Vacuum instability in slowly varying electric fields

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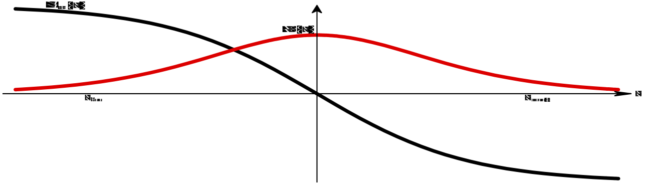
We find universal approximate representations for the vacuum means of current density and EMT components that hold true for an arbitrary strong electric field slowly varying with time. These representations do not require knowledge of corresponding solutions of the Dirac equation

Introduction

Nonperturbative methods have been well-developed for QED with the so-called t-electric potential steps (e.g., pulse electric field).



In this case a calculation technique is based on the existence of specific exact solutions (in and out solutions) of the Dirac equation.

$$i\partial_t\psi(x) = H(t)\psi(x), \quad H(t) = \gamma^0(\boldsymbol{\gamma}\mathbf{P} + m),$$
$$P_x = -i\partial_x - U(t), \quad \mathbf{P}_\perp = -i\boldsymbol{\nabla}_\perp, \quad U(t) = qA_x(t), \quad q = -e$$

However, there are only few cases when such solutions are known.

Introduction

We demonstrate that for electric fields slowly varying with time there exist physically reasonable approximations that maintain the nonperturbative character of QED calculations even in the absence of the exact solutions. Defining the slowly varying regime in general terms, we can observe a universal character of vacuum effects caused by a strong electric field for
(1) the total density of created pairs,

$$
\begin{aligned}
n^{\mathrm{cr}} &= \frac{J_{(d)}}{(2\pi)^{d-1}} \int d\mathbf{p} N_n^{\mathrm{cr}}, \quad J_{(d)} = 2^{[d/2]-1} \text{ for fermions} \\
N_n^{\mathrm{cr}} &= \left\langle 0, \mathrm{in} \middle| a_n^{\dagger}(\mathrm{out}) a_n(\mathrm{out}) \middle| 0, \mathrm{in} \right\rangle = \left| \left({}_-\psi_n, {}^+\psi_n \right) \right|^2
\end{aligned}
$$

$$
{}^{\zeta}\psi_n(x) = g\left({}_+|^{\zeta}\right) {}_+\psi_n(x) + g\left({}_-|^{\zeta}\right) {}_-\psi_n(x)
$$

$$
a_n(\mathrm{out}) = g({}^+|_+) a_n(\mathrm{in}) + g({}^+|_-) b_n^{\dagger}(\mathrm{in})
$$

Introduction

(2) for the vacuum mean values of the current density and energy-momentum tensor,

$$\langle j^{\mu}(t)\rangle = \langle 0,\mathrm{in}|j^{\mu}|0,\mathrm{in}\rangle, \quad \langle T_{\mu\nu}(t)\rangle = \langle 0,\mathrm{in}|T_{\mu\nu}|0,\mathrm{in}\rangle .$$

$$j^{\mu} = \frac{q}{2}\left[\overline{\hat{\Psi}}(x), \gamma^{\mu}\hat{\Psi}(x)\right], \quad T_{\mu\nu} = \frac{1}{2}\left(T^{can}_{\mu\nu} + T^{can}_{\nu\mu}\right),$$

$$T^{can}_{\mu\nu} = \frac{1}{4}\left\{[\overline{\hat{\Psi}}(x), \gamma_{\mu}P_{\nu}\hat{\Psi}(x)] + [P^{*}_{\nu}\overline{\hat{\Psi}}(x), \gamma_{\mu}\hat{\Psi}(x)]\right\},$$

$$P_{\mu} = i\partial_{\mu} - qA_{\mu}(x), \; \overline{\hat{\Psi}}(x) = \hat{\Psi}^{\dagger}(x)\gamma^{0}.$$

$$\hat{\Psi}(x) = \sum_{n}\left[a_n(\mathrm{in})\; {}_{+}\psi_n(x) + b^{\dagger}_n(\mathrm{in})\; {}_{-}\psi_n(x)\right]$$

Introduction

If the electric field is weak, $N_n^{\mathrm{cr}} \ll 1$, the probability of the vacuum to remain a vacuum has simple representation,

$$P_v = |c_v|^2 \approx 1 - 2\mathrm{Im}S \approx 1 - N^{\mathrm{cr}}, \quad N^{\mathrm{cr}} = \sum_n N_n^{\mathrm{cr}}.$$

$$c_v = \langle 0, \mathrm{out}|0, \mathrm{in}\rangle = e^{iS}, \quad S \textit{ is Schwinger's EA}$$

The latter relations are often used in semiclassical calculations to find N_n^{cr} and $n^{\mathrm{cr}} = N^{\mathrm{cr}}/V_{(d-1)}$.
$(-)$ When the electric field is strong, $N_n^{\mathrm{cr}} \to 1$, the sum N^{cr} cannot be considered as a small quantity.
$(+)$ **the large parameter:** For slowly varying strong electric fields

$$n^{\mathrm{cr}} \sim \Delta t/\Delta t_{\mathrm{st}}^{\mathrm{m}}, \quad \Delta t/\Delta t_{\mathrm{st}}^{\mathrm{m}} \gg 1,$$

$$\Delta t_{\mathrm{st}}^{\mathrm{m}} = \Delta t_{\mathrm{st}} \max\left\{1, m^2/e\overline{E(t)}\right\}, \quad \Delta t_{\mathrm{st}} = \left[e\overline{E(t)}\right]^{-1/2}$$

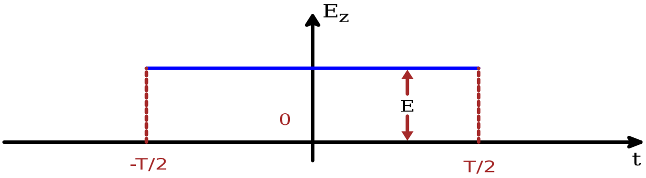
We can take into account only leading terms $\sim \Delta t/\Delta t_{\mathrm{st}}^{\mathrm{m}}$, whereas oscillation terms are disregarded.

T-constant electric field

$$N_n^{\mathrm{cr}} \approx N_n^0 = e^{-\pi\lambda}, \quad \lambda = \frac{\pi_\perp^2}{eE}, \quad \pi_\perp = \sqrt{\mathbf{p}_\perp^2 + m^2}$$

is quasiconstant over the wide range of the longitudinal momentum p_x for any given λ. Pair creation effects in such fields are proportional to large increments of the longitudinal kinetic momentum, $\Delta U = e\left|A_x(+\infty) - A_x(-\infty)\right|$.

T-const. EF turns on to E at $-T/2 = t_{in}$ and turns off to 0 at $T/2 = t_{out}$.



$A_1(t) = -Et$, $t \in [t_{in}, t_{out}]$, being constant for $t \in (-\infty, t_{in})$ and $t \in (t_{out}, +\infty)$.

Total density of created pairs

The particle energy is primarily determined by an increment of the longitudinal kinetic momentum in

$$\begin{aligned} D(t) &: \langle P_x(t)\rangle < 0, \; |\langle P_x(t)\rangle| \gg \pi_\perp, \\ \langle P_x(t)\rangle &= \langle P_x(t_{\rm in})\rangle - [U(t) - U(t_{\rm in})] \end{aligned}$$

$$D(t) \subset D(t') \subset D(t_{\rm out}) \text{ if } t < t' < t_{\rm out}$$

The leading contribution due to the T-constant field

$$\begin{aligned} n^{\rm cr} &\approx \tilde{n}^{\rm cr} = \frac{J_{(d)}}{(2\pi)^{d-1}} \int_{eEt_{\rm in}}^{eE(t_{\rm in}+\Delta t)} dp_x \int_{\sqrt{\lambda}<K_\perp} d\mathbf{p}_\perp e^{-\pi\lambda}, \\ \sqrt{eE}\Delta t &\gg K_\perp^2 \gg \max\{1, m^2/eE\}, \\ \Delta t &= t - t_{\rm in} \end{aligned}$$

Total density of created pairs

We call $E(t)$ a slowly varying electric field on a time interval Δt if

$$\left|\frac{\overline{\dot{E}(t)}\Delta t}{\overline{E(t)}}\right| \ll 1, \quad \left[e\overline{E(t)}\right]^{1/2}\Delta t \gg \max\left\{1, m^2/e\overline{E(t)}\right\},$$

In the general case it is enough to consider a finite interval $\left(t_{\rm in}^{eff}, t_{\rm out}^{eff}\right]$. We divide this interval into M intervals $\Delta t_i = t_{i+1} - t_i > 0$ and represent

$$\tilde{n}^{\rm cr} = \sum_{i=1}^{M} \Delta\tilde{n}_i^{\rm cr}, \; \sum_{i=1}^{M} \Delta t_i = t_{\rm out}^{eff} - t_{\rm in}^{eff},$$

$$\Delta\tilde{n}_i^{\rm cr} \approx \frac{J_{(d)}}{(2\pi)^{d-1}} \int_{e\overline{E}_i t_i}^{e\overline{E}_i (t_i+\Delta t_i)} dp_x \int_{\sqrt{\lambda_i}<K_\perp} d\mathbf{p}_\perp N_n^{(i)},$$

$$N_n^{(i)} = e^{-\pi\lambda_i}, \quad \lambda_i = \frac{\pi_\perp^2}{e\overline{E}_i},$$

Total density of created pairs

Representing the variable p_x as

$$p_x = U(t), \; U(t) = \int_{t_{\rm in}}^{t} dt' eE(t') + U(t_{\rm in})$$

we find the universal form in the leading-term approximation for a slowly varying, but otherwise arbitrary strong electric field

$$\tilde{n}^{\rm cr} \approx \frac{J_{(d)}}{(2\pi)^{d-1}} \int_{t_{\rm in}}^{t_{\rm out}} dt eE(t) \int d\mathbf{p}_\perp N_n^{\rm uni}, \quad N_n^{\rm uni} = e^{-\frac{\pi\pi_\perp^2}{eE(t)}}$$

After the integration over $\mathbf{p}_\perp$,

$$\tilde{n}^{\rm cr} = \frac{J_{(d)}}{(2\pi)^{d-1}} \int_{t_{\rm in}}^{t_{\rm out}} dt \left[eE(t)\right]^{d/2} e^{-\frac{\pi m^2}{eE(t)}}$$

The vacuum-to-vacuum transition probability

Using the identity
$-\kappa \ln\left(1-\kappa N_n^{\mathrm{uni}}\right) = N_n^{\mathrm{uni}} + (-1)^{(1-\kappa)/2}\left(N_n^{\mathrm{uni}}\right)^2 \ldots$, in the same manner one can derive an universal form of the vacuum-to-vacuum transition probability

$$P_v \approx \exp\left\{-\frac{V_{(d-1)}J_{(d)}}{(2\pi)^{d-1}}\sum_{l=0}^{\infty}\int_{t_{\mathrm{in}}}^{t_{\mathrm{out}}} dt\,\frac{(-1)^{(1-\kappa)l/2}\left[eE(t)\right]^{d/2}}{(l+1)^{d/2}}e^{-\frac{\pi(l+1)m^2}{eE(t)}}\right\}$$

$J_{(d)}$ is the number of the spin degrees of freedom;
for fermions $\kappa = +1$ and bosons $\kappa = -1$.
In fact, it is the possibility to adopt a locally constant field approximation which makes an effect universal.
These representations do not require knowledge of the corresponding solutions of the Dirac equation.

Total density of created pairs. Examples

One obtains precisely expressions that are found when directly adopting the approximation to the exactly solvable cases:

$$
\begin{aligned}
\text{(i) } \Delta U_{\mathrm{T}} &= eE_0 T \text{ for } T\text{-const. field;;} \\
\text{(ii) } E(t) &= E_0 \cosh^{-2}(t/T_{\mathrm{S}}), \ \Delta U_{\mathrm{S}} = 2eE_0 T_{\mathrm{S}} \text{ for Sauter-like field} \\
\text{(iii) } E(t) &= E_0 \left[\Theta(-t) e^{k_1 t} + \Theta(t) e^{-k_2 t}\right], \\
\Delta U_{\mathrm{p}} &= eE_0 \left(k_1^{-1} + k_2^{-1}\right) \text{ for a peak field.}
\end{aligned}
$$

$$
\begin{aligned}
\text{(i) } \tilde{n}^{\mathrm{cr}} &= r^{\mathrm{cr}} \frac{\Delta U_{\mathrm{T}}}{eE_0}; \\
\text{(ii) } \tilde{n}^{\mathrm{cr}} &= r^{\mathrm{cr}} \frac{\Delta U_{\mathrm{S}}}{2eE_0} \int_0^{\infty} \frac{dt t^{-1/2}}{(t+1)^{(d+1)/2}} e^{-\frac{\pi t m^2}{eE_0}}; \\
\text{(iii) } \tilde{n}^{\mathrm{cr}} &= r^{\mathrm{cr}} \frac{\Delta U_{\mathrm{p}}}{eE_0} \int_1^{\infty} \frac{ds}{s^{d/2+1}} e^{-\frac{\pi(s-1)m^2}{eE_0}}
\end{aligned}
$$

Total density of created pairs. Examples

The obtained universal forms have specially simple forms:
(1) for a weak electric field ($m^2/eE_0 \gg 1$),
$[eE(t)]^{d/2} \approx [eE_0]^{d/2}$;
(2) for a strong enough electric field ($m^2/eE_0 \ll 1$):

$$\begin{aligned}
\text{(i) } \tilde{n}^{\mathrm{cr}} &= r^{\mathrm{cr}} \frac{\Delta U_{\mathrm{T}}}{eE_0}; \\
\text{(ii) } \tilde{n}^{\mathrm{cr}} &\approx r^{\mathrm{cr}} \frac{\Delta U_{\mathrm{S}}}{2eE_0} \frac{\sqrt{\pi}\,\Gamma(d/2)}{\Gamma(d/2+1/2)}; \\
\text{(iii) } \tilde{n}^{\mathrm{cr}} &= r^{\mathrm{cr}} \frac{\Delta U_{\mathrm{p}}}{eE_0} \frac{2}{d},
\end{aligned}$$

$$r^{\mathrm{cr}} = \frac{J_{(d)} (eE_0)^{d/2}}{(2\pi)^{d-1}} e^{-\frac{\pi m^2}{eE_0}}$$

Total density of created pairs. Examples

New: the case of a strong Gauss pulse,

$$E(t) = E_0 \exp\left[-\left(t/T_G\right)^2\right], \quad T_G \to \infty$$

We do not have an exact solution and known semiclassical approximations are not applicable. However, we find

$$\tilde{n}^{\mathrm{cr}} \approx \frac{J_{(d)}\left(eE_0\right)^{d/2} T_G}{d(2\pi)^{d-2}}, \quad P_v \approx \exp\left[-V_{(d-1)}\tilde{n}^{\mathrm{cr}} \sum_{l=1}^{\infty} l^{-d/2}\right]$$

The vacuum-to-vacuum transition probability

$$P_v \approx \exp\left\{-\frac{V_{(d-1)}J_{(d)}}{(2\pi)^{d-1}}\sum_{l=0}^{\infty}\int_{t_{\rm in}}^{t_{\rm out}} dt \frac{(-1)^{(1-\kappa)l/2}\left[eE(t)\right]^{d/2}}{(l+1)^{d/2}} e^{-\frac{\pi(l+1)m^2}{eE(t)}}\right\}$$

coincides with the leading term approximation of derivative expansion results from field-theoretic calculations for $d=3$ and $d=4$ [G. Dunne and T. Hall, Phys. Rev. D 58, 105022 (1998); V. P. Gusynin and I. A. Shovkovy, J. Math. Phys. 40, 5406 (1999)],

$$P_v = \exp\left(-2\mathrm{Im}S^{(0)}\right).$$

In this approximation, P_v was derived from a formal expansion in increasing numbers of derivatives of the background field:

$$S = S^{(0)}[F_{\mu\nu}] + S^{(2)}[F_{\mu\nu}, \partial_\mu F_{\nu\rho}] + \dots$$

where $S^{(0)}$ involves no derivatives of the background field.

Time evolution of vacuum instability

The mean values

$$\langle j^{\mu}(t)\rangle = \mathrm{Re}\,\langle j^{\mu}(t)\rangle^{c} + \mathrm{Re}\,\langle j^{\mu}(t)\rangle^{p}\,,$$
$$\langle j^{\mu}(t)\rangle^{c,p} = iq\,\mathrm{tr}\left[\gamma^{\mu}S^{c,p}(x,x')\right]\Big|_{x=x'}\;;$$
$$\langle T_{\mu\nu}(t)\rangle = \mathrm{Re}\,\langle T_{\mu\nu}(t)\rangle^{c} + \mathrm{Re}\,\langle T_{\mu\nu}(t)\rangle^{p}\,,$$
$$\langle T_{\mu\nu}(t)\rangle^{c,p} = i\,\mathrm{tr}\left[A_{\mu\nu}S^{c,p}(x,x')\right]\Big|_{x=x'}\,,$$
$$A_{\mu\nu} = 1/4\left[\gamma_{\mu}\left(P_{\nu}+P_{\nu}'^{*}\right)+\gamma_{\nu}\left(P_{\mu}+P_{\mu}'^{*}\right)\right]$$

Mean values and probability amplitudes are calculated with the help of different kinds of propagators:

$$\begin{aligned} S^{c}(x,x') &= i\langle 0,\mathrm{out}|\,\hat{T}\hat{\Psi}(x)\,\hat{\Psi}^{\dagger}(x')\,\gamma^{0}|0,\mathrm{in}\rangle c_{v}^{-1}\,, \\ S^{c}_{\mathrm{in}}(x,x') &= i\langle 0,\mathrm{in}|\,\hat{T}\hat{\Psi}(x)\,\hat{\Psi}^{\dagger}(x')\,\gamma^{0}|0,\mathrm{in}\rangle \end{aligned}$$

Time evolution of vacuum instability

As a consequence of

$$D\left(t\right):\left\langle P_{x}\left(t\right)\right\rangle <0,\ \ \left|\left\langle P_{x}\left(t\right)\right\rangle \right|\gg\pi_{\perp},$$

we have a large parameter $\left\langle P_{x}\left(t\right)\right\rangle$ and

$$i\partial_{t}\ {}^{\pm}\varphi_{n}\left(t\right)\approx\pm\left|\left\langle P_{x}\left(t\right)\right\rangle \right|\ {}^{\pm}\varphi_{n}\left(t\right)$$

Leading contribution to the function

$$S^{p}(x,x')=S_{\mathrm{in}}^{c}(x,x')-S^{c}(x,x')$$

can be represented by

$$S^{p}(x,x')\approx-i\sum_{n}N_{n}^{\mathrm{cr}}\left[{}^{+}\psi_{n}(x){}^{+}\bar{\psi}_{n}(x')-{}^{-}\psi_{n}(x){}^{-}\bar{\psi}_{n}(x')\right].$$

The dominant contribution to the r.h.s. of S^{p} is from a subrange $D\left(t\right)\subset D\left(t_{\mathrm{out}}\right)$.

Time evolution of vacuum instability

Using the universal form of the differential numbers of created pairs, $N_n^{\mathrm{cr}} \approx N_n^{\mathrm{uni}}$, and performing the integration over $p_\perp$, we find the following universal behavior for any large difference $t - t_{\mathrm{in}}$:

$$\begin{aligned}
\langle j^1(t)\rangle^p &\approx \quad 2e\tilde{n}^{\mathrm{cr}}(t)\,,\ \langle T_{00}(t)\rangle^p \approx \langle T_{11}(t)\rangle^p,\\
\langle T_{00}(t)\rangle^p &\approx \frac{J_{(d)}}{(2\pi)^{d-1}}\int_{t_{\mathrm{in}}}^{t} dt'\left[U(t)-U(t')\right]\left[eE(t')\right]^{d/2} e^{-\frac{\pi m^2}{eE(t')}},\\
\langle T_{ll}(t)\rangle^p &\approx \frac{J_{(d)}}{(2\pi)^{d}}\int_{t_{\mathrm{in}}}^{t} \frac{dt'\left[eE(t')\right]^{d/2+1}}{\left[U(t)-U(t')\right]} e^{-\frac{\pi m^2}{eE(t')}},\ l = 2,\ldots,D\,.
\end{aligned}$$

For $t > t_{\mathrm{out}}$, the pair production stops, vacuum polarization effects disappear, and

$$\langle j^1(t)\rangle\big|_{t>t_{\mathrm{out}}} \approx \langle j^1(t_{\mathrm{out}})\rangle^p,\ \ \langle T_{\mu\mu}(t)\rangle\big|_{t>t_{\mathrm{out}}} \approx \langle T_{\mu\mu}(t_{\mathrm{out}})\rangle^p.$$

$\langle j^1(t_{\mathrm{out}})\rangle^p$ and $\langle T_{\mu\mu}(t_{\mathrm{out}})\rangle^p$ are the features of real pairs created from the vacuum.

Time evolution of vacuum instability. Examples

For fields admitting exactly solvable cases:
(i) the T-constant field

$$\langle T_{00}(t_{\rm out})\rangle^{p} \approx \langle T_{11}(t_{\rm out})\rangle^{p} \approx eE_0 r^{\rm cr}\left(t_{\rm out}-t_{\rm in}\right)^2 ,$$
$$\langle T_{ll}(t_{\rm out})\rangle^{p} \approx \pi^{-1} r^{\rm cr} \ln\left[\sqrt{eE_0}\left(t_{\rm out}-t_{\rm in}\right)\right], \quad l=2,\ldots,D.$$

(II) For Sauter-like field, $E(t) = E_0 \cosh^{-2}(t/T_S)$,

$$\langle T_{00}(t_{\rm out})\rangle^{p} \approx \langle T_{11}(t_{\rm out})\rangle^{p} \approx eEr^{\rm cr} T_S^2\left[\delta - G\left(\frac{d}{2},\frac{\pi m^2}{eE_0}\right)\right],$$
$$\langle T_{ll}(t_{\rm out})\rangle^{p} \approx \frac{r^{\rm cr}}{2\pi}\left[\sqrt{\pi}\Psi\left(\frac{1}{2},2-\frac{d}{2};\frac{\pi m^2}{eE_0}\right) + G\left(\frac{d}{2}-1,\frac{\pi m^2}{eE_0}\right)\right].$$

$$\delta = \int_0^\infty \frac{dt}{t^{1/2}(t+1)^{(d+1)/2}} e^{-\frac{\pi t m^2}{eE_0}}$$

Time evolution of vacuum instability. Examples

(III) For the peak field, $E(t) = E_0\left[\Theta(-t)\, e^{k_1 t} + \Theta(t)\, e^{-k_2 t}\right]$,

$$\langle T_{00}(t_{\mathrm{out}})\rangle^p \approx \langle T_{11}(t_{\mathrm{out}})\rangle^p \approx eE_0 r^{\mathrm{cr}}\left[k_2^{-1} + k_1^{-1}\right]$$
$$\times\left\{\left[k_2^{-1} - k_1^{-1}\right] G\left(\frac{d}{2}+1, \frac{\pi m^2}{eE_0}\right) + k_1^{-1} G\left(\frac{d}{2}, \frac{\pi m^2}{eE_0}\right)\right\},$$
$$\langle T_{ll}(t_{\mathrm{out}})\rangle^p \approx \frac{r^{\mathrm{cr}}}{2\pi}\left[G\left(\frac{d}{2}-1, \frac{\pi m^2}{eE_0}\right) + \frac{k_2}{k_1} G\left(\frac{d}{2}, \frac{\pi m^2}{eE_0}\right)\right].$$

$$G(\alpha, x) = \int_1^\infty \frac{ds}{s^{\alpha+1}} e^{-x(s-1)}$$

One obtains precisely expressions that are found when directly adopting the approximation to the exactly solvable cases. It is an independent confirmation of universal form.

Example. T-constant electric field in graphene

In case $\Delta t_{st}^{g} < \Delta t < \Delta t_B$, ($\Delta t_{st}^{g}$ - non-linear regime, effects of particle-creation reach their asymptotic values; Δt_B - continuous Dirac model)

$$\Delta t_{st}^{g} = \left(e\left|E\right| v_F / \hbar\right)^{-1/2} \gg t_\gamma \simeq 0.24\mathrm{fs}, \quad \Delta t_B = 2\pi\hbar\left(e\left|E\right| a\right)^{-1},$$
$$L_x \sim 1\mu\mathrm{m}, \quad \Delta t \sim T_{bal} \sim 10^{-12}\mathrm{s}, \quad V = EL_x > 7\times 10^{-4}\,\mathrm{V},$$

Δt_B is the Bloch time, $a \approx 0.142$ *nm* is the carbon-carbon distance. [Gavrilov, Gitman, Yokomizo, PRD 86 (2012)].

Example. One-loop mean current in graphene

Multiplied by a degeneracy factor of four, describe, respectively:

$$n_g^{cr} = r_g^{cr} T\,, \quad r_g^{cr} = \pi^{-2}\left(v_F \hbar^3\right)^{-1/2} |eE|^{3/2}\;;$$
$$\langle j^1(t)\rangle_g = \mathrm{sgn}(E) A \Delta t\,, \quad A = 2 e v_F r_g^{cr}\,,$$
$$\Delta t = t - t_{in}.$$

These results hold true for all t that satisfy **the stabilization condition** $\Delta t \gg \Delta t_{st}^{g}$.
$\langle j^1(t)\rangle_g \sim |E|^{3/2}$ **is a key formula in the study of the conductivity in the graphene at low carrier density beyond the linear response in dc.** It describes **the mean electric current of coherent carriers** produced by the applied electric field.

Vacuum polarization

The results [S. P. Gavrilov and D. M. Gitman, Phys. Rev. D 78, 045017 (2008); S. P. Gavrilov, D. M. Gitman, and N. Yokomizo, Phys. Rev. D **86**, 125022 (2012)] can be generalized to the case of arbitrary slowly varying electric field:
in the leading-term approximation

$$\mathrm{Re}\langle T_{00}(t)\rangle^{c} = -\mathrm{Re}\langle T_{11}(t)\rangle^{c} = E\left(t\right)\frac{\partial \mathrm{Re}\mathcal{L}\left[E\left(t\right)\right]}{\partial E\left(t\right)} - \mathrm{Re}\mathcal{L}\left[E\left(t\right)\right],$$

$$\mathrm{Re}\langle T_{ll}(t)\rangle^{c} = \mathrm{Re}\mathcal{L}\left[E\left(t\right)\right], \; l = 2, ..., D,$$

$$\mathcal{L}\left[E\left(t\right)\right] = 2^{[d/2]-2}\int_0^\infty \frac{ds}{s}\cosh\left[eE\left(t\right)s\right]\tilde{f}^{(0)}(x,x,s),$$

$$\tilde{f}^{(0)}(x,x,s) = -\frac{eE\left(t\right)s^{-d/2+1}\exp\left(-im^2 s\right)}{(4\pi i)^{d/2}\sinh\left[eE\left(t\right)s\right]}.$$

Note that $\mathcal{L}\left[E\left(t\right)\right]$ evolves in time due to the time dependence of the field $E\left(t\right)$. $\mathrm{Re}\langle T_{\mu\mu}(t)\rangle^{c}$ have been regularized and renormalized using $\mathrm{Re}\mathcal{L}\left[E\left(t\right)\right] \to \mathrm{Re}\mathcal{L}_{ren}\left[E\left(t\right)\right]$.

Vacuum polarization

In the strong-field case ($m^2/eE(t) \ll 1$), the leading contributions are

$$\mathrm{Re}\,\langle T_{\mu\mu}(t)\rangle^{c}_{ren} \sim \begin{cases} [eE(t)]^{d/2}, & d \neq 4k \\ [eE(t)]^{d/2} \ln\left[eE(t)/M^2\right], & d = 4k \end{cases} .$$

The final form of the vacuum mean components of the EMT are

$$\langle T_{\mu\mu}(t)\rangle_{ren} = \mathrm{Re}\,\langle T_{\mu\mu}(t)\rangle^{c}_{ren} + \mathrm{Re}\,\langle T_{\mu\mu}(t)\rangle^{p}$$

For $t < t_{in}$ and $t > t_{out}$ the electric field is absent such that $\mathrm{Re}\langle T_{\mu\mu}(t)\rangle^{c}_{ren} = 0$.
$\mathcal{L}_{ren}[E(t)]$ coincide with leading term approximation of derivative expansion results [G. Dunne and T. Hall, Phys. Rev. D 58, 105022 (1998); V. P. Gusynin and I. A. Shovkovy, J. Math. Phys. 40, 5406 (1999)]:

$$S^{(0)}[F_{\mu\nu}] = \int dx \mathcal{L}_{ren}[E(t)]$$

It is proof that $S^{(0)}$ is given exactly by the semiclassical WKB limit for an arbitrary strong electric fields slowly varying with time.

The main new results obtained

- Defining the slowly varying regime in general terms, we can observe the existence of universal forms for the time evolution of vacuum effects caused by strong electric field. Such universality appears when the duration of the external field is sufficiently large in comparison to the scale $\Delta t_{\rm st} = \left[e\overline{E(t)} \right]^{-1/2}$, that is, there are time intervals, inside of which the electric field potential can be approximated by a potential of a constant electric field.

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- We find universal approximate representations for the total density of created pairs and vacuum means of current density and EMT components that hold true for an arbitrary t-electric potential step slowly varying with time.

The main new results obtained

- The universality under the question is associated with the big state density that is a large parameter in the slowly varying regime. In fact, we explicitly show that the pair creation can be treated as a phase transition from the initial vacuum to a plasma of electron-positron pairs.

The main new results obtained

- The universality under the question is associated with the big state density that is a large parameter in the slowly varying regime. In fact, we explicitly show that the pair creation can be treated as a phase transition from the initial vacuum to a plasma of electron-positron pairs.
- We establish relations of these representations with leading term approximations of derivative expansion results. These results allow one to formulate some semiclassical approximations that are not restricted by smallness of differential mean numbers of created pairs.

The end