Specific properties of elastic scattering and inelastic profiles of high energy protons

I.M. Dremin

Moscow, Ginzburg100-conference, 1 June 2017

Colliding protons are not destroyed but keep their entity with increase of energy!

$$r(s) = \frac{4\sigma_{el}}{\sigma_{tot}}$$

$\sqrt{s}$, GeV	4.11	4.74	7.62	13.8	62.5	546	1800	7000
$r(s)$	0.98	0.92	0.75	0.69	0.67	0.83	0.93	1.00-1.04
					ISR			LHC
$\sigma_{inel}/\sigma_{el}$					**5**			**3**

EXPERIMENTAL FACT!

The share of elastic processes $r/4$ decreases at energies below ISR and increases (why?) up to LHC energies! NOT EXPLAINED YET!

$$r = 1 \qquad \textbf{CRITICAL!}$$

Elastic scattering variables:

$$s = 4E^2 = 4(p^2 + m^2), \quad -t = 2p^2(1 - \cos\theta) \;\; [\approx p_\perp^2; \; \theta \ll 1]$$

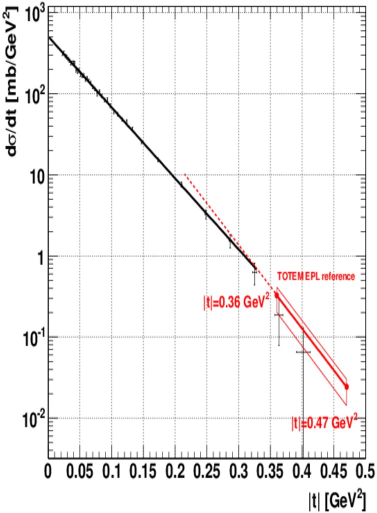
If a cap falls to the floor, it breaks up to pieces but sometimes stays intact. The stronger it hits the floor, the less chances to be unbroken.

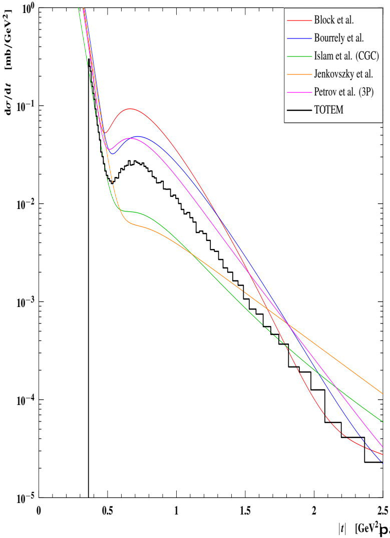
If two high energy protons collide, many new particles (mostly pions) are produced, but sometimes they scatter elastically and retain their entity. It is surprising enough that at very high energies of collisions from ISR to LHC the share of elastic processes starts increasing with increase of energy. This unexpected phenomenon and its consequences at present and higher energies are discussed in my talk.

$$\frac{d\sigma}{dt} = |f(s,t)|^2 = f_I^2 + f_R^2.$$

$$-\sqrt{d\sigma/dt} \le f_I \le \sqrt{d\sigma/dt}; \quad f_R(t=0)/f_I(t=0) = 0.1 - 0.14$$

Specific features: exponential cone (large $f_I > 0$)+ dip (zero f_I?).





The unitarity condition (connects elastic and inelastic processes)

$$SS^+ = 1 \qquad S = 1 + iT$$

$$2\mathrm{Im}\, T_{ab} = \Sigma_n \int T_{an} T^*_{nb} d\Phi_n,$$

Elastic scattering $a = b = 2, \quad n \geq 2. \qquad f_l(p,\theta) = I_2(p,\theta) + g(p,\theta) =$

$$\frac{s}{8\pi^{3/2}} \int d\theta_1 d\theta_2 \frac{\sin\theta_1 \sin\theta_2 f(p,\theta_1) f^*(p,\theta_2)}{\sqrt{[\cos\theta - \cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2) - \cos\theta]}} + g(p,\theta).$$

Optical theorem at t=0: $f_l(p,0) \propto \sigma_{tot} = \sigma_{el} + \sigma_{in}$

Transformation from transferred momenta t to impact parameter b

$$i\Gamma(s,b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s,t) J_0(b\sqrt{|t|}).$$

The unitarity condition in the b-representation (algebraic!)

$$G(s,b) = 2\mathrm{Re}\Gamma(s,b) - |\Gamma(s,b)|^2.$$

Inelastic and elastic profiles:

$$\frac{d^2\sigma_{inel}}{db^2} = \frac{d^2\sigma_{tot}}{db^2} - \frac{d^2\sigma_{el}}{db^2} \quad \rightarrow \quad \sigma_{inel} = \sigma_{tot} - \sigma_{el}.$$

Central collisions at $b = 0$

$$G(s, b = 0) = \zeta(s)(2 - \zeta(s)).$$

$\mathrm{Re}\Gamma(s,0) = \zeta(s) = (4\pi)^{-0.5}\int_0^{\infty} d|t| f_I(s,t)$
$\mathrm{Im}\Gamma(s,0)$ neglected in $|\Gamma|^2$ (i.e. $\int dt f_R = 0$)

Maximum at $\zeta = 1$; $\quad G(s,0) = 1 - \epsilon^2$ for $\zeta = 1 \pm \epsilon$

Two branches of the unitarity condition for $\zeta < 1$ and $\zeta > 1$

$$\zeta(s) = 1 \pm \sqrt{1 - G(s,0)}.$$

Branch with minus sign: $\zeta \approx G(s,0)/2$ for $G \ll 1$ -
elastic scattering interpreted as shadow of WEAK! inelastic processes - typical for electrodynamics ($ee \to ee\gamma$) and optics.
Strong interactions at present energies - large $G(s,0)$
If $\zeta(s) < 1$, conservative situation with maximum value at $b = 0$.

Critical $\zeta(s) = 1$! BRANCH WITH PLUS SIGN for $\zeta > 1$!

Similarity of $\zeta \propto \int dt f_I \propto f_I(0)/B(s) \propto const(\text{or} \ln s?)$ and $r \propto \int dt|f_I|^2/f_I(t=0) \propto f_I(0)/B(s)$ but surely $r < 4$ (!?).
$f_I(0)$ and B are the height and the slope of the diffraction cone

Increase of r from ISR to LHC! (see Table)
Slight trend of r to become larger than 1 at LHC:
TOTEM 2.76 TeV 1.03;
TOTEM+ATLAS 7 TeV 1.00–1.02; 8 TeV 1.00–1.04
(accuracy ± 0.03);
TOTEM 13 TeV (high statistics, better accuracy; end 2017).

The trends of r and ζ are similar if one approximates $f_I \approx \sqrt{d\sigma/dt}$.
$\zeta < r$ otherwise.

Model estimate ($f_I \propto (1-(t/t_0)^2)\exp(Bt/2)$):

$$\frac{r}{\zeta} = \frac{1 - \frac{4}{(Bt_0)^2} + \frac{24}{(Bt_0)^4}}{1 - \frac{8}{(Bt_0)^2}} \approx 1 + \frac{4}{(Bt_0)^2} + \frac{88}{(Bt_0)^4} > 1$$

$t_0 \approx t_{dip}$ the dip position; $r \approx \zeta$ if $Bt_0 \gg 1$ ($Bt_0 \approx 10$ at LHC).
Extrapolations of fits of ISR+LHC data
r(13 TeV)=1.05–1.06; r(95 TeV)=1.12–1.15.
See also the next talk.

The geometry of the interaction region

$$\frac{d\sigma}{dt} = \frac{\sigma_{tot}^2}{16\pi} \exp(-B(s)|t|).$$

$$i\Gamma(s,b) \approx \frac{\sigma_t}{8\pi} \int_0^\infty d|t| \exp(-B|t|/2)(i+\rho) J_0(b\sqrt{|t|}).$$

$\rho \ll 1$ except near the minimum of $d\sigma/dt$.

$$\mathrm{Re}\Gamma(s,b) = \zeta \exp(-\tfrac{b^2}{2B}) \quad (= \zeta \text{ at } b = 0)$$

Inelastic interaction region:

$$G(s,b) = \zeta \exp(-\frac{b^2}{2B})[2 - \zeta \exp(-\frac{b^2}{2B})].$$

Scaling in $b/\sqrt{2B}$. Typical $\sqrt{2B} \approx 6$ GeV$^{-1} \approx 1.2$ fm
Maximum at $b_m^2 = 2B \ln \zeta$ (for $\zeta < 1$ - maximal value at $b = 0$).
For $\zeta = 1$ maximum at $b_m = 0$ and plateau at $b^2 \ll 2B$ (LHC!)

$$G(s,b) = \zeta[2 - \zeta - \frac{b^2}{B}(1-\zeta) - \frac{b^4}{4B^2}(2\zeta - 1)].$$

For $\zeta > 1$ - maximum at $b_m > 0$. Full absorption $G(s, b_m) = 1$!
Inelastic processes stronger at periphery!

The evolution of the inelastic interaction region with increase of ζ. $\zeta = 0.7$ and 1.0 correspond to ISR and LHC energies. A further increase of ζ leads to the toroid-like shape with a dip at $b = 0$. The values $\zeta = 1.5$ and $\zeta = 1.8$ are used as possible asymptotical regimes.

No absorption at central collisions for $\zeta = 2$!