational collapse $\lesssim 10$ seconds and the supernova explosion – fast explosion in the center of presupernova during days and years.
3. To explain the supernova explosion, it is necessary to understand how an energy of $(1 - 1.5) \cdot 10^{51}$ erg, as is required for the supernova to cast o¤ its envelope, is obtained from the total energy of $(1 - 3) \cdot 10^{53}$ erg that is carried away by neutrinos.

Equations

In the Lagrangian variables $m = \int_0^r r'^2 \rho(r', t) dr'$, t:

$$\frac{\partial r}{\partial t} = v, \quad (1)$$

$$\frac{\partial v}{\partial t} + r^2 \frac{\partial}{\partial m}\left(P - \zeta \frac{\partial (r^2 v)}{\partial V}\right) = -\frac{4\pi G m}{r^2} + \frac{1}{\rho c}\sum_{\nu q}\int d\mu d\epsilon_\nu \mu (\chi_\nu^q E_\nu - \eta_\nu^q), \quad (2)$$

$$\frac{\partial \epsilon}{\partial t} + \left(P - \zeta \frac{\partial (r^2 v)}{\partial V}\right)\frac{\partial (r^2 v)}{\partial m} = \frac{1}{\rho}\sum_{\nu}\int d\mu d\epsilon_\nu (\chi_\nu^q E_\nu - \eta_\nu^q), \quad (3)$$

$$\frac{1}{c}\frac{\partial E_\nu}{\partial t} + \mu \frac{\partial r^2 E_\nu}{\partial V} + \frac{1}{r}\frac{\partial}{\partial \mu}\left\{(1-\mu^2)\left[1 + \left(\frac{3v}{c} - \frac{r}{c}\frac{\partial r^2 v}{\partial V}\right)\mu\right] E_\nu\right\}$$
$$+ \frac{1}{r}\left[\mu^2\left(\frac{3v}{c} - \frac{r}{c}\frac{\partial r^2 v}{\partial V}\right) - \frac{v}{c}\right]\frac{\partial \epsilon_\nu E_\nu}{\partial \epsilon_\nu}$$
$$+ \frac{1}{r}\left\{\frac{v}{c} + \frac{r}{c}\frac{\partial r^2 v}{\partial V} - \mu^2\left(\frac{3v}{c} - \frac{r}{c}\frac{\partial r^2 v}{\partial V}\right)\right\} E_\nu = -\chi_\nu^q E_\nu + \eta_\nu^q. \quad (4)$$

Physical processes

- $\nu + e \rightarrow \nu' + e'$, $\tilde{\nu} + e \rightarrow \tilde{\nu}' + e'$
- $\nu + n \leftrightarrow e + p$
- $\tilde{\nu} + p \leftrightarrow e^{+} + n$
- Neurtino scattering, emission and absorption by nuclei.
- $\nu + \tilde{\nu} \leftrightarrow e^{-} + e^{+}$

We evaluates exact Matrix elements in calculations, we take into account degenerate states. However, the production of neutrinos via the annihilation of electron–positron pairs was not included in the computations (the rate of the inverse reaction depends on the two angles for the two neutrinos).

The calculation of the scattering of neutrinos on electrons

Taking into account the …lled …nal electron states, one has

$$\left(\frac{\partial f_\nu(\mathbf{q},\mathbf{t})}{\partial t}\right)_{\nu e} = \int \frac{Vd\mathbf{p}\,Vd\mathbf{q}'\,Vd\mathbf{p}'}{(2\pi\hbar)^6} w_{\mathbf{q}',\mathbf{p}';\mathbf{q},\mathbf{p}}$$

$$\times\left[(1-F_e(\mathbf{p},t))f_\nu(\mathbf{q}',t)f_e(\mathbf{p}',t)-(1-F_e(\mathbf{p}',t))f_\nu(\mathbf{q},t)f_e(\mathbf{p},t)\right], \qquad (5)$$

where $F_e \equiv \frac{(2\pi\hbar c)^3}{2} f_e$, the probability of the process is

$$w_{\mathbf{q}',\mathbf{p}';\mathbf{q},\mathbf{p}} = c(2\pi\hbar)^4\frac{(\hbar c)^4}{V^3}\delta(\mathrm{q}+\mathrm{p}-\mathrm{q}'-\mathrm{p}')\frac{|M_{fi}|^2}{16\epsilon_\nu\epsilon_e\epsilon'_\nu\epsilon'_e}, \qquad (6)$$

$$|M_{fi}|^2 = 32\left(\frac{G_F}{(\hbar c)^3}c^2\right)^2\left[(C_V+C_A)^2(\mathrm{pq})(\mathrm{p'q'})\right.$$

$$\left.+(C_V-C_A)^2(\mathrm{p'q})(\mathrm{pq'})-(C_V^2-C_A^2)m_e^2c^2(\mathrm{qq'})\right], \qquad (7)$$

where $C_V = 1+\frac{a}{2m_Z^2}\frac{m_W^2}{g^2} = \frac{1}{2}+2\sin^2\theta_W = 1.2$, $C_A = 1-\frac{b}{2m_Z^2}\frac{m_W^2}{g^2} = \frac{1}{2}$,
$m_W = 37.3/\sin\theta_W$ GeV, $m_Z = 74.6/\sin(2\theta_W)$ GeV, $\tan\theta_W = g'/g$,
$\frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{8}\frac{g^2}{m_W^2c^4} = \frac{1.015\cdot10^{-5}}{m_p^2c^4}$.

Equation of state and initial model

The equation of state takes into account the equilibrium radiation of photons, the electron–positron gas, and a mix of nuclei in equilibrium with free nucleons. We used the equation of state of an ideal gas for the nuclei. Fermi statistics were used for the nucleons in the equation of state in a non-relativistic approximation.
We take in the initial model the politrope $n = 3$, $P \propto \rho^{4/3}$ (degenerate relativistic pairs) for the total mass $1.4 M_{\odot}$. The collapse of a massive stellar core ($\gtrsim 2M_{\odot}$) occurs near the stability boundary, $\langle \Gamma \rangle \approx 4/3$ due to neutrino losses.

Computational method

Grid in the phase space (m, ϵ, μ). Lines method for the solving of evolutionary equations.

Conservative scheme is important in non-transparent regions. Upwind scheme for Boltzmann equations with variable 1–2 order for a transport.

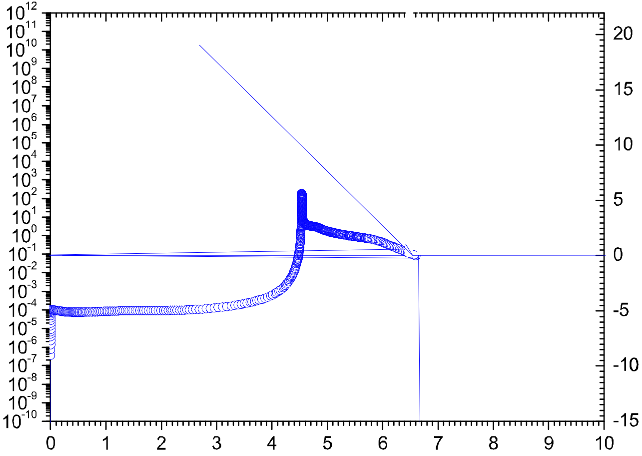
Implicit Gear method for the sti¤ ODEs system. To inverse Jacobi matrix we use cycle reduction method.

Calculations of the collapse of the stellar core. Initial and …nal states.

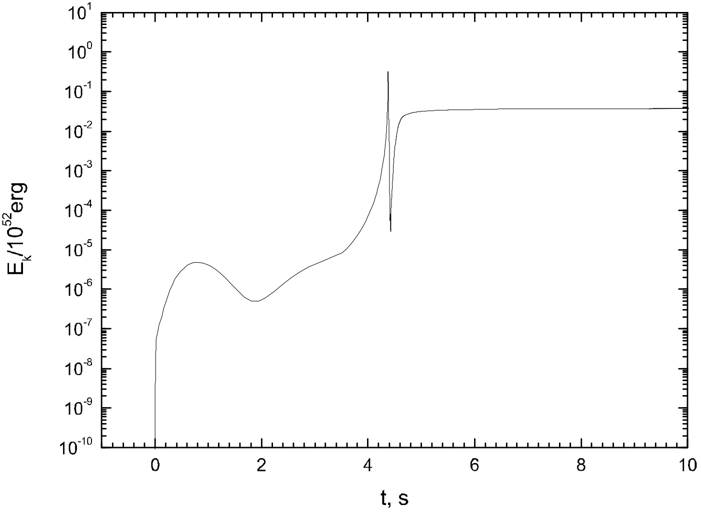
time, s	E_{tot}, erg	E_{gr}	E_{k}	ρ_c, g cm^{-3}	T_c, K
$t = 0$	$-5.1 \cdot 10^{50}$	$-3.7 \cdot 10^{51}$	0	$4.0 \cdot 10^{9}$	$7.2 \cdot 10^{9}$
$t = 8$	$-8.4 \cdot 10^{52}$	$-1.9 \cdot 10^{53}$	$3.4 \cdot 10^{50}$	$2.9 \cdot 10^{14}$	$7.7 \cdot 10^{10}$
	R_{star}, cm				
$t = 0$	$1.9 \cdot 10^{8}$				
$t = 8$	$2.7 \cdot 10^{6}$				
	$M = 1.353 M_{\odot}$				

Table: Characteristics of the stellar core at the beginning and the hot NS

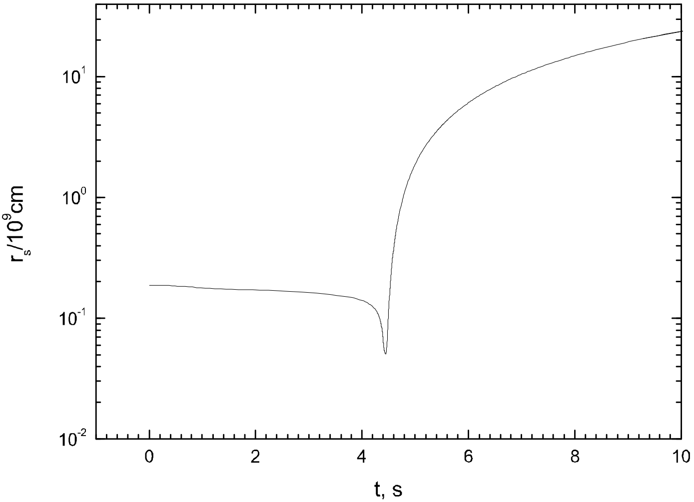
Luminosity, mean energy, and the total energy of the star at various times



Kinetic energy at various times



Radius of the star at various times



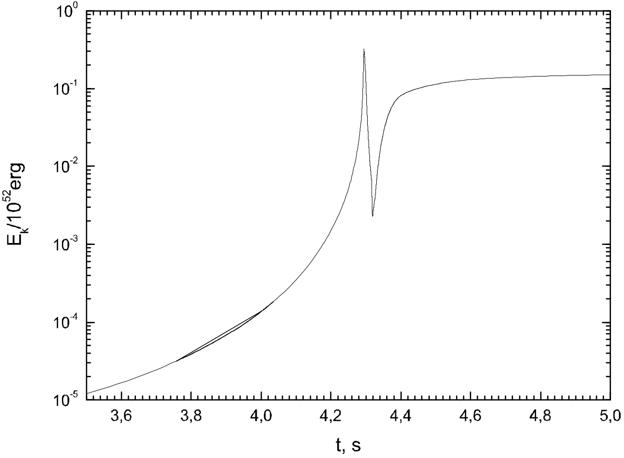
Checking of the kinetic energy in 1D calculations

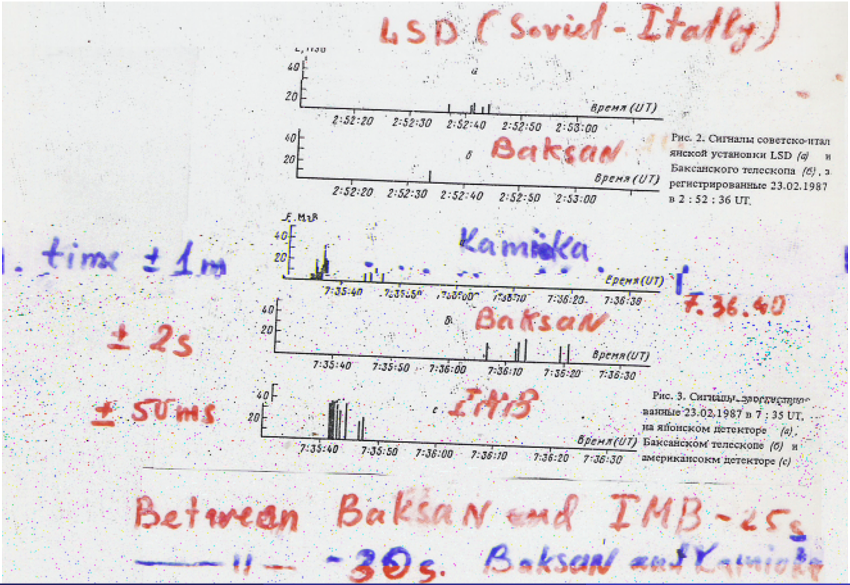
The cross section for scattering on electrons increases with increasing neutrino energy $\sigma_{e\nu} = 1.7 \cdot 10^{-44}\mathrm{cm}^2 \frac{\epsilon_\nu}{m_e c^2} \frac{E_F}{m_e c^2}$, for $E_F \ll \epsilon_\nu$, where the Fermi energy of the electrons is $E_F = (3\pi^2)^{1/3}\hbar c n_e^{1/3}$. in a collision with a stationary electron $\epsilon_e = E_F$ the neutrino transfers a fraction of its energy $\frac{\epsilon_\nu - \epsilon'_\nu}{\epsilon_\nu} = \frac{\epsilon_\nu}{E_F + \epsilon_\nu} \approx 1$, for $E_F \ll \epsilon_\nu$. Thus, a shell with thickness l_{sh} and with a density of degenerate electrons nsh absorbs a fraction of the neutrino energy

$$\frac{E_{\mathrm{k}}}{E_\nu} = \sigma_{e\nu} l_{\mathrm{sh}} n_{\mathrm{sh}}.$$

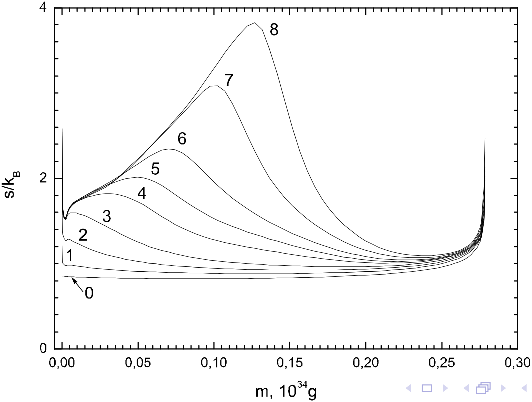
As an estimate, we take $l_{\mathrm{sh}} = R_{\mathrm{star,min}}$ from calculations, and to estimate the number density, we use the mean density for the separated out mass $0.047 M_\odot$ from Table. Thus, the number of electrons in the shell is $N_{\mathrm{sh}} = 2.8 \cdot 10^{55}$, $n_e = 5.3 \cdot 10^{31}\mathrm{cm}^{-3}$, $E_F/(m_e c^2) = 4.5$. For an absorbed fraction of the neutrino energy $E_{\mathrm{k}}/(E_{\mathrm{total}}(t=0) - E_{\mathrm{total}}(t_{\mathrm{fin}})) = 0.0041$ we should obtain a mean energy for the neutrinos of ~ 10 MeV in agreement with the computations.

Kinetic energy of the shell for corrected energy of neutrino $10 \longrightarrow 30$ MeV in 1D

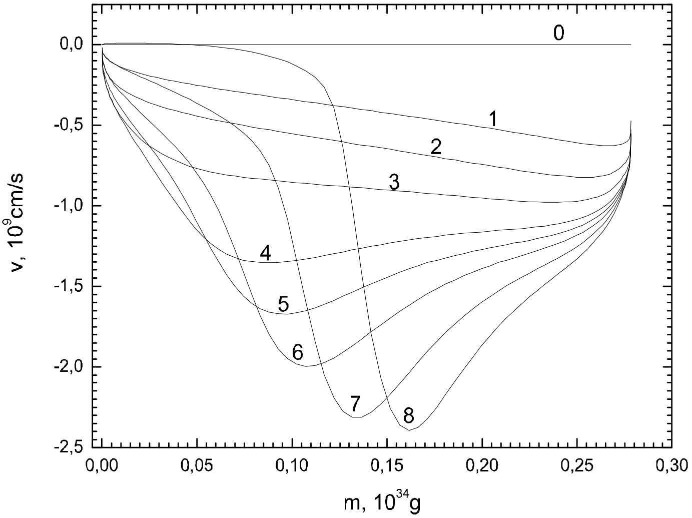



LSD (Soviet - Italy)
Время (UT)
2:52:20 2:52:30 2:52:40 2:52:50 2:53:00
Baksan
Время (UT)
2:52:20 2:52:30 2:52:40 2:52:50 2:53:00
Рис. 2. Сигналы советско-итал
янской установки LSD (а) и
Баксанского телескопа (б), з
регистрированные 23.02.1987
в 2 : 52 : 36 UT.
time ± 1m
Kamioka
Время (UT)
7:35:40 7:36:20 7:36:30
7.36.40
± 2s
Baksan
Время (UT)
7:35:40 7:35:50 7:36:00 7:36:10 7:36:20 7:36:30
± 50ms
IMB
Время (UT)
7:35:40 7:35:50 7:36:00 7:36:10 7:36:20 7:36:30
ванные 23.02.1987 в 7 : 35 UT,
на японском детекторе (а),
Баксанском телескопе (б) и
американсокм детекторе (с)
Between Baksan and IMB - 25s
~30s.

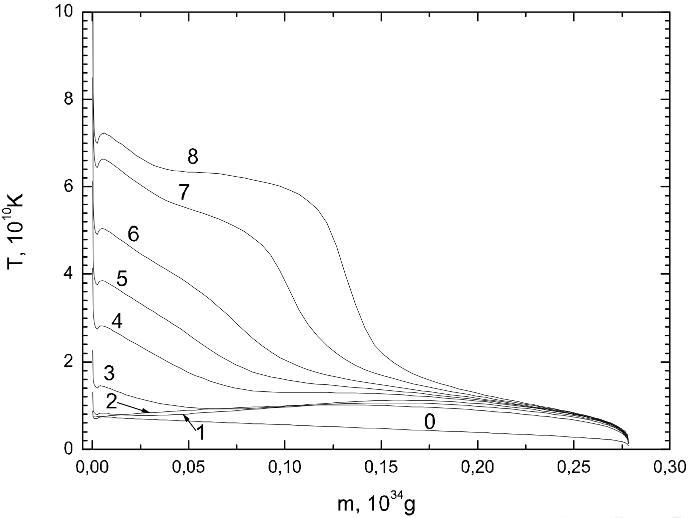
The dependence of the speci…c entropy from the mass coordinate at moments $t = 8 \cdot 10^{-3}$ (0), 0.7642 (1), 0.7721 (2), 0.7752 (3), 0.7770 (4), 0.7776 (5), 0.7782 (6), 0.7789 (7), 0.7796 (8) s.



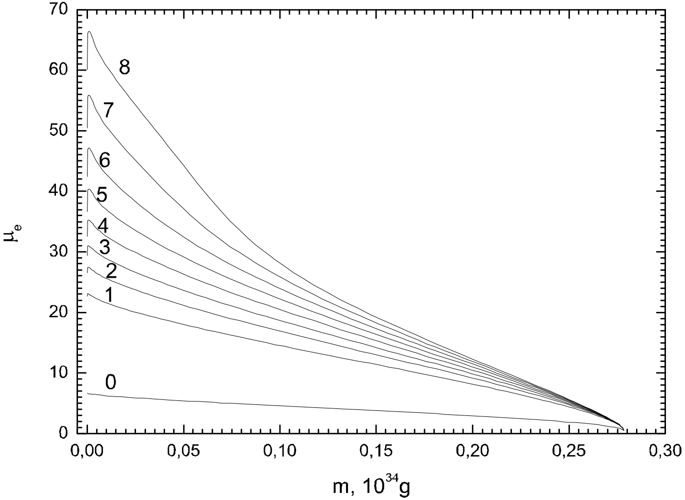
The dependence $v(m)$ at di¤erent time moments.



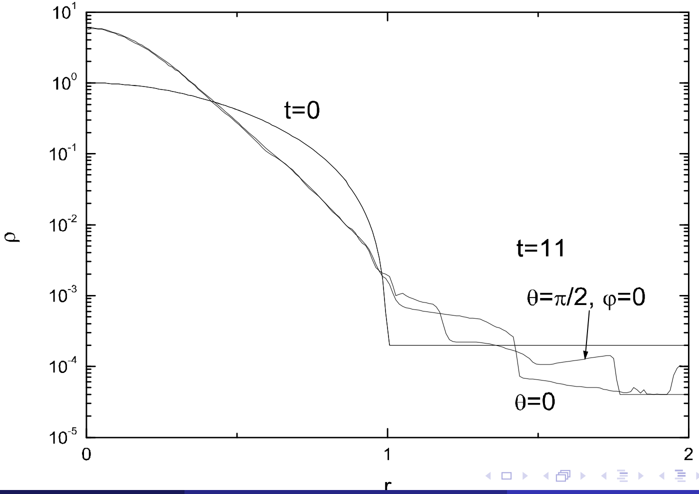
The dependence $T(m)$ at di¤erent time moments.



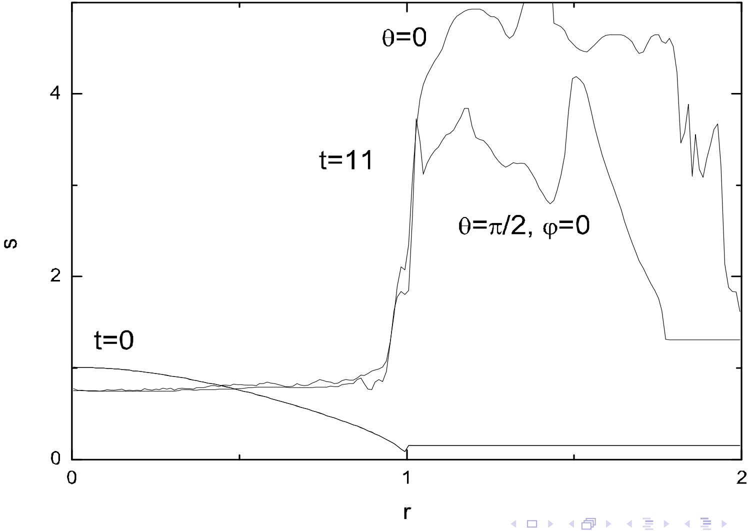
The dependence $\mu_e(m)$ at di¤erent time moments.



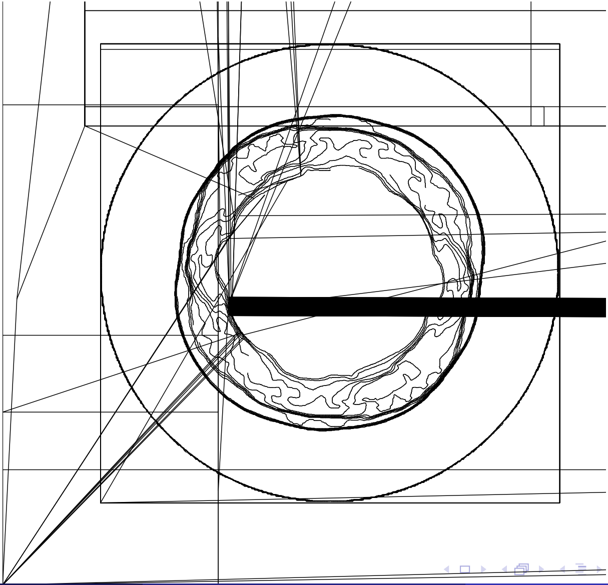
Convection in 3D hydrodynamic with unstable $ds/dr < 0$. $\rho(r, \theta = \pi/2, \phi = 0)$, $\rho(r, \theta = 0)$, $P = (4/3 - 1)\rho\epsilon$



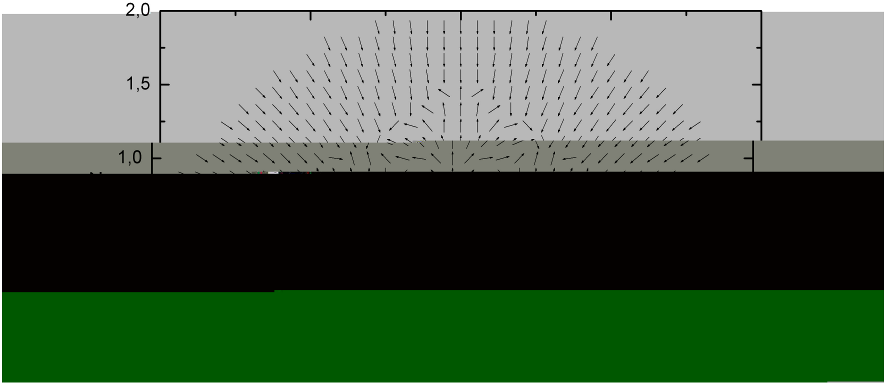
$s(r, \theta = \pi/2, \phi = 0)$, $s(r, \theta = 0)$



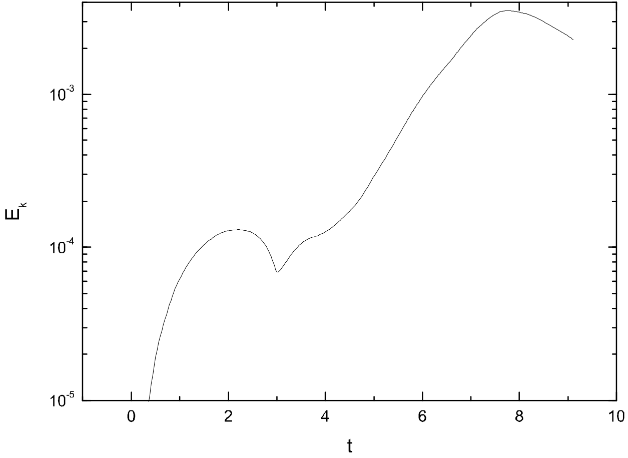
Contours of constant $\lg \rho$ in the equatorial plane



Velocities in the plane $y = 0$



Dependence of the kinetic energy from the time



Criterium of the stability

1. The Schwarzschild criterion for the one component gas

$$\frac{ds}{dr} < 0 \tag{8}$$

2. The Ledoux criterion for convection of the gas with the chemistry is

$$\left(\frac{\partial P}{\partial s}\right)_{\rho Y_l} \frac{ds}{dr} + \left(\frac{\partial P}{\partial Y_l}\right)_{\rho s} \frac{dY_l}{dr} < 0, \tag{9}$$

where l is the number of leptons can be applied if one expects the neutrino are trapped.

Multi component hydrodynamics with the ‡ux limiters

The developed multi-temperature multi-dimensional code operates with the concentrations of components ρ_i and their speci…c energies ϵ_i. The matter has the density $\rho_i = \rho$ and the temperature T_i, while neutrino $\rho_\nu = 0$, and T_ν. The equation of state includes the di¤erence of electrons and positrons per nucleon Y_e: $\epsilon_i = \epsilon_i(\rho, T_i, Y_e)$, $P_i = P_i(\rho, T_i, Y_e)$. The code is based in the Riemann problem solver.

In Eulerian coordinates we have the equation for baryons concentrations

$$\frac{\partial \rho/m_p}{\partial t} + \mathrm{div}(\rho/m_p \mathbf{v}) = 0, \tag{10}$$

the equation for the electrons di¤erence

$$\frac{\partial \Delta n_e}{\partial t} + \mathrm{div}(\Delta n_e \mathbf{v}) = \dot{Y}_e \rho/m_p, \tag{11}$$

the equation for the momentum of matter

$$\frac{\partial \rho v_j}{\partial t} + \nabla_i \Pi^{\mathrm{m}}_{ij} = \rho g_j + \rho f_\nu, \tag{12}$$

the equation for the matter energy

$$\frac{\partial \rho E_{\mathrm{m}}}{\partial t} + \mathrm{div}(E_{\mathrm{m}}\rho + P_{\mathrm{m}})\mathbf{v} = \rho \mathbf{v}\mathbf{g} + \rho q_{\mathrm{m}}, \tag{13}$$

$\Pi^{\mathrm{m}}_{ij} = \rho v_i v_j + P_{\mathrm{m}} \delta_{ij}$, $E_{\mathrm{m}} = \varepsilon_{\mathrm{m}} + \mathbf{v}^2/2$. The gravity $\mathbf{g} = -\mathrm{grad}\Phi$, $\Delta\Phi = 4\pi G\rho$. The transport of the neutrino energy is

$$\frac{\partial \rho \varepsilon_\nu}{\partial t} + \mathbf{v}\nabla\left(\rho\varepsilon_\nu\right) = \mathrm{div}\mathbf{F}_\nu - \rho q_{\mathrm{m}}, \tag{14}$$

$\mathbf{F}^{\mathrm{thick}}_\nu = -\int d\varepsilon \frac{1}{3\chi} \mathrm{grad} U_\nu$, $\mathbf{F}_\nu = \frac{\mathbf{F}^{\mathrm{thick}}_\nu}{|\mathbf{F}^{\mathrm{thick}}_\nu|/F^{\mathrm{max}}_\nu + 1}$, $F^{\mathrm{max}}_\nu = c\int d\varepsilon_\nu U_\nu$.

Shafranov test strong shock wave in the hydrogen plasma

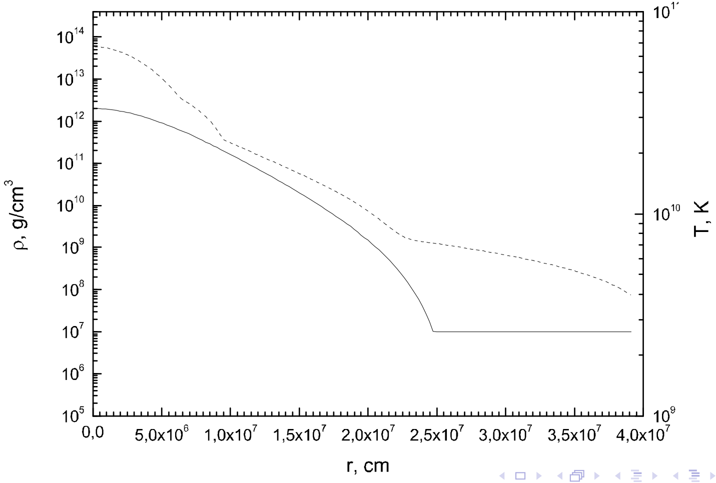
2D collapse with the neutrino transport

The initial state is the equilibrium polytrope $P \propto \rho^{1+1/n}$ with $n = 3$ and the central density $\rho_c = 2 \cdot 10^{12}$ g $\cdot$ cm^{-3}. For the total mass $1.4 M_\odot$ the star radius is $2.6 \cdot 10^7$ cm. We need to calculate T in the initial state to provide the exact equilibrium at $t = 0$ (without the neutrino looses) and $Y_e = Y_e^{\beta-\mathrm{eq}}$:

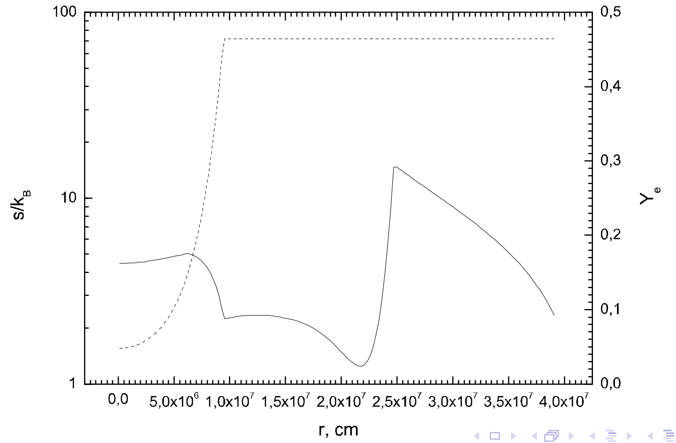
$$e^- + p \to n + \nu, \;\; e^+ + n \to p + \tilde{\nu}.$$

Selected initial conditions can be considered as the reconstruction of the solution of the 1D task about the collapse from the previous section, because this equilibrium initial state has the neutronization region in the center and unstable entropy pro…le also in the center. We reproduced the pro…le $ds/dr < 0$ in the central region like as in detailed spherically symmetric calculations with the neutrino transport. Such entropy pro…le is the result of the neutronization. This process reduces the number of electrons Y_e, the speci…c energy transforms from electrons to nucleons.

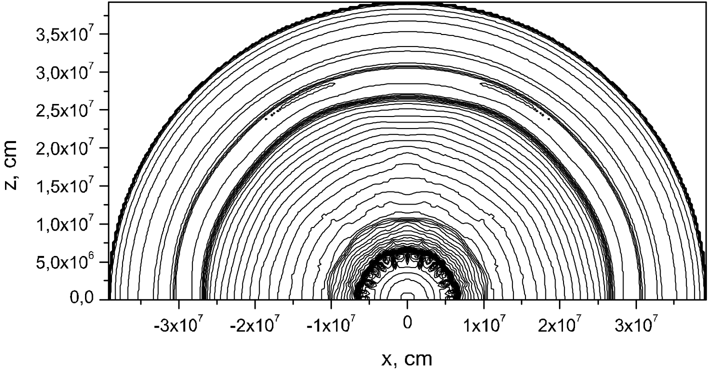
Dependence of ρ (solid line) and T (dashed line) in the star on the radius r at time $t = 0$



Dependence of s/k_B (solid line) and Y_e (dashed line) in the star on radius at time $t = 0$ in 2D calculations at $t = 0$.



Contours of the constant densities $\lg \rho_{\min} = 5$, $\lg \rho_{\min} = 13$, $\Delta \lg \rho = 0.1$ at $t = 11$ ms in 2D calculations.



Conclusions

- We studied spherically-symmetric collapse with full physics. We considered convective instability for the corresponding pro…le of $s(\mathbf{r})$ in 2D with taking into account self gravity and the neutrino transport. The timescale of the convection is 10 ms.
- The convection has large scale. The large average energy of neutrino $\sim$ 30 MeV due to the convection is enough to explain SN. The neutrino energy is the open question for the neutrino registration. For SN1987A: 20–40 MeV IMB 1987PhRvL..58.1494B, 9–35 MeV Kamiokande-II 1987PhRvL..58.1490H, 20 MeV Baksan-LSD 1987JETPL..45..589A, 1987Natur.330..142S. *Ginzburg, Syrovaskiy 1960: SN is the source of particles. Now the neutrino registration helps specify the SN mechanism.*
- We should to check the hypothesis in multidimensional simulations with transport in the multi-group di¤usion approximation with the ‡ux limiters. The magnetic …eld is also important for the collapse and in the non spherically-symmetric accretion connected with GRB.