Cosmological structure formation in mildly–nonlinear regime:

from IR to UV

Mikhail Ivanov
EPFL & INR & MSU

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

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w/ D. Blas, G. D'Amico, M. Garny, S. Sibiryakov, R. Scoccimarro

Large Scale Structure people's mantra

Connect the statistical properties of
the distribution of mass on large scales
to the physics and the constituents of the universe
(pNG, DM/gravity properties, neutrino masses, DE,...)

Equal time correlation functions are particularly important

$$\rho_n(x,t) = \bar{\rho}_n(t)(1+\delta_n(x,t)) \longleftarrow$$ density contrast

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}' + \mathbf{k}) P(k)$$

$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)$$

$$\ldots$$

CMB vs. LSS

We've already learned a lot thanks to CMB

Will get much more in future !

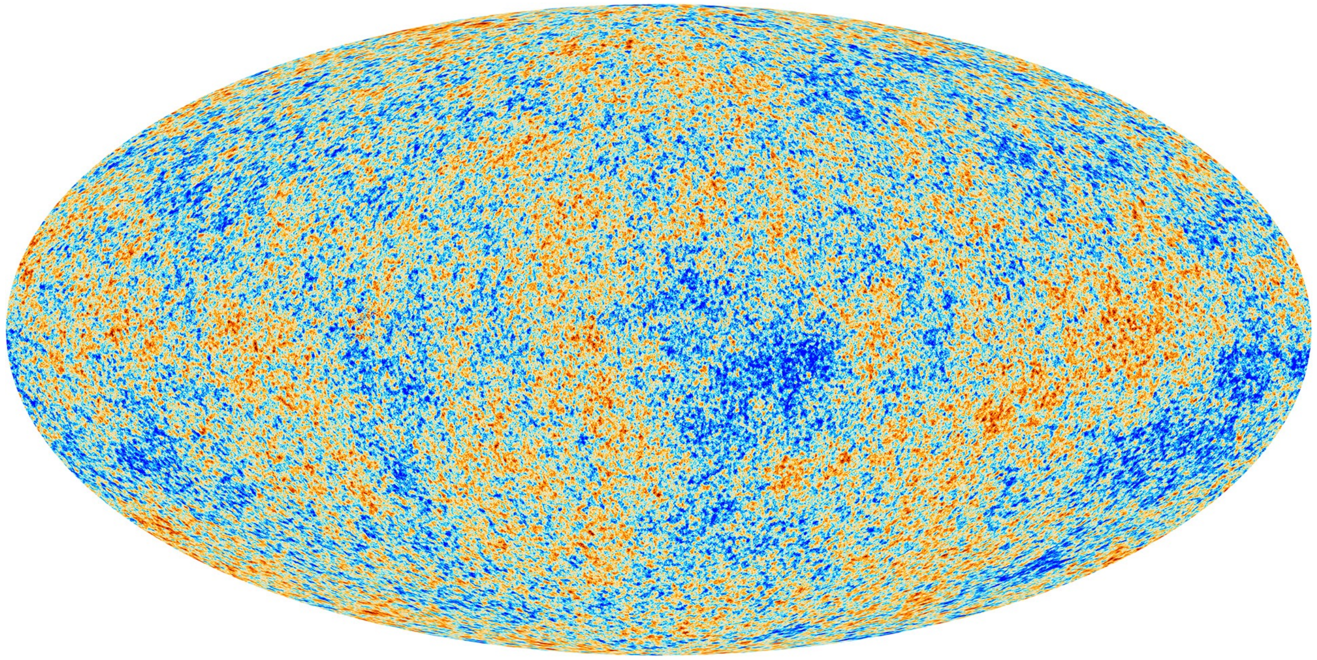
$$\sigma \sim \frac{1}{\sqrt{N}}$$

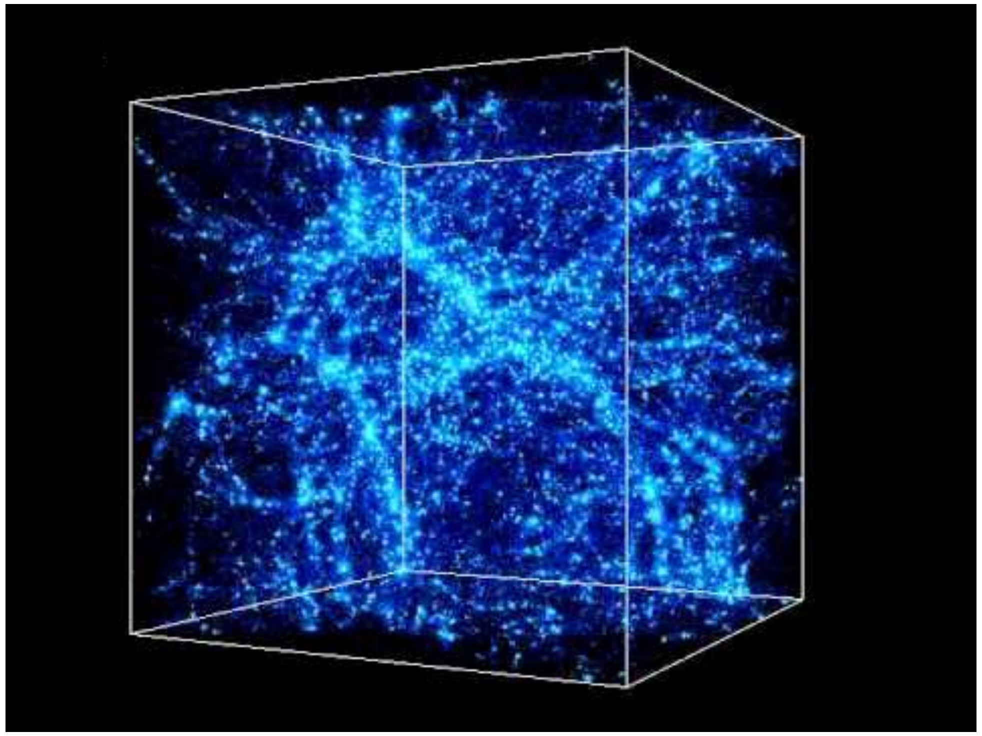
error bar

# of modes

$$N_{\rm CMB} \sim l_{max}^2 \sim 10^7$$

$$N \sim \left(\frac{\lambda_{max}}{\lambda_{min}}\right)^3 \sim 10^9$$





Existing surveys
(biased sample)

SDSS

DARK ENERGY
SURVEY

2MASS
2 MICRON ALL SKY SURVEY

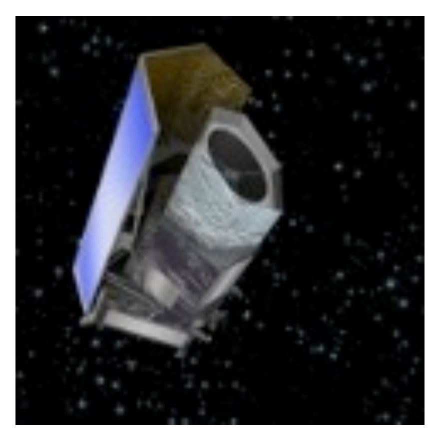
PAU

Future surveys
(biased sample)

LSST

SKA
SQUARE KILOMETRE ARRAY

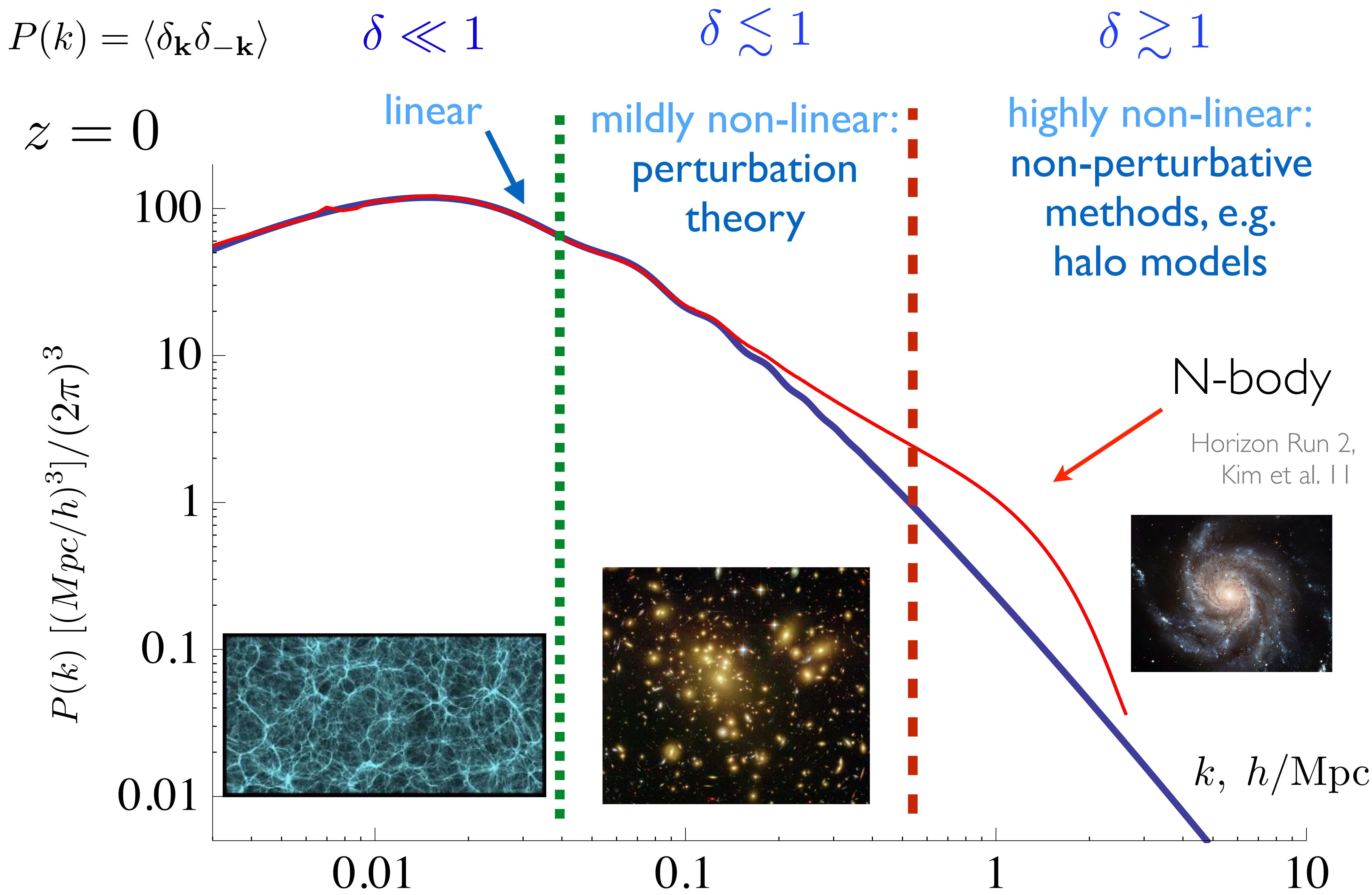
Euclid



Dark Energy Spectroscopic Instrument

DESI

Non-linearities come into play



Large Scale Structures: basics

I) Initial distribution

$$\langle \delta_0(\mathbf{k})\delta_0(\mathbf{k}')\rangle = P_0(k)\delta^{(3)}(\mathbf{k}'+\mathbf{k})$$

δ random stochastic variable

II) Time evolution: Eulerian pressureless perfect fluid description

$$\frac{\partial \delta}{\partial \tau} + \nabla\cdot(1+\delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla\Phi$$

Poisson equation:

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

$\delta = \rho/\bar{\rho} - 1$	density conrast
$\vec{v}$	pecular velocity
H	Hubble param.
Φ	Newton's potential

III) Fluctuations are small on large scales

$$\delta \ll 1$$

Standard Perturbation Theory

$$\frac{\partial \delta(\mathbf{k})}{\partial t} - \theta(\mathbf{k}) = \int d\mathbf{q}_1 d\mathbf{q}_2 \delta_D^{(3)}(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \frac{(\mathbf{q}_1 + \mathbf{q}_2) \cdot \mathbf{q}_2}{q_2^2} \delta(\mathbf{q}_1) \theta(\mathbf{q}_2)$$

$$\frac{\partial \theta(\mathbf{k})}{\partial t} - \frac{3}{2}\delta(\mathbf{k}) + \frac{1}{2}\theta(\mathbf{k}) = \int d\mathbf{q}_1 d\mathbf{q}_2 \delta_D^{(3)}(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \frac{(\mathbf{q}_1 + \mathbf{q}_2)^2 (\mathbf{q}_1 \cdot \mathbf{q}_2)}{2 q_1^2 q_2^2} \theta(\mathbf{q}_1) \theta(\mathbf{q}_2)$$

$$\theta \equiv -\frac{\nabla \vec{v}}{aH}$$

$$t = \log a(\tau)$$

Perturbative Ansatz:

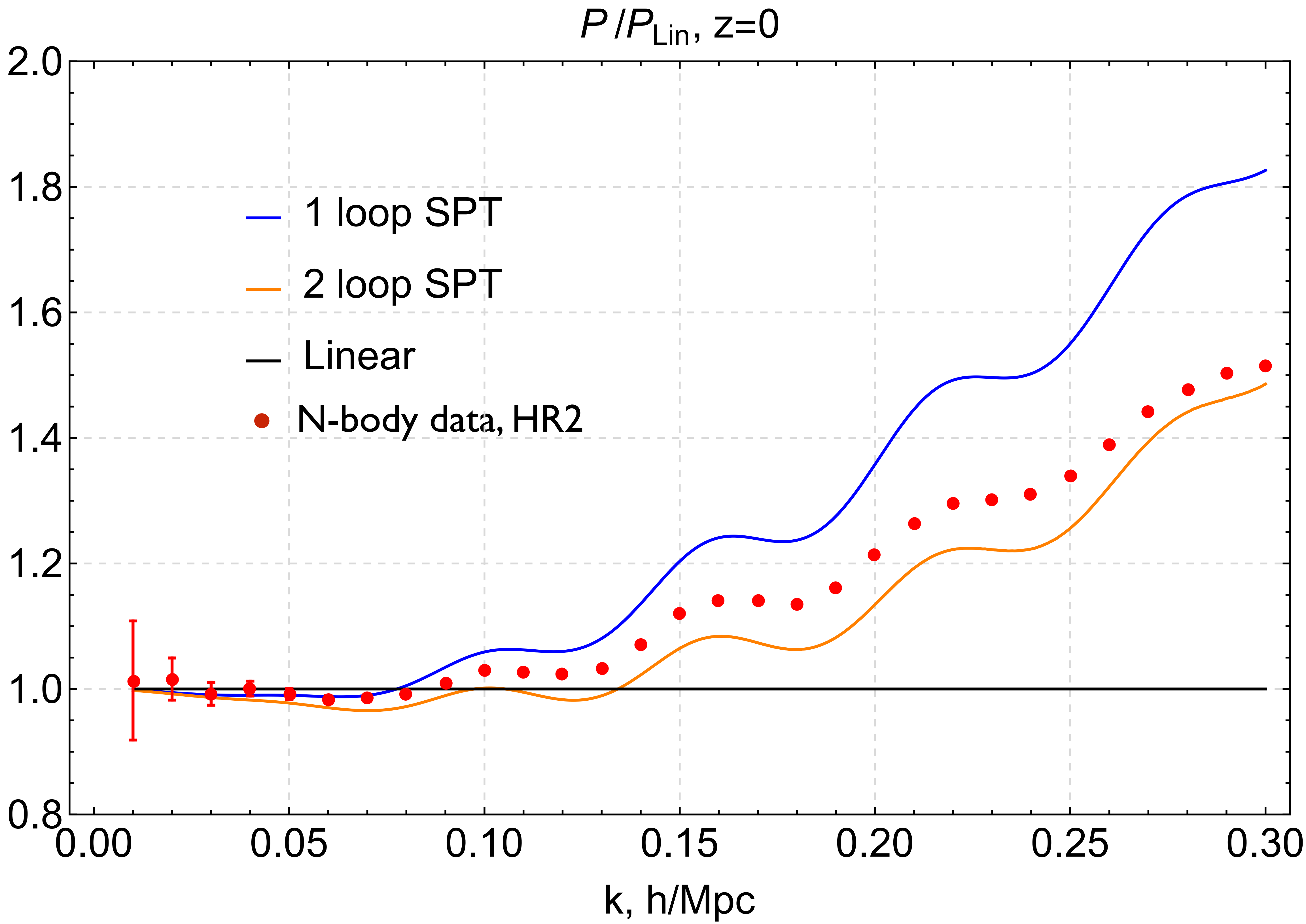
$$\theta(\mathbf{q}) = \sum_n \int G_n(\mathbf{q}_1, ..., \mathbf{q}_n) \delta_L(\mathbf{q}_1) ... \delta_L(\mathbf{q}_n)$$

$$\delta(\mathbf{q}) = \sum_n \int F_n(\mathbf{q}_1, ..., \mathbf{q}_n) \delta_L(\mathbf{q}_1) ... \delta_L(\mathbf{q}_n)$$

Loop corrections:

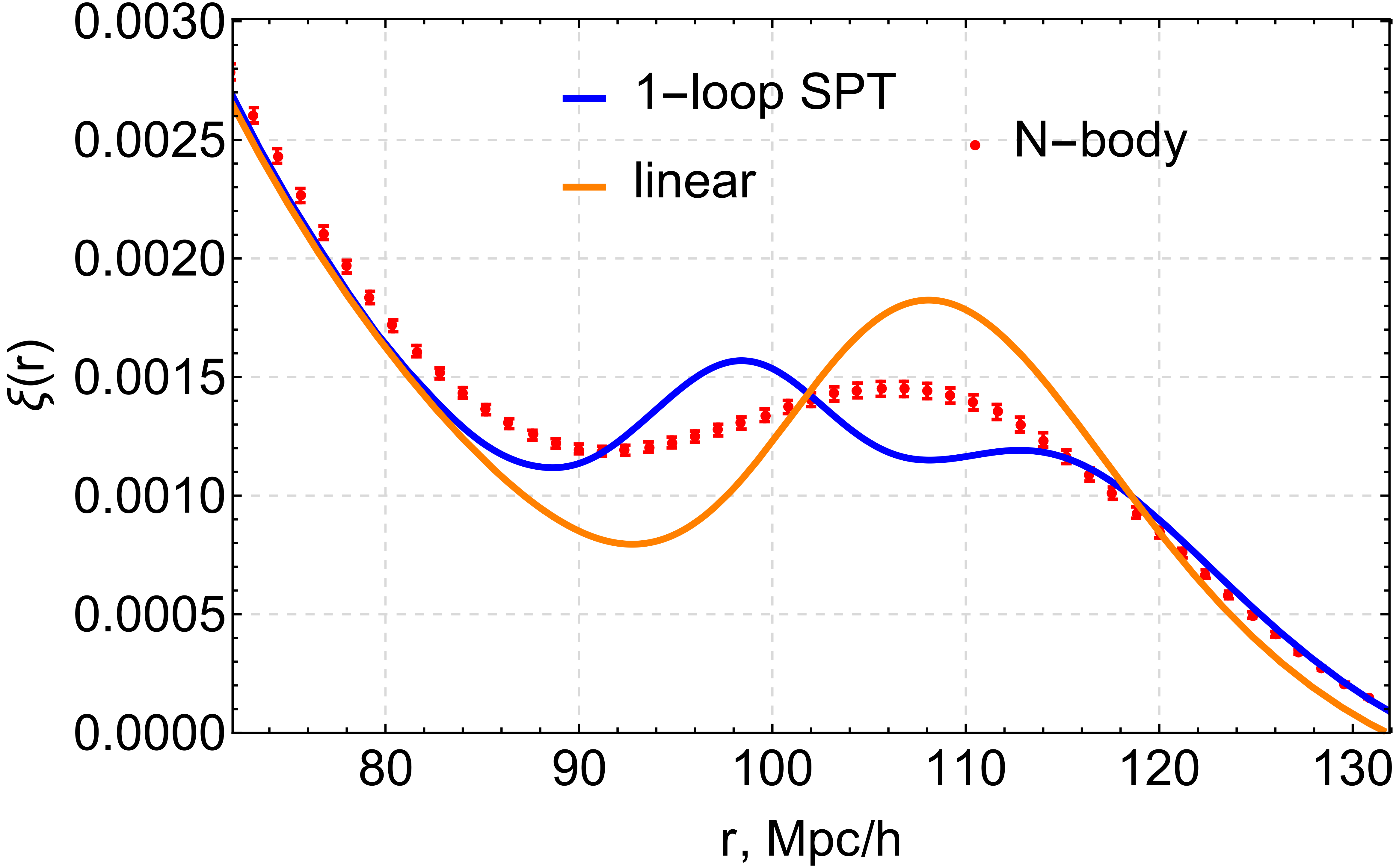
$$\langle |\delta(\mathbf{k})|^2 \rangle = P_L(k) + \int_0^\infty P_L^2 (2F_2^2 + 6F_3) \quad + ...$$

Performance of SPT loop expansion: UV

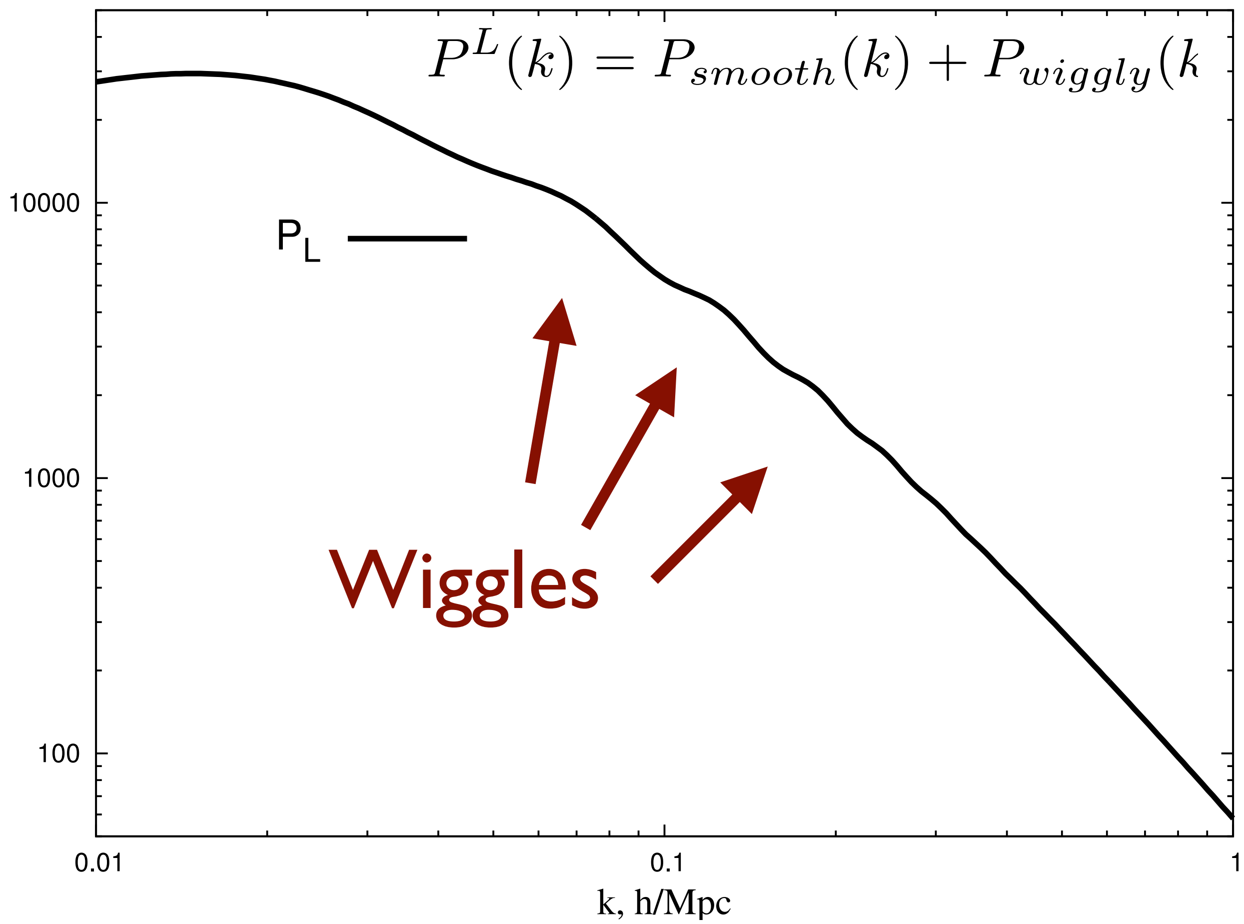


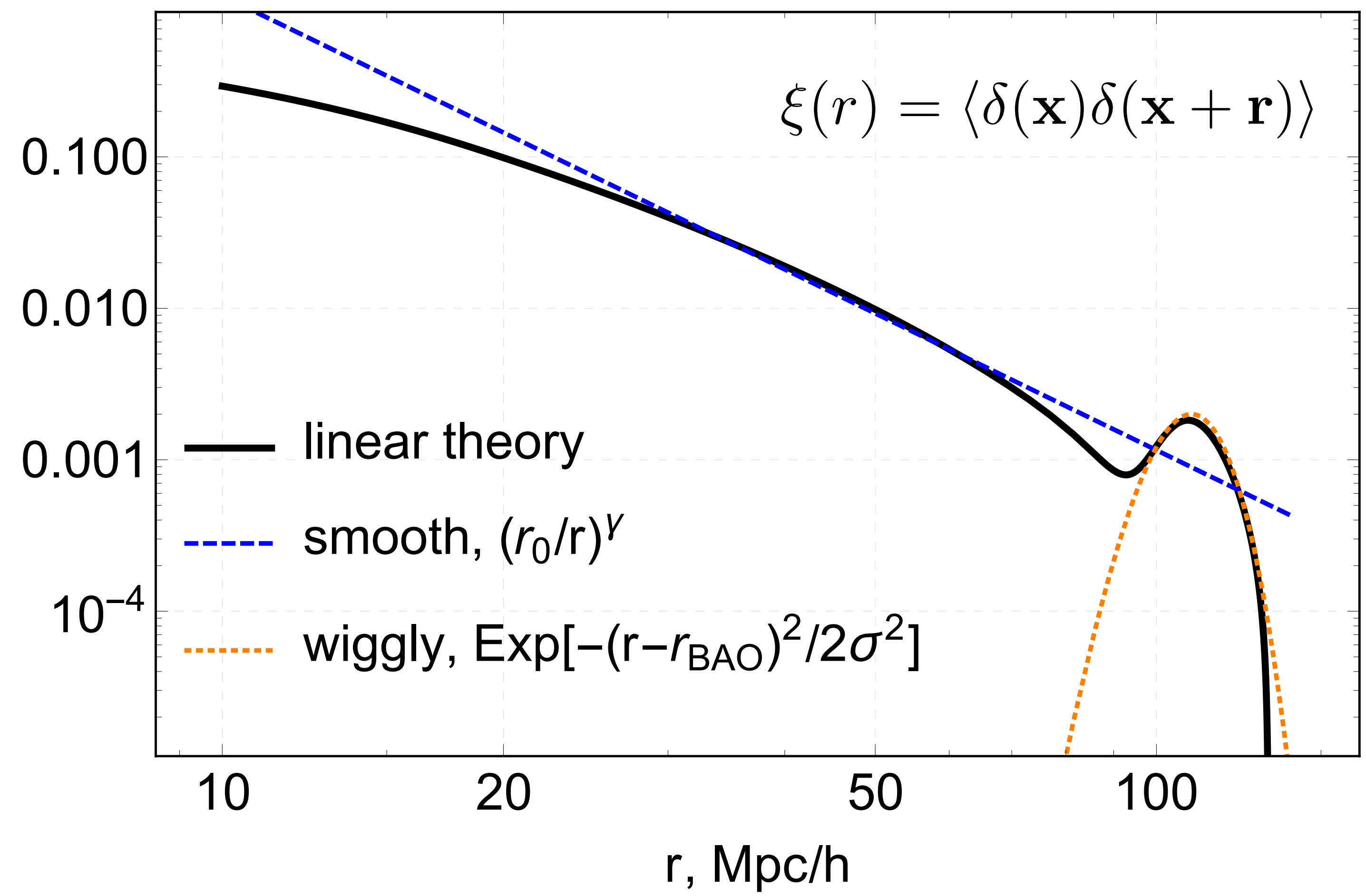
Performance of SPT loop expansion: IR

$$\xi(r) = \int d^3k e^{i\vec{k}\cdot\vec{r}} P(k)$$



What went wrong?



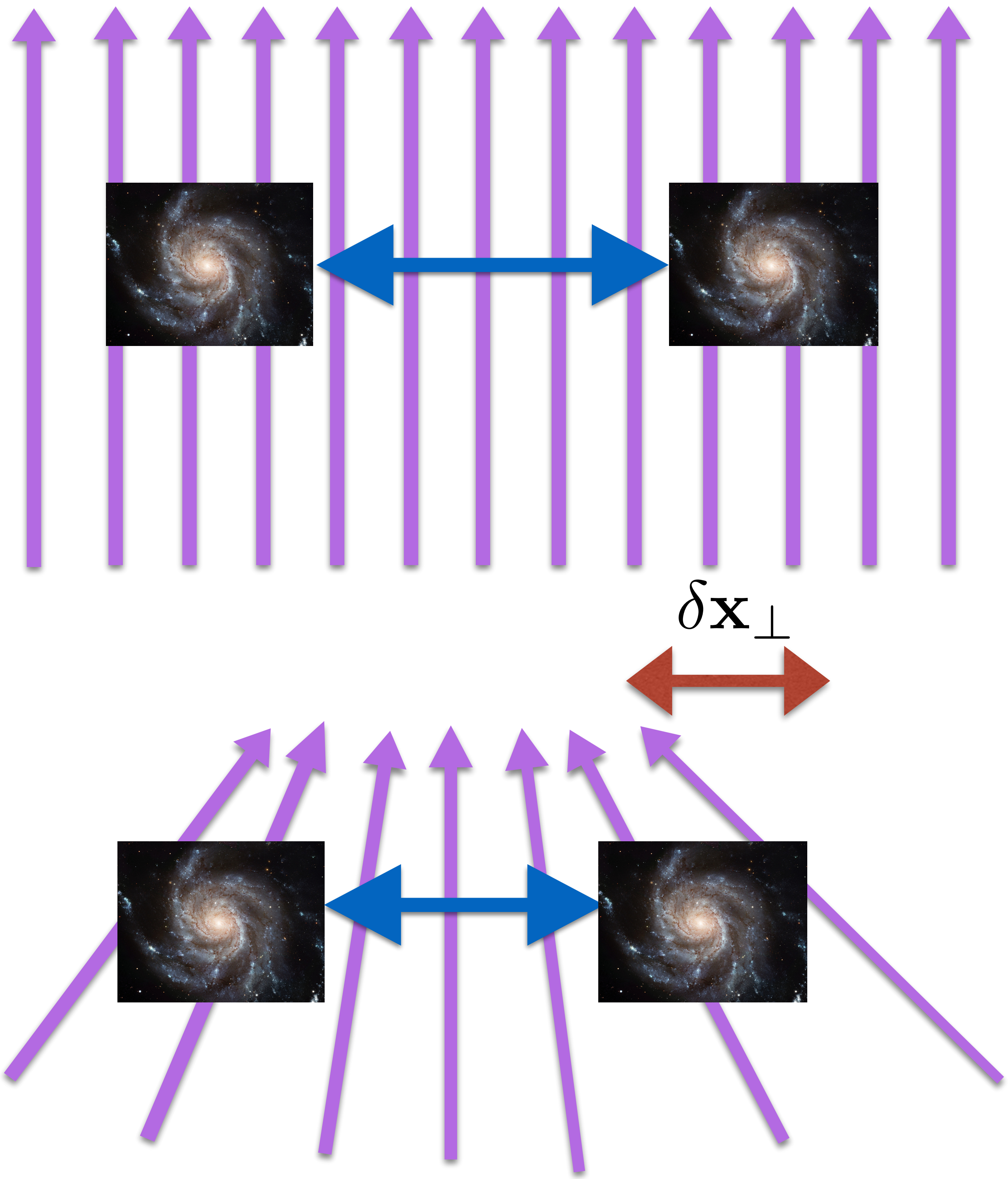


$$P_{lin} = P_{smooth} + P_{wiggly}$$

UV - sensitive

IR - sensitive

What went wrong? IR story 1



Uniform motion -

No effect!

Gradient in the flow

$$\xi(|\mathbf{x}+\delta\mathbf{x}_\perp|)$$

$$= \int [dk] P(k) e^{i\mathbf{k}(\mathbf{x}+\delta\mathbf{x}_\perp)}$$

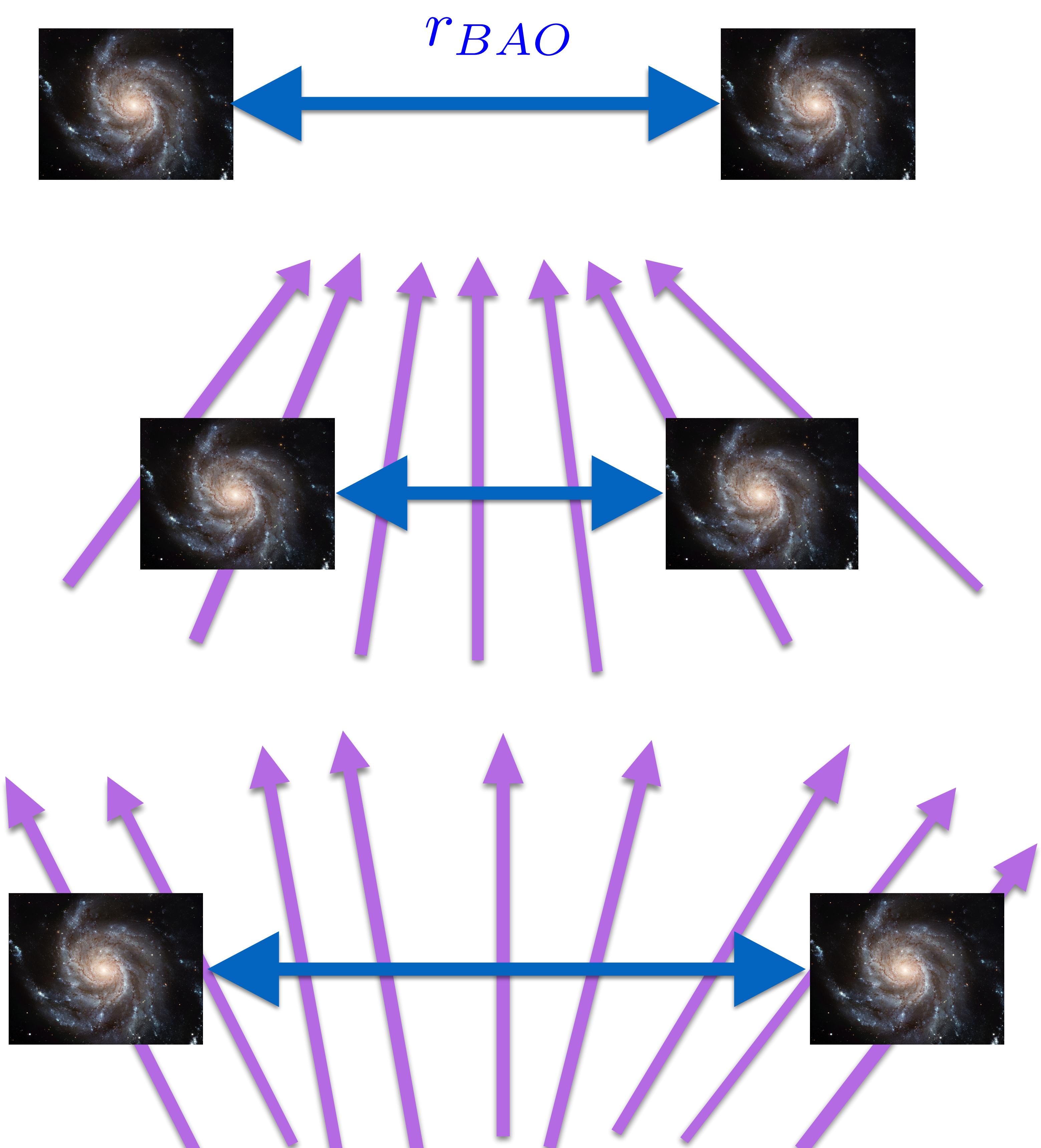
$$= e^{\delta\mathbf{x}_\perp\cdot\nabla} \int [dk] e^{i\mathbf{k}\cdot\mathbf{x}} P(k)$$

$$= e^{\delta\mathbf{x}_\perp\cdot\nabla} \xi(x)$$

$$\xi_{smooth} = \left(\frac{r_0}{r}\right)^\gamma$$

$$\delta\mathbf{x}_\perp \nabla \sim \frac{\delta x_\perp}{r} \sim 0.1$$

What went wrong? IR story 2



$$\xi_{wiggly} \sim e^{-\frac{(r-r_{BAO})^2}{2\sigma^2}}$$

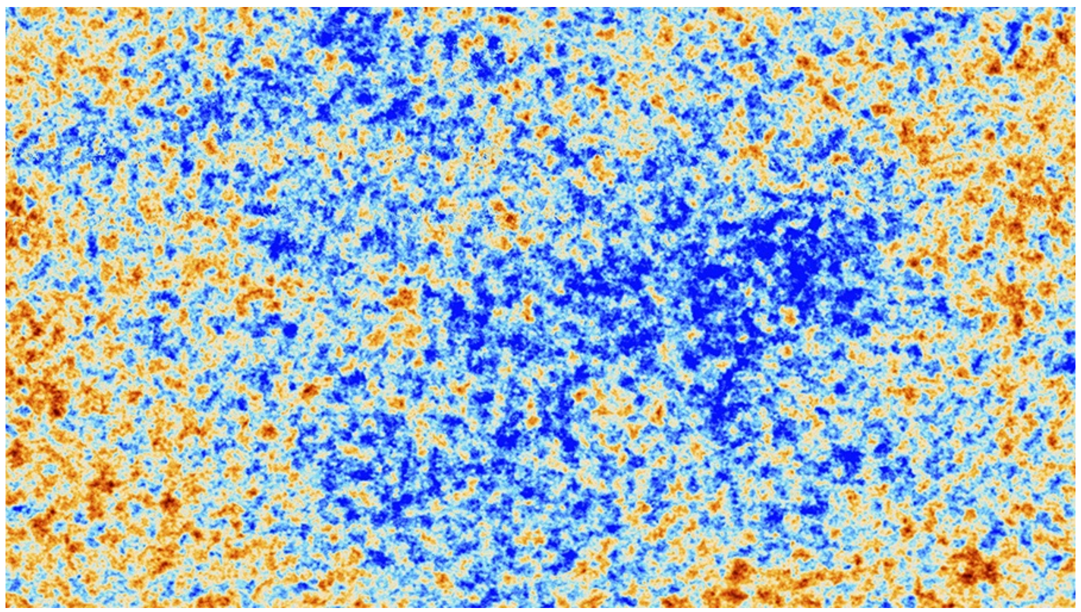
$$\delta \mathbf{x}_\perp \nabla \sim \frac{\delta x_\perp}{\sigma} \sim 1$$

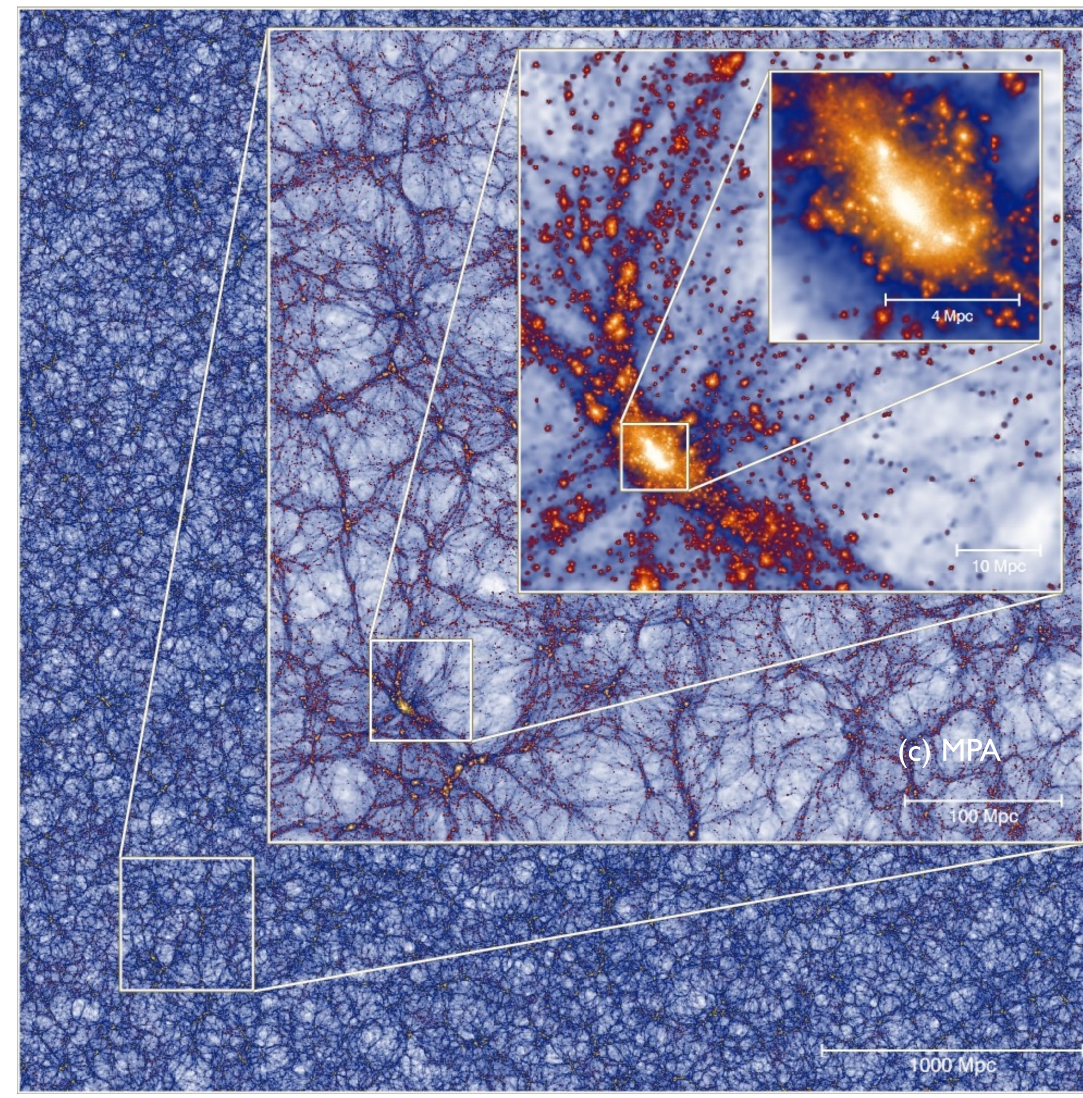
$$\xi_{IR} = e^{\langle \delta x_\perp^2 \rangle \nabla^2} \xi_{lin}(x)$$

$$\delta x_\perp \propto \delta$$

IR resummation needed !

What went wrong? UV story I





✓ SPT integrals are UV sensitive
- large contribution from short modes, where fluid approx. breaks down

✓ Halos are not taken into account

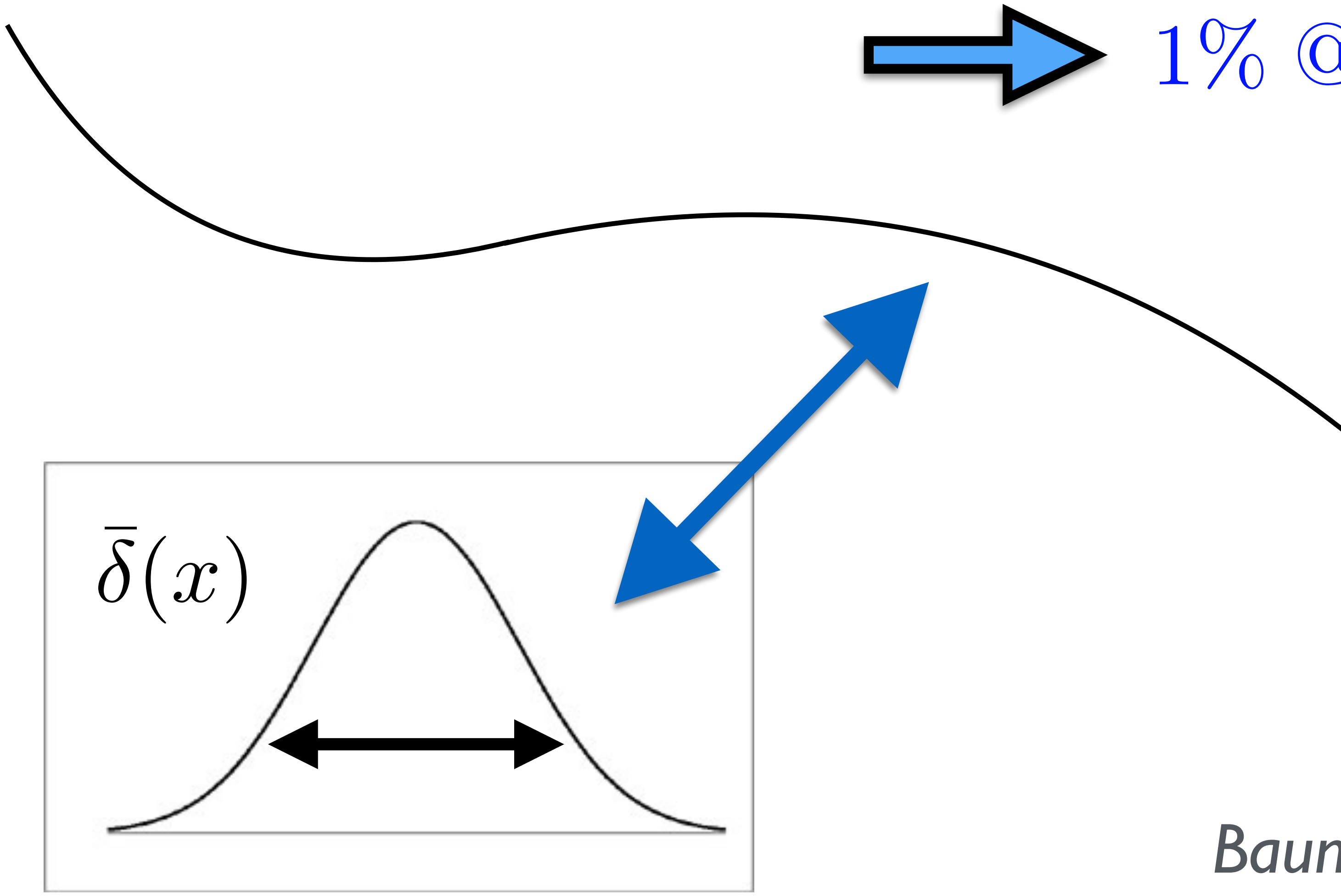
What went wrong? UV story 2

$$\delta(\mathbf{k}) = \int d^3x\, \bar{\delta}(x) e^{i\mathbf{k}\cdot\mathbf{x}} = \int d^3x\, \bar{\delta}(x)\left(\underbrace{\cancel{1}}_{0} + \underbrace{\cancel{i\mathbf{k}\cdot\mathbf{x}}}_{0} - \frac{1}{2}(\mathbf{k}\cdot\mathbf{x})^2 + \ldots\right) \propto (k\, R_{\rm vir})^2$$

$$R_{\rm vir} \sim 1\ \mathrm{Mpc}$$

$$P(k) \supset k^2 R_{\rm vir}^2\, P_L(k)$$

$\Rightarrow$ $1\%\ @\ k \sim 0.1\mathrm{Mpc}^{-1}$



$x \sim R_{\rm vir}$

EFT to rescue!

describe dynamics of
long-wavelength dofs
through effective operators

Baumann, Nicolis, Senatore, Zaldarriaga' 2010
Carrasco, Hertzberg, Senatore' 2012
...

Recap: UV and IR issues have to be fixed

IR - eqs are OK, perturb. theory is wrong

UV - complicated physics: non-linear back reaction

Can we fix these issues while working directly with correlation functions (only they are of interest)?

Can there be a true QFT description?

Time - sliced perturbation theory (TSPT)

Time evolution of fields $\longleftrightarrow$ **Time evolution of PDF**

SPT, EFT,…

Blas, Garny, MI, Sibiryakov, 1512.05807

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot (1+\delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla\Phi + \ldots$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

$$\frac{d}{d\tau}\left(\mathcal{D}\delta\ \mathcal{P}[\delta;\tau]\right) = 0$$

$$\boxed{\begin{gathered}\partial_t \mathcal{P} + \frac{\partial}{\partial \delta}(\dot{\delta}\mathcal{P}) = 0 \\ \mathcal{P}[\delta]\Big|_{t=0} = \mathcal{P}[\delta_0]\end{gathered}}$$

Gaussian ICs:

Liouville eqn.

$$\langle \delta_0(\mathbf{k})\delta_0(\mathbf{k}')\rangle = P_0(k)\delta^{(3)}(\mathbf{k}'+\mathbf{k})$$

$$Z[J] = \mathcal{N}^{-1}\int \mathcal{D}\delta_0\ \exp\left\{-\frac{\delta_0^2}{2P_0} + J\cdot\delta_t\right\}$$

$$Z[J] = \mathcal{N}^{-1}\int \mathcal{D}\delta_t\ \mathcal{P}[\delta_t]\ \exp\left\{J\cdot\delta_t\right\}$$

$$\delta_t = G(t)\delta_0 + G^2(t)F_2\delta_0^2 + \ldots$$

Time - sliced perturbation theory (TSPT)

Perturbative solution for Gaussian i.c.

$$\mathcal{P}(\delta_0, t_0) = \mathcal{N}^{-1} \exp\left\{ -\frac{1}{2} \int \frac{\delta_0(k)\delta_0(-k)}{P(k,t_0)} \right\}$$

$$\Rightarrow \quad \mathcal{P}(\delta, t) = \mathcal{N}^{-1} \exp\left\{ -\sum_{n=1} \frac{1}{n!} \int [dq]^n \Gamma_n(t, \{q_j\})\, \delta^n \right\}$$

vertices

EOM $\quad \dot{\delta}_k(t) = \mathcal{I}_k(\delta, t) = \sum_n \frac{1}{n!} \int [dq]^n I_n(k, \{q_j\})\, \delta^n$

$$\partial_t \mathcal{P} + \frac{\partial}{\partial \delta}(\dot{\delta}\mathcal{P}) = 0 \quad \blacktriangleright \quad \dot{\Gamma}_n^{tot} = -n\Gamma_n^{tot} - \sum_{m=2}^{n} C_n^m I_m \Gamma_{n-m+1}^{tot} + \int I_{n+1}$$

✓ **Consistent hierarchical equations (compare to BBGKY):**

Γ_n^{tot} from Γ_m^{tot} with $m \leq n$

TSPT diagrams

We solve **exactly** for $$\mathcal{P}(\delta, t) = \frac{e^{-\sum_{n=2}\frac{1}{n!}\int[dk_i]\Gamma_n(t,k_n)\Pi\delta^n_{k_n}}}{\mathcal{N}}$$

and **perturbatively** for $$Z[J,t] = \int[\mathcal{D}\delta]\,\mathcal{P}[\delta,t]e^{\int[dk]J_k\delta_{-k}}$$

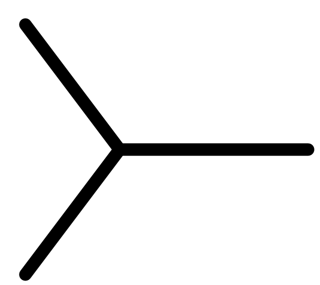
Gaussian integral: Wick theorem + Feynman rules as in QFT

Propagator = $P^L = \dfrac{1}{\Gamma_2} =$

k

QFT: Γ_n - 1PI n - p.f.

Vertex $\langle\delta^3\rangle \sim \Gamma_3 =$



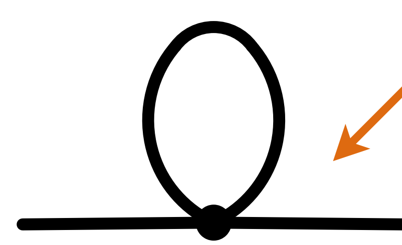
$$\delta_L = D_+\delta_0$$

$$D_+^2 \equiv g^2$$

$D_+^2(t)$ $D_+^4(t)$

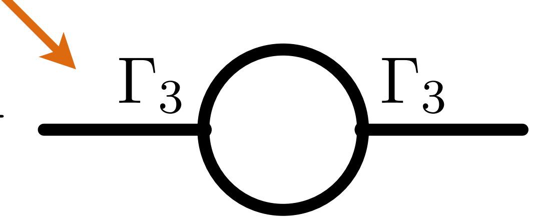
$P_{1-loop} =$

$+$



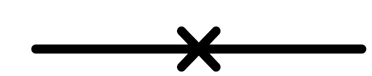
Γ_4

$+$ Γ_3



Γ_3

+ C_2



TSPT in a nutshell

$$Z[J, J_\Theta] = \int \mathcal{D}\delta \; \exp\left\{ \underbrace{-\int \frac{\delta^2}{2g^2 P_0(k)} - \sum_{n=3}^{\infty} \frac{1}{n!} \int \frac{\Gamma_n \delta^n}{g^2}}_{\Gamma[\delta;\tau]} + J\delta + J_\Theta \Theta[\delta] \right\}$$

$+$ counterterms

3d Euclidean QFT:

- ★ $\Gamma[\delta;\tau]$ 1PI effective action
- ★ $g^2 \equiv D_+^2$ coupling constant
- ★ τ external parameter (~temperature)
- ★ $\Theta[\delta] = \sum_{n=1}^{\infty} K_n \delta^n$ composite source (only one statistically independent variable!)

BAO IR - resummation

Step I: smooth + wiggly decomposition

$$P^L(k) = P_{smooth}(k) + P_{wiggly}(k) \quad \Rightarrow \quad \Gamma_n = \Gamma_n^{smooth} + \Gamma_n^{wiggly}$$

Step II: identify enhanced diagrams

$$\Gamma_n^{smooth}(\mathbf{k}, -\mathbf{k}, \mathbf{q}_1, ..., \mathbf{q}_{n-2}) = \mathcal{O}(1) \qquad q_i \to 0$$

$$\boxed{\Gamma_n^{wiggly}(\mathbf{k}, -\mathbf{k}, \mathbf{q}_1, ..., \mathbf{q}_{n-2}) = \mathcal{O}(1) \cdot \left(\frac{k}{k_{osc}}\right)^{n-2} \gg 1} \qquad \frac{k}{k_{osc}} \sim 10$$

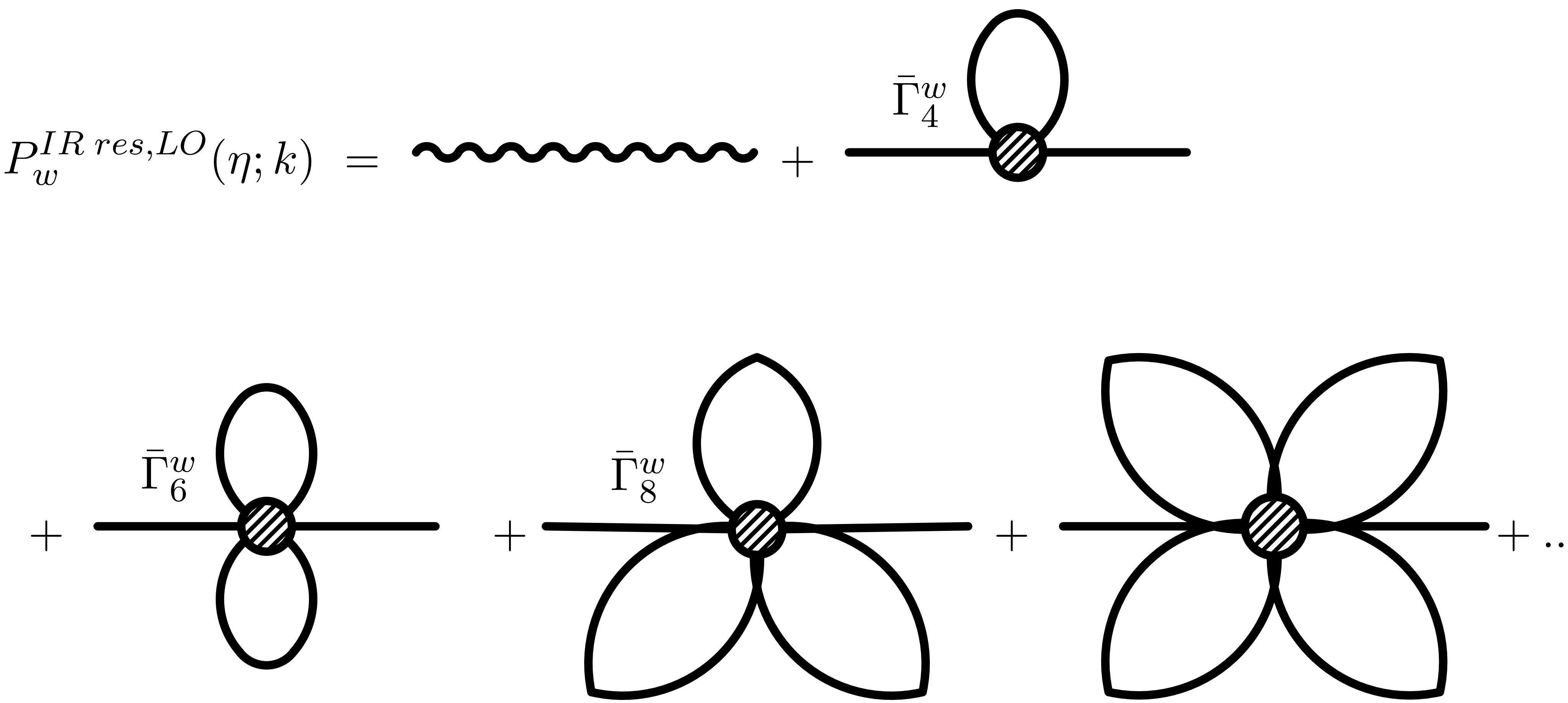
Step III: resumm them at a desired order of precision

NB. Valid for any correlation function

IR - resummation in TSPT

Daisy graphs

$$P_w^{IR\,res,LO}(\eta;k) =$$

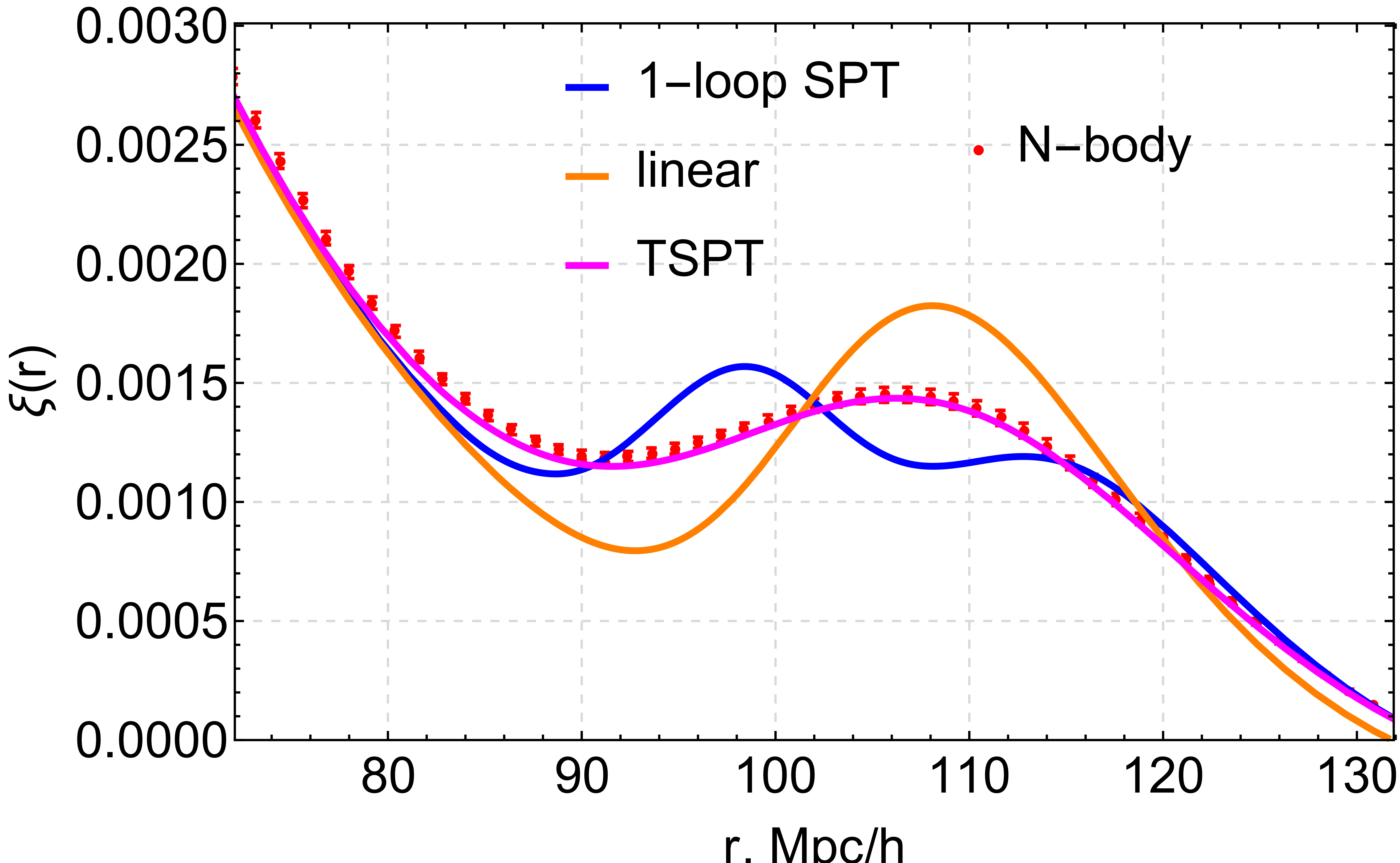


$$= \underline{e^{-\Sigma^2 k^2}} P_w(k)$$

$$= \exp\left\{ -\frac{k^2}{3}\cdot 4\pi \int_0^{k_S} dq P_s^L(q)(1 - j_0(qr_{BAO}) + 2j_2(qr_{BAO}))\right\} P_w(k)$$

$BAO\ scale$

DM correlation function in TSPT



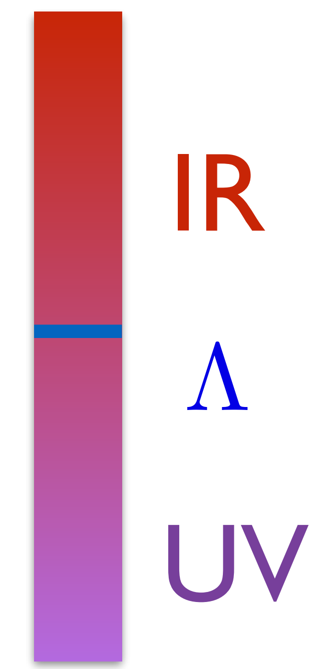
Data: Horizon Run

Blas, Garny, M.I., Sibiryakov 1605.02149

Short scales in TSPT

✓ path integral formulation allows for Wilsonian Renormalization and fixes the RG flow

$$Z[J] = \int \mathcal{D}\delta \ \exp\left\{ -\int \frac{\delta^2}{2\bar{P}} - \sum_{n=3}^{\infty} \frac{1}{n!} \int \Gamma_n \delta^n + J\delta \right\}$$



Step 1 Coarse grain fluctuations at scales smaller than Λ^{-1}

Step 2 Renormalize interaction vertices to ensure that physical observables are independent of Λ

$$\Gamma_n \to \Gamma_n^\Lambda$$

Wilsonian/Polchinski RG: $$\frac{d}{d\log\Lambda}\Gamma_n^\Lambda = \mathcal{F}(\Gamma_\Lambda)$$

Step 3 "Integration constants": finite counterterms encapsulating effects of short scales $C_n(\mathbf{k}_1, ..., \mathbf{k}_n)$

Fit or not to fit?

- Counterterms are unknown: match or measure (fit from data)
- EFT parameters are not fundamental: have to be marginalized over
- In this case, the "cheapest" prescription is to introduce them at the level of correlation functions, which TSPT does
- How many parameters do we need in practice? Degeneracies?

to be understood in future…

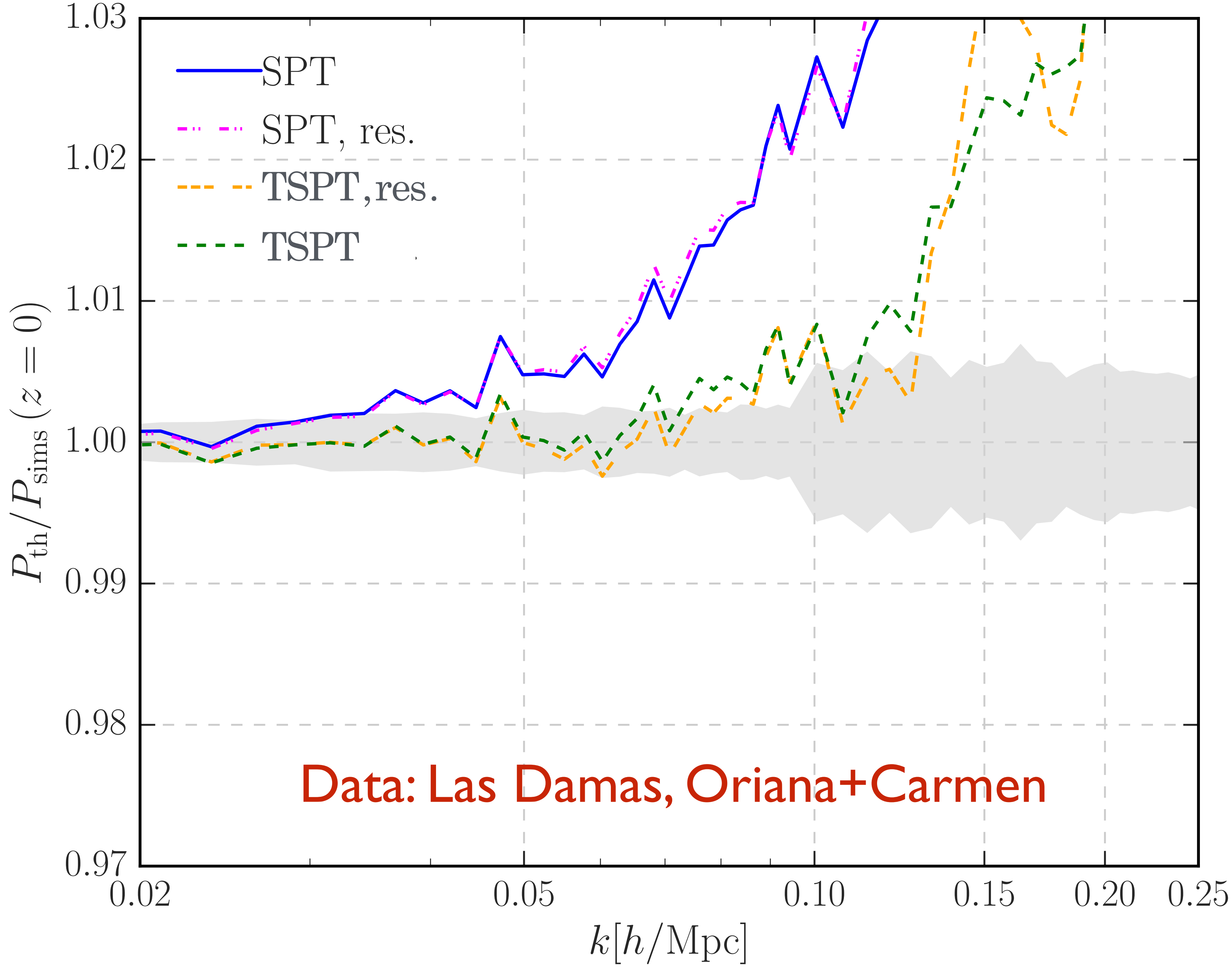
TSPT with only 1 parameter (at 1 loop) $\sim$ 1% fit to N-body up to ~ 0.15 h/Mpc both PS and BS

TSPT power spectrum

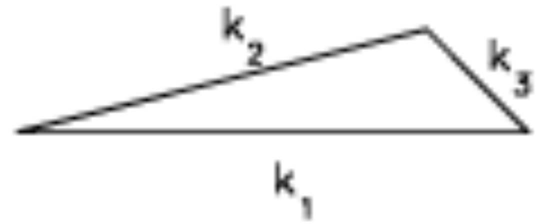
Preliminary

$$P^{1\text{ loop, ren}}(k) = P^{L}(k) + P^{1\text{ loop, SPT}}(k) - 2\gamma\, k^2 P^{L}(k)$$

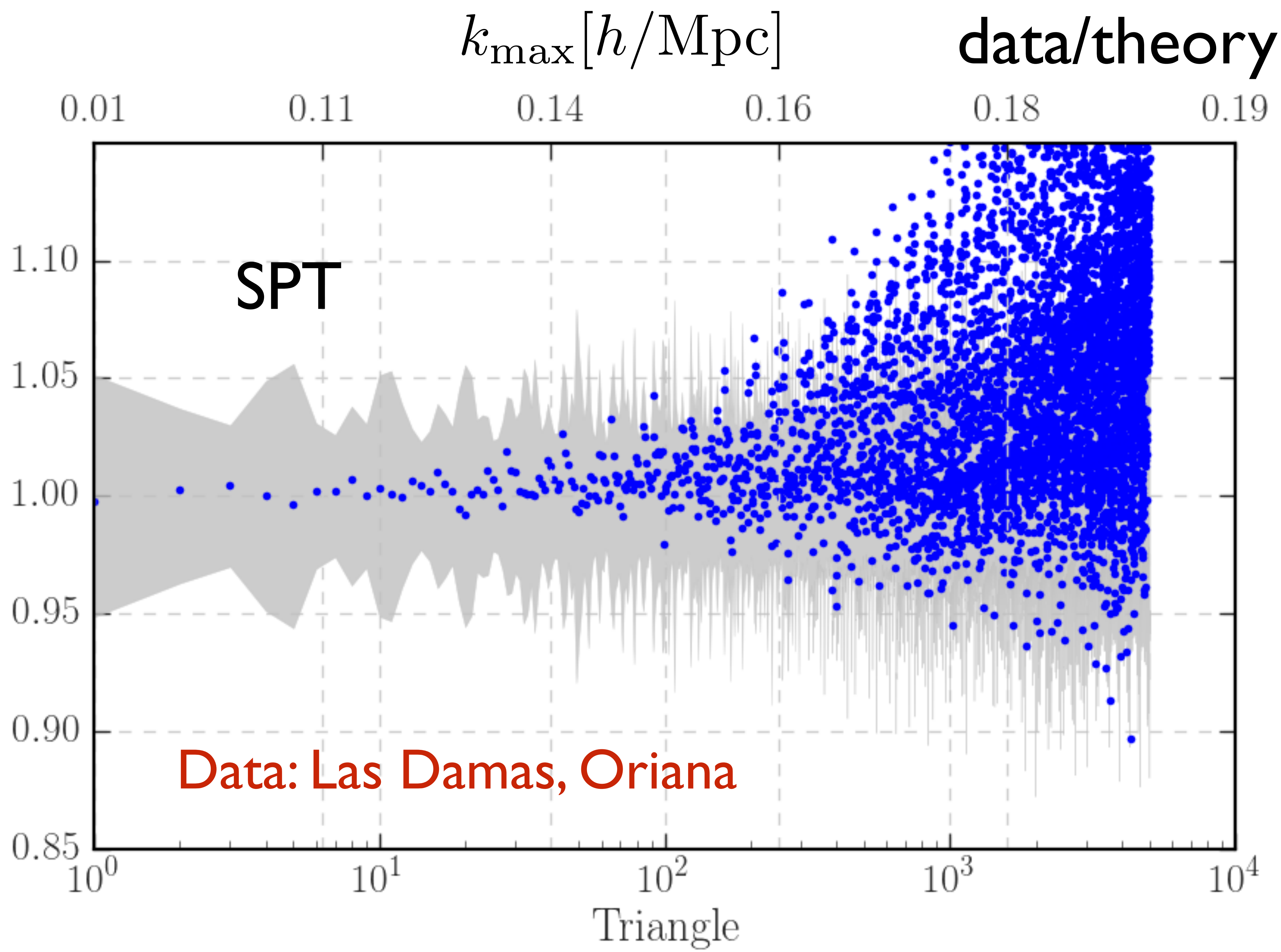
similar to EFT



TSPT Bispectrum: 0 parameter fit

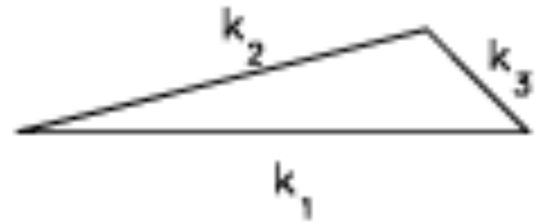


$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle = B(k_1, k_2, k_3)$$

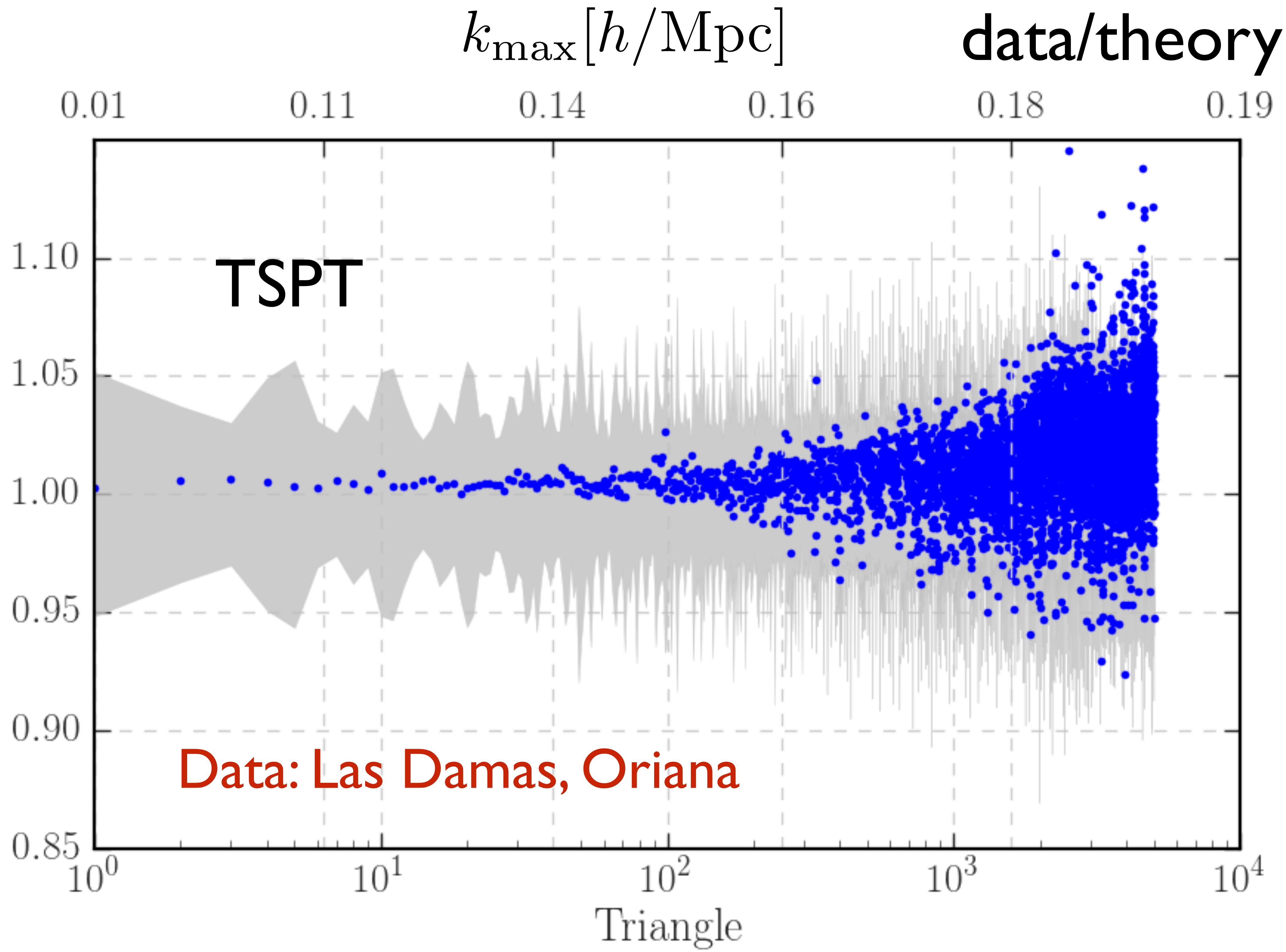


Preliminary

TSPT Bispectrum: 0 parameter fit



$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle = B(k_1, k_2, k_3)$$



Summary

- LSS - key observable for future cosmology
- TSPT: QFT way to deal with mildly non-linear effects
- IR effects are accounted for by systematic IR resummation

  ⇒ percent accuracy for the BAO

- UV effects are accounted for by Wilsonian EFT

  ⇒ k-reach at first order ~ 0.15 h/Mpc
  ~ 8 x more modes than SPT

Thank you for your attention !

Backup slides

TSPT vertices

$$\dot{\Gamma}_n^{tot} = -n\Gamma_n^{tot} - \sum_{m=2}^{n} C_n^m I_m \Gamma_{n-m+1}^{tot} + \int I_{n+1}$$

- ✓ Consistent hierarchical equations (compare to BBGKY):

  Γ_n^{tot} from Γ_m^{tot} with $m \leq n$

- ✓ time dependence factorizes for *t* independent I_n

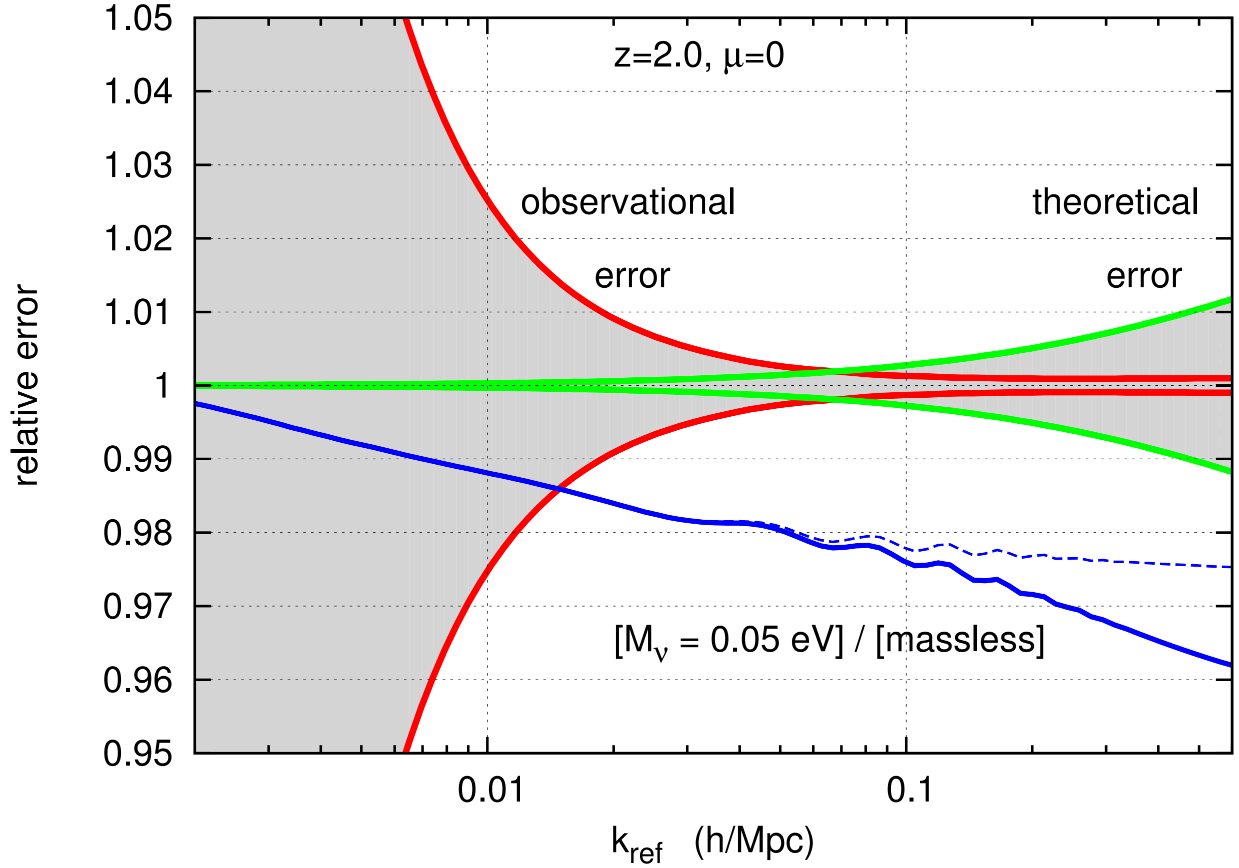
$$\Gamma_n^{tot}(t, \{k_n\}) = \frac{1}{D_+^2(t)} \bar{\Gamma}_n(\{k_n\}) + \bar{C}_n(\{k_n\})$$

$$\bar{\Gamma}_n(\{k_n\}) = -\frac{1}{n-2} \sum_{m=2} C_m^n I_m \bar{\Gamma}_{n-m+1} \qquad \bar{C}_n(\{k_n\}) = \ldots$$

- ✓ Perfect fluid equations as in SPT + EDs (dev. are small)
- ✓ Adiabatic IC: only one statistically independent variable

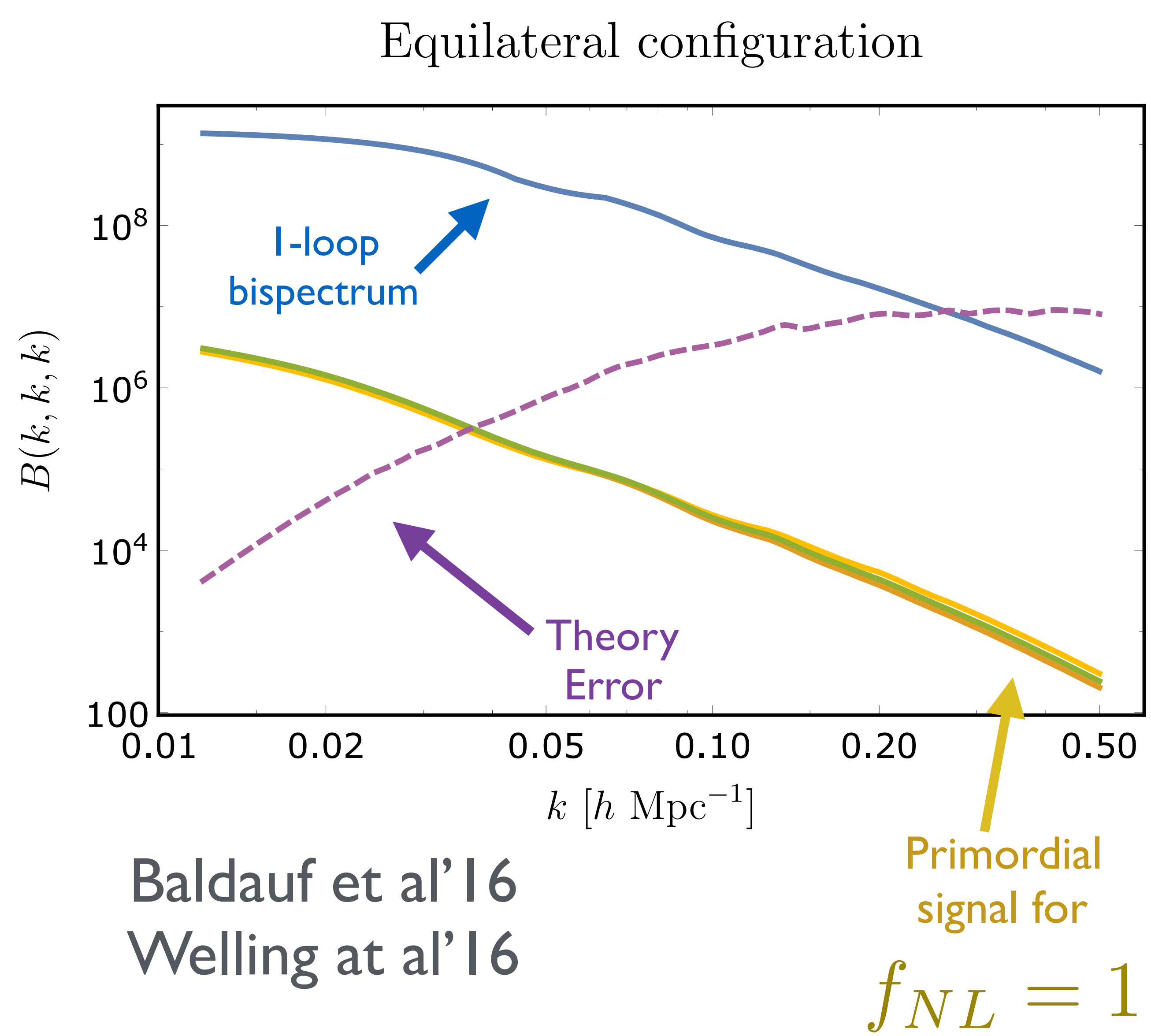
Examples

Neutrino masses



Audren et al'12
Baldauf et al'16

PNG



Baldauf et al'16
Welling at al'16

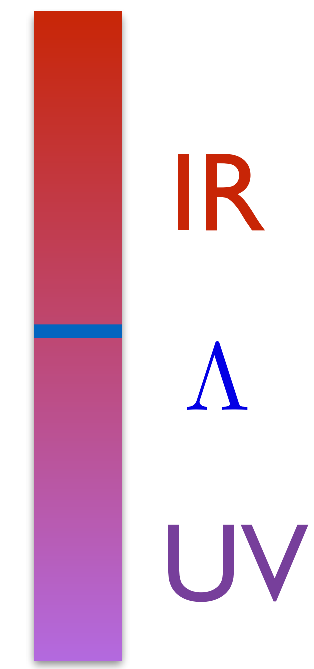
$$\Delta M_\nu = O(0.02\ \mathrm{eV}) \qquad \Delta f_{NL}^{\rm loc} = O(1) \qquad \Delta f_{NL}^{\rm equil} = O(10)$$

Planck: $\Delta M_\nu = O(0.1\ \mathrm{eV})$ $\Delta f_{NL}^{\rm loc.} = O(5)$ $\Delta f_{NL}^{\rm eq.} = O(50)$

Short scales in TSPT

✓ path integral formulation allows for Wilsonian Renormalization and fixes the RG flow

$$Z[J] = \int \mathcal{D}\delta \ \exp\left\{ -\int \frac{\delta^2}{2\bar{P}} - \sum_{n=3}^{\infty} \frac{1}{n!} \int \Gamma_n \delta^n + J\delta \right\}$$



Step 1 $\quad P^{\Lambda}(\eta; k) = \bar{P}(|\mathbf{k}|)\theta(\Lambda - k) = \begin{cases} \bar{P}(|\mathbf{k}|), & k < \Lambda \\ 0, & k > \Lambda \end{cases},$

Step 2 $\quad \Gamma_n \to \Gamma_n^{\Lambda}$

Wilsonian/Polchinski RG: $\dfrac{d}{d\log\Lambda}\Gamma_n^{\Lambda} = \mathcal{F}(\Gamma_{\Lambda})$

Step 3 "Integration constants": finite counterterms $C_n(\mathbf{k}_1, ..., \mathbf{k}_n)$ encapsulating effects of short scales

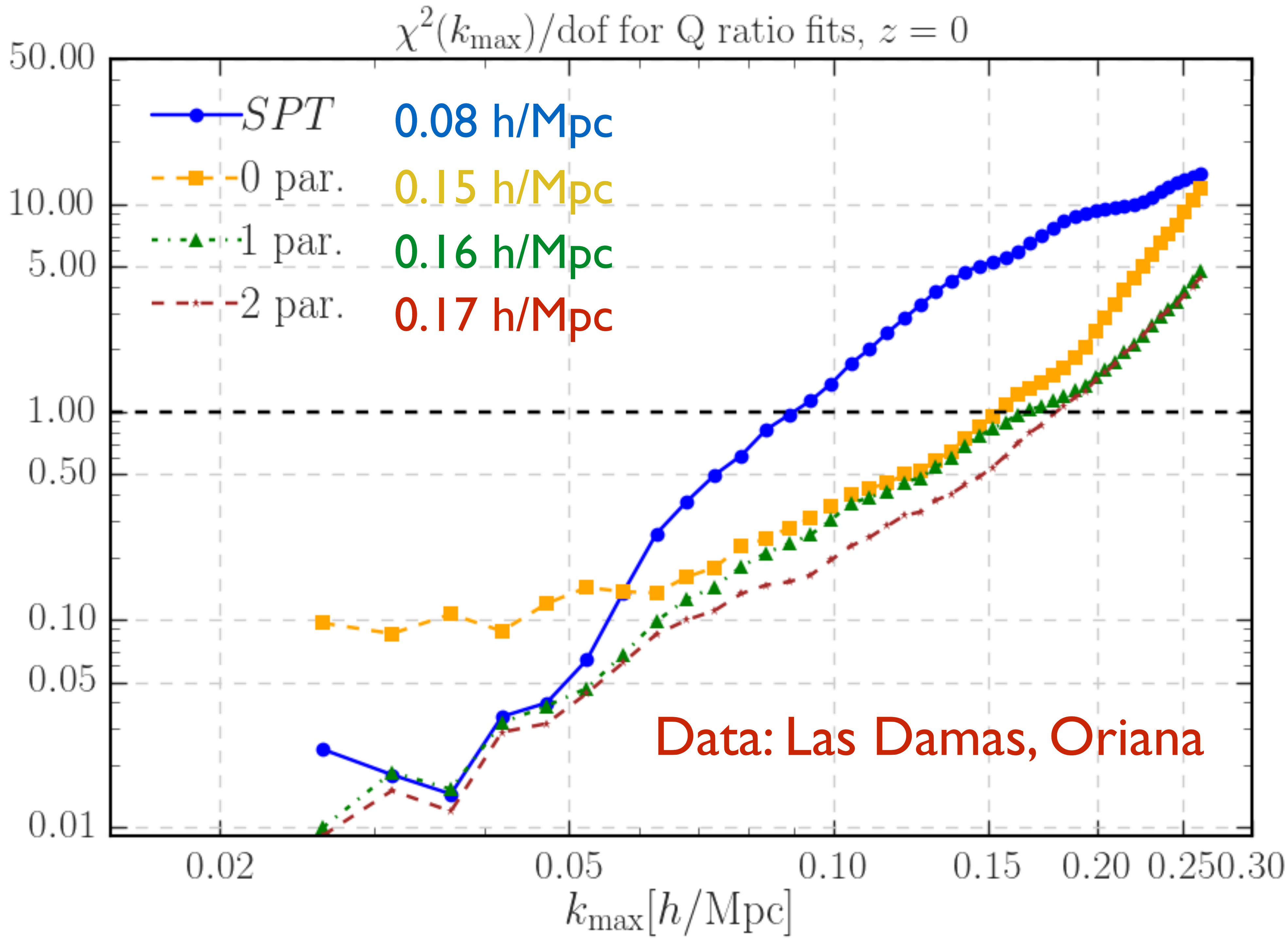
Analogy: QED $\quad \dfrac{d}{d\log\Lambda}\alpha = -b\alpha^2 \qquad \alpha(m_e) = 1/137$

Preliminary

TSPT Bispectrum: minimal models

$$\langle \delta_{\mathbf{k}_1}\delta_{\mathbf{k}_2}\delta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

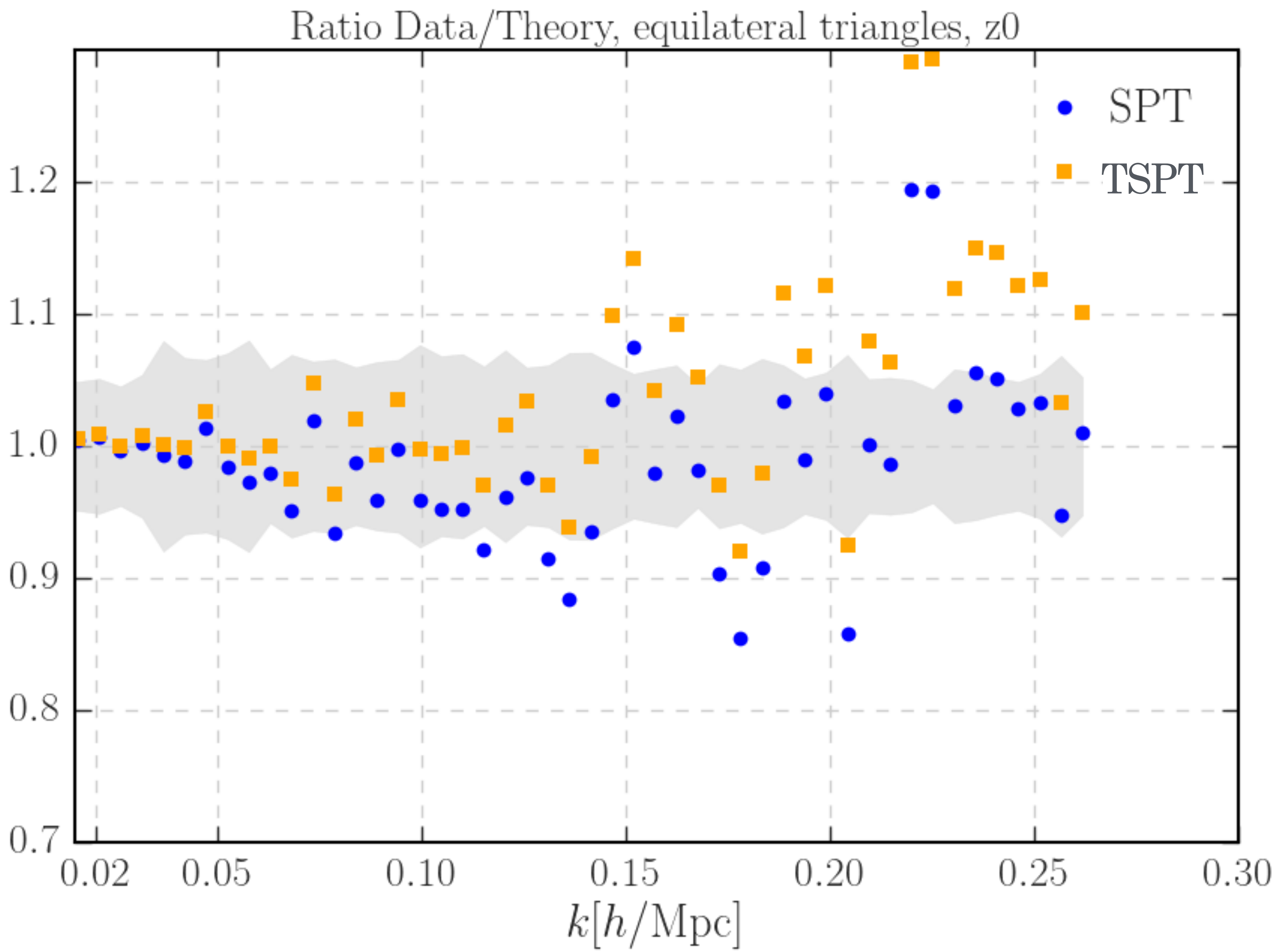
$$B^{\text{1-loop, ren}} = B^{\text{tree}} + B^{\text{1-loop, SPT}} + \gamma C_2 B^{\text{tree}} + \beta C_3$$



TSPT Bispectrum: 0 parameter fit

Preliminary

$$B^{\text{1-loop, ren}} = B^{\text{tree}} + B^{\text{1-loop, SPT}} + \gamma C_2 B^{\text{tree}}$$

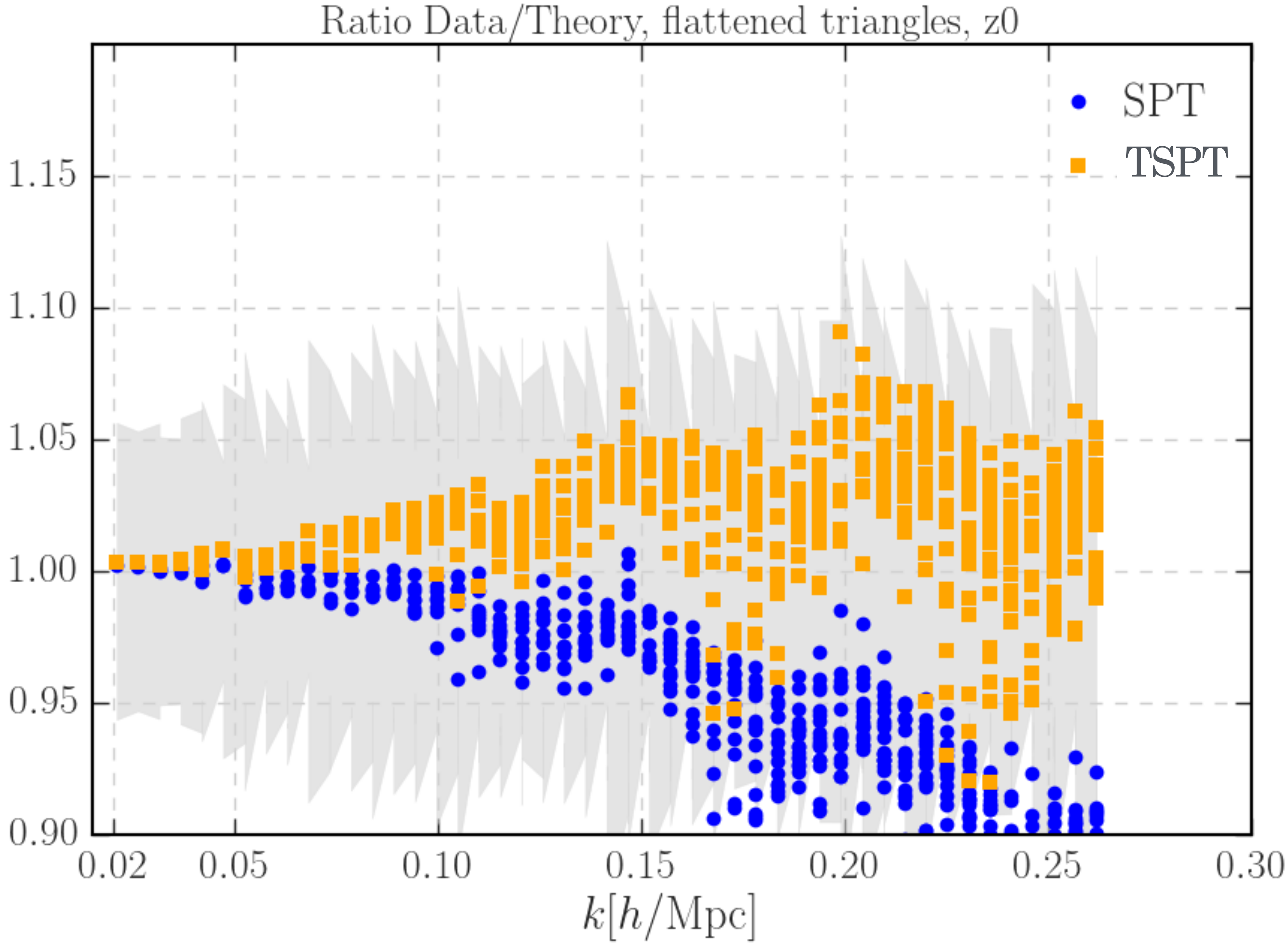


Data: Las Damas, Oriana

Preliminary

TSPT Bispectrum: 0 parameter fit

$$B^{\text{1-loop, ren}} = B^{\text{tree}} + B^{\text{1-loop, SPT}} + \gamma C_2 B^{\text{tree}}$$

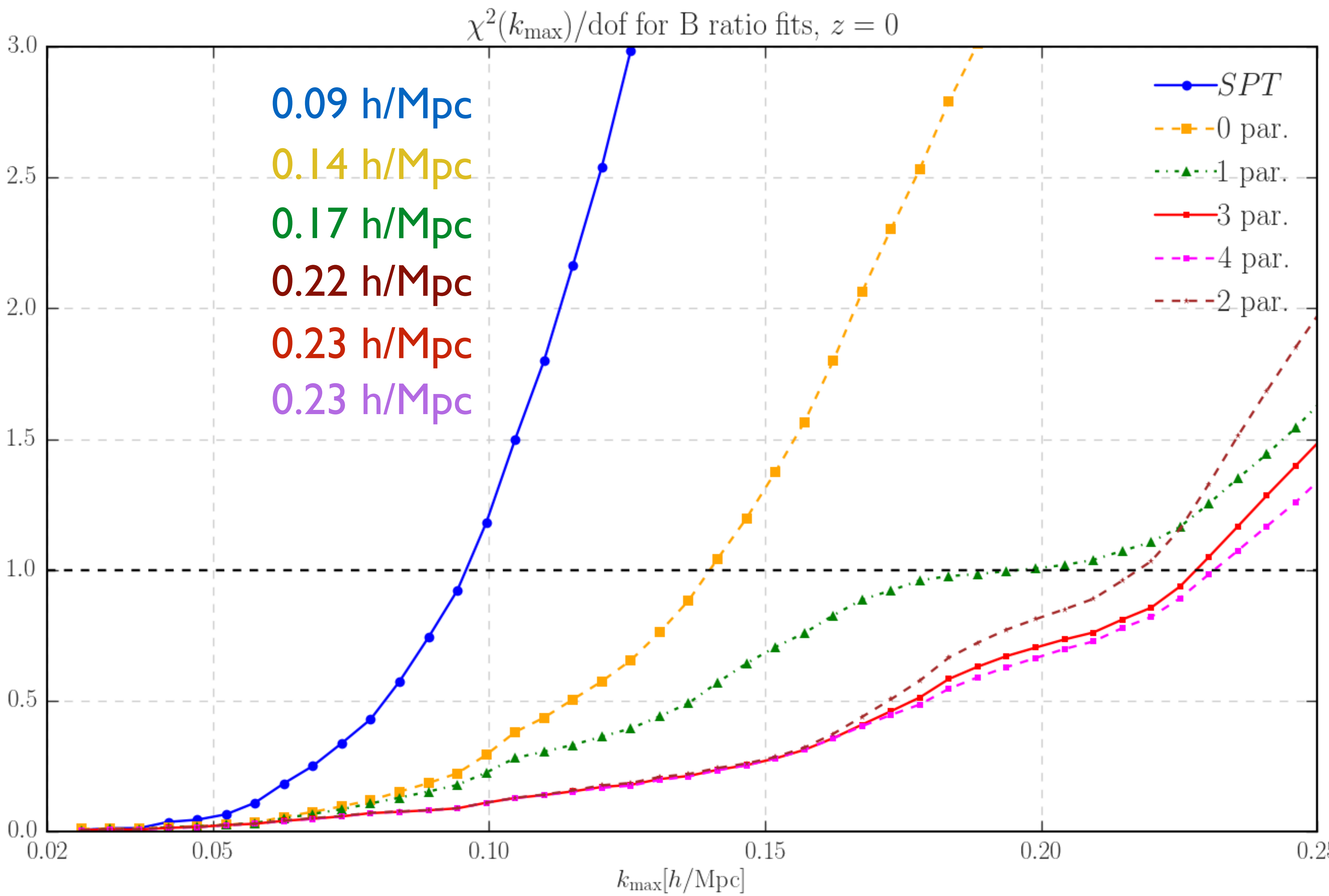


Data: Las Damas, Oriana

Preliminary

TSPT vs EFT Bispectrum

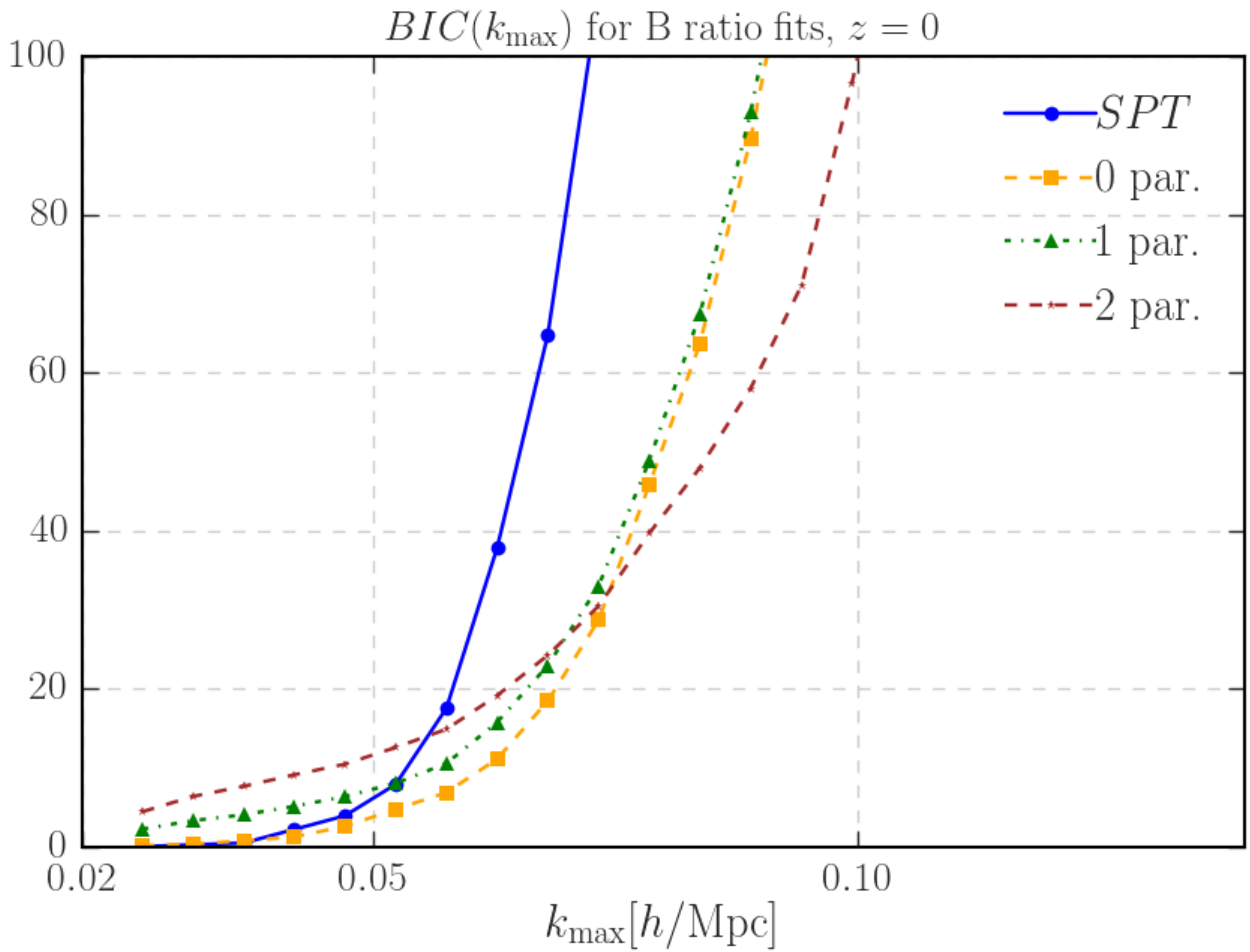
Baldauf, Mercolli, Mirbabayi, Pajer'15



Data: Las Damas, Oriana

Blas, D'Amico, Garny, MI, Sibiryakov, Scoccimarro

TSPT vs EFT Bispectrum



Data: Las Damas, Oriana

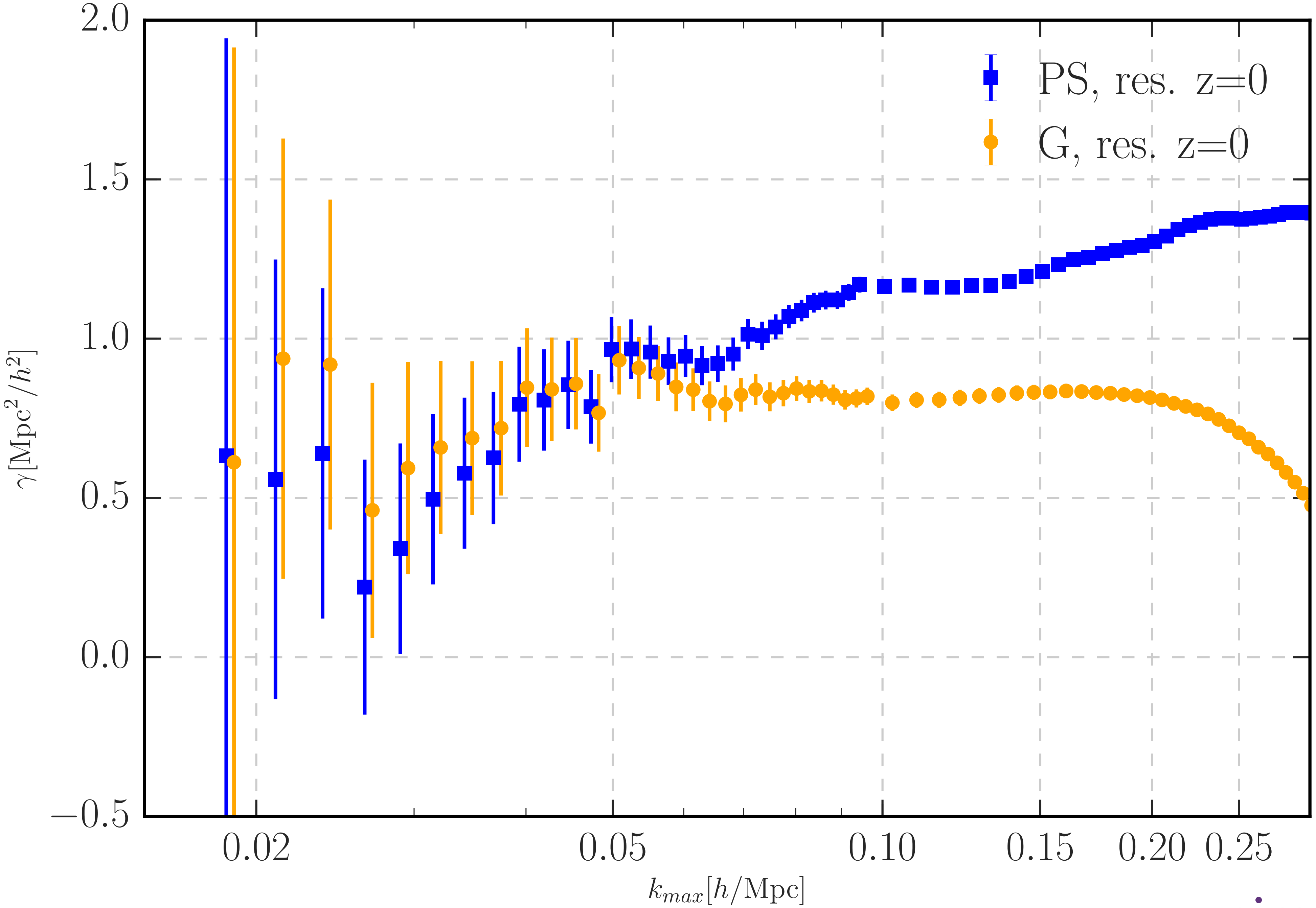
Blas, D'Amico, Garny, MI, Sibiryakov, Scoccimarro

TSPT power spectrum

Preliminary

$$P^{1\text{ loop, ren}}(k) = P^L(k) + P^{1\text{ loop, SPT}}(k) - 2\gamma\, k^2 P^L(k)$$

$\equiv \gamma C_2$



Data: Las Damas, Oriana

similar to EFT

Blas, D'Amico, Garny, MI, Sibiryakov, Scoccimarro

TSPT power spectrum

Preliminary

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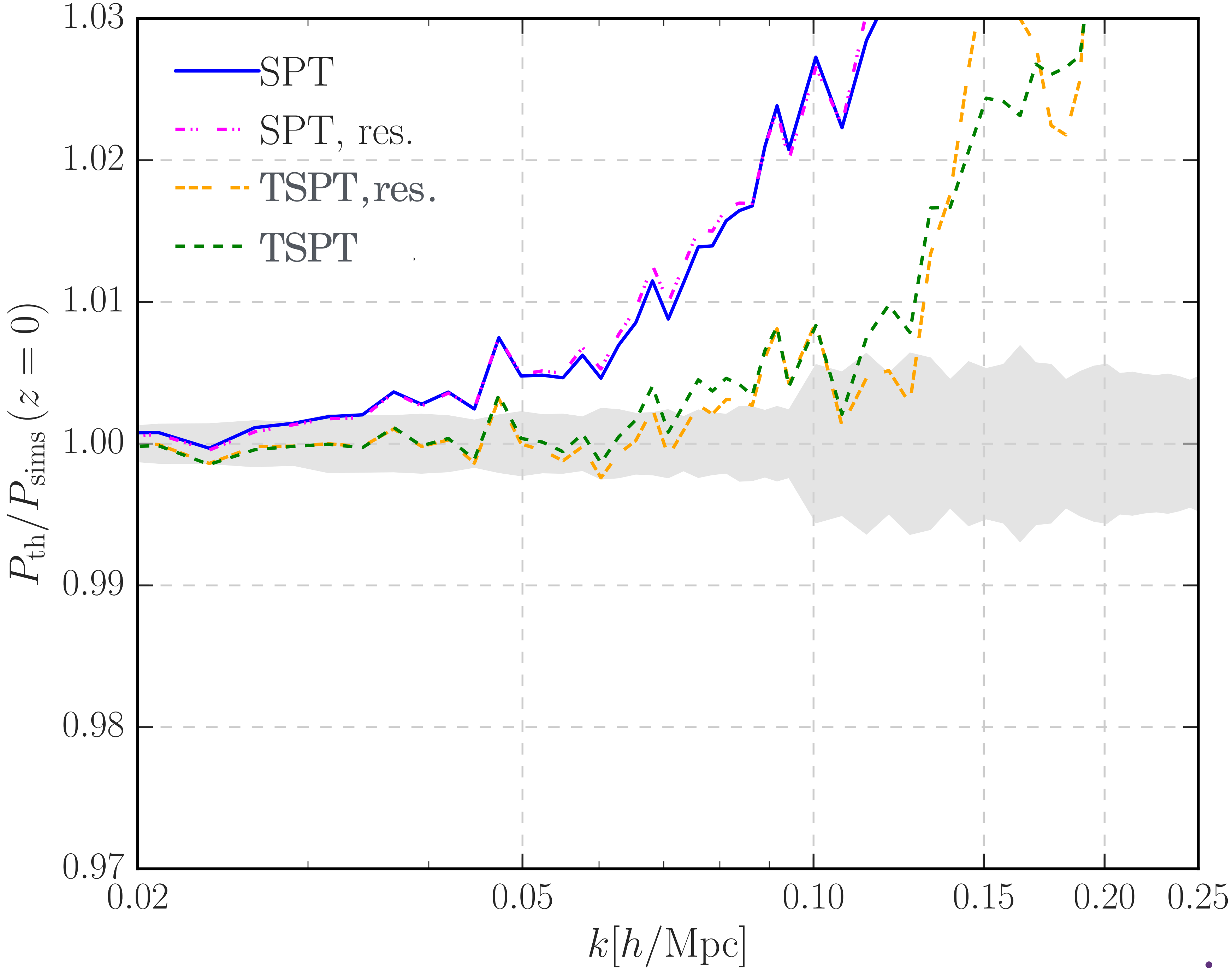


Data: Las Damas, Oriana

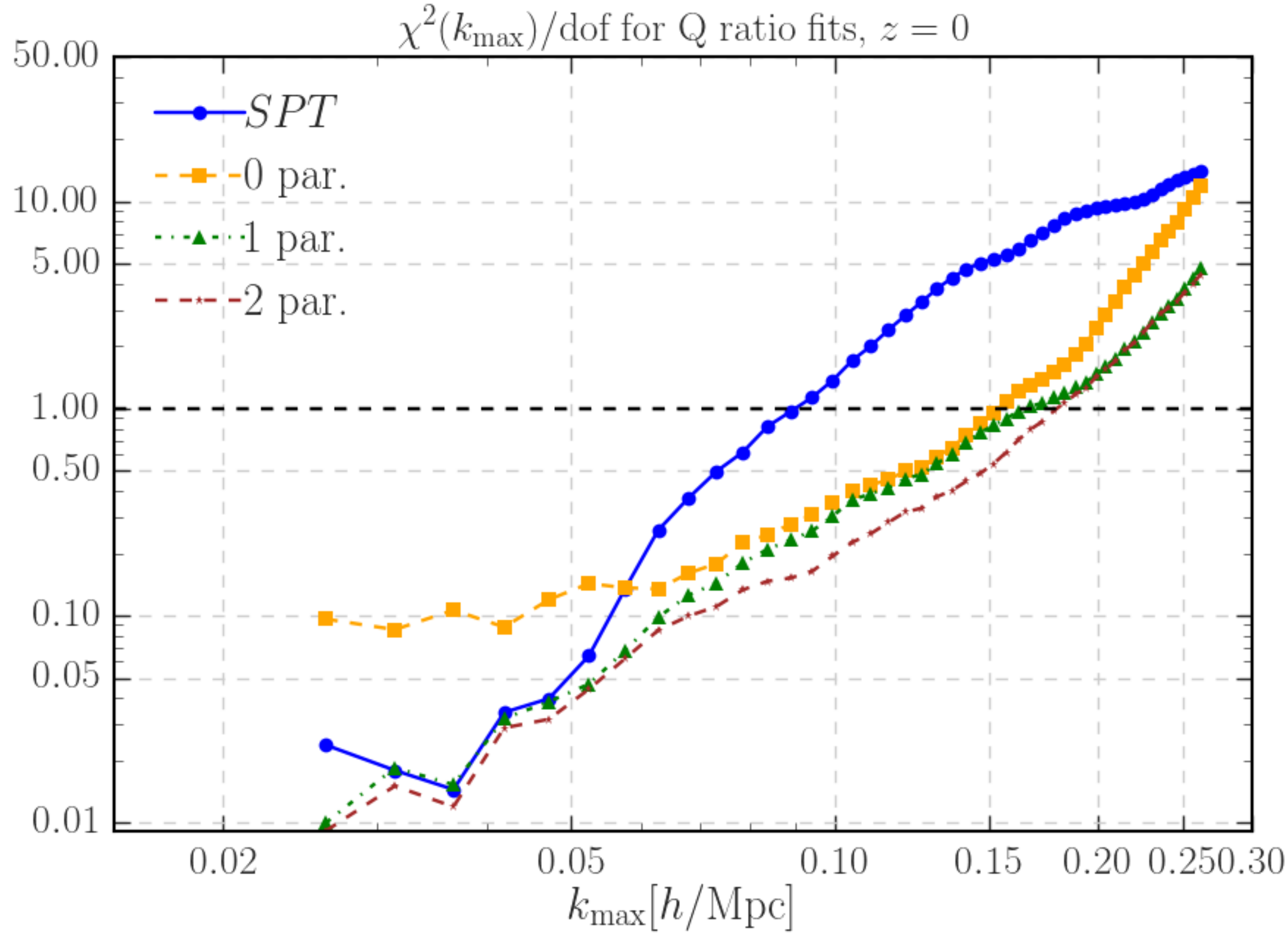
similar to EFT

Blas, D'Amico, Garny, MI, Sibiryakov, Scoccimarro

Preliminary

TSPT Bispectrum

$$B^{\text{1-loop, ren}} = B^{\text{tree}} + B^{\text{1-loop, SPT}} + \gamma C_2 B^{\text{tree}} + \beta C_3$$



Data: Las Damas, Oriana

Blas, D'Amico, Garny, MI, Sibiryakov, Scoccimarro