Asymptotic Fragility, $T\bar{T}$ deformation and Flat Space Holography

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arxiv:1706.xxxxx

Expanded and accurate version of the title

Integrable Asymptotic Fragility, $T\bar{T}$ deformation, Flat Space Limit of Near AdS_2 Holography and QCD Strings

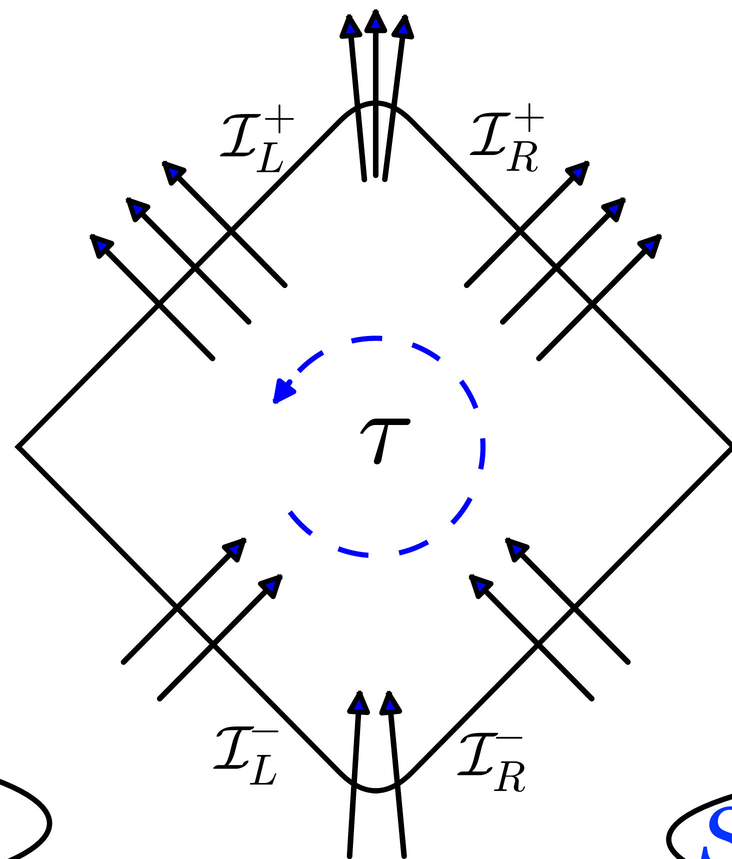
Main takeaway message:

$$IAF = T\bar{T} = \lim_{L\to\infty} N AdS_2$$

and provide an integrable approximation to the QCD string

Integrable Asymptotic Fragility

SD, Gorbenko, Mirbabayi
1305.6939



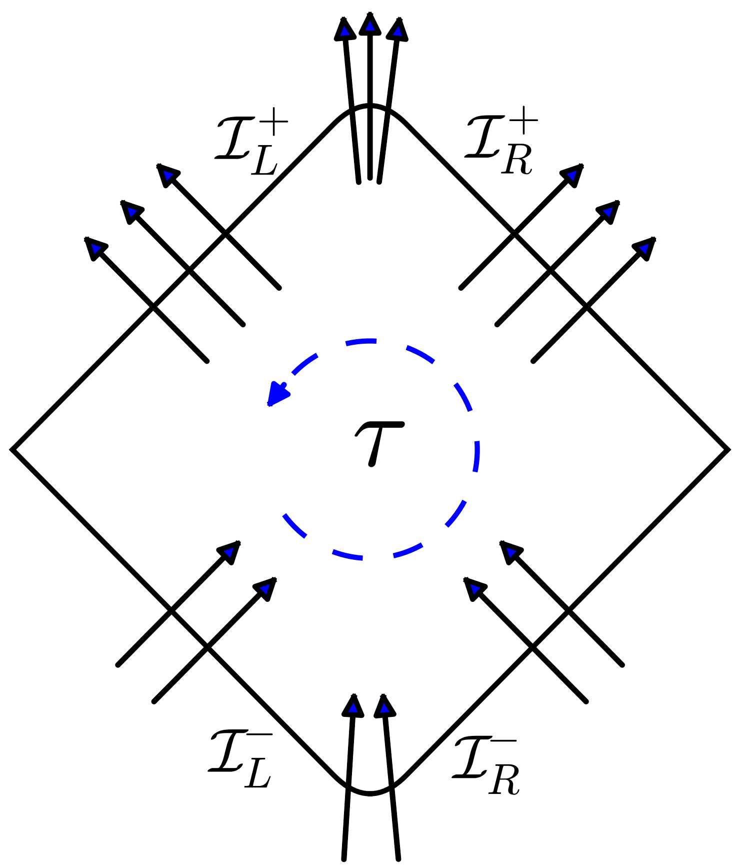
"dressed" *S*-matrix

S-matrix of *any* 2D QFT

$$\hat{S}_n(p_i) = e^{i\ell^2/4 \sum_{i<j} p_i * p_j} S_n(p_i)$$

$$p_i * p_j = \epsilon_{\alpha\beta} p_i^\alpha p_j^\beta$$

“Holographic” Form of the Dressing Factor



$$e^{i\ell^2/4\sum_{i<j} p_i * p_j} = \int \mathcal{D}X^\alpha e^{iS_{CS}[X^\alpha]+\sum_i p_{i\alpha}X^\alpha}$$

Chern-Simons boundary quantum mechanics

$$S_{CS} = \ell^{-2}\oint d\tau \epsilon_{\alpha\beta} X^\alpha \partial_\tau X^\beta$$

$T\bar{T}$ deformation

Smirnov, Zamolodchikov, 1608.05499
Cavaglia, Negro, Szecsenyi, Tateo, 1608.05534

Deformation of *any* 2D QFT by an irrelevant operator

$$T\bar{T} \equiv \frac{1}{2}\left(T_{\alpha\beta}T^{\alpha\beta} - T_{\alpha}^{\alpha 2}\right)$$

is solvable: the deformed finite volume spectrum satisfies

$$\partial_t E_t(n,P,R) = E_t(n,P,R)\partial_R E_t(n,PR) + \frac{P(R)^2}{R}$$

R — circle size $\quad$ P — spatial momentum

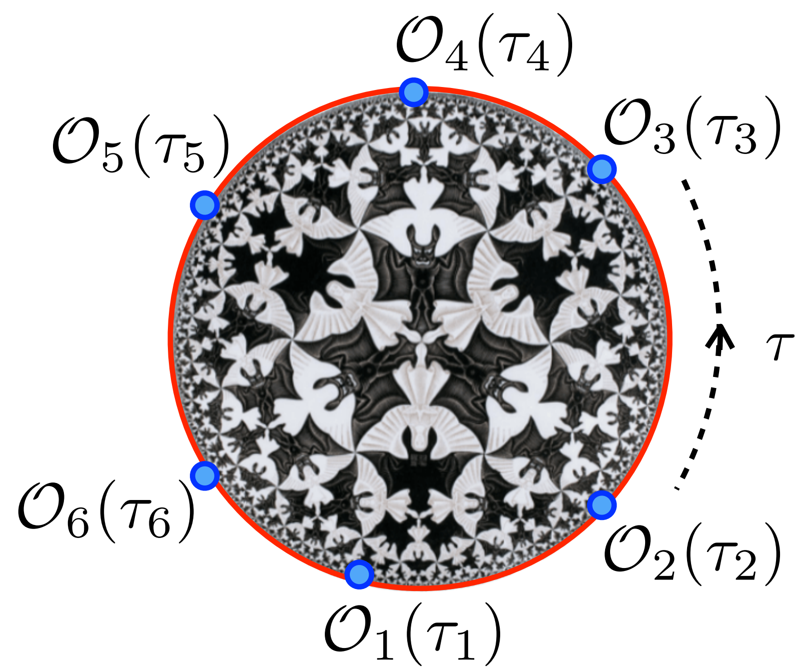
t — deformation parameter

$N\,AdS_2$ Holography

- consider *any* 2D QFT in a rigid AdS_2 (=Poincare disc)

Jensen, 1605.06098
Maldacena, Stanford, Yang, 1606.06098
Engelsoy, Mertens, Verlinde, 1606.03438

$$ds^2 = dr^2 + L^2 \sinh^2 \frac{r}{L} d\tau^2$$



- calculate generating functional $Z_0[\lambda_{\mathcal{O}_a}(\tau)]$ for boundary conformal correlators
- introduce dynamical (Jackiw—Teitelboim) gravity

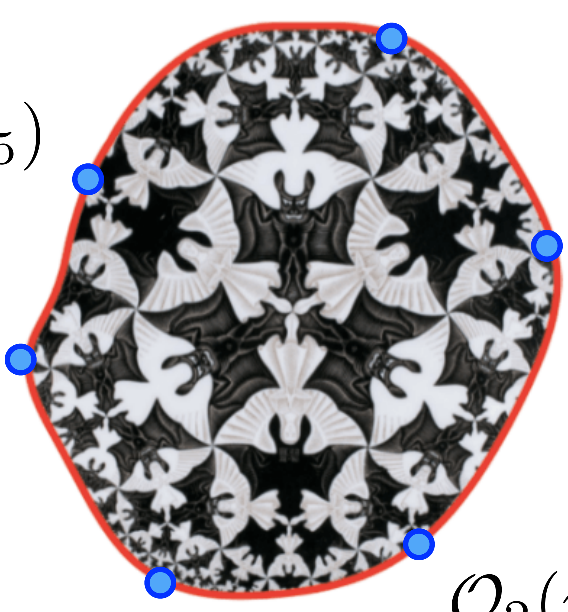
$$S = \int \sqrt{-g} \left(\phi(R + \frac{2}{L^2}) - \Lambda + \mathcal{L}_m(g, \psi) \right)$$

Dressed generating functional

$$Z_{JT}[\lambda_{\mathcal{O}_a}(\tau)] = \int \mathcal{D}\tau \, e^{\Lambda L^2 \int du Sch(\tau(u))} Z[\lambda_{\mathcal{O}_a}(\tau(u))]$$

Schwarzian boundary quantum mechanics

$$Sch(\tau(u)) = \frac{\partial_u^3 \tau}{\partial_u \tau} - \frac{3}{2}\left(\frac{\partial_u^2 \tau}{\partial_u \tau}\right)^2$$



Main takeaway message:

$$IAF = T\bar{T} = \lim_{L\to\infty} NAdS_2$$

Solves (flat space) Jackiw —Teitelboim gravity

$$S = \int \sqrt{-g}\,(\phi R - \Lambda + \mathcal{L}_m(g,\psi))$$

with

$$\frac{1}{\ell^2} = -\frac{\Lambda}{2} = t$$

$$IAF = T\bar{T}$$

- Obvious to anybody who knows both constructions
- True for CFT's and integrable QFT's, where one can calculate finite volume spectrum with TBA
- First and simplest example: one starts with 24 free massless bosons and ends up getting a critical bosonic string

SD, Flauger, Gorbenko, 1205.6805

$$e^{i2\delta(s)} = e^{is/4\ell^2} \quad \xleftrightarrow{\text{TBA}} \quad E = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}(N+\tilde{N}-2)}$$

- Provides reason d'être for IAF and rigorous definition for $T\bar{T}$
- Should be possible to derive E's from the JT partition function in a generic case *(work in progress)*

$T\bar{T}$ from linearized JT gravity

Minkowski vacuum

$$g_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \qquad \phi = -\frac{\Lambda}{4}\eta_{\alpha\beta}\sigma^\alpha\sigma^\beta = \frac{\Lambda}{2}\sigma^+\sigma^-$$

Quadratic action

$$\begin{aligned} S^{(2)}_{JT} = \int \varphi\left(\partial_+^2 h_{--} + \partial_-^2 h_{++} - 2\partial_+\partial_- h_{+-}\right) + \frac{\Lambda}{4}(h_{++}h_{--} - 2h_{+-}^2) \\ + \frac{\Lambda}{4}\left(\sigma^+ h_{++}(2\partial_- h_{+-} - \partial_+ h_{--}) + \sigma^- h_{--}(2\partial_+ h_{+-} - \partial_- h_{++})\right) \\ + \frac{1}{2}h_{++}T_{--} + \frac{1}{2}h_{--}T_{++} + h_{+-}T_{+-} \ . \end{aligned}$$

Solution

$$h_{\alpha\beta} = -\frac{2}{\Lambda}(T_{\alpha\beta} - \eta_{\alpha\beta}T^\gamma_\gamma)$$

$$\partial_+\phi = \frac{1}{2}(\sigma^+ T_{++} - \sigma^- T_{+-})$$

$$\partial_-\phi = \frac{1}{2}(\sigma^+ T_{--} - \sigma^+ T_{+-})$$

Plug back into action

$$S_{T\bar{T}} = -\frac{1}{2\Lambda}\int (T_{\alpha\beta}T^{\alpha\beta} - T^{\gamma\,2}_\gamma)$$

Heuristics of IAF=JT

$$S = \int \sqrt{-g}\,(\phi R - \Lambda + \mathcal{L}_m(g,\psi))$$

JT dilaton is a Lagrange multiplier, which forces metric to be flat, hence we can write

$$Z = \int \mathcal{D}X^a \mathcal{D}\psi e^{i\int d^2\sigma\sqrt{-g_f}\left(-\Lambda + \mathcal{L}_m(\psi, g_f)\right)}$$

with

$$g_{f\alpha\beta} = \partial_\alpha X^a \partial_\beta X^b \delta_{ab}$$

gives

$$-\Lambda \int d^2\sigma\sqrt{-g_f} = -\frac{\Lambda}{2}\oint d\tau \epsilon_{ab} X^a \partial_\tau X^b$$

Chern-Simons boundary quantum mechanics

Proof of IAF=JT

Minkowski vacuum

$$g_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \qquad \phi = -\frac{\Lambda}{4}\eta_{\alpha\beta}\sigma^\alpha\sigma^\beta = \frac{\Lambda}{2}\sigma^+\sigma^-$$

Appears inhomogeneous. Why do we expect Poincare invariant *S*-matrix in the first place?

Conformal gauge

$$g_{\alpha\beta} = e^{2\Omega}\eta_{\alpha\beta}$$

JT action

$$S_{JT} = \int d\sigma^+ d\sigma^- \left(4\phi\partial_+\partial_-\Omega - \Lambda e^{2\Omega}\right)$$

is invariant under

$$\sigma^\pm \to \sigma^\pm + a^\pm \qquad \phi \to \phi - \frac{\Lambda}{2}(a^+\sigma^- + a^-\sigma^+)$$

which is the symmetry of the vacuum as well

Dynamical coordinates of JT gravity

$$X^{\pm} = 2\frac{\partial_{\mp}\phi}{\Lambda} \equiv \sigma^{\pm} + Y^{\pm}$$

Field equations

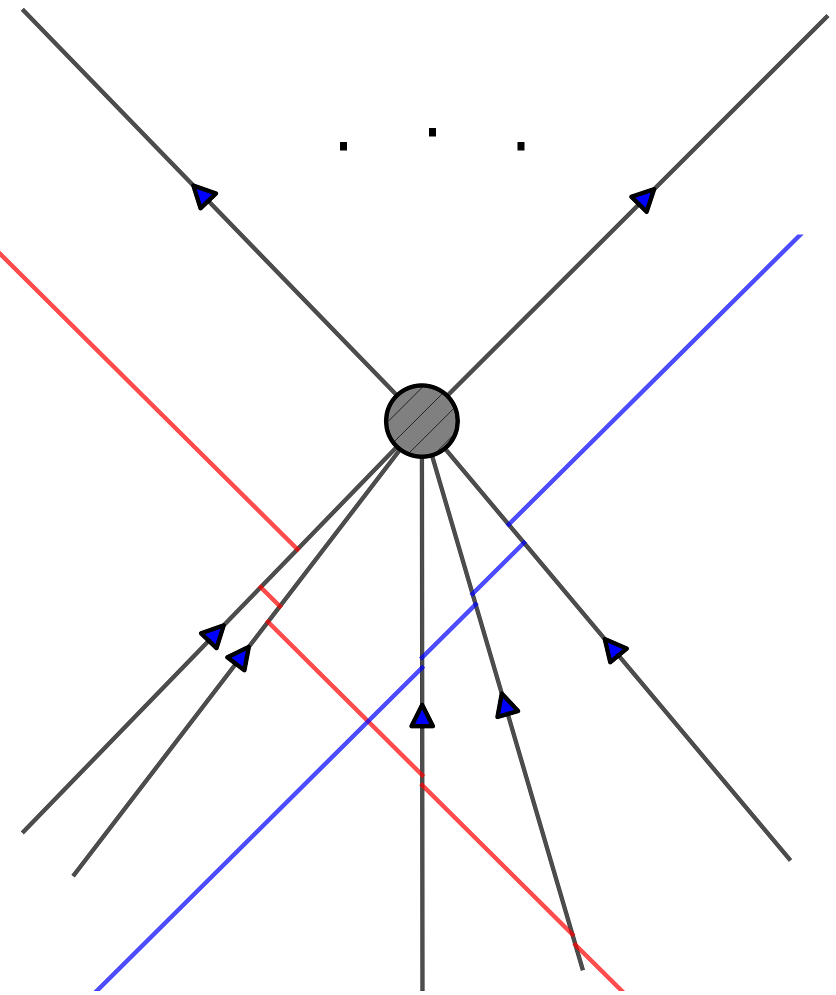
$$\partial_+ Y^- = -\frac{T_{++}}{\Lambda} \ ,$$

$$\partial_- Y^+ = -\frac{T_{--}}{\Lambda} \ ,$$

$$\partial_+ Y^+ = \partial_- Y^- = \frac{T_{+-}}{\Lambda}$$

Matter is unperturbed in σ- coordinates

Let's focus on the asymptotic *in*-region



$$Y^-(p_i) = \frac{1}{2\Lambda}\left(\mathcal{P}^-_<(p_i) - \mathcal{P}^-_>(p_i)\right)$$

$$Y^+(p_i) = \frac{1}{2\Lambda}\left(\mathcal{P}^+_>(p_i) - \mathcal{P}^+_<(p_i)\right)$$

$\mathcal{P}^\alpha_<(p_i)$ $(\mathcal{P}^\alpha_>(p_i))$ are *operators* which add up momenta of all particles with smaller (larger) rapidities

Before gravity

$$\psi = \int_{-\infty}^{\infty} \frac{dp}{\sqrt{2\pi}} \frac{1}{\sqrt{2E}} \left(a_{in}^{\dagger}(p) e^{-ip_\alpha \sigma^\alpha} + h.c. \right)$$

After gravity

$$A_{in}^{\dagger}(p) = a_{in}^{\dagger}(p) e^{ip_\alpha Y^\alpha(p)} = a_{in}^{\dagger}(p) e^{-i\left(p^+ Y^-(p) + p^- Y^+(p)\right)}$$

$$[A_{in}^{\dagger}(p), A_{in}^{\dagger}(p')] = 0$$

Dressed *in*-states

$$|\{p_i\}, in\rangle_{dressed} = \prod_{i=1}^{n_{in}} A_{in}^{\dagger}(p_i)|0\rangle = e^{-\frac{i}{2\Lambda}\sum_{i<j} p_i * p_j} |\{p_i\}, in\rangle$$

Dressed *out*-states

$$|\{q_i\}, out\rangle_{dressed} = \prod_{i=1}^{n_{out}} A_{out}^{\dagger}(q_i)|0\rangle = e^{\frac{i}{2\Lambda}\sum_{i<j} q_i * q_j} |\{q_i\}, out\rangle$$

Dressed *S*-matrix

$$\hat{S} \equiv {}_{dressed}\langle out, \{q_i\} | \{p_i\}, in\rangle_{dressed} = e^{-\frac{i}{2\Lambda}\sum_{i<j} p_i * p_j}$$

Q.E.D.

The same result can be obtained by taking the flat space limit of the Schwarzian dressing. However, it needs to be modified to reproduce the correct (and unitary) S-matrix,

$$S_b = \frac{\Lambda L^2}{2}\left(e^{\frac{R-R_0}{L}} + e^{-\frac{R-R_0}{L}} S_{Sch}\right)$$

R is an overall *dynamical* total length of the boundary. This action appears non-local (cf Euclidean wormholes?). However, it becomes local in the static gauge,

$$S_b = \Lambda L^2 \oint du \left(\cosh\frac{r-R_0}{L} - \frac{1}{2}e^{\frac{R_0-r}{L}} r'^2\right)$$

A link to QCD strings

SD, Gorbenko, 1511.01908
SD, Hernandez-Chifflet, 1611.09796

An integrable theory of a single massless boson with

$$e^{i2\delta(s)} = e^{is/4\ell^2}$$

is invariant under 3D Poincare group and provides a promising zeroth order approximation for confining strings in 3D Yang-Mills. A 4D generalization exists as well.

JT gravity have good chances to provide a useful analogue of the Polyakov formalism for these non-critical strings.

$$S_{3D} = \int \sqrt{-g}\left(\phi R + 2\ell_s^{-2} - \frac{1}{2}(\partial X)^2\right)$$

Future Directions

- Calculate JT partition function
- In very similar setups people discussed black holes, wormholes, baby universes and other beasts. It is not clear at the moment whether it is a matter of interpretation, small modifications are needed or drastically different setup is needed.
- Relation to QCD strings gives us a number of solid well posed questions and experimental data. Lattice simulations of large N gluodynamics provide a non-perturbative definition of a non-integrable gravitational theory!