Local Current Interactions from Nonlinear Higher-Spin Equations

O.A.Gelfond

NIISI & Lebedev Institute , Moscow

Ginzburg 2017, Moscow

Main results

Reconstruction of local current interactions in the gauge sector from nonlinear HS equations

Resulting gauge invariant interactions reproduce the two types of $4d$ cubic vertices found by Metsaev (2005)

OG, M. Vasiliev, to appear

AdS_4 background connections

In two-component spinor notations flat $sp(4)$ connection

$w = (\omega^L{}_{\alpha\beta}, \overline{\omega}^L{}_{\dot\alpha\dot\beta}, h_{\alpha\dot\beta})$:

Lorentz connection $\omega^L{}_{\alpha\beta}$, $\overline{\omega}^L{}_{\dot\alpha\dot\beta}$ + vierbein $h_{\alpha\dot\beta}$ $\qquad \alpha = 1,2\,, \quad \dot\alpha = 1,2$

$H^{\alpha\beta} = H^{\beta\alpha} := h^{\alpha\dot\alpha}h^{\beta}{}_{\dot\alpha}\,, \qquad \overline{H}^{\dot\alpha\dot\beta} = \overline{H}^{\dot\beta\dot\alpha} := h^{\alpha\dot\alpha}h_{\alpha}{}^{\dot\beta}$ the basis 2-forms

Space time coordinates $x^{\alpha\dot\beta} = x^{\underline{n}}\sigma_{\underline{n}}^{\alpha\dot\beta}$, σ - Hermitian 2×2 matrices

Spinorial (twistor) variables

$Y^A = (y^\alpha, \bar{y}^{\dot\alpha}),\ Z^A = (z^\alpha, \bar{z}^{\dot\alpha})$

Fields of the theory: $B(Z;Y;K|x) = C(Y;K|x) + Z(\ldots)$ and

$W(Z;Y;K|x) = \omega(Y;K|x)\,dx + Z(\ldots) + dZ(\ldots)$, $\qquad K = (k, \bar{k})$

Spin s dynamical field $C(Y;K|x) = C^{1,0}(Y|x)k + C^{0,1}(Y|x)\bar{k}$

$C^{kj}(y,\bar{y}|x) = \frac{1}{2i}\sum_{|m-n|=2s}\frac{1}{m!n!}C^{kj}{}_{\alpha_1\ldots\alpha_n,\dot\beta_1\ldots\dot\beta_m}(x)y^{\alpha_1}\ldots y^{\alpha_n}\bar{y}^{\dot\beta_1}\ldots\bar{y}^{\dot\beta_m}$

Spin s dynamical field ω (for $s\geq 1$) even degree in $k, \bar{k}$

$\omega(y,\bar{y};K|x) = \frac{1}{2i}\sum_{n+m=2(s-1)}\frac{1}{m!n!}\omega_{\alpha_1\ldots\alpha_n,\dot\beta_1\ldots\dot\beta_m}(K|x)y^{\alpha_1}\ldots y^{\alpha_n}\bar{y}^{\dot\beta_1}\ldots\bar{y}^{\dot\beta_m}$

Nonlinear higher-spin equations in AdS_4

$$\mathrm{d}W + W * \wedge W = -i\theta_\alpha \wedge \theta^\alpha (1 + \eta B * \kappa * k) - i\bar{\theta}_{\dot\alpha} \wedge \bar{\theta}^{\dot\alpha} \left(1 + \bar\eta B * \bar\kappa * \bar k\right)$$

$$\mathrm{d}B + W * B - B * W = 0$$

$\mathrm{d} = dx^{\underline{m}} \frac{\partial}{\partial x^{\underline{m}}}$ **space-time de Rham differential**, $\theta = dz$, $\bar\theta = d\bar z$

$$(f * g)(Z;Y) = \int d^4U d^4V f(Z+U; Y+U)\, g(Z-V; Y+V)\, \mathrm{e}^{iU_A V^A},$$

$U_A V^A = U^A V^B \epsilon_{AB} = u^\alpha v^\beta \epsilon_{\alpha\beta} + \bar u^{\dot\alpha} \bar v^{\dot\beta} \epsilon_{\dot\alpha\dot\beta}$

$sp(4)$**-invariant symplectic form** $\epsilon_{AB} = (\epsilon_{\alpha\beta}, \epsilon_{\dot\alpha\dot\beta})$. $f * 1 = f$

κ **and** $\bar\kappa$ **-inner Klein operators**

$$\kappa := \exp(iz_\alpha y^\alpha), \qquad \kappa * \kappa = 1, \qquad \kappa * f(z^\alpha; y^\alpha; dz^\alpha) = f(-z^\alpha; -y^\alpha; dz^\alpha) * \kappa$$

k **and** $\bar k$ **-outer Klein operators**

$$k * k = 1, \qquad k * f(z^\alpha; y^\alpha; dz^\alpha) = f(-z^\alpha; -y^\alpha; -dz^\alpha) * k\,.$$

Central on-shell theorem

Free unfolded equations Vasiliev (1989)

$$\begin{cases} D^{ad}\omega(y,\bar{y}|x) = i\Big(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\ \bar{C}(0,\overline{y}|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0|x)\Big) \\ D^{tw}C(y,\bar{y}|x) = 0 \end{cases}$$

$$D^{ad}\omega(y,\bar{y}|x) := D^{L}\omega(y,\bar{y}|x) + \lambda h^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial\overline{y}^{\dot\beta}} + \frac{\partial}{\partial y^{\alpha}}\overline{y}_{\dot\beta}\Big)\omega(y,\bar{y}|x)\,,$$

$$D^{tw}C(y,\bar{y}|x) := D^{L}C(y,\bar{y}|x) - i\lambda h^{\alpha\dot\beta}\Big(y_\alpha\overline{y}_{\dot\beta} - \frac{\partial^2}{\partial y^{\alpha}\partial\overline{y}^{\dot\beta}}\Big)C(y,\bar{y}|x)\,,$$

$$D^{L}f(y,\bar{y}|x) := \mathsf{d}f(y,\bar{y}|x) + \Big(\omega^{L\,\alpha\beta}y_\alpha\frac{\partial}{\partial y^{\beta}} + \overline{\omega}^{L\,\dot\alpha\dot\beta}\overline{y}_{\dot\alpha}\frac{\partial}{\partial\overline{y}^{\dot\beta}}\Big)f(y,\bar{y}|x)\,.$$

η **and** $\bar\eta$ **complex conjugated free parameters**

$\lambda^{-1} = \rho$ **radius of** AdS_4

Current equations and current deformations

Rank-two unfolded equations in AdS_4 **= current equations**

$$D_{cur}{}^{tw}\,\mathcal{J}(y,\bar{y}|x) = 0$$

OG, Vasiliev (2003)

$$D^{tw}_{cur} = D^L + \lambda e^{\alpha\dot{\beta}}\Big(y^1{}_\alpha\,\bar{y}^1{}_{\dot{\beta}} - y^2{}_\alpha\,\bar{y}^2{}_{\dot{\beta}} - \frac{\partial^2}{\partial y^{1\alpha}\partial\bar{y}^{1\dot{\beta}}} + \frac{\partial^2}{\partial y^{2\alpha}\partial\bar{y}^{2\dot{\beta}}}\Big)$$

Solved by $\mathcal{J}(y_1,y_2,\bar{y}_1,\bar{y}_2|K;x) = C_1(y_1,\bar{y}_1|K;x)C_2(y_2,\bar{y}_2|K;x)$

In the unfolded dynamics approach current interactions result from a nontrivial mixing between fields of ranks one and two

Schematically for the flat connection $D = \mathrm{d} + w$

$$\begin{cases} D\omega + L(C,\overline{C},w) = 0 \\ DC = 0 \\ D_2\mathcal{J} = 0 \end{cases} \Rightarrow \begin{cases} D\omega + L(C,\overline{C},w) + \Gamma_{cur}(w,\mathcal{J}) = 0 \\ DC + \mathcal{H}_{cur}(w,\mathcal{J}) = 0 \\ D_2\mathcal{J} = 0 \end{cases}$$

$\Gamma_{cur}(w,\mathcal{J})$ **and** $\mathcal{H}_{cur}(w,\mathcal{J})$ **glue rank-one and rank-two modules**

Gauge invariant current interactions

Let $\mathbf{s}$ **- spin of** C,

s_1 **and** s_2 **spins of the constituent fields of** $\mathcal{J} \sim C_1 C_2$.

ω **-dependent terms can be non-zero at** $\mathbf{s} < s_1 + s_2$.

$\Rightarrow$ ω**-independent current interactions are in the region**

$$\mathbf{s} \geq s_1 + s_2 \,.$$

Quadratic corrections from nonlinear equations

Quadratic corrections in the zero-form sector

$$D^{tw}C + [\omega\,, C]_* + \mathcal{H}_\eta(w, \mathcal{J}) + \mathcal{H}_{\bar\eta}(w, \mathcal{J}) \quad = 0$$

$\mathcal{H}_\eta$ contains arbitrary degrees of $\quad \partial_{1\alpha}\partial_2{}^\alpha\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha} \quad \Rightarrow \quad$ non-local

Modulo field redefinition $C =: C + \Phi_\eta(\mathcal{J}) + \bar\Phi_{\bar\eta}(\mathcal{J})$ M. Vasiliev (2015)

$$\widetilde{\mathcal{H}}_\eta(w, \mathcal{J}) \;=\; \mathcal{H}_\eta(w, \mathcal{J}) + D^{tw}\Phi_\eta(\mathcal{J})$$

$$\Phi_\eta(\mathcal{J}) \;=\; \frac{1}{2}\eta\int\frac{dSdT}{2\pi^4}\exp iS_A T^A \int d\tau_i \prod_{i=1}^{3}\theta(\tau_i)\delta'\Big(1-\sum_{i=1}^{3}\tau_i\Big)$$
$$\mathcal{J}(\tau_3 s + \tau_1 y, t - \tau_2 y; \bar y + \bar s, \bar y + \bar t; K) * k$$

$\widetilde{\mathcal{H}}_\eta(w, \mathcal{J})$ is local, cc is analogous

Quadratic corrections in the one-form sector

$$\mathcal{D}_{ad}\omega + [\omega\,,\omega]_* - L(C) = \Gamma(w,\mathcal{J}) + Q(C,\omega),$$

$$\Gamma(w,\mathcal{J}) = \Gamma_{\eta\eta}(w,\mathcal{J}) + \Gamma_{\bar\eta\bar\eta}(w,\mathcal{J})\ , \qquad \Gamma_{\eta\bar\eta}(w,\mathcal{J}) = 0$$

$$\Gamma_{\eta\eta} = \ \mathcal{D}_{ad}\Psi \quad -\frac{i\eta^2}{8}\bar H^{\dot\alpha\,\dot\beta}\int_0^1 d\tau \int dS\, dT \int \exp(is_\alpha t^\alpha + i\bar s_{\dot\gamma}\bar t^{\dot\gamma})$$
$$(\bar t - \bar s)_{\dot\alpha}(\bar t - \bar s)_{\dot\beta}\mathcal{J}(-\tau s,\, t,\, \bar y + \bar s,\, \bar y + \bar t)$$

for some Ψ

OG, Vasiliev (2016)

In different form deformations in one-form sector were obtained by Boulanger, Kessel, Skvortsov and Taronna (2015)

η^2,$\bar{\eta}^2$-independence

Field redefinition $\omega \to \omega - \Psi$

$$\widetilde{\Gamma}_{\eta\eta}(\mathcal{J}) =: \Gamma_{\eta\eta} - \mathcal{D}_{ad}\Psi$$

cancels

$$i\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\overline{y}^{\dot{\alpha}}\partial\overline{y}^{\dot{\beta}}}\,\Phi_\eta(\mathcal{J})(0,\overline{y}|x)$$

resulting from the field redefinition in the zero-form sector via Central on-shell Theorem. **cc is analogous**

η^2,$\bar{\eta}^2$**-independence is in accordance with the result obtained for lower-spin currents from analysis in the zero-form sector**

Is in agreement with the old conjecture on self-dual HS theory

$\eta\bar\eta$- dependence

Resulting quadratic correction in the one-form sector $\sim \eta\bar\eta$:

$$\Gamma_{\eta\bar\eta} =: i\bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\Phi_\eta(\mathcal{J})(y,0|x) + i\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial \bar y^{\dot\alpha}\partial \bar y^{\dot\beta}}\bar\Phi_{\bar\eta}(\mathcal{J})(0,\bar y|x)$$

from the field redefinition in the zero-form sector

As a result

$$\Gamma_{\eta\bar\eta} = -\frac{i}{8}\eta\bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\int dS dT \exp i[s_\beta t^\beta + \bar s_{\dot\beta}\bar t^{\dot\beta}]\int d\tau_i \prod_{i=1}^{3}\theta(\tau_i)$$
$$\delta'(\Big(1-\sum_{i=1}^{3}\tau_i\Big))\mathcal{J}(\tau_3 s + \tau_1 y, t - \tau_2 y; \bar\tau_3 \bar s + \bar\tau_1 \bar y, \bar t - \bar\tau_2 \bar y; K)\Big|_{y=0} + cc$$

Nonlocal deformation should be shifted to a local one modulo exact forms

Nonlocal $\to$ Local

$$X(\mathcal{J}) \;=\; \frac{i}{8}\eta\bar\eta\int d^3\tau d^3\bar\tau\,\Upsilon\,\delta(1-\tau_3-\tau_2)\delta(1-\bar\tau_3-\bar\tau_2)\delta'(1-\tau_1-\bar\tau_1)\,h(\partial,\bar\partial)\frac{(1-\tau_3\bar\tau_3)}{\tau_2\bar\tau_2}$$
$$\exp i\Big(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha}+\bar\tau_3\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}\Big)\mathcal{J}(\tau_2\tau_1 y,-\tau_2\bar\tau_1 y,\bar\tau_2\bar\tau_1\bar y,-\bar\tau_2\tau_1\bar y;K|x)\,,$$
$$\Upsilon=\prod_{i=1,2,3}\theta(\tau_i)\bar\theta(\tau_i)$$

$$\mathcal{D}_{ad}X(\mathcal{J})=\Gamma_{\eta\bar\eta}-\Gamma^{loc}_{\eta\bar\eta}(\mathcal{J})$$

with

$$\Gamma^{loc}_{\eta\bar\eta}(\mathcal{J}) \;=\frac{i}{8}\eta\bar\eta\int dSdT\exp iS_AT^A\int d^3\bar\tau d^3\tau\,\theta(\tau_3)\theta(\bar\tau_3)$$

$$\Big\{\theta(\bar\tau_1)\theta(\bar\tau_2)\delta(x)\delta'(\bar x)\delta(\tau_1)\delta(\tau_2)\bar H^{\dot\alpha\dot\beta}\bar\partial_{\dot\alpha}\bar\partial_{\dot\beta}+\theta(\tau_1)\theta(\tau_2)\delta'(x)\delta(\bar x)\delta(\bar\tau_1)\delta(\bar\tau_2)H^{\alpha\beta}\partial_\alpha\partial_\beta\Big\}$$

$$\mathcal{J}(\tau_3 s+\tau_1 y;t-\tau_2 y,\bar\tau_3\bar s+\bar\tau_1\bar y;\bar t-\bar\tau_2\bar y;K)$$

Contains torsion, does not admit flat limit $\Rightarrow$ gauge invariant part should be shifted to a canonical one modulo local exact forms

Local gauge invariant → Canonical

Decomposing

$$\Gamma^{loc}_{\eta\bar{\eta}} = \Gamma^{\geq} + \Gamma^{<}$$

$$\Gamma^{\geq}(\mathcal{J}) = \Gamma^{loc}_{\eta\bar{\eta}}(\mathcal{J})\Big|_{\mathrm{s}\geq s_1+s_2}, \qquad \Gamma^{<}(\mathcal{J}) = \Gamma^{loc}_{\eta\bar{\eta}}(\mathcal{J})\Big|_{\mathrm{s}<s_1+s_2}$$

There exists local zero-form $\Lambda(f_-, f_+, \mathcal{J})$ such that Vasiliev, OG (2017)

$$\mathcal{D}_{ad}\Lambda = \Gamma^{can} - \Gamma^{\geq} + B$$

where B does not contribute to the dynamical equations if $\mathrm{s} > 1$

reproduce our results of 2010 in the one-form sector

$f_+ = y^{1\nu}y^2{}_\nu - \frac{\partial^2}{\partial\bar{y}^{1\dot{\nu}}\partial\bar{y}^2{}_{\dot{\nu}}}$, $f_- = \overline{f_+}$, generate Howe dual algebra acting on solutions of current equations, *i.e.*, on a space of conserved currents

Current contribution to dynamical equations from the nonlinear HS equations

For integer spin $s \geq 2$

$$D^L \omega_{\mathbf{s}-1\,,\mathbf{s}-1} = h(\partial, \bar{y})\omega_{\mathbf{s}\,,\mathbf{s}-2} + h(y, \bar{\partial})\omega_{\mathbf{s}-2\,,\mathbf{s}}$$

$$D^L_{\alpha\dot{\gamma}} \omega_{\mathbf{s}-2\,,\mathbf{s}\alpha}{}^{\dot{\gamma}} = -\bar{y}_{\dot{\beta}} \partial_\alpha \omega_{\mathbf{s}-1\,,\mathbf{s}-1\alpha}{}^{\dot{\beta}} - y_\alpha \bar{\partial}_{\dot{\beta}} \omega_{\mathbf{s}-3\,,\mathbf{s}+1\alpha}{}^{\dot{\beta}} + \partial_\alpha \partial_\alpha \widetilde{\mathcal{J}}_{\mathbf{s},\mathbf{s}}$$

$$\mathcal{A}_{n,m}(y, \bar{y}) =: \mathcal{A}_{\alpha(n),\dot{\alpha}(m)} (y^\alpha)^n \left(\bar{y}^{\dot{\alpha}}\right)^m$$

$$\widetilde{\mathcal{J}}_{\mathbf{s},\mathbf{s}} = i\eta\bar{\eta}\frac{(\mathbf{s}-2)!}{8(2\mathbf{s})!} \sum_{k,m\in[0,\mathbf{s}]} \frac{(m+k)!(2\mathbf{s}-m-k)!}{(\mathbf{s}-k)!k!(\mathbf{s}-m)!m!} \left(\mathcal{N}_1\right)^m \left(-\mathcal{N}_2\right)^{\mathbf{s}-m} \left(-\overline{\mathcal{N}}_2\right)^k \left(\overline{\mathcal{N}}_1\right)^{\mathbf{s}-k}$$

$$\Big\{ \sum_{0\leq\mathbf{n}\leq\mathbf{s}} \frac{1}{(\mathbf{s}+\mathbf{n}-1)!} (i\partial_{1\gamma}\partial_2{}^\gamma)^{\mathbf{n}} \sum_{j,l=0,1} C^{j,1-j}(Y^1|x)k^j\bar{k}^{1-j} C^{l,1-l}(Y^2|x)k^l\bar{k}^{1-l}$$

$$+ \sum_{0<\mathbf{n}\leq\mathbf{s}} \frac{1}{(\mathbf{s}+\mathbf{n}-1)!} (i\bar{\partial}_{1\dot{\gamma}}\bar{\partial}_2{}^{\dot{\gamma}})^{\mathbf{n}} \sum_{j,l=0,1} C^{j,1-j}(Y^1|x)k^j\bar{k}^{1-j} C^{l,1-l}(Y^2|x)k^l\bar{k}^{1-l} \Big\}\Big|_{Y^j=0}$$

$$\mathcal{N}_j = y^\alpha \partial_{j\alpha}\,, \qquad \overline{\mathcal{N}}_j = \bar{y}^{\dot{\alpha}} \bar{\partial}_{j\dot{\alpha}}$$

Although $\widetilde{\mathcal{J}}_{\mathbf{s},\mathbf{s}} \sim \eta\bar{\eta}$ the current contribution to Fronsdal equtions depends on the phase of η, because by Central on-shell theorem.

Number of derivatives

Let helicities of constituent fields be h_1 and h_2.
The number of space-time derivatives in the respective vertices by virtue of unfolded equations is

$$\sharp\partial_x = \mathrm{s} + |h_1 + h_2|$$

Maximal number of derivatives $\sharp\partial_x = \mathrm{s} + s_1 + s_2$ if $h_1 h_2 > 0$

Minimal number of derivatives $\sharp\partial_x = \mathrm{s} + s_1 + s_2 - 2min(s_1, s_2)$ if $h_1 h_2 < 0$

just reproducing the results of Metsaev of 2005 since $\mathrm{s} \geq s_1 + s_2$

Conclusion

Modulo field redefinitions quadratic corrections in nonlinear equations in the one-form sector are independent of η^2 and $\bar{\eta}^2$

Canonical quadratic corrections do not contribute to torsion-like terms

Our result reproduces all types of vertices with fixed coefficients.

All improvements are removed, allowing a flat limit.