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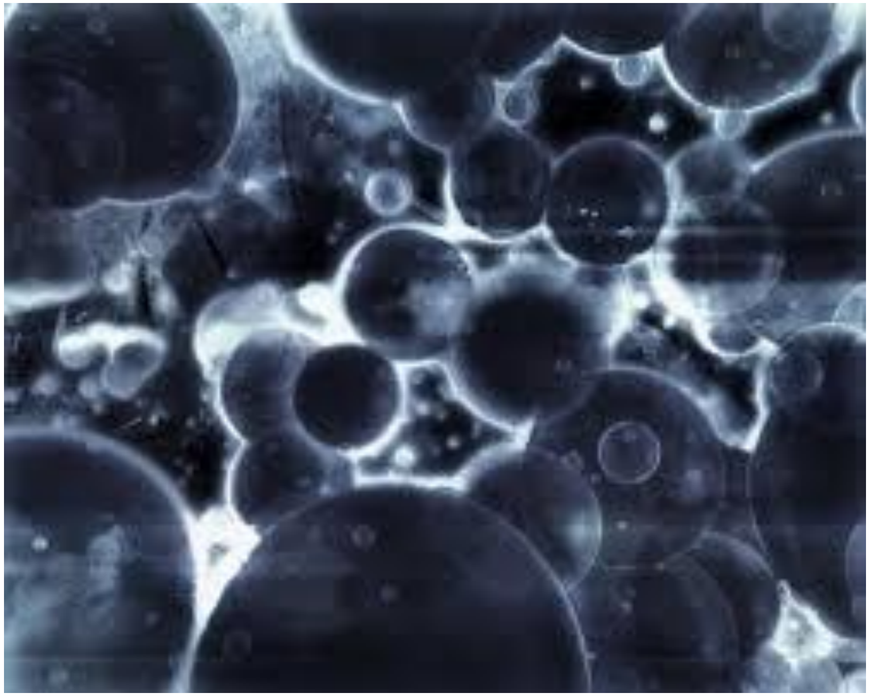

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A new approach to cosmological models of very early universe: on general solution of the dynamical equation in theories with scalar or vector fields coupled only to gravity.

A.T.Filippov. (JINR, Dubna)

The dynamics of any spherical cosmology with a scalar field **(`scalaron')** coupling to gravity is described by the nonlinear second-order differential equations for two metric functions and the scalaron depending on the `time' parameter. The equations depend on the scalaron potential and on the arbitrary gauge function that describes the time parameterizations.

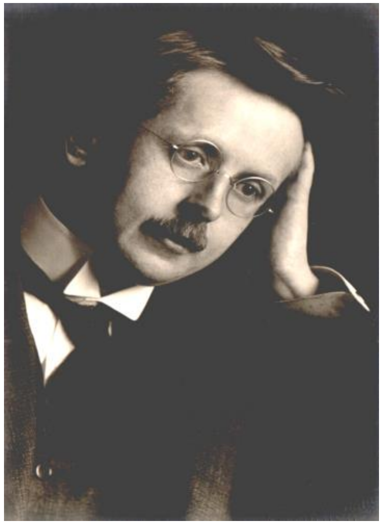
In the standard approach, this dynamical system can be **explicitly integrated** for flat isotropic models **with very special potentials.** But, replacing the time variable by one of the metric functions allows us **to completely integrate the general isotropic model in any gauge and with apparently arbitrary potentials**.

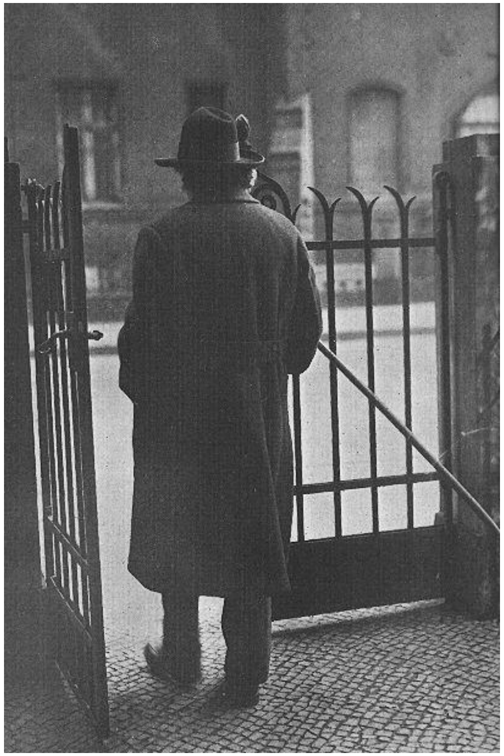
The main restrictions on the potential arise from positivity of the derived analytic expressions for the solutions, which are essentially the squared canonical momenta. An interesting consequence is emerging of **classically forbidden** regions for these analytic solutions. It is also shown that **in this rather general model the inflationary solutions can be identified, explicitly derived, and analytically compared to the standard approximate expressions.**

This approach is being applied to intrinsically anisotropic models with a massive vector field **(`vecton')** as well as to some non-inflationary models. The models are based on WEE ideas on affine generalizaton of gravity nonsymmetric connection).

Weyl, Eddington and Einstein ideas

1919 - 1923





arXiv:1011.2445 v1 (gr-qc) vecton as a relativistic particle in gravity field

arXiv:0812.2616 v2 (gr-qc) the first paper on new interpretation of **Einstein** 3 papers of **1923**; simplified model, static solutions, existence of horizons, non-ntegrability, approximate solutions by various power series expansions.

see also:1003.0782, 1008.2333

Main principles (suggested by Einstein's approach)

1. **Geometry:** **dimensionless** *`action'* constructed of a ***scalar*** *density*; its variations give the geometry and main equations ***without complete specification of the analytic form of the Lagrangian .***
2. **Dynamics:** a concrete Lagrangian constructed of the ***geometric variables*** - homogeneous function of order *D* (e.g. , the square root of the determinant of the curvature) produces a physical **effective Lagrangian**.
3. **Duality** between the geometrical and physical variables and Lagrangians.

NB: This looks more artificial than the first two principles and works for rather special models (actually giving *exotic fields*, *tachyons* etc.) (Einstein. did not know this! He was looking for unified theory of EM and Gravity.)

GEOMETRY OF SYMMETRIC CONNECTIONS

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + a^i_{jk}$$

$$\Gamma^i_{jk}[g] = \frac{1}{2} g^{il}(g_{lj,k} + g_{lk,j} - g_{jk,l})$$

$$r^i_{jkl} = -\gamma^i_{jk,l} + \gamma^i_{mk}\gamma^m_{jl} + \gamma^i_{jl,k} - \gamma^i_{ml}\gamma^m_{jk}$$

NONSYMMETRIC RICCI CURVATURE

$$r_{jk} = -\gamma^i_{jk,i} + \gamma^i_{mk}\gamma^m_{ji} + \gamma^i_{ji,k} - \gamma^i_{mi}\gamma^m_{jk}$$

Symmetric part of the Ricci curvature

$$s_{ij} \equiv \frac{1}{2}(r_{ij} + r_{j\,i})$$

Anti-symmetric part of the Ricci curvature

$$a_{ij} \equiv \frac{1}{2}(r_{ij} - r_{j\,i}) = \frac{1}{2}(\gamma^{m}_{jm,i} - \gamma^{m}_{im,j})$$

$$a_{ij,\,k} + a_{jk,i} + a_{ki,j} \equiv 0$$

VECTON: $a_i \equiv a^{m}_{im}$

$$a_i \equiv \gamma^{m}_{mi} - \Gamma^{m}_{mi} \equiv \gamma_i - \partial_i \ln \sqrt{|g|}$$

$$a_{ij} \equiv -\frac{1}{2}(a_{i,j} - a_{j,i}) \equiv -\frac{1}{2}(\gamma_{i,j} - \gamma_{j,i})$$

$\alpha\beta$ - CONNECTION

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha(\delta^i_j \, \hat{a}_k + \delta^i_k \, \hat{a}_j) - (\alpha - 2\beta) g_{jk} \, \hat{a}^i$$

Weyl: $\beta = 0$

geo-Riemannian: $\alpha = 2\beta$

Einstein $\alpha = -\beta = \frac{1}{6}$

NB: the mass of the vecton depends on these parameters

Using a simple dimensional reduction to the dimension 1+1 (similar to spherical or cylindrical reductions in the metric case) we can prove the relation between the **conjugate Lagrangians**:

$$\mathcal{L} = -\frac{1}{2}\sqrt{|\det(s + \lambda^{-1} a)|} = -2\Lambda\sqrt{|\det(\mathbf{g} + \lambda\mathbf{f})|} = \mathcal{L}^*$$

Λ having the dimension L^{-2}

Using the above definitions, → we can then write the **generalized Einstein eqs.**

$$s_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{g}^{ij}}, \qquad a_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{f}^{ij}}$$

In **dimension D** we can similarly derive the relation

$$\mathcal{L}^* \equiv \sqrt{-\det(s_{ij} + \nu a_{ij})} \sim \sqrt{-g}\,[\det(\delta_i^j + \lambda f_i^j)]^{1/(D-2)}$$

The generalized Einstein –Eddington-Weyl model in dimension D

$$\mathcal{L}_{eff} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^{1/(D-2)} + R(g) + c_a\, g^{ij} a_i a_j\right]$$

Restoring the dimensions and expanding the root term up to the second order in the vector and scalar fields

$$\mathcal{L}_{eff} \cong \sqrt{-g}\left[R[g] - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i + g^{ij}\partial_i\psi\,\partial_j\psi + m^2\psi^2\right)\right]$$

$$A_i \sim a_i,\; F_{ij} \sim f_{ij},\; \kappa \equiv G/c^4$$

NB: $\partial_i\psi$ Is proportional to F_{ij} for i < 4, j=4

WEE-type models have unusual properties that may by of interest for cosmology

$$-2e^{2\beta}\left[e^{\alpha-\gamma}(\dot{\beta}^2+2\dot{\beta}\dot{\alpha})+\Lambda\sqrt{e^{2(\alpha+\gamma)}-\lambda^2\dot{A}^2}+\frac{1}{2}\mu^2A^2e^{-\alpha+\gamma}\right]$$

In the gauge $\gamma=-\alpha$, and with notation $\alpha=\rho-2\sigma$ and $\beta=\rho+\sigma$.

$$\mathcal{L}_c=-2e^{2\beta}\left[3e^{2\alpha}(\dot{\rho}^2-\dot{\sigma}^2)+\Lambda\sqrt{1-\lambda^2\dot{A}^2}+\frac{1}{2}\mu^2A^2e^{-2\alpha}\right]$$

$$\mathcal{H}=\bar{c}\sqrt{p_A^2+M_A^2\,\bar{c}^2}+\mu^2A^2e^{2(\beta-\alpha)}+\frac{1}{24}e^{2(\beta+\alpha)}\,(p_\sigma^2-p_\rho^2)$$

Which is zero if there are no other fields. $M_A\equiv 2\lambda^2\Lambda e^{2\beta}$ $\lambda^{-1}\equiv\bar{c}$,

In a sense, the WEE vecton looks like a massive particle in a **giant gravitational accelerator**. Just for fan, let us call it **GGA**

Spherical symmetry

$$ds_4^2 = e^{2\alpha}dr^2 + e^{2\beta}d\Omega^2(\theta,\phi) - e^{2\gamma}dt^2 + 2e^{2\delta}dr\,dt$$

1+1 dimensional Lagrangian for the EW linear model (plus scalar)

$$e^{2\beta}[e^{-\alpha-\gamma}(\dot{A}_1 - A_0')^2 - e^{-\alpha+\gamma}(\psi'^2 + \mu^2 A_1^2) + e^{\alpha-\gamma}(\dot{\psi}^2 + \mu^2 A_0^2) - e^{\alpha+\gamma}(V + 2\Lambda)] + \mathcal{L}_{gr}$$

$$\mathcal{L}_{gr} \equiv e^{-\alpha+2\beta+\gamma}(2\beta'^2 + 4\beta'\gamma') - e^{\alpha+2\beta-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha}) + 2ke^{\alpha+\gamma}$$

Reduction to cosmological or static solutions

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' \; = \; \frac{1}{2}[\dot{\psi}\psi' + A_0 A_1]$$

This is one of Einstein's equations corresponding to delta-variations

Separation of variables (a sort of dimensional reduction)

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r),$$

NB: To get FRW cosmology we should take $\dot{\alpha} = \dot{\beta}\,, \quad \gamma' = 0$

Cosmological Lagrangian with scalaron

$$6\bar{k}e^{\alpha+\gamma} - e^{2\beta}[e^{\alpha+\gamma}(V + 2\Lambda) - e^{\alpha-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha} - \dot{\psi}^2)]$$

Notation

$$\rho \equiv \frac{1}{3}(\alpha + 2\beta)\,, \qquad \sigma \equiv \frac{1}{3}(\beta - \alpha)\,,$$

$$A_{\pm} = e^{-2\rho+4\sigma}(\dot{A}^2 \pm \mu^2 e^{2\gamma} A^2)\,, \qquad \bar{V} \equiv V(\psi) + 2\Lambda$$

Cosmological Lagrangian with scalaron and vecton **(k=0)**

$$e^{2\rho-\gamma}(\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2) + e^{3\rho-\gamma} A_{-} - e^{3\rho+\gamma}\,\bar{V}(\psi)$$

The energy constraint

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2 + A_{-} + e^{2\gamma}\,\bar{V} = 0$$

Equations of motion for scalaron and vecton (k=0)

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\mu^2 A = 0\,,$$

$$4\ddot{\rho} + 6\dot{\rho}^2 - 4\dot{\rho}\dot{\gamma} - 6\dot{\sigma}^2 + \frac{1}{3}A_- + \dot{\psi}^2 - e^{2\gamma}\,\bar{V} = 0\,,$$

$$\ddot{\sigma} + 3\dot{\sigma}\dot{\rho} - \dot{\sigma}\dot{\gamma} - \frac{1}{3}A_- = 0\,.$$

$$\ddot{\psi} + (3\dot{\rho} - \dot{\gamma})\dot{\psi} + \frac{1}{2}e^{2\gamma}\,\bar{V}_{\psi} = 0\,,$$

Anisotropic scalaron cosmology

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2 + e^{2\gamma}\,\bar{V} = 0\,,$$

$$4\ddot{\rho} + 6\dot{\rho}^2 - 4\dot{\rho}\dot{\gamma} - 6\dot{\sigma}^2 + \dot{\psi}^2 - e^{2\gamma}\,\bar{V} = 0\,,$$

$$\ddot{\sigma} + 3\dot{\sigma}\dot{\rho} - \dot{\sigma}\dot{\gamma} = 0\,,$$

$$\ddot{\psi} + (3\dot{\rho} - \dot{\gamma})\dot{\psi} + \frac{1}{2}e^{2\gamma}\,\bar{V}_{\psi} = 0\,.$$

$$\ddot{\sigma} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\sigma} = k\,e^{2\gamma - 2(\rho+\sigma)} + A_- \,,$$

$$\ddot{\psi} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\psi} + e^{2\gamma}\,v'(\psi)/2 = 0;$$

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\,m^2 A = 0\,,$$

The energy (Hamiltonian) constraint

$$\dot{\psi}^2 - 6\dot{\rho}^2 + 6\dot{\sigma}^2 + e^{2\gamma}\,v(\psi) + 6k\,e^{2\gamma - 2(\rho+\sigma)} + 3\,A_+ = 0\,.$$

The canonical Hamiltonian

$$\mathcal{H}_c^{\mathrm{can}} = \frac{1}{24}(6p_\psi^2 + p_\sigma^2 - p_\rho^2 + 6p_A^2\,e^{2\rho - 4\sigma})\,e^{\gamma - 3\rho} + v(\psi)\,e^{\gamma + 3\rho} + 6k\,e^{\gamma + \rho - 2\sigma} + m^2 A^2\,e^{\gamma + \rho + 4\sigma}\,.$$

`rho'-equation is omitted

1605.03948 hep-th

see also: 1506.01664

A fresh view of cosmological models describing very early Universe: general solution of the dynamical equations.

$$(\dot{\rho},\, \dot{\psi},\, \dot{\sigma}) \equiv [\,\xi(\rho)\,, \eta(\rho)\,, \zeta(\rho)\,] = [\,\xi(\rho),\, \xi\,\psi'(\rho),\, \xi\,\sigma'(\rho)\,] \equiv \xi(\rho)[1,\, \chi(\rho),\, \omega(\rho)]$$

$\chi(\rho) \equiv \eta/\xi = \psi'(\rho)$ and $\omega(\rho) \equiv \zeta/\xi = \sigma'(\rho)$ are gauge invariant

$$v(\psi) = \bar{v}[\rho(\psi)] \quad \text{for arbitrary } \bar{v}(\rho)$$

$$v'(\psi) = \frac{dv}{d\psi} = \frac{dv}{d\rho}\frac{d\rho}{d\psi} = \bar{v}'(\rho)\frac{\xi}{\eta} = \bar{v}'(\rho)/\chi(\rho)$$

Gauge invariant Anzatz for solving all equations for vanishing anisotropy

$$[\,x(\rho),\, y(\rho),\, z(\rho)\,] \equiv \exp\,(6\rho - 2\gamma)\,[\,\xi^2(\rho)\,, \eta^2(\rho),\, \zeta^2(\rho)\,]$$

$$y'(\rho) + V'(\rho) - 6V(\rho) = 0\,, \qquad V \equiv e^{6\rho}\,\bar{v}(\rho)$$

$$x'(\rho) - V(\rho) = 4k\,e^{4\rho-2\sigma}\,, \qquad z'(\rho) = 2k\,e^{4\rho-2\sigma}\,\sigma'(\rho).$$

$$6\,x(\rho) \;=\; y(\rho) + V(\rho) \;+\; 6z(\rho) \;+\; 6k\,e^{4\rho-2\sigma}\,.$$

$$y(\rho) = 6\left(C_y + \int V(\rho)\right) - V(\rho)$$

The main solution of the psi eq.

$$x(\rho) = \left(C_x + \int V(\rho)\right) + 4k\int e^{4\rho-2\sigma(\rho)}$$

$$x(\rho)\,\sigma'^2(\rho) \;\equiv\; C_x - C_y + 2k\int \sigma'(\rho)\,e^{4\rho-2\sigma(\rho)}\,.$$

The fundamental expressions for the solution with vanishing anisotropy

$$\hat{r}(\rho) \equiv \dot{\psi}^2 e^{-2\gamma} / v(\psi) \quad = \quad \chi^2 (1 + 6k\, e^{-2\rho}/\bar{v})\, [\, 6(1-\omega^2) - \chi^2]^{-1}$$

$$\hat{r}(\rho) \;=\; \frac{6\, C_y}{V(\rho)} + \frac{6}{V} \int V(\rho) \;-\; 1 = \frac{6\, C_y}{V(\rho)} + \sum_1^{\infty} (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\, \bar{v}(\rho)}$$

$$\chi^2 = 6(1-\omega^2) \left[\sum_1^{\infty} (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\, \bar{v}(\rho)} + \frac{6\, C_y}{V} \right] \times$$

$$\left[1 + \sum_1^{\infty} (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\, \bar{v}(\rho)} + \frac{6}{V} \left(k\, e^{4\rho - 2\sigma} + C_y \right) \right]^{-1}$$

All these formulas are exact,
the second is independent on anisotropy

The first terms of the exact expression for the transition function

$$\chi^2 = (1-\omega^2)\left[\left(-\bar{l}' + \mathrm{o}(\bar{l}')\right) + 36\, C_y\, \frac{e^{-6\rho}}{\bar{v}(\rho)}\right] \times$$

$$\left[\left(1 - \frac{1}{6}\,\bar{l}' + \mathrm{o}(\bar{l}')\right) + 6\,\frac{e^{-2\rho}}{\bar{v}(\rho)}\left(k\, e^{-2\sigma} + C_y\, e^{-4\rho}\right)\right]^{-1}$$

$$\chi^2 = -\bar{l}'(\rho) + \mathrm{o}(\bar{l}') = -\chi\, v'(\psi)/v(\psi) + \ldots$$

$$\chi = -v'(\psi)/v(\psi) + \ldots \equiv -l'(\psi) + \mathrm{o}(l')$$

This provides the direct relation to approximate standard formulas.
The small corrections for anisotropy are derived in asymptotic domain.

Remarks on the vecton theory:

1. The structure of the linearized theory is similar to the scalaron case but anisotropy requires additional efforts.

2. With zero anisotropy, the equations can be solved, otherwise they give a sort of very useful `sum rules'. Small anisotropy approximation can be as effective as in the scalaron case.

3. The most difficult 'large vecton momentum' case can be treated asymptotically. It is very interesting for transition from inflation to particle production processes, or else, for description of bouncing phenomena.

THE

END

FROM GEOMETRY TO DYNAMICS

REQUIREMENTS TO LAGRANGIAN DENSITIES

1. IT IS INDEPENDENT OF DIMENSIONAL CONSTANTS.

2. ITS INTEGRAL OVER SPACE-TIME IS DIMENSIONLESS.

3. IT CAN DEPEND ON TENSOR VARIABLES HAVING
a DIRECT GEOMETRIC MEANING and
a NATURAL PHYSICAL INTERPRETATION.

4. THE RESULTING GENERALIZED THEORY MUST AGREE WITH ALL ESTABLISHED EXPERIMENTAL CONSEQUENCES OF EINSTEIN'S THEORY.

r_{ij}, s_{ij}, a_{ij}, and $a_k \equiv a^i_{ik}$ satisfy requirement **3**.

Einstein's choice is $\mathcal{L} = \mathcal{L}(s_{ij}, a_{ij})$