Spatial structure of the modified Coulomb potential in a superstrong magnetic field

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Influence of magnetic field

- $a_B \sim a_H$: $\quad B_1 = \dfrac{m^2 e^3 c}{\hbar^3} \approx 2.4 \times 10^9$ G

  $$U(r) = -\frac{e^2}{r} + \dots$$

- $\hbar\omega_B \sim mc^2$: $\quad B_2 = \dfrac{m^2 c^3}{\hbar e} \approx 4.4 \times 10^{13}$ G

  $$U(r) = -\frac{e^2}{r} + \dots$$

- $B_3 = \dfrac{m^2 c^4}{e^3} \approx 6 \times 10^{15}$ G

  $U(r)$ changes

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- $\hbar\omega_B \sim mc^2$: $\quad B_2 = \dfrac{m^2 c^3}{\hbar e} \approx 4.4 \times 10^{13}$ G $\qquad = B_0$

  Schwinger field

  $$U(r) = -\frac{e^2}{r} + \ldots$$

- $B_3 = \dfrac{m^2 c^4}{e^3} \approx 6 \times 10^{15}$ G $\qquad$ – superstrong magnetic field

  $U(r)$ changes $\qquad$ $U(r) = ?$

The modification of Coulomb potential

Coulomb potential is modified due to the enchancement of the vacuum polarization at one loop:

$$\Phi\left(\rho, z\right) = 4\pi e \int \frac{d^2 k_\perp d k_\parallel}{(2\pi)^3} \frac{e^{-i\vec{k}_\perp \vec{\rho}} e^{-i k_\parallel z}}{k_\parallel^2 + k_\perp^2 - \Pi^{(2)}\left(k_\perp, k_\parallel\right)}$$

Polarization operator:

$$\Pi^{(2)}\left(k_\perp, k_\parallel\right) = -\frac{2e^3 B}{\pi} \exp\left(-\frac{k_\perp^2}{2eB}\right) T\left(t\right),$$

$$T(t) = 1 - \frac{1}{\sqrt{t(1+t)}} \log\left(\sqrt{1+t} + \sqrt{t}\right), \quad t \equiv k_\parallel^2 / 4m^2.$$

- One–loop calculation
- Lowest Landau Level

$\hbar = c = 1$
$e^2 = \alpha = 1/137.0..$

Numerical evaluation

Coulomb

$$\Delta\Phi(\rho, z) \equiv \boxed{\frac{e}{\sqrt{\rho^2 + z^2}}} - \Phi(\rho, z) =$$

$$= \frac{e}{\pi} \int_{-\infty}^{\infty} dk_{\parallel} e^{-ik_{\parallel} z} \int_{0}^{\infty} dk_{\perp} k_{\perp} J_0(k_{\perp}\rho) \times$$

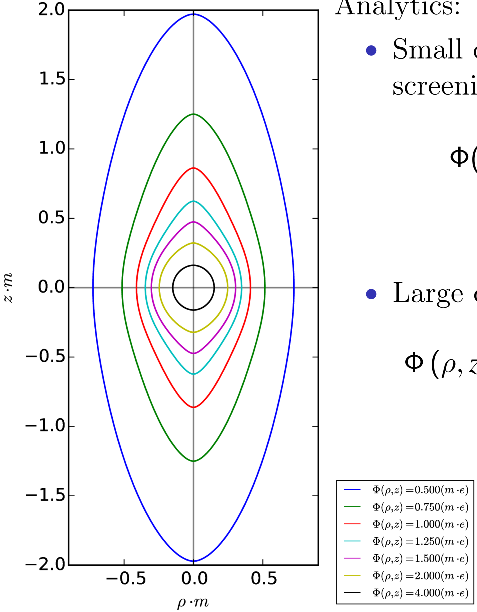
$$\times \frac{\frac{2e^3 B}{\pi} e^{-k_{\perp}^2/2eB} T(k_{\parallel}^2/4m)}{\left(k_{\perp}^2 + k_{\parallel}^2\right)\left(k_{\perp}^2 + k_{\parallel}^2 + \frac{2e^3 B}{\pi} e^{-k_{\perp}^2/2eB} T(k_{\parallel}^2/4m)\right)}.$$

Two–step integration:

1. Integration over $k_{\perp}$: Gnu Scientific Library (GSL)
2. Integration over $k_{\parallel}$: Fast Fourier Transformation (FFTW)

The absolute error for any (ρ, z) is $< 10^{-6}(m \cdot e)$.

Potential structure for $B = 10^4 B_0$



Analytics:

- Small distances: Yukawa screening

$$\Phi(\rho, z) = \frac{e}{r} \cdot e^{-r\sqrt{2e^3B/\pi}},$$

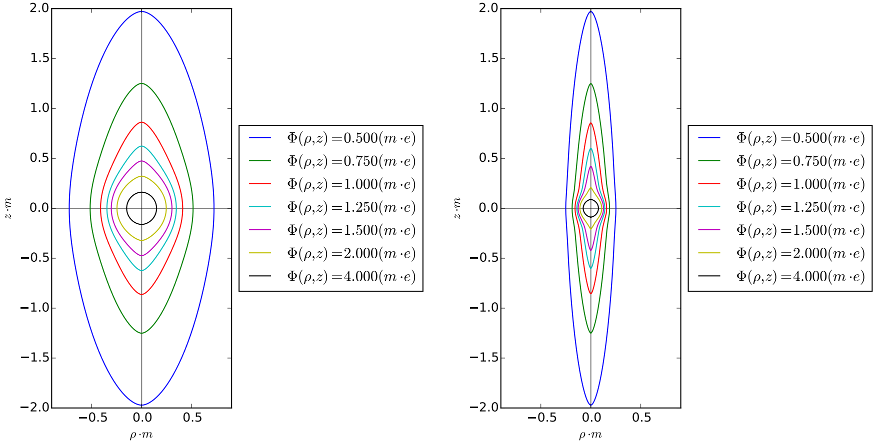
$$r = \sqrt{\rho^2 + z^2}$$

- Large distances: ellipses

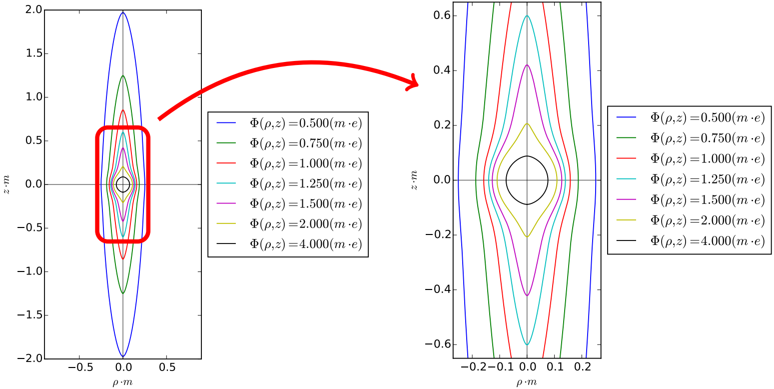
$$\Phi(\rho, z) = \frac{e}{\sqrt{z^2 + \rho^2\left(1 + \frac{e^3B}{3\pi m^2}\right)}}$$

Shabad, Usov (2007), (2008)
Vysotsky (2010)
Machet, Vysotsky (2011)
Godunov, Machet, Vysotsky (2012)

Potential structure: $B = 10^4 B_0$ vs $B = 10^5 B_0$



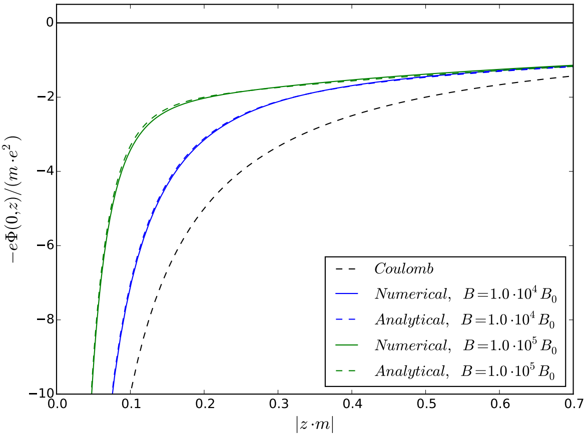
Internal structure for $B = 10^5 B_0$



Equipotential lines for $1/\sqrt{e^3 B} \lesssim z \lesssim 1/m$ are eye–shaped rather than ellipses

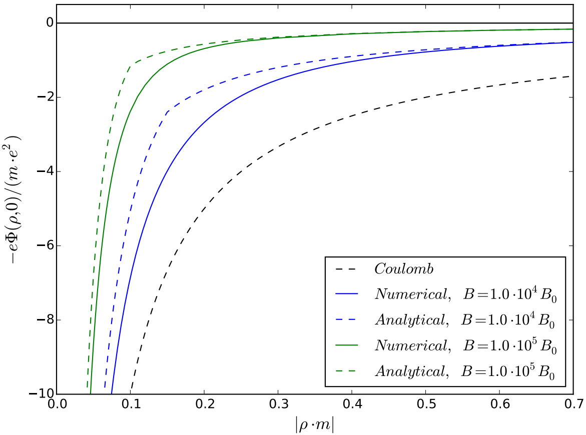
Potential for $(0, z)$

$$\Phi(0,z) = \frac{e}{|z|}\left(1 - e^{-|z|\sqrt{6m^2}} + e^{-|z|\sqrt{(2/\pi)e^3 B + 6m^2}}\right)$$



Potential for $(\rho, 0)$

$$\Phi(\rho, 0) = \begin{cases} \dfrac{e}{\rho} \cdot \exp\left(-\rho\sqrt{(2/\pi)e^3 B}\right), & \rho < l_0, \\ \dfrac{e}{\rho} \cdot \sqrt{\dfrac{3\pi m^2}{e^3 B}}, & \rho > l_0, \end{cases}$$



Possible applications

- Extreme magnetic fields in nature, e.g. for magnetars $B \sim 10^{15}$ G

- Condenced matter physics: $B_3 = \dfrac{m_{\rm eff}^2}{e_{\rm eff}^3}$.

- Critical charge problem

  Oraevskii et al. (1977), Godunov et al. (2012), ...

Conclusions

- Superstrong magnetic field $B > m^2/e^3$ modifies the Coulomb potential (it becomes screened)

- We numerically calculated the modified potential in all space with high precision

- The simulations showed a new feature: at mid–range distances $1/\sqrt{e^3 B} \lesssim z \lesssim 1/m$ equipotential lines are eye–shaped

- Such a feature may be important for some problems, e.g. with spatially distributed charges (a direction for further study)

Thank you!