Holographic Entanglement Entropy in BCFT

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D.V. Fursaev
Dubna University & BLTP JINR

(in collaboration with S.N. Solodukhin, A.F. Astaneh, C.Berthiere)

Lebedev Institute, Moscow
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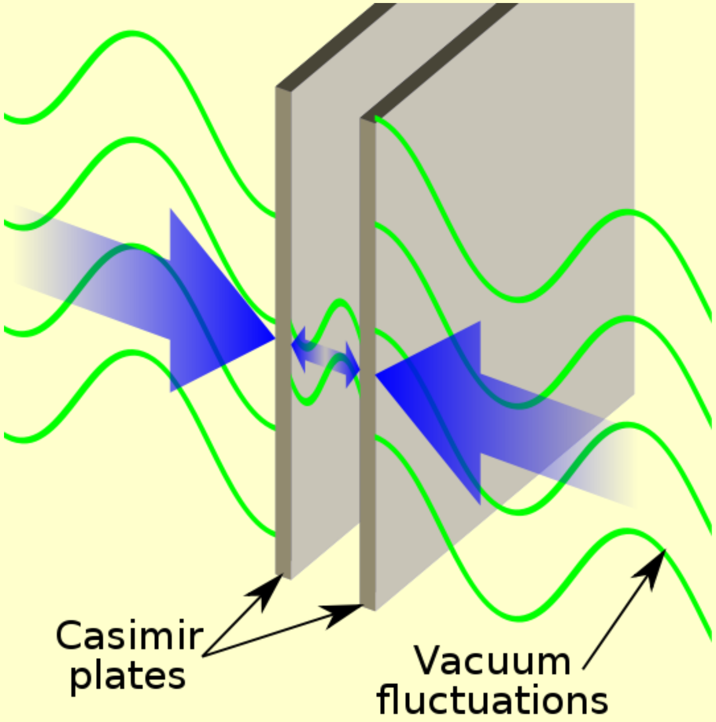
Motivations:



● Boundaries result in observable effects in QFT (the Casimir forces);

● Boundaries change single-particle spectra, we expect that the entangle-ment entropy (EE) is sensitive to the boundaries;

● EE carries a new piece of information about physics of boundaries in QFT (how states are entangled across the boundary): importance for condensed matter

We consider EE when an entangling surface crosses the boundary

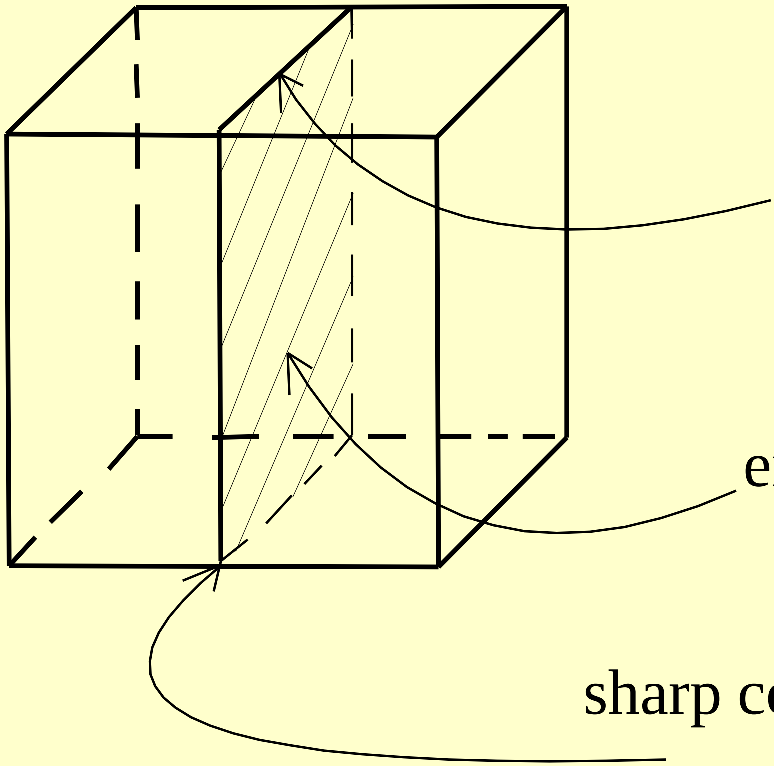
Finite size effects of EE in 2D CFT's

J. L. Cardy, "Boundary Conditions, Fusion Rules and the Verlinde Formula," Nucl. Phys. B 324, 581 (1989);

I. Affleck and A. W. W. Ludwig, "Universal non-integer 'ground state degeneracy' in critical quantum systems," Phys. Rev. Lett. 67, 161 (1991);

and other works

first studies of boundary effects in 4D QFT's



Boundary of entangling surface B,
P is its perimeter

entangling surface B of area $A(B)$

sharp corners

$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + s_{\log} \ln \varepsilon, \quad s_{\log} = C(\alpha_i), \qquad \varepsilon \text{ is UV cutoff}$$

Fursaev, PRD73, 124025 (2006)

Wilczek, Hertzberg, PRL 106, 050404 (2011)

Boundary terms appear in

$s_{\log}$ - the 'logarithmic part' of EE

$$S(B) \sim \frac{A(B)}{\varepsilon^2} + \frac{P}{\varepsilon} + s_{\log} \ln \varepsilon,$$

This may be important:

we expect that the logarithmic part of EE is related to the conformal anomaly and may have a holographic description

EE and trace anomaly in d=4:

local conformal anomaly

$$\left\langle T^{\mu}_{\mu} \right\rangle = -2aE - cI - \frac{c'}{24\pi^2}\nabla^2 R$$

$$E = \frac{1}{16\pi^2}\left(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$$ – "density" of the Euler n.

$$I = -\frac{1}{16\pi^2}C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}, \quad C_{\mu\nu\lambda\rho}$$ – the Weyl tensor

"bulk charges" a, c

a- monotonically decreases under RG flow from UV to IR

suggested by J. Crardy, PLB 215, 749-752 (1988),

proved by Z.Komargodski and A.Schwimmer, JHEP 12 (2011)099

3 invariants on a smooth entangling surface *B* in d=4 (no boundaries)

$$F_a = -\frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x\, R(B) \quad , \qquad R(B) - \text{scalar curvature of } B$$

$$F_c = \frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x\, C_{\mu\nu\lambda\rho} n_i^\mu n_j^\nu n_i^\lambda n_j^\rho \quad , \quad C_{\mu\nu\lambda\rho} - \text{Weyl tensor of } M \text{ at } B,$$

$$F_b = \frac{1}{2\pi}\int_B \sqrt{\sigma}\, d^2x \left(\frac{1}{2}\mathrm{Tr}(k_i)\mathrm{Tr}(k_i) - \mathrm{Tr}(k_i k_i) \right),$$

$(k_i)_{\mu\nu}$ – extrinsic curvatures of B, n_i – normal vectors

F_a, F_b, F_c – are invariant with respect to the Weyl

transformations $g_{\mu\nu}'(x) = e^{2\omega(x)} g_{\mu\nu}(x)$

Logarithmic term in EE in d=4

$$s_{\log} = aF_a + cF_c + bF_b$$ (no boundaries)

- Ryu,Takayanagi, JHEP 0608, 045 (2006),
- Solodukhin, PLB 665, 305 (2008)
- Fursaev, Patrushev, Solodukhin, PRD 88, 044054 (2013)

$$c = b$$ for CFT's

conformal charges in the trace anomaly of a CFT uniquely fix the logarithmic term in EE (no boundaries) !

Holographic entanglement entropy (Ryu-Takayanagi formula)

volume of a holographic surface $\tilde{B}$ in AdS

$$A(\tilde{B}) = \frac{1}{2\varepsilon^2} A(B) + \frac{\pi}{2}(F_a + F_c + F_b)\ln\frac{\mu}{\varepsilon} + ...$$

$z = \varepsilon -$ position of the boundary (a UV cutoff in CFT)

(expansion for $A(\tilde{B})$ first found by A.Schwimmer and S.Theisen, arXiv:0802.1017)

$$S(B) = \frac{A(\tilde{B})}{4G_5} \sim \frac{N^2\Lambda^2}{4\pi} A(B) + \frac{1}{4} N^2 (F_a + F_c + F_b)\ln \mu\Lambda + ...$$

use AdS/CFT dictionary: $\frac{1}{G_5} = \frac{2N^2}{\pi}, \quad \varepsilon = 1/\Lambda$

one reproduces correctly the structutre of the leading divergences and exact value

of the logarithmic part of the entropy

the rest of the talk:

We study effects of boundaries in the conformal anomaly and in the entropy of entanglement, when the entangling surface crosses the boundary

- "boundary charges" in the integrated conformal anomaly BCFT. relation between bulk and boundary charges ;

- logarithmic terms in EE for BCFT, "boundary charges" in the conformal anomaly and in EE;

- AdS/CFT description of boundary terms in the anomaly and EE.

New parameters of BCFT from the integrated conformal anomaly

If a classical theory is scale invariant :

$$g'_{\mu\nu}(x) = e^{2\sigma(x)} g_{\mu\nu}(x),$$

the trace of the stress - energy tensor is zero, $T^{\mu}_{\mu} = 0;$ classical property is

broken for quantum everages of the corresponding (renormalized) operators

$$\left\langle \hat{T}^{\mu}_{\mu} \right\rangle \neq 0 -$$ local (trace) anomaly

the property is known as the conformal or scale anomaly;

we also use the integrated anomaly

$$\mathrm{A} = \partial_{\sigma} W[e^{2\sigma} g_{\mu\nu}]_{\sigma=0} = \int_M \left\langle \hat{T}^{\mu}_{\mu} \right\rangle \sqrt{g}\, d^n x + \mathrm{b.t.}$$

of the effective action W

Boundary terms in d=4:

a general structure of the integrated anomaly in the presence of boundaries

$$\mathrm{A} = -2a\chi_4 - ci_4 + q_1 j_1 + q_2 j_2 \quad , \; i_4 = \int_M I$$

$$\chi_4 = \int_M E + \frac{1}{32\pi^2}\int_{\partial M} Q$$ – Euler characteristic of M;

$$Q = -8\left[\det K_{ab} + (\hat{R}_{ab} - \frac{1}{2} g_{ab}\hat{R})K^{ab}\right]$$

$$j_1 = \frac{1}{16\pi^2}\int_{\partial M} C_{\mu\nu\lambda\rho} n^\nu n^\rho \hat{K}^{\mu\lambda} \quad , \quad j_2 = \frac{1}{16\pi^2}\int_{\partial M} \mathrm{Tr}(\hat{K}^3)$$

$\hat{K}^{\mu\lambda}$ – traceless part of the extrinsic curvature of the boundary ∂M,

conformal structure of A has been studied first for a scalar field

with the Dirichlet boundary condition (Dowker & Schofield, 1990)

Results for boundary charges in d=4

(DF, JHEP 1512, 112 (2015))

- boundary "charges" q_k are calculated for CFT's, spins 0, 1/2, 1

- a relation between boundary q_k and bulk "charges" a, c is established

Results for d=4

CFT	a	c	q1	q2	b.cond.
Scalar	1 / 360	1 / 120	1 / 15	2 / 35	Dirichlet
Scalar	1 / 360	1 / 120	1 / 15	2 / 45	Robin
Spinor	11 / 360	1 / 20	2 / 5	2 / 7	Mixed
Maxwell	31 / 180	1 / 10	12 / 15	16 / 35	Absolute
Maxwell	31 / 180	1 / 10	12 / 15	16 / 35	Relative

- For an Abelian gauge field "charges" do not depend on the boundary conditions:

$$\vec{E}_{\parallel} = \vec{B}_{\perp} = 0 \quad \text{or} \quad \vec{E}_{\perp} = \vec{B}_{\parallel} = 0$$

Properties of boundary chargers in d=4

- $q_1 = 8c,$

- as consequence, integrated anomaly has a correct Gibbons-Hawking type

boundary term: the functional

$$c\int_M C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho} + q_1 \int_{\partial M} C_{\mu\nu\lambda\rho} n^\nu n^\rho \hat{K}^{\mu\lambda},$$

under variations has no normal derivatives of the bulk metric on the boundary

(Solodukhin, PLB 752, 131 (2016))

- Boundaries yield a single independent boundary charge q_2 (at $\int \mathrm{Tr}\,\hat{K}^3$)

- q_2 is sensitive to boundary conditions

- q_2 appears in RG equation for 3-point correlation function of the stress-energy

tensor near the boundary (Kuo-Wei Huang (2016), 1604.02138[hep-th])

Computations are based on conformal invariance of the heat coefficient

- Let the classical action be invariant

$$I[\phi, g] = \int d^d x \sqrt{g}\, \phi(x) L \phi(x)$$

under conformal transformations:

$$g_{\mu\nu}'(x) = e^{2\omega(x)} g_{\mu\nu}(x), \quad \phi'(x) = e^{k\omega(x)} \phi(x),$$

$$I[\phi, g] = I[\phi', g']$$

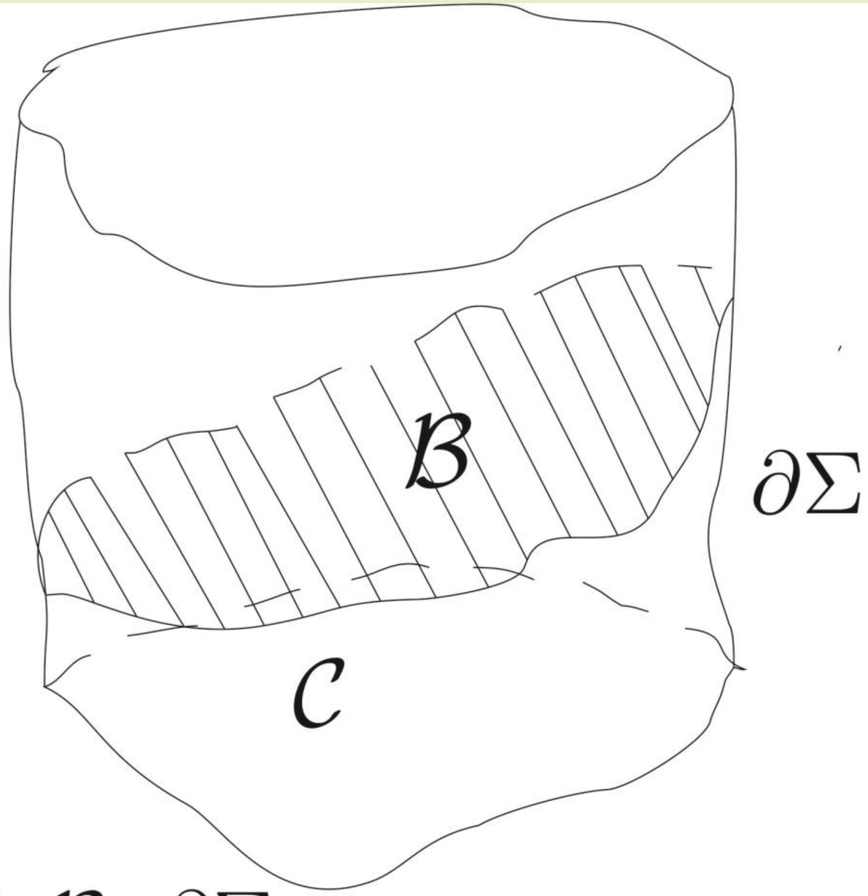
- Let boundary conditions respect the conformal invariance,

for an example: $\phi\big|_{\partial\Sigma} = 0$ (the Dirichlet condition)

Then the heat coefficient $A_{p=d}$ is a conformal invariant:

$$A_{p=d}[g] = A_{p=d}[g']$$

EE for entangling surface crossing the boundary



$\mathcal{C}=\mathcal{B}\cap\partial\Sigma$ - boundary entangling curve

Logarithmic terms in EE in CFT's (d=4)

$$s_{\log}(B) = aF_a + cF_c + bF_b + dF_d + eF_e$$

terms on $C = B \bigcap \partial M$

$$F_a = -\frac{1}{2\pi}\left(\int_B \sqrt{\sigma} d^2x\, R(B) + \int_C ds\, k\right) = -2\chi_2(B) \quad ,$$

$\chi_2(B) -$ Euler characteristics of B

$F_c, F_b -$ are not modified in the presence of boundaries

$F_d = F_d(C), \quad F_e = F_e(C) -$ terms of a new type (pure boundary effects)

$F_d, F_e -$ are dimensionless Weyl invariant (for CFT's) integrals on C

$d, e -$ are boundary coefficients in the entropy

Do d, e are related to charges in the integrated conformal anomaly?

Invariants and coefficients

$$F_d = \frac{3}{2\pi} \int_C ds\, \psi_1\, \hat{K}_{\mu\nu} u^\mu u^\nu\ , \quad u^\nu - \text{tangent vector to } C$$

$$F_e = \frac{1}{\pi} \int_C ds\, \psi_2\, (N \cdot p_i)(\hat{k}_i)_{\mu\nu} u^\mu u^\nu \quad ,$$

$(\hat{k}_i)_{\mu\nu}$ − traceless part of extrinsic curvature of B,

$\psi_1(\alpha), \psi_2(\alpha)$ − are unknown functions of α - a tilt angle of B and ∂M (between normal vector to ∂M and a normal vector to ∂M in B)

coefficient d at F_d can be calculated when B is orthogonal to ∂M ($\psi_1(0) \equiv 1$)

Fursaev, JHEP 1307, 119 (2013), Fursaev, Solodukhin, Berthiere, Astaneh, PRD (2017)

Results for d=4 (orthogonal configuration)

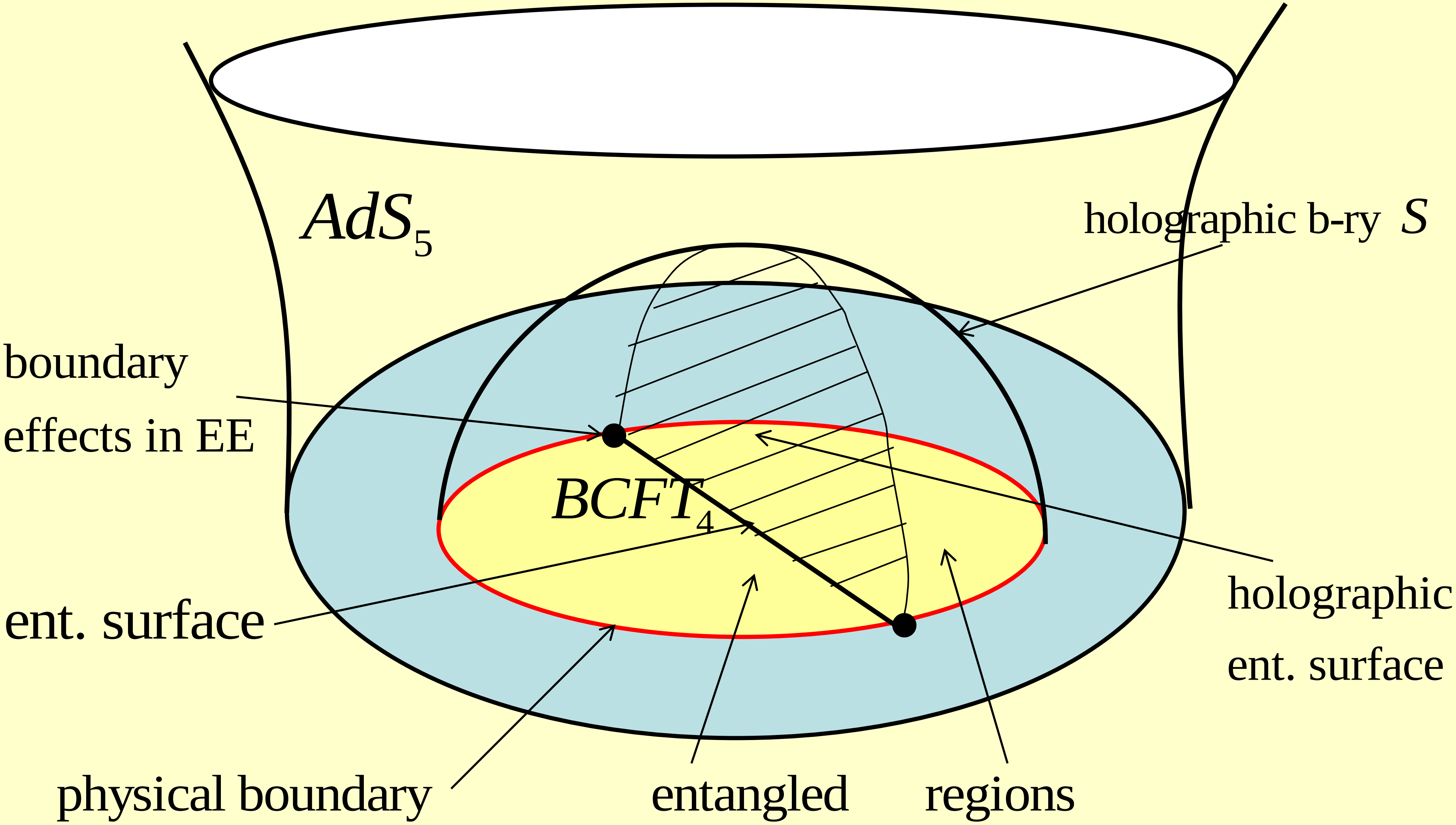
CFT	a	c	q2	d	b.cond.
Scalar	1 / 360	1 / 120	2 / 35	1/60	Dirichlet
Scalar	1 / 360	1 / 120	2 / 45	-1/90	Robin
Spinor	11 / 360	1 / 20	2 / 7	1/60	Mixed
Maxwell	31 / 180	1 / 10	16 / 35	7/60	Absolute
Maxwell	31 / 180	1 / 10	16 / 35	7/60	Relative

- For gauge fields extra arguments are needed
- A new 'magic' relation !

$$d = 3\,a - 14\,c + 35/12\;q_2$$

- d depends on boundary conditions

Holographic BCFT



BCFT in D=4:

$\mathrm{N}=4, \quad SU(N)$ super YM at weak coupling with b.c. which break 1/2 of supersymmetries

boundary effects we can calculate at a weak coupling:

- boundary terms in the integarted conformal anomaly
- boundary terms in EE

Integrated anomaly in 4D BCFT

$\mathrm{N} = 4, \quad SU(N)$ super YM at weak coupling, 1/2 of susy's are broken

$$\mathrm{A} = -2a\chi_4 - ci_4 + 8cj_1 + q_2 j_2$$

$$a = c = \frac{N^2 - 1}{4}, \quad q_2 = \frac{4}{3}(N^2 - 1)$$

see Astaneh, Solodukhin PLB 769 (2017) 25

Log-term in EE in 4D BCFT

$\mathrm{N}=4, \quad SU(N)$ super YM

$$s_{\log}=\frac{N^2-1}{8\pi}\left[\left(\int_B R_B+2\int_C k_B\right)+\int_B \mathrm{Tr}\,k_i^2-2\int_C \hat{K}_{\mu\nu}u^\mu u^\nu\right]$$

M is flat, B is orthogonal to ∂M $(\psi_1(0)\equiv 1)$, $C=\partial M\cap B$,

see Astaneh, Berthiere, Fursaev, Solodukhin, PRD (2017)

Definition of the 'holographic boundary' (HB)?:

- Takayanagi, PRL107 (2011) 101602, (restricted version – Miao, Chu, Guo): HB is determined by properties of boundary terms in gravity action

$$I_{AdS} = I_{\text{bulk}} + I_{\text{bound}}$$

$$I_{\text{bound}} = -\frac{1}{8\pi G}\int_S \left(K_S + T\right) \quad , \; T \text{- a free parameter}$$

$$HB \text{ equation} \quad K_S = -\frac{d}{d-1}T \quad , \quad \text{consistent with varaitional principle}$$

- Astaneh and Solodukhin, PLB 769 (2017) 25: HB is a kind of brane governed by Nambu-Goto eqs

$$I_{\text{bound}} = -\frac{\lambda}{8\pi G}\int_S \quad , \; \lambda \text{- is a constant}$$

$$HB \text{ equation} \quad K_S = 0 \quad , \quad \text{minimal syrface equation}$$

Prescription for the holographic EE:

$$S = \frac{A(\tilde{B})}{4G_5}$$ – Ryu-Takayanagi formula

$\tilde{B}$ – holographic surface in the bulk,

$\tilde{B}$ – is extended in AdS till the holographic boundary S

Results:

- minimal HB surface (Astaheh-Solodukhin prescription, Takayanagi, Miao et al prescription) reproduce exactly weak coupling results for the integrated anomaly and EE in 4D BCFT with ½ susy's,
  - if correct, it implies that new boundary charges in the anomaly and EE do not receive quantum corrections (same as for the bulk charges) ;

- for non-minimal HB surface (in restricted Takayanagi's prescription) boundary charges differ from charges at weak couplings:
  - the charges are not protected from ?
  - BCFT has different b.c.

GOOD NEWS: Holography seems to be able to deal with boundary effects

MORE WORK is to be done to fix prescriptions and draw conclusions

Geometric configuration

bulk metric

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}(-dt^2 + dr^2 + (\gamma_{ij} - k_{ij} r)^2 dx^i dx^j)$$

M is flat, $\partial M: \ \ r = 0,$ holographic boundary: $r = f(\rho)$

entangling surface $B: \ \ x^1 = 0$

holographic entangling surface $\tilde{B}: \ \ x^1 = f(r, \rho)$

Comments:

- computations were also done in D=3;

- boundary terms in EE for gauge fields are to further studied;

- curvature effects are important to learn the full structure of boundary terms in EE (have not been calculated so far by other methods);

- there can be other versions of Ryu-Takayanagi formula for holographic EE with boundaries

Thank you for attention