Accretion Rings at the Outermost Part of Accretion Disks and their Observational Appearances

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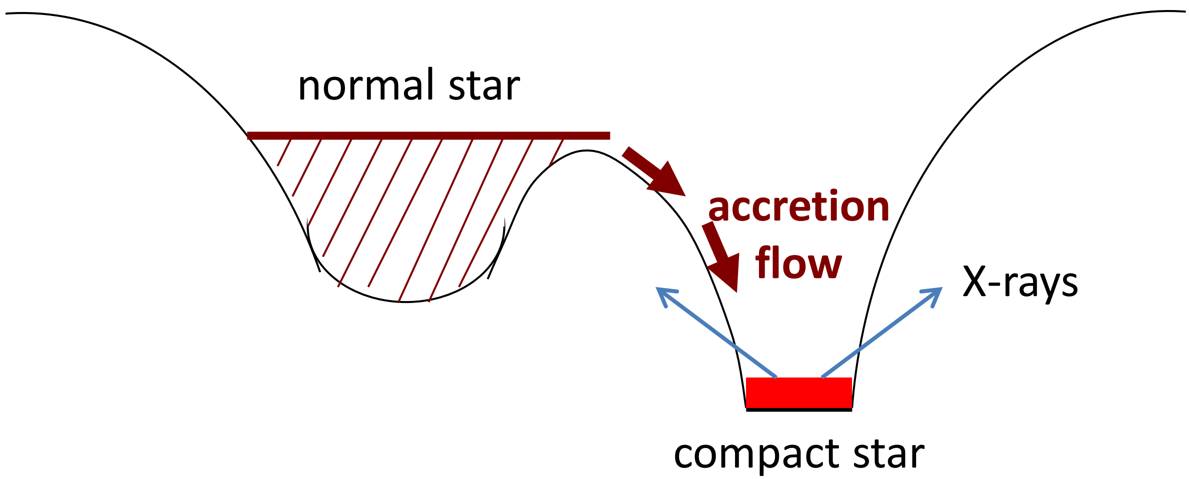
Early results of X-ray astronomy in 1960s-1970s

Most of bright X-ray sources = <u>Compact stars</u> in close binaries
(neutron stars or black holes)

X-ray binaries (close binaries)



X-ray emission:

Release of gravitational energy of matter accreted by a compact star
released energy per mass ~ 0.1 c^2

Super-orbital periods in X-ray binaries

Several X-ray binaries sometimes exhibit periodic variations of X-ray fluxes with periods 10 ~ 100 times longer than the orbital periods.

Typical examples

	binary period	super-orbital period	object character
Her X-1	1.7 d	35 d	X-ray pulsar
SMC X-1	3.9 d	55 d	X-ray pulsar
LMC X-4	1.4 d	30 d	X-ray pulsar
SS433	13.1 d	162.5 d	jet object
Cyg X-1	5.6 d	294 d	black hole object

Super-orbital period in Her X-1

<u>Her X-1</u>

X-ray pulsar period 1.24 sec
close binary period 1.7 days

X-ray on-off period ~ 35 days

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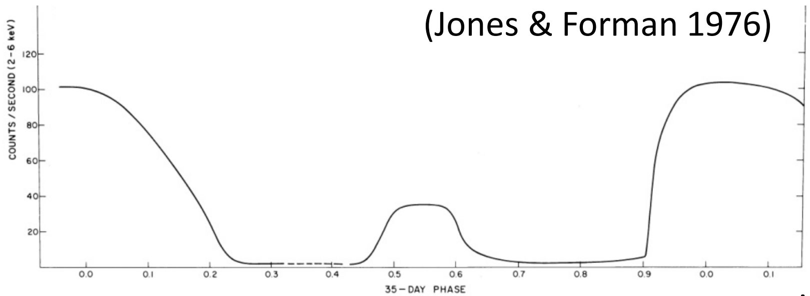
a precession of the accretion disk

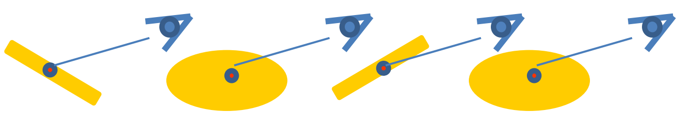
X-ray flux

(Jones & Forman 1976)



time

35 days



(Giacconi et al. 1973)

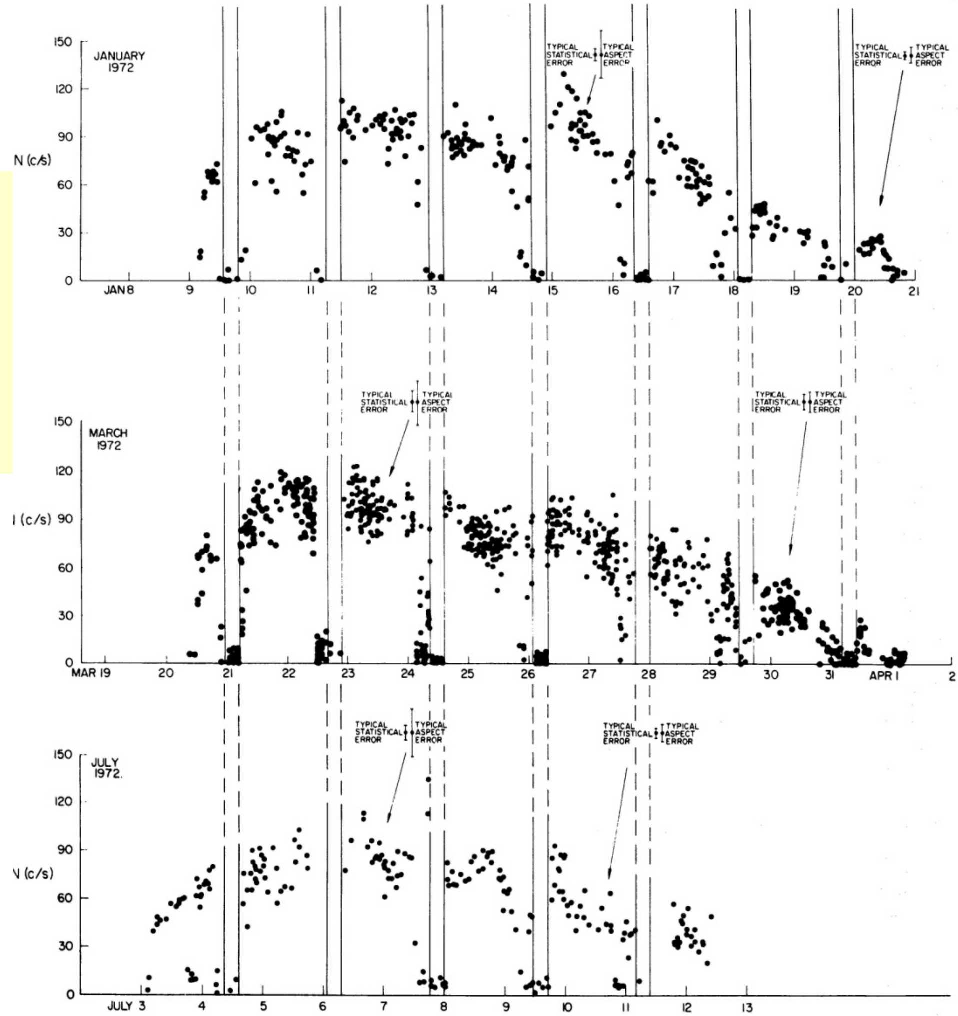


FIG. 2.—Hercules intensity data (2–6 keV) during three ON states. The vertical lines represent the orbital eclipses, whose positions are determined accurately from the pulsation Doppler analysis. Typical errors appropriate for different groups of data are shown; the statistical error bar is relevant for point-to-point comparisons, and the aspect error bar is relevant for day-to-day comparisons. It should be noted that most intensity points below about 10 counts s^{-1} are upper limits.

Super-orbital period in SS433

SS433

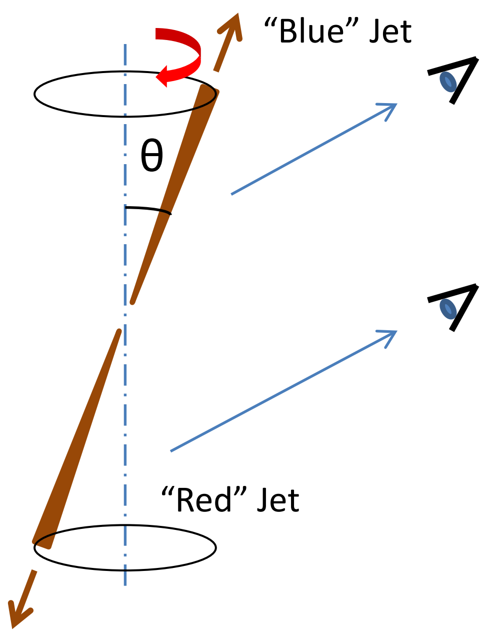
close binary period 13.1 days

Relativistic jets

velocity ～0.26 C

precession tilt angle ～20°

period ～162.5 days



(Margon 1984)

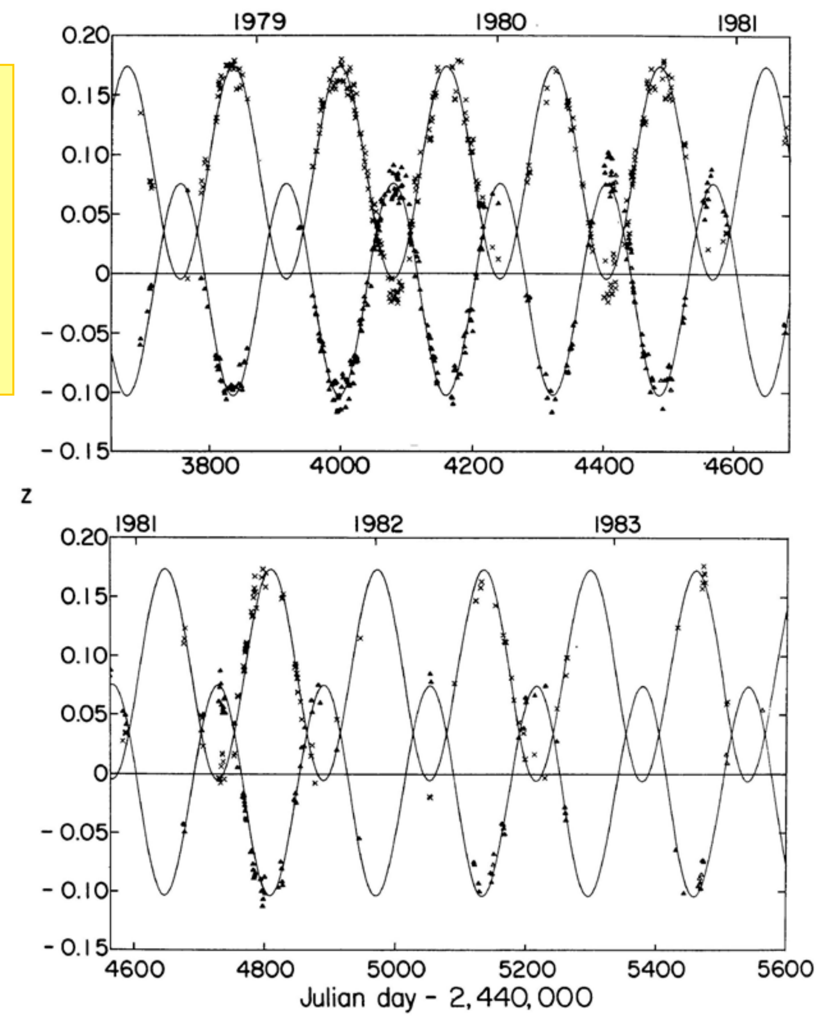
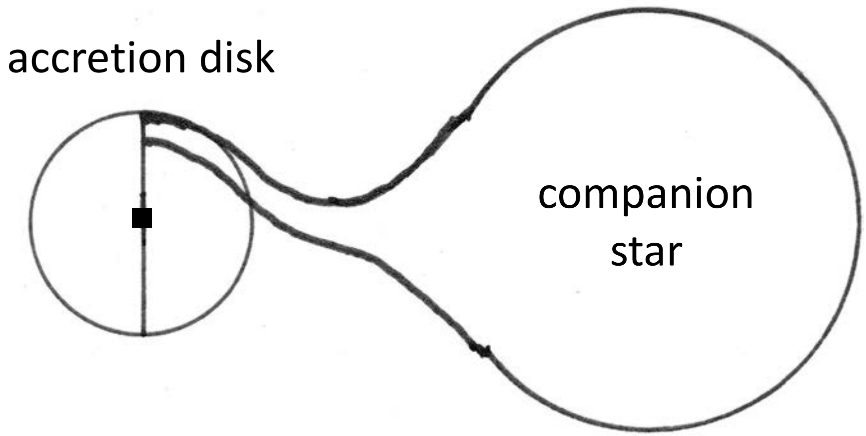
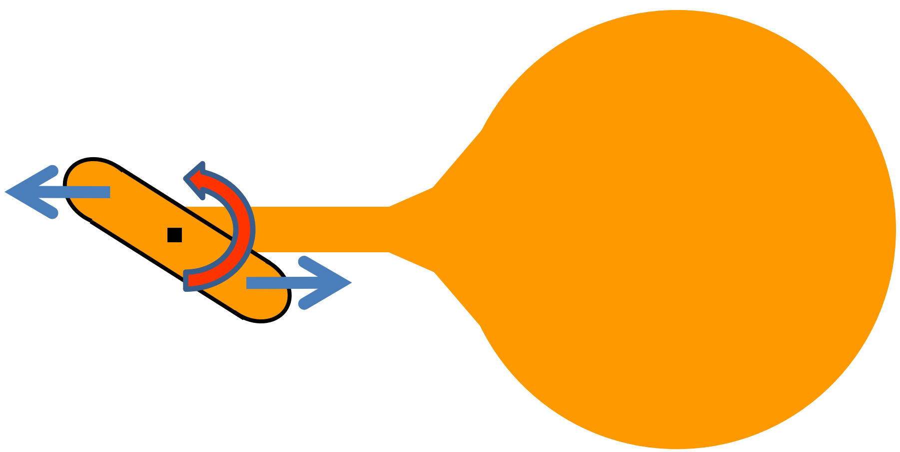
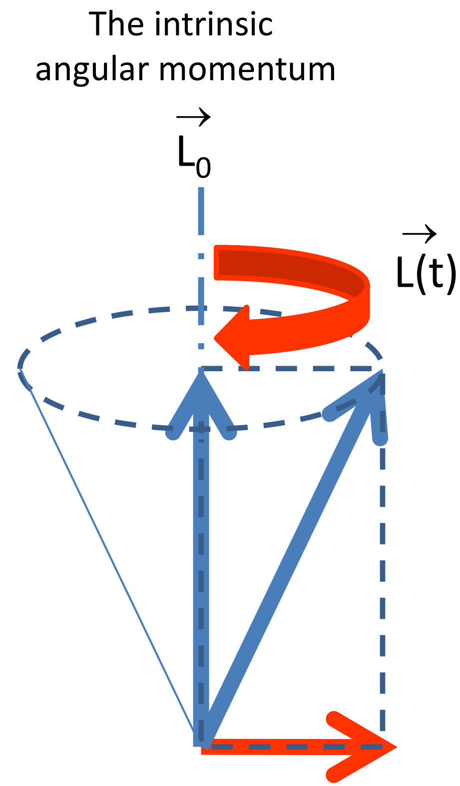


Figure 1 Doppler shifts of SS 433 on 450 nights in the period 1978–83. The majority of these data were obtained by the author and colleagues, supplemented by sources cited in (105). The solid curve is a least-squares best fit to the simple "kinematic model" (1). The free parameter values and their associated 1σ uncertainties (notation as in 105) for this fit are $v/c = 0.2601 \pm 0.0014$, $\theta = 19.80° \pm 0.18°$, $i = 78.82° \pm 0.11°$, $t_0 =$ JD 2,443,562.27 $\pm$ 0.39, $P = 162.532 \pm 0.062$ days.

A precession of an accretion disk induced by tidal force from a companion star



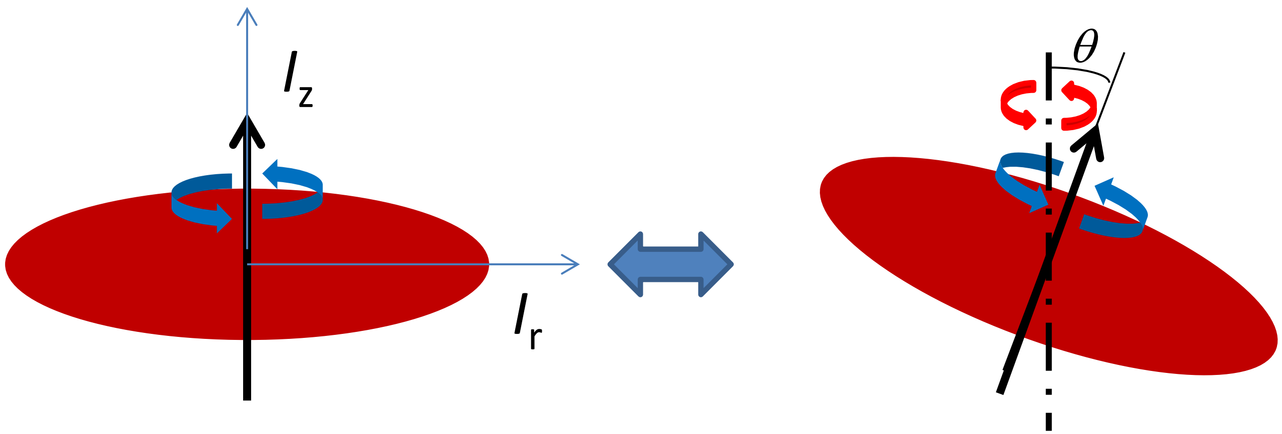




When the accretion disk tilts, the tidal force from the companion star induces a torque on it.

The torque causes a preccession of the disk.

Stability of free rotating rigid bodies



$$E = \frac{L^2}{2\, I_z} \left[1 + \sin^2 \theta \left(\frac{I_z - I_r}{I_r} \right) \right]$$

E: Rotational energy
L: Angular momentum
I : Inertial moment

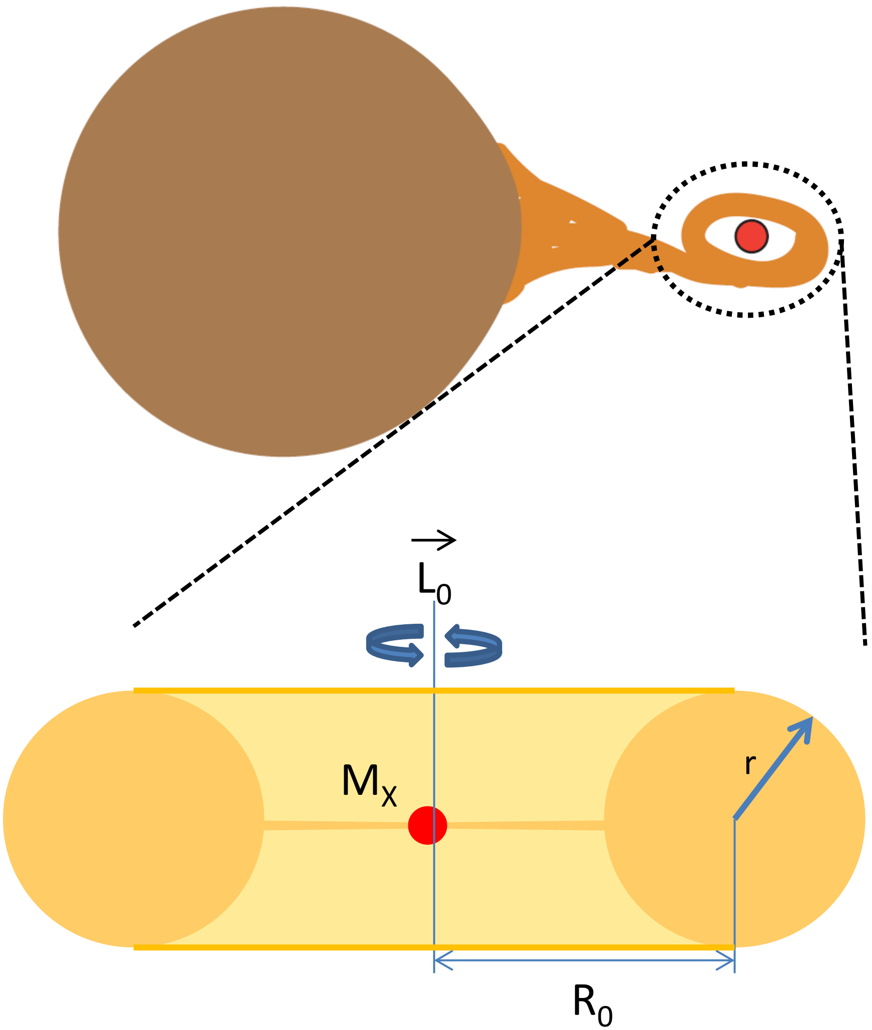
The minimum of the rotational energy is realized at $\theta = 0$ when $I_z > I_r$.

A free spin around the largest principal axis of inertia is stable against a precession.

What happens in accretion flow in X-ray binaries ?

Accretion ring at the outermost part of an accretion disk

Matter flowing from the companion star should tend to circulate around the compact star according to the Coriolis force in the rotating binary system.



If we approximate that all the fluid particles of the inflow have the same angular momentum per mass around the compact star,

$$l_0 = (R_0 G M_X)^{1/2},$$

a ring should be formed with the average radius, R_0 and the average angular velocity,

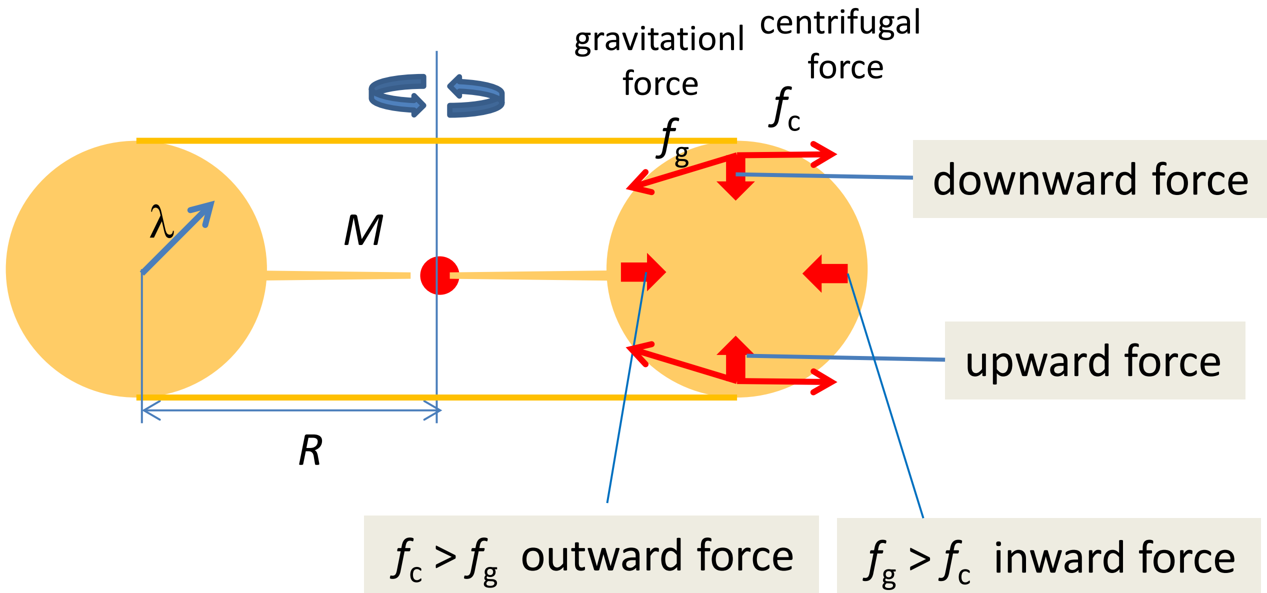
$$\Omega_0 = \frac{(GM_X)^{1/2}}{R_0^{3/2}}.$$

The total angular momentum of the ring, L_0, is

$$L_0 = M_R R_0^2 \Omega_0.$$ (M_R: the ring mass)

Hydrostatic balance of the matter in the ring-tube

When all the fluid particles have the same angular momentum per mass around the compact star, contracting force toward the ring-tube center balances with the pressure gradient in the approximately circular cross section of the ring-tube.



Hydrostatic equation

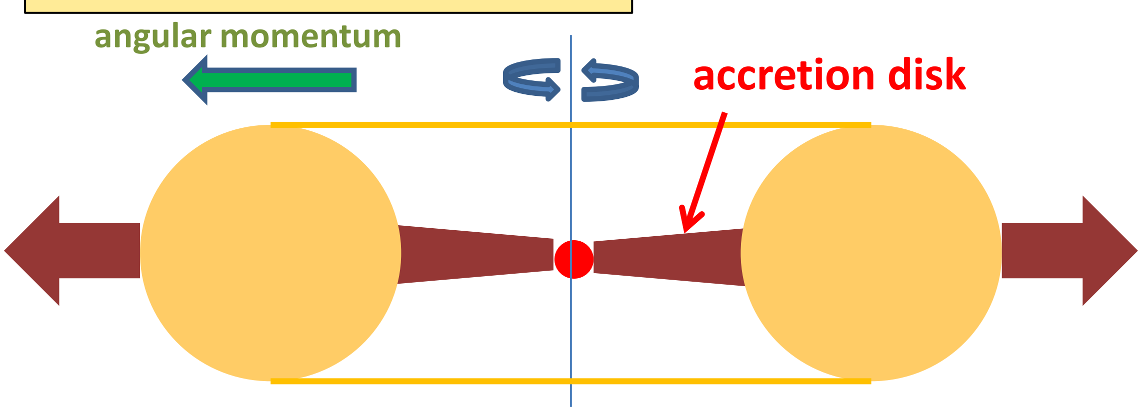
$$\frac{dP}{d\lambda} = -\mu m_H n \frac{GM_X}{R^3} \lambda$$

isothermal approximation

$$n \propto \exp[-\lambda^2 / 2a^2],$$

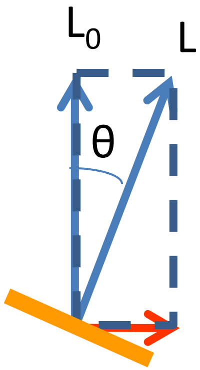
$$\text{where} \quad a = \left[\frac{kTR^3}{\mu m_H GM_X} \right]^{1/2}$$

Accretion disk inside the ring



Angular momentum should gradually be transferred in the ring and the inner side should extend inward to be an accretion disk onto the compact star.

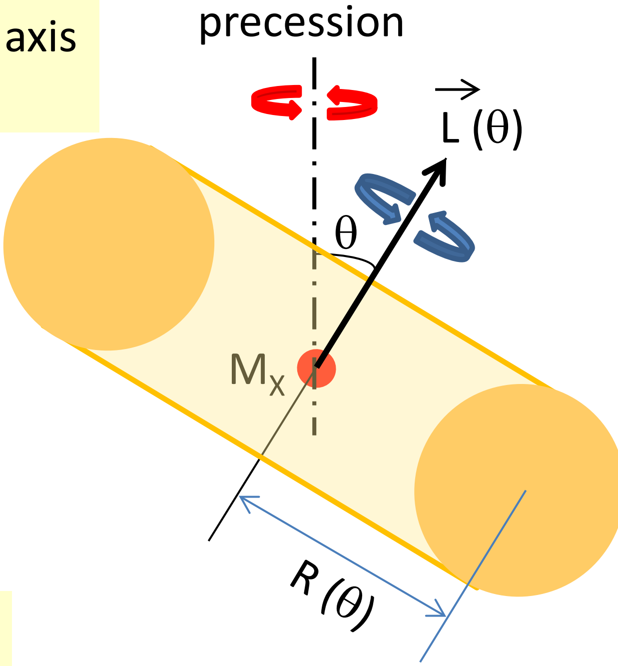
Let us consider a case in which the angular momentum axis of the ring tilts from the intrinsic axis by angle, θ.



Then, $L = L_0 \cos^{-1}\theta$.

(Subscript 0 indicates parameters without tilt)

This angular momentum vector should precesses around the original axis owing to the tidal force from the companion star.



The total angular momentum of the ring, L, is given by

$$L = M_R\, R^2\, \Omega \propto R^{1/2}, \quad (M_R\text{: the ring mass})$$

where

$$\Omega = \frac{(GM_X)^{1/2}}{R^{3/2}} \propto R^{-3/2} .$$

$$\boxed{R = R_0 \cos^{-2}\theta}$$

When the tilt angle θ gets larger, the ring radius R increases.

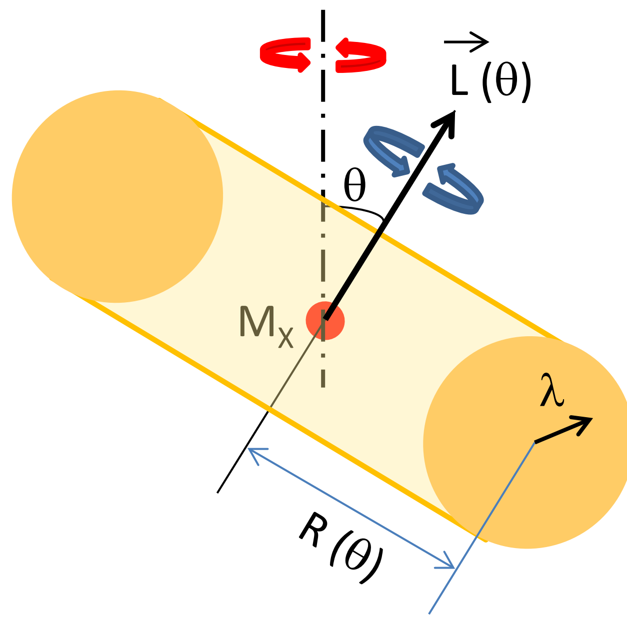
- rotational energy

$$E_K = (1/2)\, M_R\, R^2\, \Omega^2$$
$$= M_R(GM_X/2R_0) \cos^2\theta$$

- gravitational energy

$$E_G = -\, M_R(GM_X/R_0) \cos^2\theta$$

Hydrostatic balance in the meridian cross section of the ring-tube $\frac{dP}{d\lambda} = -\mu m_H n \frac{GM_X}{R^3} \lambda$



On an iso-thermal approximation,

$n \propto \exp[- \lambda^2 / 2 a^2]$,

where $\boxed{a = \left[\frac{kTR^3}{\mu m_H GM_X} \right]^{1/2} \propto T^{1/2} R^{3/2}}$.

Adiabatic expansion of the ring-tube

$T/T_0 = (V/V_0)^{-2/3}$

$V \propto Ra^2$

$T/T_0 = (R/R_0)^{-8/5} \propto \cos^{16/5}\theta$

When the tilt angle θ gets larger, the ring temperature T decreases.

・thermal and effective potential energy for the hydrostatic balance in the meridian cross section of the ring-tube

$E_T = (5/2)\,(M_R/\mu m_H)\,kT$

$= M_R(GM_X/2R_0)\,\alpha \cos^{16/5}\theta$

$\alpha = (5kT_0/2\mu m_H) / (GM_X/2R_0)$

When the tilt angle θ gets larger, the ring thermal energy decreases.

The energetics of the gas ring-tube as a function of the tilt angle

- rotational energy

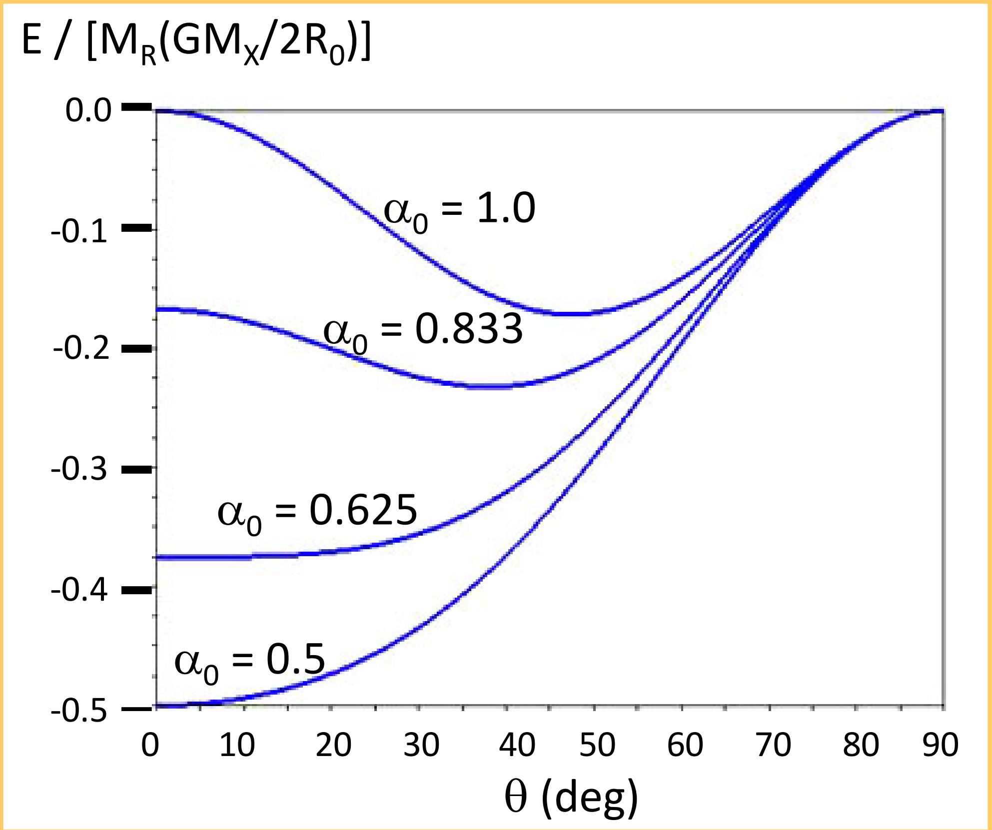
$$E_K = (1/2)\, M_R\, R^2\, \Omega^2$$
$$= M_R(GM_X/2R_0) \cos^2\theta$$

- gravitational energy

$$E_G = -\,2\, E_K$$

- thermal and effective potential energy

$$E_T = (5/2)\,(M_R/\mu m_H)\, kT$$
$$= M_R(GM_X/2R_0)\, \alpha_0 \cos^{16/5}\theta$$
$$[\ \alpha_0 = (5kT_0/2\mu m_H) / (GM_X/2R_0)\ ]$$



- total energy of the ring

$$E = E_K + E_G + E_T + (\text{minute terms on } \theta)$$
$$\approx M_R(GM_X/2R_0)\,(-\cos^2\theta + \alpha_0 \cos^{16/5}\theta)$$

⇩

$$\cos\theta_{min} = [\ 5/(8\alpha_0)\ ]^{5/6}$$

When the ring-tube has enough thermal energy ($\alpha_0 > 5/8$), the total energy has the minimum at a certain tilt angle.

Precessions of gas rings around compact stars are possible to take place from the energetics point of view.

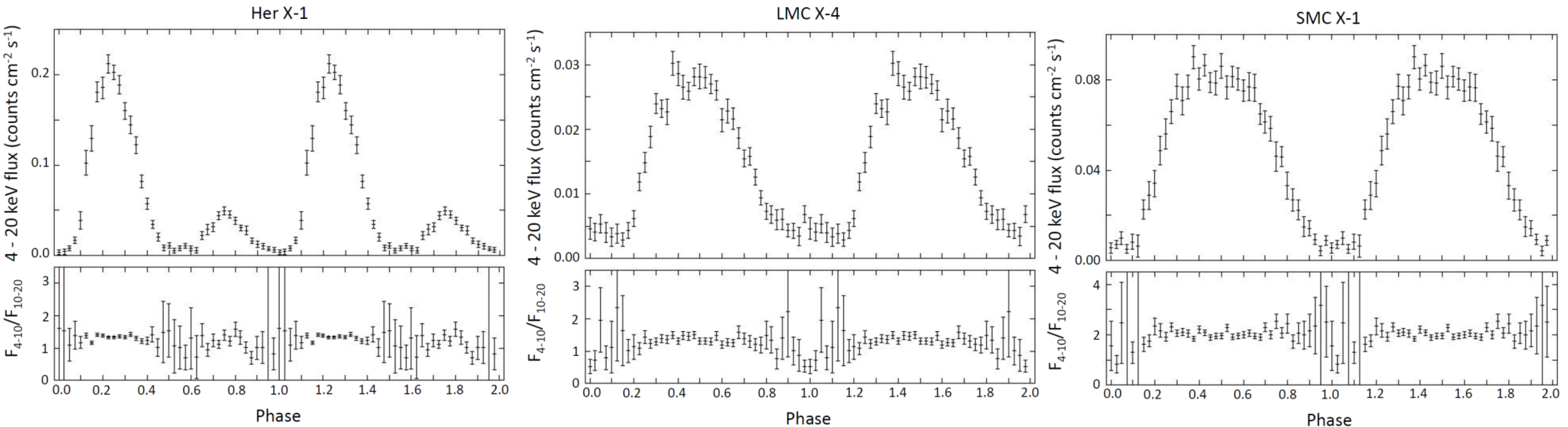
Observational confirmations of the ring precession

Reproduction of X-ray light curves of three X-ray pulsars, folded with the respective super-orbital periods, by a precessing ring model

We try to reproduce the X-ray light curves folded with the respective super-orbital periods of three X-ray pulsars, Her X-1, SMC X-1 and LMC X-4, observed with MAXI by a model that X-rays from a compact object are periodically modulated through electron scatterings by hot gas in an accretion ring precessing around the central object with the super-orbital period.

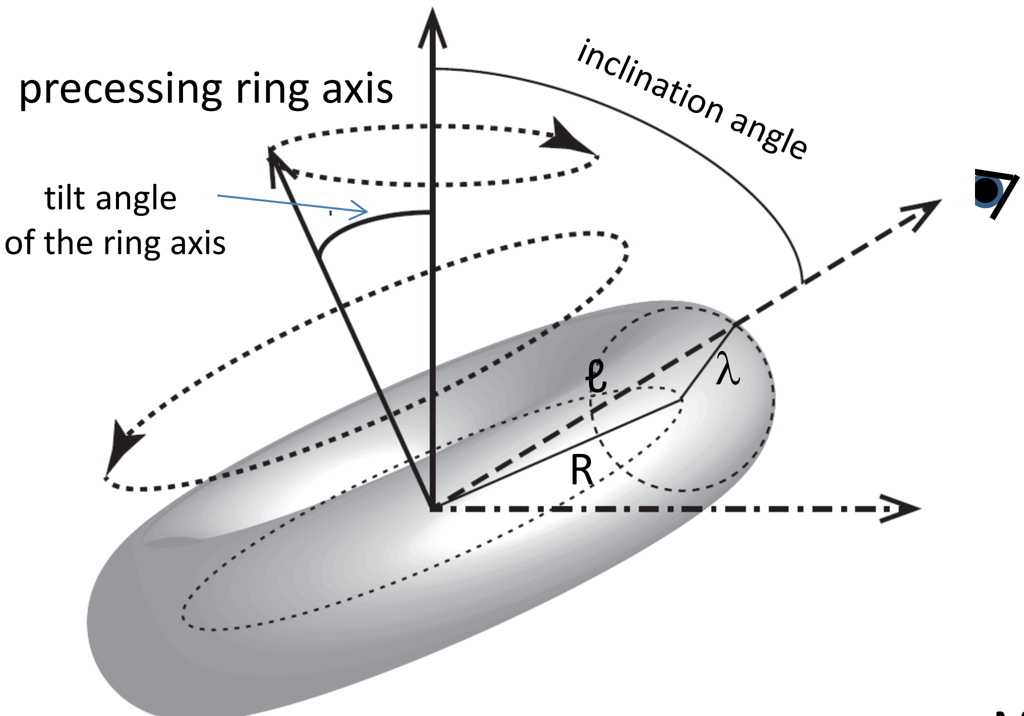
	binary period	super-orbital period	object character
Her X-1	1.7 d	35 d	X-ray pulsar
LMC X-4	1.4 d	30 d	X-ray pulsar
SMC X-1	3.9 d	55 d	X-ray pulsar

Folded light curves (2 cycles) of the three X-ray pulsars observed with MAXI



The precessing ring model

X-ray flux modulation is due to periodic variation of optical depth for the electron scattering through the precessing accretion ring on the line of sight



<u>number density distribution</u>

$$n = n_0 \exp[- \lambda^2 / 2 a^2],$$

⬇

<u>optical depth for the electron scattering</u>

$$\tau = \sigma_T \int n \, d\ell$$

(σ_T : Thomson scattering cross section
$\int$: integration along the line of sight)

⬇

<u>X-ray flux modulation</u>

$$F_D = F_0 \exp(- \tau)$$

<u>Assumption on the observed X-ray flux, *F*</u>

$$F = F_D + F_S$$

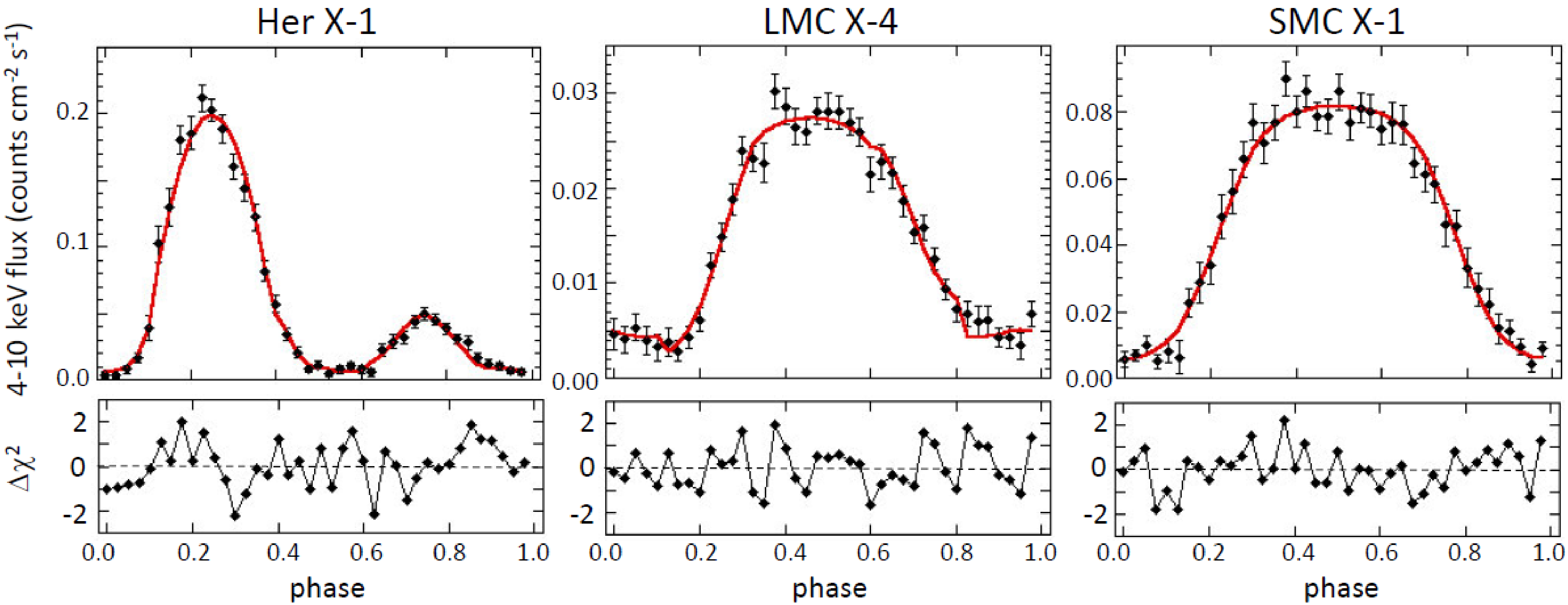
F_D : directly observed component suffering from the molulation

F_S : scattered component by the ring with the constant flux

<u>Model parameters</u>

inclination angle	i
tilt angle of the ring axis	θ
scale height indicator	a / R
column density indicator	$n_0 a$
direct component flux	F_D
scattered component flux	F_S
initial phase angle	ϕ_0

Best fit solutions



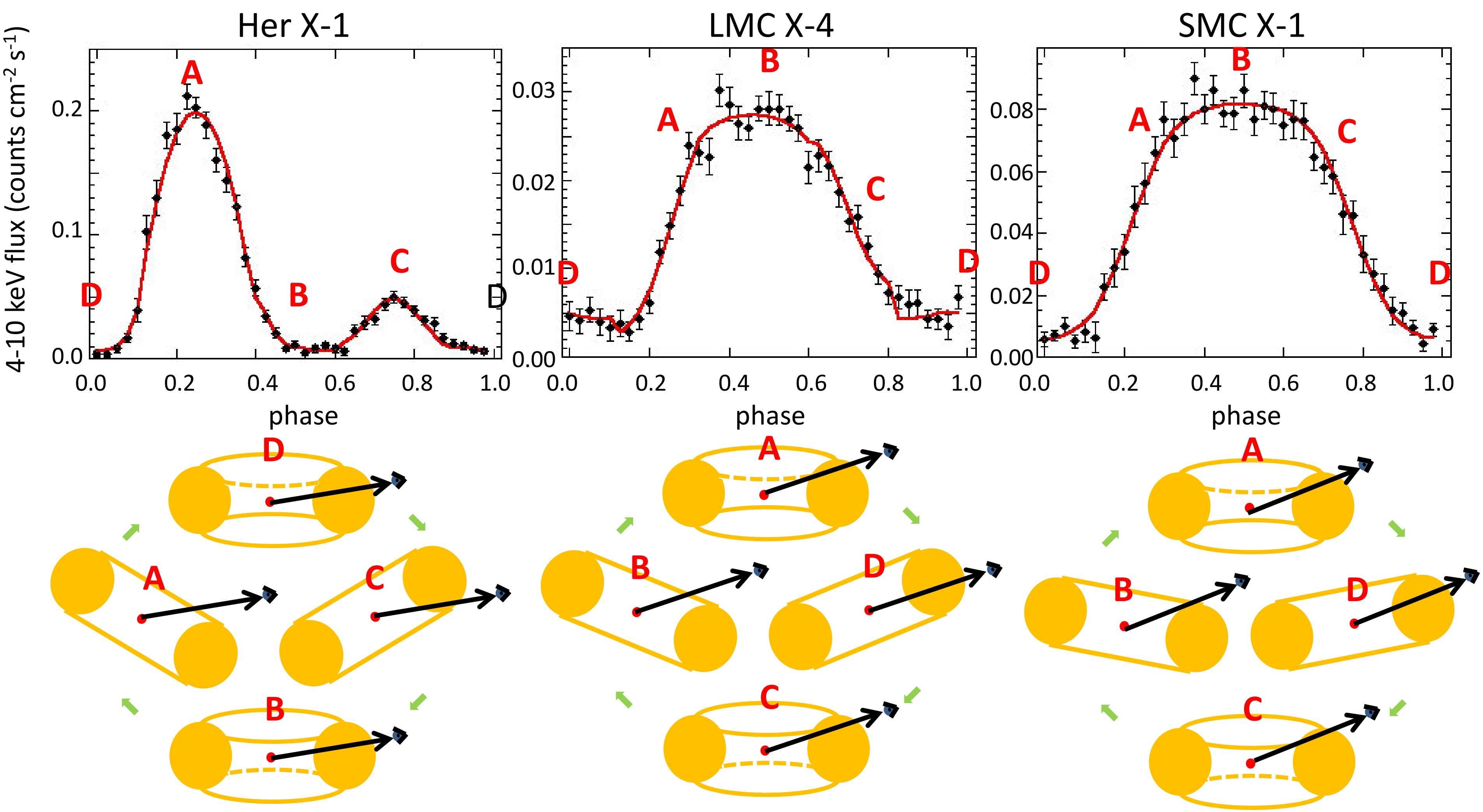
reduced χ^2 = 1.24
(D.O.F = 33)

reduced χ^2 = 1.06
(D.O.F = 34)

reduced χ^2 = 0.92
(D.O.F = 35)

The precessing ring model well reproduce the observed light curves folded with the super-orbital periods.

	Her X-1	LMC X-4	SMC X-1	
inclination angle of the line of sight	83°	59°	62°	fixed to values obtained from other Obs.
tilt angle of the ring axis	$30.3°^{+1.5°}_{-1.2°}$	$38.7° \pm 3.8°$	$13.3°^{+7.7°}_{-6.4°}$	



Sizes of the rings in the binary systems

Kepler motion → Separation of the two stars : *D*

$$D = [\,G\,(M_X + M_C)\,(2\pi)^2/P_B^{\,2}\,]^{1/3}$$

Approximate formula of the Roche lobe radius → Average radius of the Roche lobe : R_L

$$R_L/\,D = 0.49/[0.6+q^{2/3}\ln(1+q^{-1/3})]$$

Eggeleton (1983)

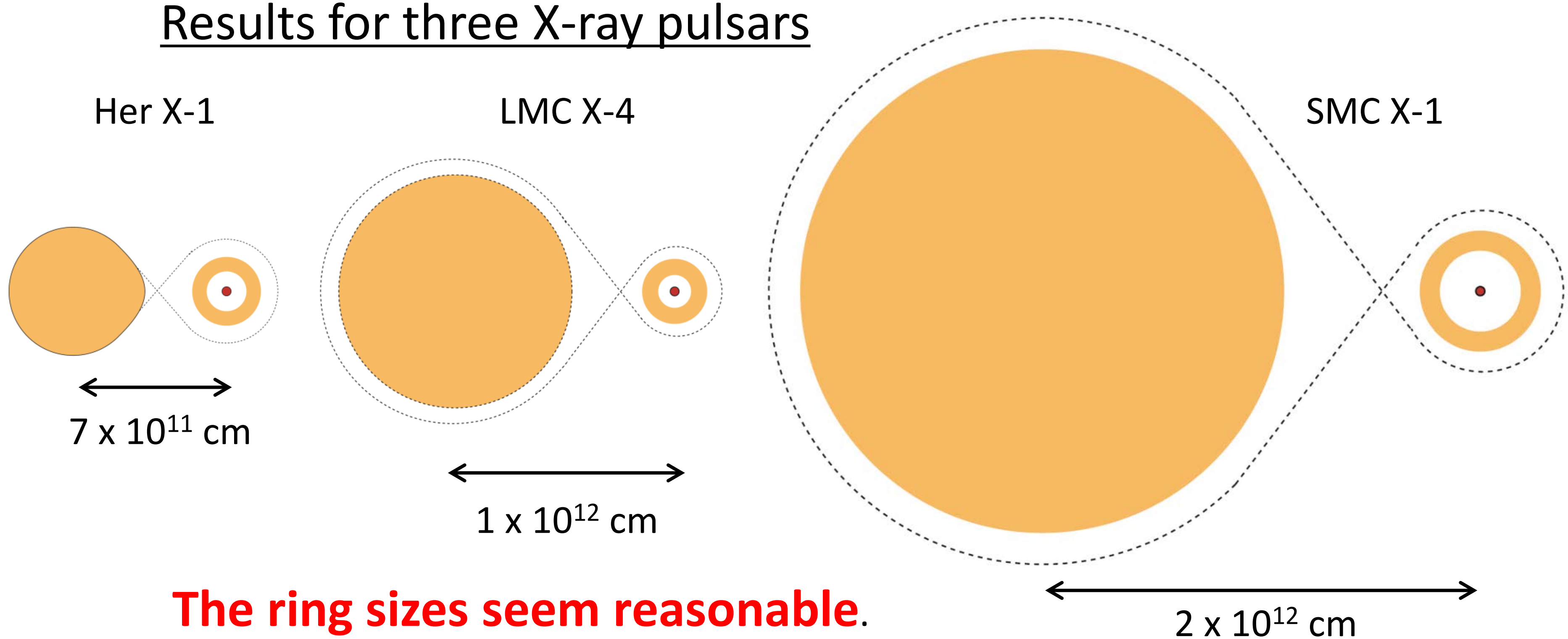
The precessing motion of a ring → Ring radius : *R*

$$R\,/\,D = \{\,2\,[(1 + q\,)^{1/2}/\,q]\,[P_B/P_P]\,\cos^{-1}\theta\,\}^{2/3} \qquad q \equiv M_C/M_X$$

Inoue (2012)

Model parameter, *a*/*R* → Ring-tube thickness : *a*

Results for three X-ray pulsars



The ring sizes seem reasonable.

Geometrical thickness of the ring-tube

Density distribution in the ring-tube on an iso-thermal approximation: $n = n_0 \exp[- \lambda^2 / 2 a^2]$

a : a scale parameter of the ring-tube thickness, given as $\dfrac{a}{R} = \left[\dfrac{kT}{\mu m_H} \Big/ \left(\dfrac{GM_x}{R}\right) \right]^{1/2}$

Condition for the ring matter to be bound in the gravitational potential of the compact star

$$\underbrace{\frac{5}{2}\frac{kT}{\mu m_H}}_{\text{thermal enthalpy}} + \underbrace{\frac{GM_x}{2R}}_{\text{kinetic energy}} - \underbrace{\frac{GM_x}{R}}_{\text{potential energy}} < 0 \Rightarrow \frac{5}{2}\frac{kT}{\mu m_H} < \frac{GM_x}{2R} \Rightarrow \frac{a}{R} < \left(\frac{1}{5}\right)^{1/2} \approx 0.45$$

Results of the model fits

	Her X-1	LMC X-4	SMC X-1
$\dfrac{a}{R}$	0.26 ± 0.03	0.33 ± 0.03	$0.20^{+0.03}_{-0.20}$

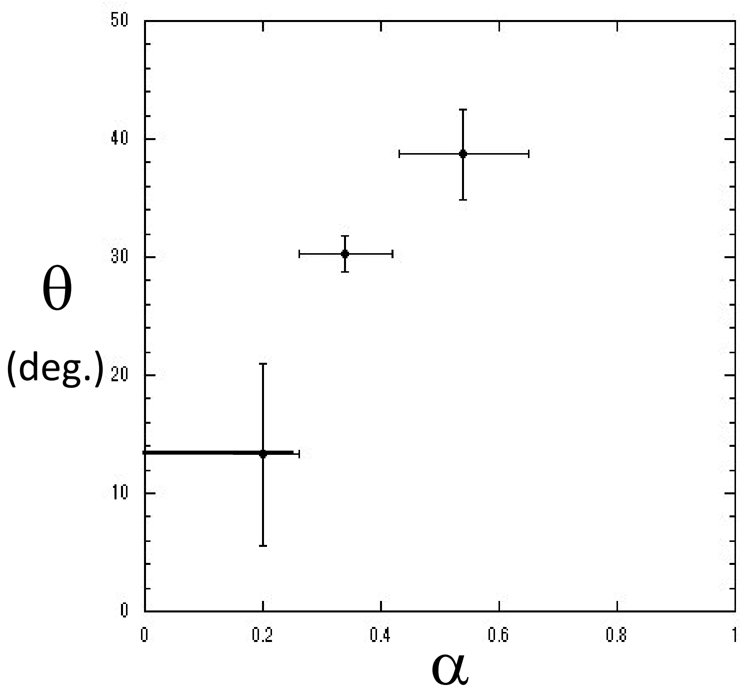
$\dfrac{a}{R} < 0.45$ is satisfied.

The ring matter is well gravitationally bound.

Tilt angle vs. ring-tube thickness

	Her X-1	LMC X-4	SMC X-1
tilt angle of the ring axis, θ	$30.3^\circ\,^{+1.5^\circ}_{-1.2^\circ}$	$38.7^\circ \pm 3.8^\circ$	$13.3^\circ\,^{+7.7^\circ}_{-6.4^\circ}$
ring-tube thickness (a / R)	0.26 ± 0.03	0.33 ± 0.03	$0.20\,^{+0.03}_{-0.20}$
relative amount of the ring thermal energy, α	0.34 ± 0.08	0.54 ± 0.10	$0.20\,^{+0.06}_{-0.20}$

$$\alpha = (\text{enthalpy})/(\text{kinetic energy}) = (5kT/2\mu m_H) / (GM_X/2R) = 5\,(a/R)^2$$



The larger α exhibits the larger tilt angle.

This observational result is qualitatively consistent with the theoretical consideration that adiabatic cooling of the ring matter induced by tilting could excite the ring precession.

Accumulation time & cooling time of the ring matter

Physical quantities evaluated from the best fit parameters

	Her X-1	LMC X-4	SMC X-1	
Typical column density of the tube, $n_0 a$	3.5×10^{24}	1.1×10^{24}	3.5×10^{24}	cm^{-2}
Typical optical depth of the tube, τ	2.3	0.7	2.3	for Thomson scattering
Total ring mass, M_R	9.4×10^{23}	3.3×10^{23}	2.7×10^{24}	g
Number density at the center of the tube, n_0	1.2×10^{14}	3.1×10^{13}	7.8×10^{13}	cm^{-3}
Temperature of the ring matter, T	9.0×10^{5}	1.6×10^{6}	7.8×10^{5}	K
Average X-ray luminosity, L	3.1×10^{37}	1.6×10^{38}	6.5×10^{38}	$erg\ s^{-1}$
Average accretion rate, $\dot{M}$	1.7×10^{17}	8.6×10^{17}	3.5×10^{18}	$g\ s^{-1}$

Two important time scales

	Her X-1	LMC X-4	SMC X-1	
Accumulation time, $t_A = M_R / \dot{M}$	5.5×10^{6}	3.8×10^{5}	7.7×10^{5}	s
Radiative cooling time, $t_C \approx 3kT/(n_0 \Lambda)$ ($\Lambda = 1 \times 10^{-22}$ erg $cm^3\ s^{-1}$ is assumed.)	3.2×10^{-2}	2.1×10^{-1}	1.2×10^{-2}	s

$t_A >> t_C$

Accreted matter is kept present in the ring for the accumulate time, t_A without suffering from radiative cooling, even though the cooling time, t_C is much shorter than t_A.

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Some heating with a rate high enough to compensate the high cooling rate is needed.

X-ray heating on the accretion ring

A possible heating mechanism of the ring matter : X-ray heating

Effect of X-ray heating relative to radiative cooling is represented with the so-called ionization parameter, ξ defined as

$$\xi = L / (nr^2)$$

L : luminosity of an X-ray source
n : number density of irradiated matter
r : distance of irradiated matter from an X-ray source

$\xi \geq 10^4$ erg cm s^{-1} : Strong X-ray heating so that k$T \approx$ mean X-ray photon-energy

10 erg cm s$^{-1} < \xi < 10^4$ erg cm s^{-1} :

T is determined by a balance between X-ray heating and radiative cooling.

$T \approx 10^5 \sim 10^6$ K when $\xi \approx 10^2 \sim 10^3$ erg cm s^{-1} (e.g. Kallman & McCray 1982)

$\xi \leq 10$ erg cm s^{-1} : Effect of X-ray heating is negligible.

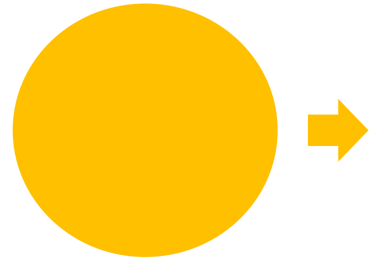
Typical ξ values of the rings evaluated from the best fit parameters

	Her X-1	LMC X-4	SMC X-1	
$\xi = L / (\bar{n}R^2)$ ($\bar{n} = n_0\, e^{-1}$)	7.2×10^1	1.2×10^3	4.5×10^2	erg cm s^{-1}
T	9.0×10^5	1.6×10^6	7.8×10^5	K

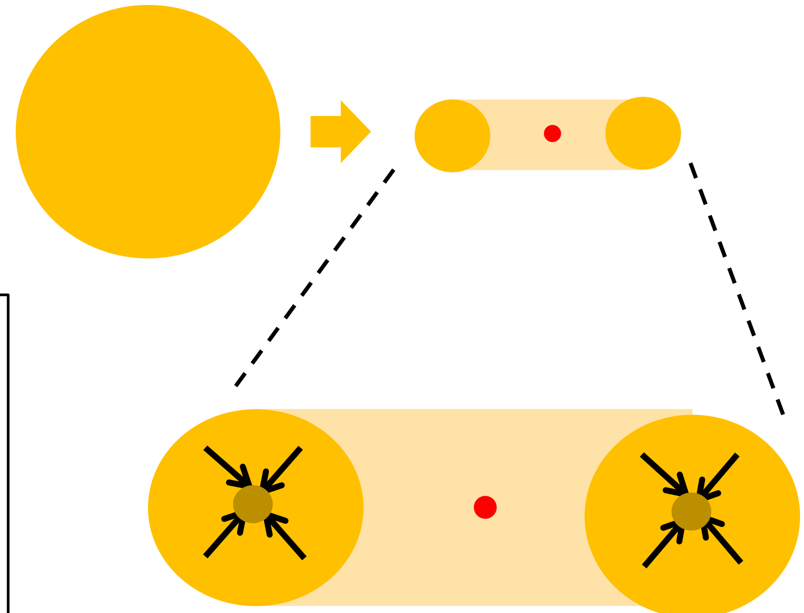
These ξ-values are consistent with a picture that X-ray heating plays the main role to keep accreted matter accumulate in the ring for the time much longer than the radiative cooling time.

Possible scenario on establishment of a steady flow from an accretion ring to a compact star: 1

Matter starts flowing from a companion star to a compact star.



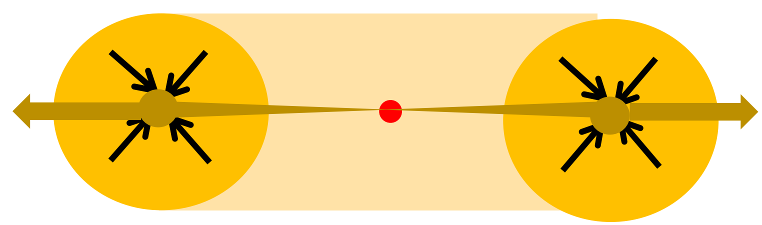
An accretion ring is formed around the compact star.



Radiative cooling induces a cooling flow across the ring tube toward the tube center.

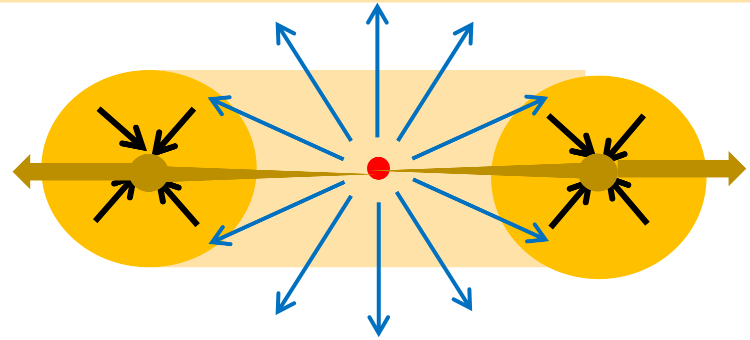
The cooling flow across the laminar rotational flow in the ring-tube causes turbulent motions and efficient angular momentum transfer takes place in the central part of the ring.

The efficient angular momentum transfer makes an accretion disk extension to the central compact star.



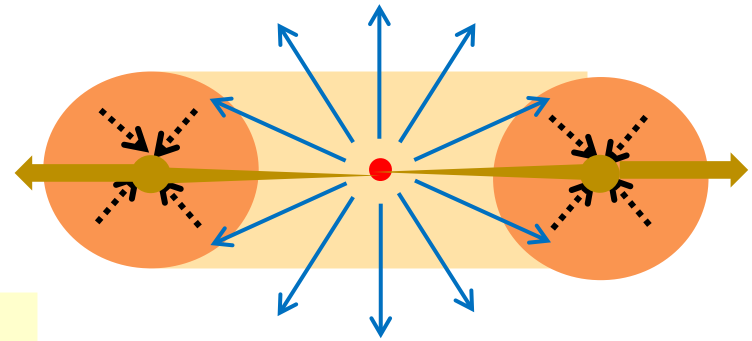
Possible scenario on establishment of a steady flow from an accretion ring to a compact star: 2

X-ray emission from the compact star starts.



X-ray heating suppresses the cooling flow in the ring-tube.

Matter accumulation in the ring starts.



Ring matter, M_R ↗ → Density, n ↗

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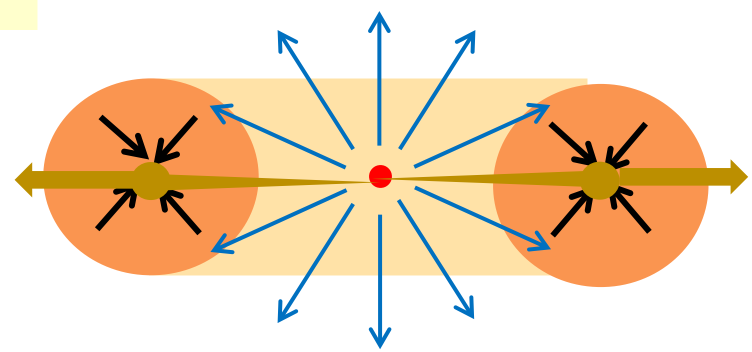
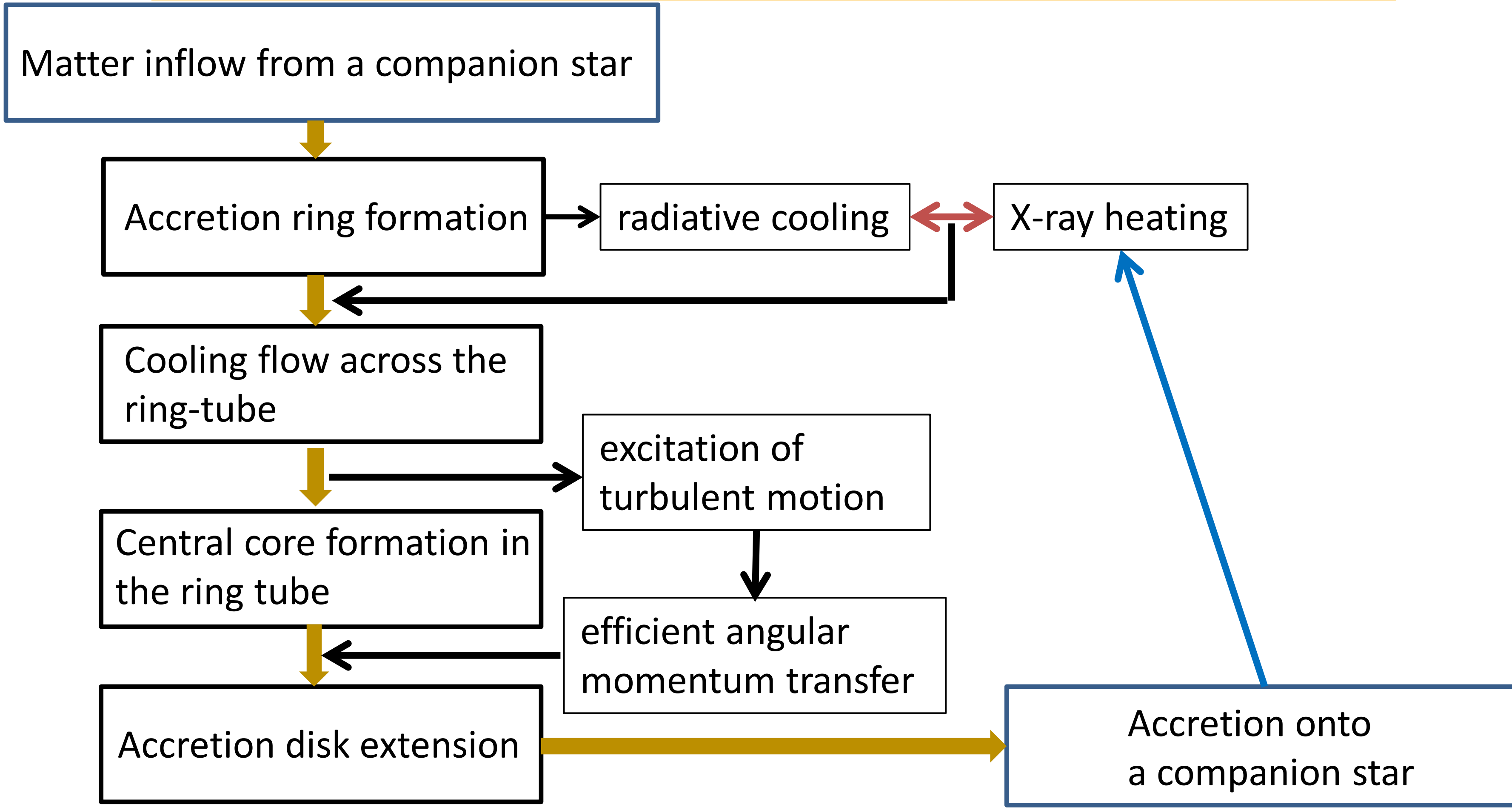
Ionization parameter, ξ ↘ and Optical depth, τ ↗

Conditions on ξ and/or τ for a steady cooling flow across the ring-tube to take place are realized.



A steady flow from the companion star to the compact star is established.

Roles of the accretion ring in the steady accretion flow



More detailed study of accretion rings is necessary.

- Overall understanding of accretion processes in binary X-ray sources
- Instabilities in the ring in relation to long term variabilities of X-ray binaries
- Links to broad line regions in AGNs

Summary

- X-ray variations with super-orbital periods 10 ~ 100 times longer than respective orbital periods are observed from several X-ray binaries.
- Energetics considerations show that a precession of an accretion ring around a compact star can be excited by a tidal force from a companion star.
The precession of the accretion ring is possible to explain the super-orbital periodicities.
- The X-ray light curves of three X-ray pulsars, Her X-1, LMC X-4 and SMC X-1, folded with their respective super-orbital periods are fit to a model light curve which is calculated from a model that the X-ray flux modulation is due to periodic variation of optical depth for the electron scattering through the precessing accretion ring on the line of sight.
- It is shown that the model statistically well reproduces the observed light curves, and that best fit parameters are reasonable from several physical points of view.
- One interesting finding is that the best fit value of the typical electron scattering optical depth across the ring-tube is around unity and that the total ring mass needs an accumulation time of accreted matter of $\sim 10^6$ s which is much longer than the radiative cooling time calculated from the best fit parameters.
- Heating of the accretion ring by X-rays from the central compact star is suggested to play a key role to elongate the accumulation time against the high cooling rate. In fact, the ionization parameter, ξ of the ring, calculated from the best fit parameters supports this suggestion.
- A scenario for a steady flow from the companion star to the compact star, and important roles of the accretion ring in it are discussed.