Asymptotic Symmetries in Gravity: the Hamiltonian Approach

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One approach to the study of asymptotic symmetries is based on the Hamiltonian formulation, where the evolution of the system is studied on spacelike hypersurfaces.

We shall review the Hamiltonian formulation here, in the specific case of gravity.

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where n is normal to the spacelike hypersurfaces and $\frac{\partial}{\partial x^k}$ form a basis of tangent vectors.

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and "surface term" $\equiv Q[\xi] = \oint_{\partial\Sigma}$ (local expression) .

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There are matter contributions to $\mathscr{H}$ and $\mathscr{H}_k$ in the presence of sources.

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If there is no boundary, there is no surface term to be added to the Hamiltonian and the charges are zero.

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We examine first the question of the surface terms.

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where the volume term contains only undifferentiated variations

$$\int d^d x \left(A^{ij}(x)\, \delta g_{ij}(x) + B_{ij}(x)\, \delta \pi^{ij}(x) + \begin{matrix}\text{contributions from} \\ \text{other fields}\end{matrix} \right)$$

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one must impose

$$M[\xi] + \delta Q[\xi] = 0.$$

Gravitational Hamiltonian

The condition $M[\xi] + \delta Q[\xi] = 0$ implies that the variation $\delta H[\xi]$ reduces to the volume term, so that the Hamiltonian action principle (written here for pure gravity)

$$\delta\left(\int dt\, d^d x \pi^{ij} \dot{g}_{ij} - \int dt H[\xi]\right) = 0$$

yields under these conditions the correct equations of motion.

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- This constant is usually taken to be zero for some natural “background" configuration.
- Which boundary conditions to impose depends on the context.

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Two asymptotic symmetries that have the same asymptotic behaviour have the same charge and should be identified.

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Note that the asymptotic symmetries may not always be realized as background isometries.

Algebra of charges : A useful theorem

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Hence $[H[\xi], H[\eta]]$ and $H\big[[\xi, \eta]\big]$ differ, up to trivial terms, by a c-number $K[\xi, \eta]$ (which fulfills cocycle condition etc...).

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Indeed, one can prove that if the asymptotic symmetry group can be realized as isometry group of some background, then $K_{AB} = 0$.

Theories exist, however, with $K_{AB} \neq 0$.

Asymptotically anti-de Sitter spaces - Background metric

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The background metric is (the universal cover of) AdS,

$$ds^2 = -\left(1+\left(\frac{r}{l}\right)\right)dt^2 + \left(1+\left(\frac{r}{l}\right)\right)^{-1} dr^2 + r^2\, d\omega^2,$$

$$D=3: \; d\omega^2 = d\phi^2, \quad D=4: \; d\omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2, \quad \text{etc}$$

with $D=d+1$.

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For $D=3$, the boundary is the cylinder $\mathbb{R}\times S^1$ with metric $-dt^2 + d\phi^2$ and the conformal group at infinity is infinite-dimensional and contains $O(2,2)$, which is a proper finite-dimensional subgroup.

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One finds that the asymptotic symmetry group is in all cases the conformal group at infinity.

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The AdS group $O(D-1,2)$ is the asymptotic symmetry group in $D \geq 4$ dimensions. But it is the infinite-dimensional conformal group $\mathscr{C}^{(2)}$ that is the asymptotic symmetry group in $D = 3$. In that latter case, central charges are allowed and do, in fact, occur.

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the boundary conditions of Brown + MH (1986) are invariant under diffeomorphisms η^α of the form

$$\begin{aligned} \eta^+ &= T^+ + \frac{l^2}{2r^2}\partial_-^2 T^- + \cdots , \\ \eta^- &= T^- + \frac{l^2}{2r^2}\partial_+^2 T^+ + \cdots , \\ \eta^r &= -\frac{r}{2}\left(\partial_+ T^+ + \partial_- T^-\right) + \cdots , \end{aligned}$$

where $T^\pm = T^\pm(x^\pm)$.

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This is the full conformal group in 2 dimensions, generated by $T^+(x^+)$ and $T^-(x^-)$.

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In terms of Fourier modes,

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In the 1960's, Bondi, Metzner and Sachs (BMS) found that reasonable asymptotic conditions defining asymptotically flat spaces at null infinity

were invariant under an infinite-dimensional group containing the Poincaré group.

This infinite-dimensional symmetry was somewhat viewed then by many authors (but not all !) as an embarrassment. Various attempts were made to reduce the asymptotic symmetry group to the Poincaré group by strengthening the boundary conditions.

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For instance, they make the angular momentum generically diverge (see below).

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($O(\log r)$ terms are in fact permitted but will be not considered here.)

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The parity conditions are fulfilled by all known solutions.

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The asymptotic symmetry algebra is then infinite-dimensional and contains BMS_4 (Compère and Dehouck 2011, Troessart 2017).

This yields an infinite number of conserved charges, in agreement with the analysis performed at null infinity.

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Also not discussed here : inclusion of higher spin gauge fields with their higher spin gauge symmetries - analysis can be performed following the same ideas.

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THANK YOU !