Global geometry of the Vaidya space-time

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Prahalad Chunnilal Vaidya space-time

The exact spherically symmetric nonstationary solution of the Einstein equations

$$ds^2 = \left[1 - \frac{2Gm(z)}{c^2 r}\right] dz^2 + 2dzdr - r^2 d\Omega^2$$

$m(z)$ — arbitrary varying mass-function due to the radial influx at $z = -v$ (or outflux at $z = u$) of null particles

$\frac{dm}{dz}$ — influx or outflux (accretion ot emission rate)

u — advanced time null coordinate

v — retarded time null coordinate

r — radial coordinate

$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ — line element on the 2D unit sphere

Vaidya metric in the diagonal coordinates

Simple physical interptretation

The classical Vaidya metric is transformed to the special diagonal coordinates in the case of the linear mass function allowing rather easy treatment. We find the exact analytical expressions for metric functions in this diagonal coordinates. Using these coordinates, we elaborate the maximum analytic extension of the Vaidya metric with a linear growth of the black hole mass and construct the corresponding Carter-Penrose diagrams for different specific cases. The derived global geometry seemingly is valid also for a more general behavior of the black hole mass in the Vaidya metric.

Transformation to the diagonal coordinates

$$ds^2 = f_0(\eta, y)d\eta^2 - \frac{dy^2}{f_1(\eta, y)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$f_0(\eta, y)$ and $f_1(\eta, y)$ — two principal metric functions
η — new "time" coordinate,
$y = 1 - 2Gm(z)/c^2 r$ — new "space" coordinates

Crucial ansatz: $m(z) = -\alpha z + m_0$, $dm/dz = -\alpha = const$
$\alpha > 0$ — mass accretion or emission rate parameter

Analytic solution:

$$m = C(\eta)\Phi(y),\ r = \frac{2C(\eta)}{1-y}\Phi(y),\ C(\eta) = \alpha\eta + C_0,\ C_0 = const,$$

$$f_0 = -\frac{(y^2 - y + 4\alpha)}{1-y}\frac{C_{,\eta}^2}{\alpha^2}\Phi^2, \quad f_1 = -\frac{(1-y)^3(y^2 - y + 4\alpha)}{(2C\Phi)^2},$$

$$\Phi(y) = \exp\left[-2\alpha\int\frac{dy}{(1-y)(y^2 - y + 4\alpha)}\right]$$

Specific cases of the Vaidya space-time

"Powerful" accretion at $\alpha > 1/16$:

$$\Phi = \frac{\sqrt{1-y}}{(y^2 - y + 4\alpha)^{1/4}} \exp\left[-\frac{1}{2\sqrt{16\alpha - 1}}\left(\arctan\frac{2y-1}{\sqrt{16\alpha-1}} + \frac{\pi}{2}\right)\right]$$

Transient case at $\alpha = 1/16$:

$$\Phi = \sqrt{\left|\frac{y-1}{y-(1/2)}\right|} \exp\left\{\frac{1}{4[y-(1/2)]}\right\}$$

"Weak" accretion at $\alpha < 1/16$:

$$\Phi = \sqrt{1-y}\,|y - y_3|^{y_3/[2(y_4-y_3)]}\,|y - y_4|^{-y_4/[2(y_4-y_3)]}$$

where $y_3 = (1 - \sqrt{1-16\alpha})/2$ and $y_4 = (1 + \sqrt{1-16\alpha})/2$

Transformation to the diagonal coordinates $(\eta, y, \theta, \phi) \quad \Rightarrow \quad (t, r, \theta, \phi)$

$$ds^2 = f_0(t,r)dt^2 - \frac{dr^2}{f_1(t,r)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Differential transformation to the new coordinates

$$dz(t,r) = f_0^{1/2} f_1^{-1/2} dt - f_1^{-1} dr$$

Construction of the global geometries. I

We define the invariant

$$Y = \gamma^{ik} y_{,i} y_{,k} = -f_1 = \frac{1}{4m^2}(1-y)^3(y^2 - y + 4\alpha)$$

γ^{ik} — 2-dim part of the Vaidya metric (without angle variables)

R^*-regions if $Y < 0$
η is the time coordinate, y is a space coordinate

T^*-regions if $Y > 0$
η is a space coordinate, y is a time coordinate

Construction of the global geometries. II

Carter-Penrose diagrams in the coordinates $\log C(\eta)$ and $\log \Phi(y)$

Reduction to the simple form by conformal transformation

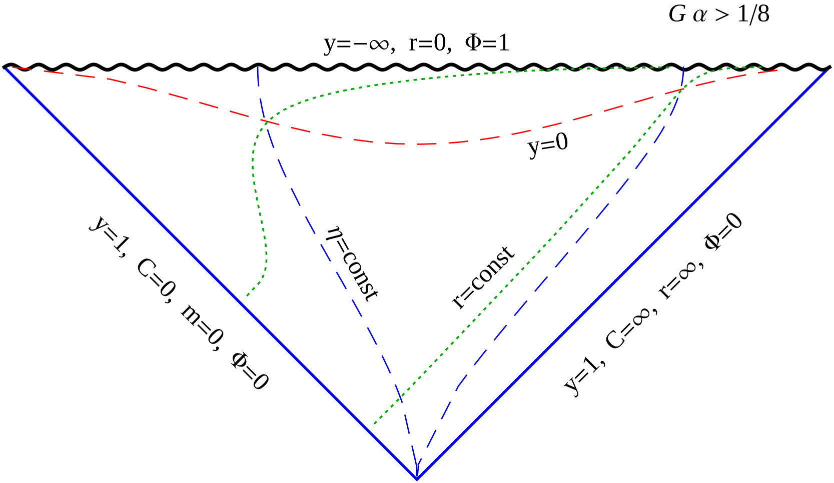
$$ds^2 = \frac{C^2}{\alpha^2} \frac{\Phi^2}{(1-y)} \left\{ (y^2 - 2y + 4\alpha) \left[(d \log \Phi)^2 - (d \log C)^2 \right] \right\} - r^2 d\Omega^2$$

Carter-Penrose diagrams in the coordinates $\log C(\eta)$ and $\log \Phi(y)$ under the standard **arctan**-type transformation

$$t = \arctan\left[\log C + \log \Phi(y)\right] - \arctan\left[\log C - \log \Phi(y)\right]$$
$$x = \arctan\left[\log C + \log \Phi(y)\right] + \arctan\left[\log C - \log \Phi(y)\right]$$

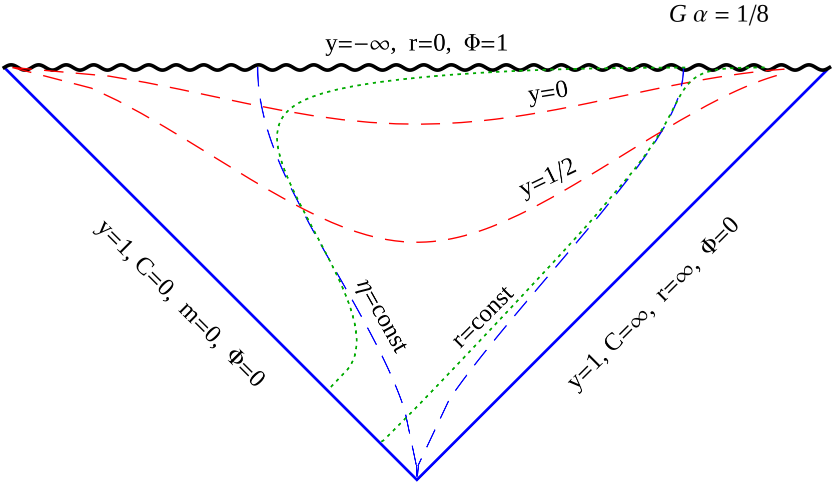
Carter-Penrose diagrams: $G\alpha > 1/8$

"Superpowerful accretion"



Carter-Penrose diagrams: $G\alpha = 1/8$

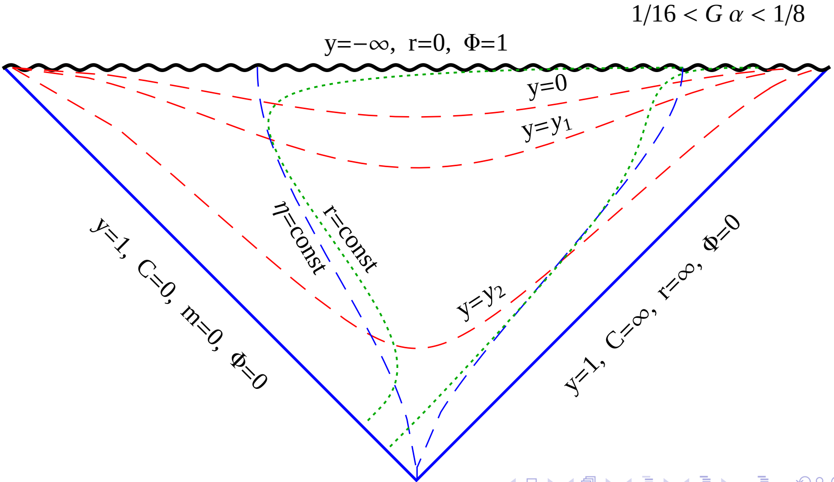
Transition case from the "superpowerful" to "just-powerful" accretion



Carter-Penrose diagrams: $1/16 < G\alpha < 1/8$

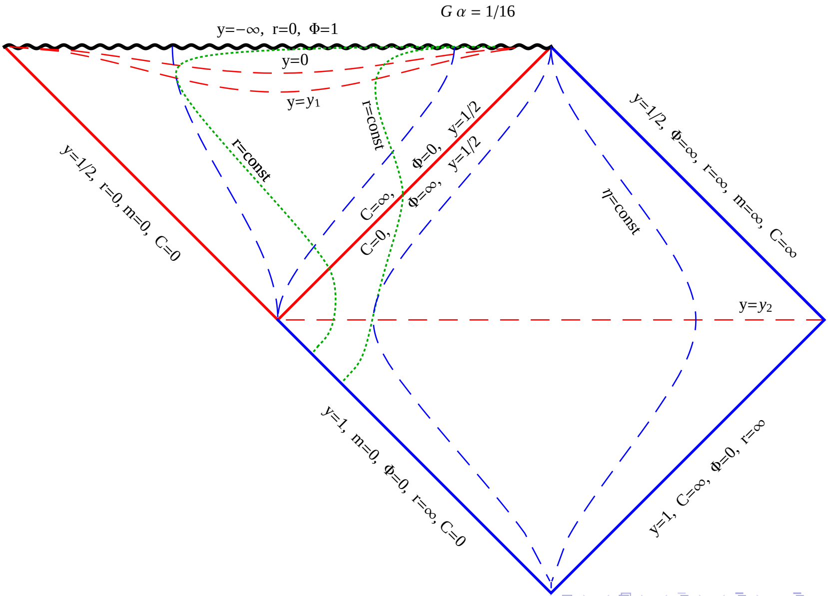
"Powerful" accretion

$$y_1 = \frac{1}{2}(1 - \sqrt{1-8\alpha}), \quad y_2 = \frac{1}{2}(1 + \sqrt{1-8\alpha})$$



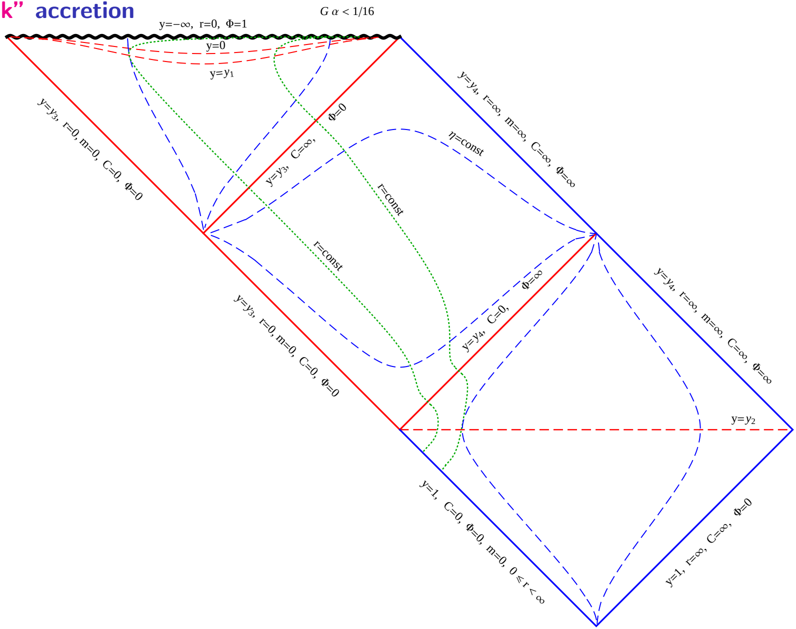
Carter-Penrose diagrams: $G\alpha = 1/16$

Transition case from the “powerful” to “weak” accretion



Carter-Penrose diagrams: $G\alpha < 1/16$

"Weak" accretion



Results: JETP 124, 446 (2017)

- We find the exact analytical expressions for metric functions of the Vaidya space-time with a linear growth of the black hole mass in the diagonal coordinates
- We construct the corresponding Carter-Penrose diagrams for different specific cases
- There are narrow region outside the apparent horizon of the black hole where Vaidya metric is qualitatively different from the Schwarzschild metric and cannot be described as a small perturbation of the Schwarzschild black hole. There are singular coordinate surfaces where the energy-momentum tensor is diverged, while tidal forces remain the finite. For this reason these singularities are not physical but only coordinate ones. We call these coordinate singularities the "false firewalls"