On the interactions of Maxwell-like higher spins

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Exploring the higher-spin interaction problem

➡ *Charting the space of possible (Q)RFT & (Q)GT*

➡ *Connection to Strings (➡ hard to close the case for hsp in flat space)*

➡ *Crucial outstanding issue: higher-spin symmetry breaking*

Motivations

Having this general perspective in mind we aimed at:

Exploring alternative bases of fields, leading to
explicit, ``simple'' forms of unbroken higher-spin theories

Exploring different types of spectra, including string-like ones
(no string-like hsp theory ever built so far)

Concretely:

study Lagrangian interactions both in Minkowski and in (A)dS spaces, leading to non-linear eqs of the form

$$\Box\,\varphi^{(s_i)} - \partial\,\partial\cdot\varphi^{(s_i)} = J\,(\varphi^{(s_j)},\varphi^{(s_k)})$$

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X. Bekaert, N. Boulanger, D.F. `15

D.F., S. Lyakhovic, A. Sharapov `14

D.F., `10 - `12

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- generalise unimodular GR
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Main outcomes

- cubic vertices simpler than Fronsdal
- fully off-shell (A)dS vertices
- new role for gauge deformation in the Noether procedure
- single vertices encoding multi-particle interactions (all possible unitary spectra)

Plan

I. Higher spins & the Noether procedure

II. The cubic vertex

III. Noether deformation of the constraints

IV. Outlook

Higher spins & the Noether procedure

~

The Noether procedure I: basics

Basic idea: construct interactions perturbatively
while keeping

locality

number of independent gauge symmetries

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$$S\,[\varphi] \,=\, S_0\,[\varphi] \,+\, g\,S_1\,[\varphi] \,+\, g^2\,S_2\,[\varphi]\,\ldots\,,$$
$$\delta\,\varphi \quad=\, \delta_0\,\varphi \,+\, g\,\delta_1\,\varphi \,+\, g^2\,\delta_2\,\varphi\,\ldots\,,$$

The Noether procedure I: basics

Basic idea: construct interactions perturbatively
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$$S\,[\varphi] \;=\; S_0\,[\varphi] + g\,S_1\,[\varphi] + g^2\,S_2\,[\varphi] \ldots\,,$$
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Solving order by order for $\delta\,S[\varphi] \;=\; 0$

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$$\delta_2\,S_0\,[\varphi] \,+\, \delta_1\,S_1\,[\varphi] \,+\, \delta_0\,S_2\,[\varphi] \;=\; 0\,,$$
$$\ldots$$

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$$\ldots$$

Berends, Burgers and
van Dam, 1985

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$$\ldots$$

Berends, Burgers and van Dam, 1985

First step: building the free theory

$$\mathcal{L}_0 = \frac{1}{2}\,\varphi\,\mathcal{K}\,\varphi$$

$$\delta_0\,\varphi : \; \delta_0\,\mathcal{L}_0 = 0$$

$\sim$

Maxwell-like hsp

Any covariant free theory for massless particles must imply systems of the form

[*Fierz 1939*]

$$\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \Lambda_{\mu_2 \cdots \mu_s)}$$

$$\Box \varphi_{\mu_1 \cdots \mu_s} = 0 \,, \qquad \Box \Lambda_{\mu_1 \cdots \mu_{s-1}} = 0 \,,$$

$$\partial^{\alpha} \varphi_{\alpha \mu_2 \cdots \mu_s} = 0 \,, \qquad \partial^{\alpha} \Lambda_{\alpha \mu_2 \cdots \mu_{s-1}} = 0 \,,$$

$$\varphi^{\alpha}{}_{\alpha \mu_3 \cdots \mu_s} = 0 \,, \qquad \Lambda^{\alpha}{}_{\alpha \mu_3 \cdots \mu_{s-1}} = 0 \,,$$

single particle - spin s

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single particle - spin s

Maxwell-like hsp

More unitary options: reducible spectra

$$\delta\,\varphi_{\,\mu_1\,\cdots\,\mu_s} \;=\; \partial_{\,(\mu_1}\,\Lambda_{\,\mu_2\,\cdots\,\mu_s)}$$

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$$\varphi^{\,\alpha\beta}{}_{\,\alpha\beta\,\mu_5\,\cdots\,\mu_s} \;=\; 0\,,$$

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two particles - spin s, s-2

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$$\varphi^{\,\prime\prime\prime}{}_{\,\mu_7\,\cdots\,\mu_s} \;=\; 0\,, \qquad \Lambda^{\,\prime\prime\prime}{}_{\,\mu_7\,\cdots\,\mu_{s-1}} \;=\; 0\,,$$

three particles - spin s, s-2, s-4

Maxwell-like hsp

More unitary options: reducible spectra

$$\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \Lambda_{\mu_2 \cdots \mu_s)}$$

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no trace constraints at all

full unitary spectrum of particles - spin s, s-2, s-4, ... 1 or 0

outcome of tensionless limit of the free string

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Bengtsson, Ouvry-Stern '86 Henneaux-Teitelboim '88
D.F.-Sagnotti '02, Sagnotti-Tsulaia '03
Buchbinder-Galajinsky-Krykhtin '07
Fotopoulos-Tsulaia '08, Sorokin-Vasiliev '09,
D.F. '10, Agugliaro-Azzurli-Sorokin '16 . . .

Maxwell-like hsp

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$$\cdots \qquad \cdots$$

Possible to encompass all of these options
in one and the same Lagrangian

Maxwell-like hsp

to go off-shell...

$$\Box\,\varphi_{\,\mu_1\cdots\,\mu_s} \;=\; 0\,,$$
$$\partial^{\,\alpha}\varphi_{\,\alpha\,\mu_2\cdots\,\mu_s} \;=\; 0\,,$$
$$[\ldots]$$

$$\Box\,\Lambda_{\,\mu_1\cdots\,\mu_{s-1}} \;=\; 0\;,$$
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Maxwell-like hsp

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$$\Box\,\varphi_{\,\mu_1\,\cdots\,\mu_s} \not\equiv 0\,,$$
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consistency condition

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consistency condition

[. . .] [. . .]

$\Box\, \delta\, \varphi_{\,\mu_1 \cdots \mu_s}$ *can be compensated only by* $\delta\, \partial_{\,(\mu_1} \partial^{\,\alpha}\, \varphi_{\,\alpha\, \mu_2 \cdots \mu_s)}$

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$$M = \Box\, \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial^{\alpha} \varphi_{\alpha\, \mu_2 \cdots \mu_s)}$$

minimal building-block for any gauge theory

Maxwell-like hsp

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Free, k-th reducible Maxwell-like theory

$$\mathcal{L} = \frac{1}{2}\varphi\left(\Box - \partial\,\partial\cdot\right)\varphi$$

→

Lagrangian for all possible unitary spectra: from the single spin s to the tensionless string one

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$$\mathcal{L} = \frac{1}{2}\,\varphi\,(\Box - \partial\,\partial\cdot)\,\varphi$$

→ *Lagrangian for all possible unitary spectra: from the single spin s to the tensionless string one*

- Trace conditions enforce a projection of the corresponding eom:

$$\Box\varphi - \partial\,\partial\cdot\varphi + \frac{2\,k}{\prod_{i=1}^{k}[D+2(s-k-i)]}\,\eta^{k}\,\partial\cdot\partial\cdot\varphi^{[k-1]} = 0\,,$$

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Absence of traces in $\mathcal{L}$ crucial to our cubic vertex construction

Higher-spin gauge theories go together with constrained gauge symmetry

Maxwell-like

$$\partial \cdot \Lambda^{[k]} = 0$$

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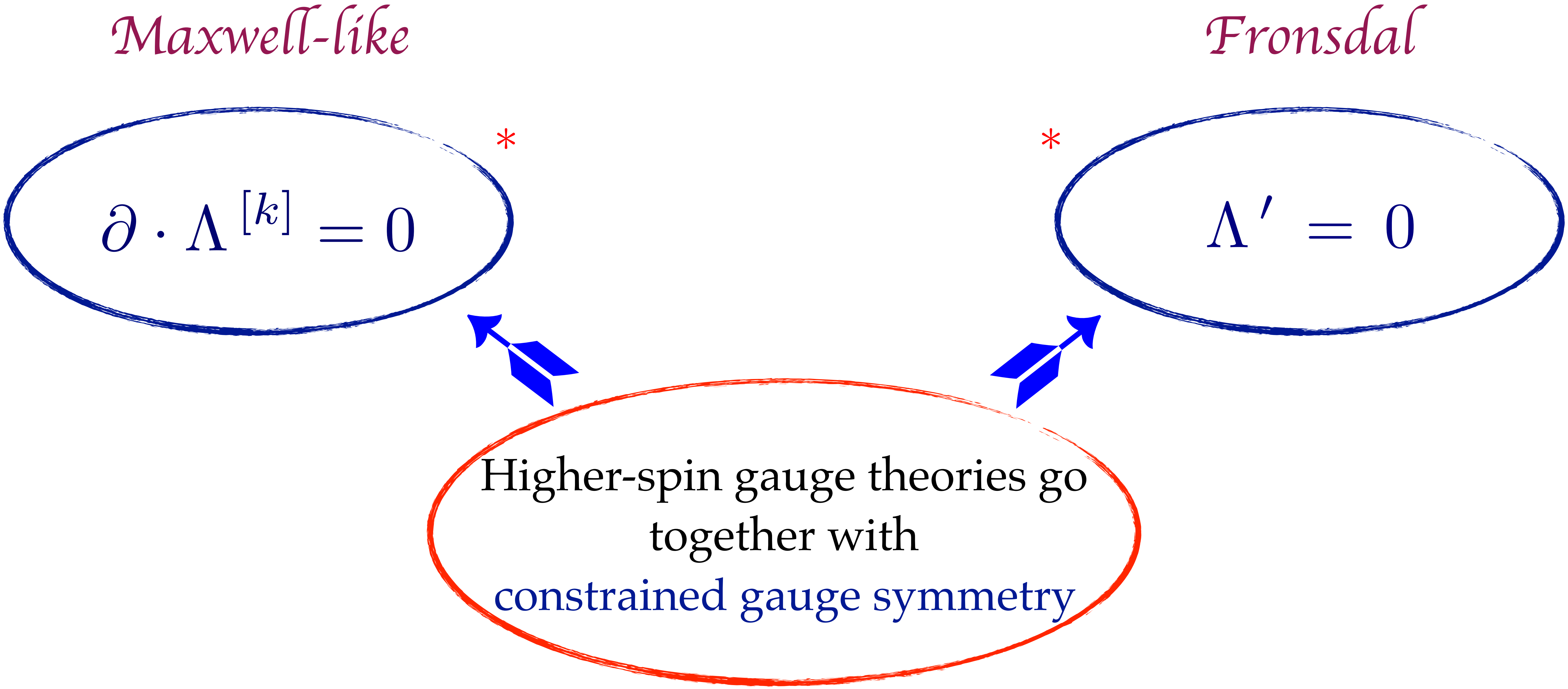
Maxwell-like

$$\partial \cdot \Lambda^{[k]} = 0$$

Fronsdal

$$\Lambda' = 0$$

Higher-spin gauge theories go together with constrained gauge symmetry



* can be removed in various ways: solving, introducing pure-gauge non-local terms, via auxiliary fields

Maxwell-like *

$$\partial \cdot \Lambda^{[k]} = 0$$

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Higher-spin gauge theories go together with constrained gauge symmetry

Assuming that we keep them,
which role do they play in the Noether procedure?

*can be removed in various ways: solving, introducing pure-gauge non-local terms, via auxiliary fields

The Noether procedure II: constrained gauge symmetry

add an equation to the system

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then, in principle at least, they might get corrected:

$$S\,[\varphi] \;=\; S_0\,[\varphi] \,+\, g\,S_1\,[\varphi] \,+\, g^2\,S_2\,[\varphi]\;\ldots\,,$$

$$\delta\,\varphi \;=\; \delta_0\,\varphi \,+\, g\,\delta_1\,\varphi \,+\, g^2\,\delta_2\,\varphi\;\ldots\,,$$

$$\mathcal{O}\Lambda \,+\, g\,\mathcal{O}_1\,(\Lambda\,,\varphi) \,+\, g^2\,\mathcal{O}_2(\Lambda\,,\varphi^{\,2}) \,+\,\ldots \,=\, 0$$

The Noether procedure II: constrained gauge symmetry

1st consequence

each $\delta_k\varphi$ admits an expansion:

$$\delta_k\,\varphi \,=\, \delta_k^{(0)}\varphi \,+\, \delta_k^{(1)}\varphi \,+\, \delta_k^{(2)}\,\varphi \,+\, \dots\,,$$

$$\delta_k^{(l)}\varphi \,:=\, o\,(\varphi^{\,k+l})\,\begin{cases} \textit{explicitly} \text{ on } \sim \varphi^{\,k}, \\ \textit{implicitly} \text{ on } \sim \varphi^{\,l}, \text{ via its } \Lambda\text{−dependence.} \end{cases}$$

The Noether procedure II: constrained gauge symmetry

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$$\delta_k \varphi = \delta_k^{(0)} \varphi + \delta_k^{(1)} \varphi + \delta_k^{(2)} \varphi + \dots ,$$

$$\delta_k^{(l)} \varphi := o\,(\varphi^{\,k+l}) \begin{cases} \textit{explicitly} \text{ on } \sim \varphi^{\,k}, \\ \textit{implicitly} \text{ on } \sim \varphi^{\,l}, \text{ via its } \Lambda\text{-dependence.} \end{cases}$$

for instance, in the Maxwell-like case:

- $\delta_0^{(0)} \varphi = \partial \Lambda \quad s.t. \quad \partial \cdot \Lambda = 0$
- $\delta_0^{(1)} \varphi = \partial \Lambda \quad s.t. \quad \partial \cdot \Lambda + g\, \mathcal{O}_1 (\Lambda, \varphi) = 0$

The Noether procedure II: constrained gauge symmetry

2nd consequence

the Noether system changes:

$$o(\epsilon,\varphi) : \delta\,\mathcal{S} = \int \frac{\delta\mathcal{L}_0}{\delta\varphi}\,\delta_0^{(0)}\varphi$$

$$o(\epsilon,\varphi^2) : \delta\,\mathcal{S} = \int \left\{ \frac{\delta\mathcal{L}_0}{\delta\varphi}\,(\delta_0^{(1)}\varphi \,+\, \delta_1^{(0)}\varphi) \,+\, \frac{\delta\mathcal{L}_1}{\delta\varphi}\,\delta_0^{(0)}\varphi \right\}$$

$$o(\epsilon,\varphi^3) : \delta\,\mathcal{S} = \int \left\{ \frac{\delta\mathcal{L}_0}{\delta\varphi}\,(\delta_0^{(2)}\varphi + \delta_1^{(1)}\varphi + \delta_2^{(0)}\varphi) + \frac{\delta\mathcal{L}_1}{\delta\varphi}(\delta_0^{(1)}\varphi + \delta_1^{(0)}\varphi) + \frac{\delta\mathcal{L}_2}{\delta\varphi}\delta_0^{(0)}\varphi \right\}$$

$$\ldots$$

The Noether procedure II: constrained gauge symmetry

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$\ldots$

thus, to first order

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$\ldots$

thus, to first order

- no differences for $\mathcal{L}_1$

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$\ldots$

thus, to first order

- no differences for $\mathcal{L}_1$
- two contributions to the deformation of the gauge transformation

The cubic vertex

The cubic vertex

General scheme:

- select three fields: $\varphi^{(s_1)}, \varphi^{(s_2)}, \varphi^{(s_3)}$
- select a total number of derivatives $n = n_1 + n_2 + n_3$
- consider first transverse-traceless fields and compute

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$$\delta\left(\mathcal{L}_{TT} + \ldots\right) \sim \Lambda\left(\quad\right) + \Box\,\Lambda\left(\quad\right) + \partial \cdot \Lambda\left(\quad\right)$$

fixes the coefficients
of the TT vertex

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$$\delta\left(\mathcal{L}_{TT} + \ldots\right) \sim \Lambda\left(\quad\right) + \Box\Lambda\left(\quad\right) + \partial\cdot\Lambda\left(\quad\right)$$

Fronsdal: $\mathcal{D} = \partial\cdot\varphi - \frac{1}{2}\partial\varphi'$ *s.t.* $\delta\mathcal{D} = \Box\Lambda$

The cubic vertex

General scheme:

- ➙ select three fields: $\varphi^{(s_1)}, \varphi^{(s_2)}, \varphi^{(s_3)}$
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$\varphi' \downarrow\,,\ \mathcal{D} \rightarrow$	0	1	2	3
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1	$\varphi'\,\varphi\,\varphi$	$\varphi'\,\mathcal{D}\,\varphi$	$\mathcal{D}\,\mathcal{D}\,\varphi'$	
2	$\varphi'\,\varphi'\,\varphi$	$\varphi'\,\varphi'\,\mathcal{D}$		
3	$\varphi'\,\varphi'\,\varphi'$			

Building blocks for Fronsdal's cubic vertices

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Building blocks for Maxwell-like cubic vertices

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Building blocks for Maxwell-like cubic vertices

Related work: Fotopoulos-Tsulaia 2010 flat-space triplet cubic vertex (fully reducible case)

Noether deformation of the constraints

$\sim$

Deformation of the constraint

The full Maxwell-like cubic vertex has the schematic form

$$\mathcal{L}_1 = \mathcal{L}_{TT} + \mathcal{L}_{1,\mathcal{D}} + \mathcal{L}_{1,\mathcal{D}\mathcal{D}} + \mathcal{L}_{1,\mathcal{D}\mathcal{D}\mathcal{D}}$$

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$$\delta\,\mathcal{L}_1 \sim \Delta_1\,M + \Delta_2\,\partial\cdot\mathcal{D}$$

expected, proportional to free eom

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$$\delta\,\mathcal{L}_1 \sim \Delta_1\,M + \Delta_2\,\partial\cdot\mathcal{D}$$

vanishes on the free eom, but not locally proportional to M

Deformation of the constraint

Consider the full variation up to cubic order:

$$\delta\{\mathcal{L}_0 + \mathcal{L}_1\} \sim \partial \cdot \Lambda \, \partial \cdot \mathcal{D} + \Delta_1 \, M + \Delta_2 \, \partial \cdot \mathcal{D}$$

$\delta \, \mathcal{L}_0$

$\delta_1^{(0)} \, \varphi$

non-locally proportional to the free eom

can be combined together to restore gauge invariance

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spin 2: Maxwell-like ➡ unimodular gravity

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$$\partial^{\,\mu} \Lambda_{\,\mu} \, = \, 0 \quad \rightarrow \quad \mathcal{D}^{\,\mu} \Lambda_{\,\mu} \, = \, 0$$

could be avoided at cubic order!

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✳ Traces can be consistently included, if wanted, thus providing the freedom to selectively rearrange part of the cross couplings.

✳ Full (A)dS extension achieved via the ambient space approach.

Twofold simplification:

→ *No traces*

→ *Flat, commuting derivatives to be then projected to the embedded (A)dS manifold*

Summary & Outlook

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- (Would-be) fully interacting theories would comprise a variety of spectra, possibly with infinitely-many particles for each spin (including scalars):
  - *different from usually considered Vasiliev's theories*
  - *different (maybe) from tensionless string spectrum*
  - *any candidate holographic dual?*