Effects of gas compressibility on the dynamics of premixed flames in long narrow adiabatic channels

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Relevance

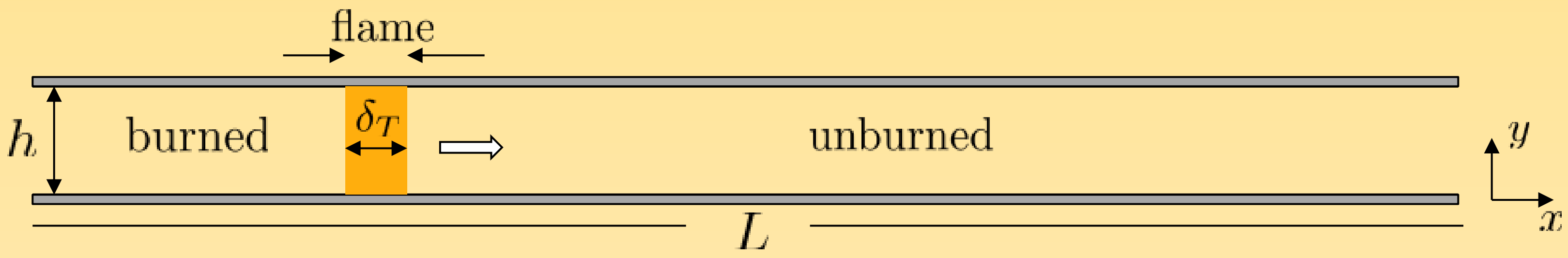
- application to micro-propulsion devices
- safety considerations
- deflagration-to-detonation transition

some of the earlier studies associated with flame acceleration

- H. Guenoche (Markstein's monograph)
  extended experimental studies.
- Kurdyumov & Matalon (PCI 2013, 2015)
  parametric study considering long channels with h/δ_T small and O(1)
- Bychkov et al. (PRE 2005), Demirgok et al. (PCI, 2015)
  used model-type equations

zero
Mach number

- Ott, Oran & Anderson and Gamezo & Oran (AIAA J. 2003, 2006)
  DNS of acetylene-air mixture in a narrow channel/tube
- Kagan, Gordon & Sivashinsky (PCI 2015)
  examined effect of gas compressibility in channels with h/δ_T small



$$x = x'/\delta_T, \quad y = y'/h, \quad t = S_L t'/\delta_T, \quad u = u'/S_L, \quad v = v'/aS_L,$$
$$\rho = \rho'/\rho_u, \quad p = a^2(p' - p_u)/\rho_u S_L^2, \quad Y = Y'/Y_u, \quad \theta = (T' - T_u)/(T_a - T_u),$$

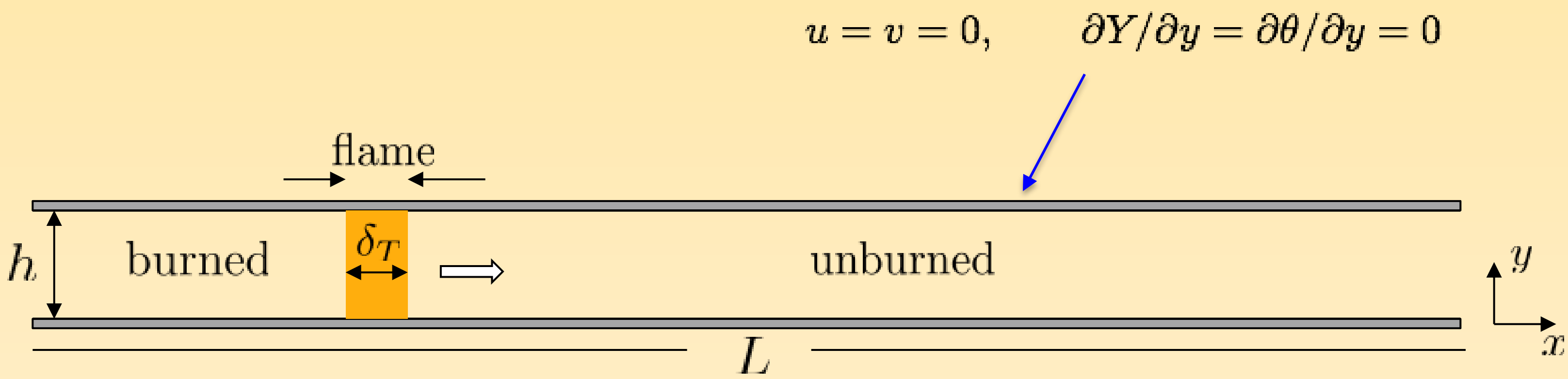
$$\boxed{a \equiv \frac{h}{\delta_T}}$$

Consider first zero Mach number equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,$$
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{1}{a^2}\frac{\partial p}{\partial x} + Pr\left[\frac{1}{a^2}\frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{3}\frac{\partial^2 v}{\partial x \partial y}\right]$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{1}{a^4}\frac{\partial p}{\partial y} + Pr\left[\frac{1}{a^2}\left(\frac{4}{3}\frac{\partial^2 v}{\partial y^2} + \frac{1}{3}\frac{\partial^2 u}{\partial x \partial y}\right) + \frac{\partial^2 v}{\partial x^2}\right]$$
$$\rho\left(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) - \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 \theta}{\partial y^2}\right) = \omega$$
$$\rho\left(\frac{\partial Y}{\partial t} + u\frac{\partial Y}{\partial x} + v\frac{\partial Y}{\partial y}\right) - \frac{1}{Le}\left(\frac{\partial^2 Y}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 Y}{\partial y^2}\right) = -\omega$$
$$1 = \rho(1 + q\theta)$$

$$\boxed{q, Pr, Le}$$

Boundary conditions



$$\text{at } x = 0,\ \ell : \quad \begin{cases} p = 0 & \text{open end} \\ u = 0 & \text{closed end} \end{cases}$$

$$\boxed{\ell \equiv L/\delta_T}$$

Consider first $a << 1$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{1}{a^2}\frac{\partial p}{\partial x} + Pr\left\{\frac{1}{a^2}\frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{3}\frac{\partial^2 v}{\partial x \partial y}\right\}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{1}{a^4}\frac{\partial p}{\partial y} + Pr\left\{\frac{1}{a^2}\left(\frac{4}{3}\frac{\partial^2 v}{\partial y^2} + \frac{1}{3}\frac{\partial^2 u}{\partial x \partial y}\right) + \frac{\partial^2 v}{\partial x^2}\right\}$$

$$\boxed{\begin{gathered} p = p(x,t) \\ u = 6Uy(1-y) \\ U(x,t) = \int_0^1 u\, dy = -\frac{1}{12Pr}\frac{\partial p}{\partial x} \end{gathered}}$$

continuity $\Rightarrow$ $v = \dfrac{1}{\rho}\dfrac{\partial(\rho U)}{\partial x}\left[y(2y^2 - 3y + 1)\right]$

mean velocity U determined by the combustion intensity and BCs.

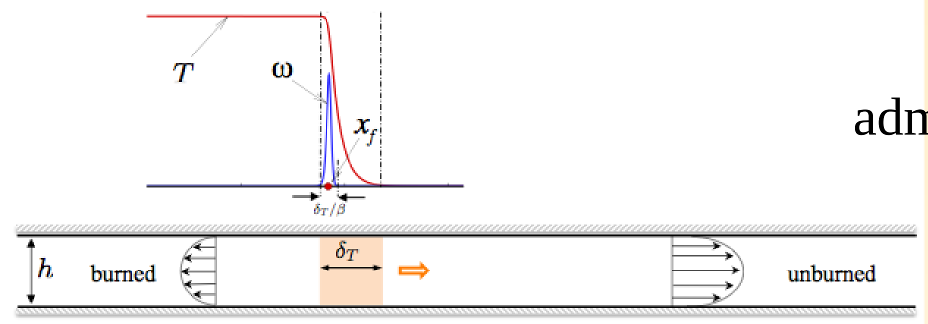
$$\rho\left(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial^2\theta}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2\theta}{\partial y^2}\right)=\omega$$

$$\rho\left(\frac{\partial Y}{\partial t}+u\frac{\partial Y}{\partial x}+v\frac{\partial Y}{\partial y}\right)-\frac{1}{Le}\left(\frac{\partial^2 Y}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2 Y}{\partial y^2}\right)=-\omega,$$

$$\Rightarrow \qquad \theta=\theta(x,t), \quad Y=Y(x,t)$$

$$\boxed{\begin{gathered}\frac{\partial\rho}{\partial t}+\frac{\partial(\rho U)}{\partial x}=0,\\ \rho\frac{\partial Y}{\partial t}+U\frac{\partial Y}{\partial x}-\frac{1}{Le}\frac{\partial^2 Y}{\partial x^2}=-\omega,\\ \rho(\frac{\partial\theta}{\partial t}+U\frac{\partial\theta}{\partial x})-\frac{\partial^2\theta}{\partial x^2}=\omega\\ 1=\rho(1+q\theta),\end{gathered}}$$

eigenvalue problem for the determination of *U*



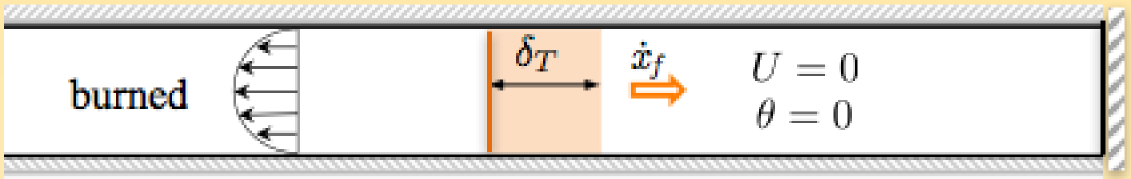
admits quasi-steady (wave-like) solutions

$$\boxed{U=\dot{x}_f-(1+q\theta)}$$

$$\boxed{U = \dot{x}_f - (1 + q\theta)}$$

in units of S_L

propagation towards a closed end

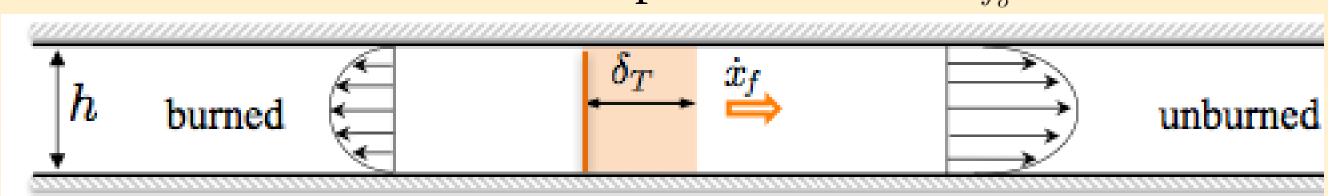


$\boxed{x_f = t}$

propagation from a closed end

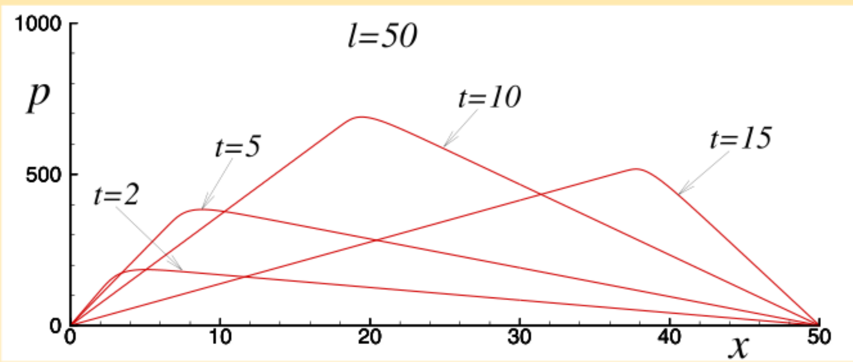
$\boxed{x_f = (1+q)t}$

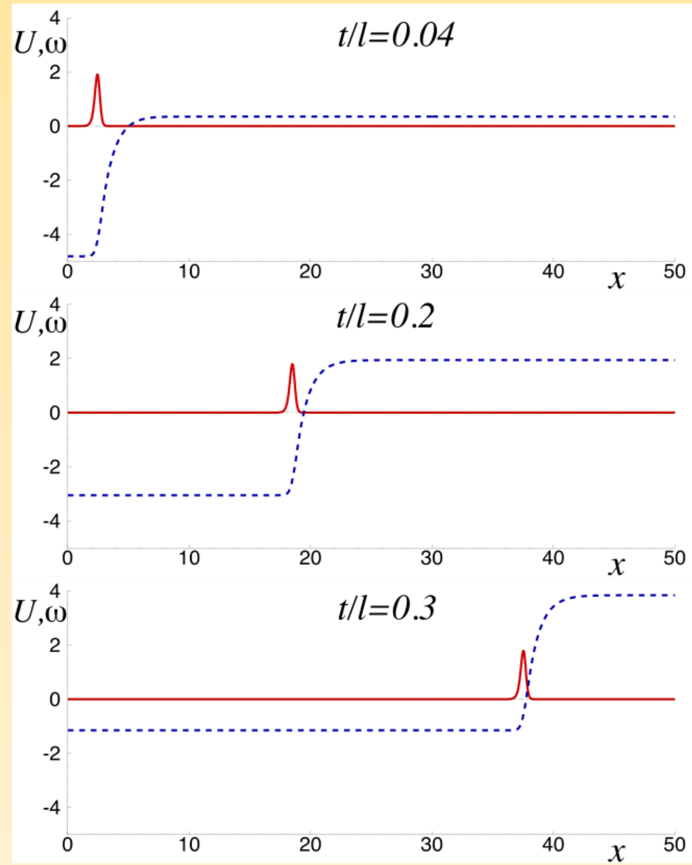
channel open at both ends $\int_o^L U\,dx = 0$



$\boxed{x_f \sim e^{qt/L}}$

flame accelerates when traveling down the channel

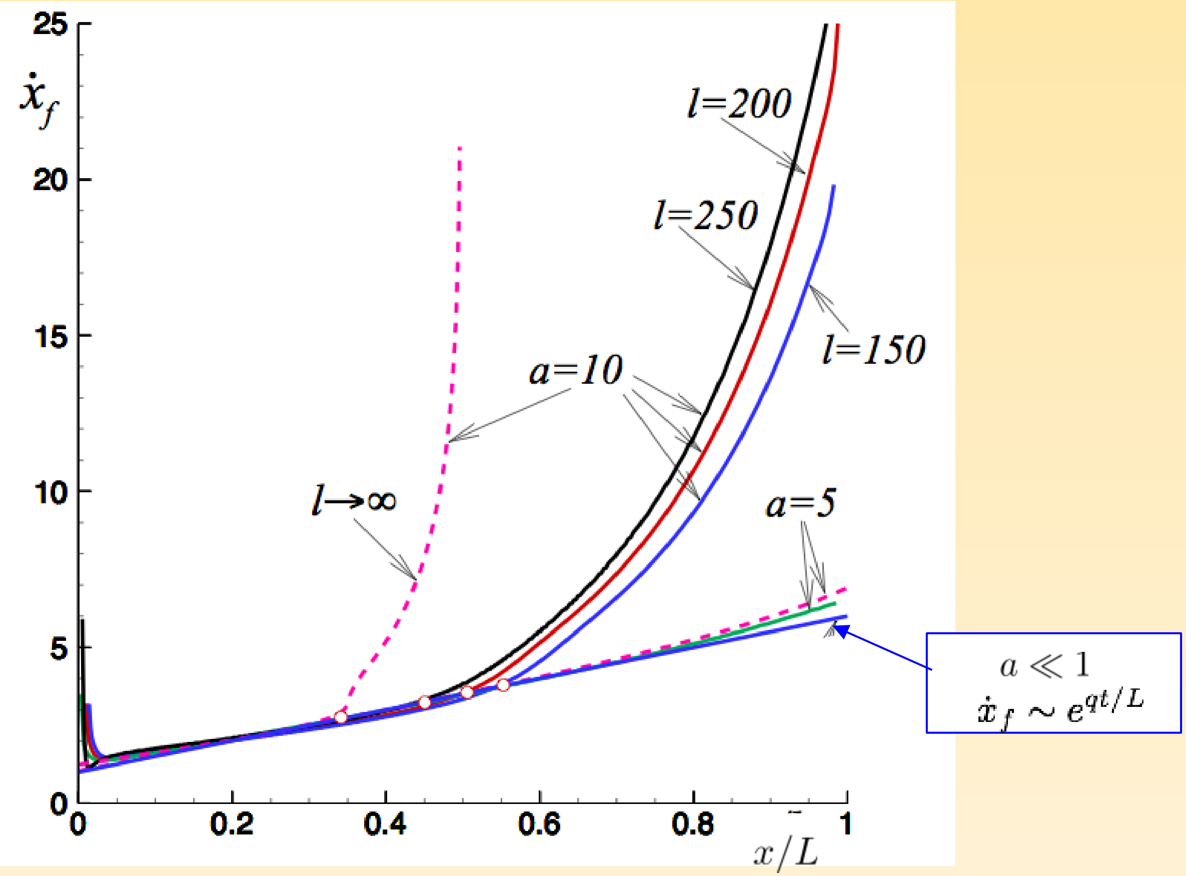

l=50
p
1000
500
0
t=2
t=5
t=10
t=15
0
10
20
30
40
50
x


t/l=0.04
t/l=0.2
t/l=0.3
U,ω
4
2
0
-2
-4
0
10
20
30
40
50
x

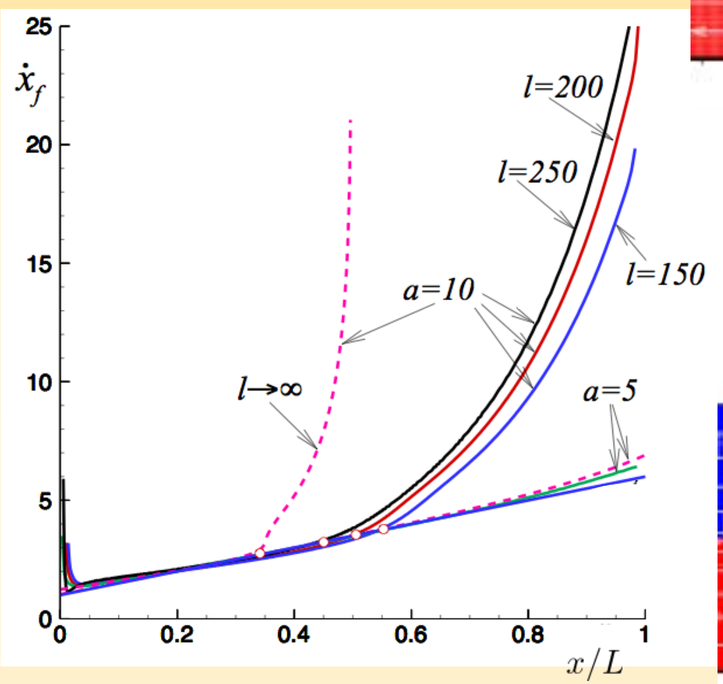
$$x_f \sim e^{qt/L}$$

Results for for a << 1 were corroborated with 2D numerical calculations, with $a = \mathcal{O}(1)$

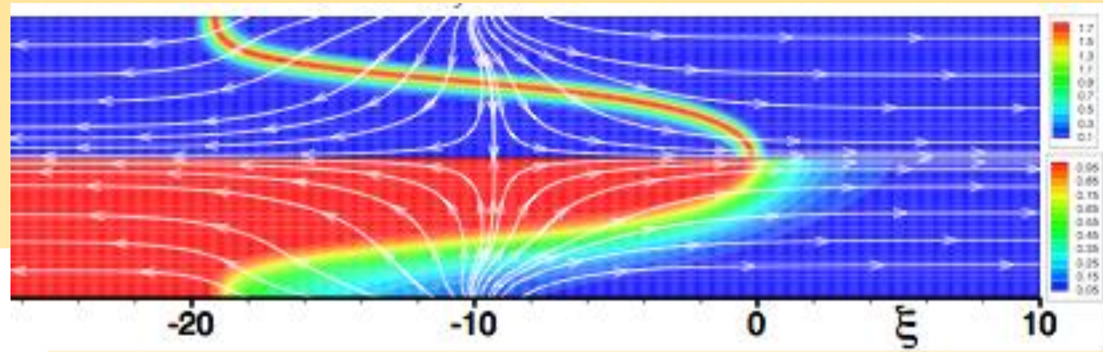
channel heights a = 5, 10 and various lengths $\ell = L/\delta_T$



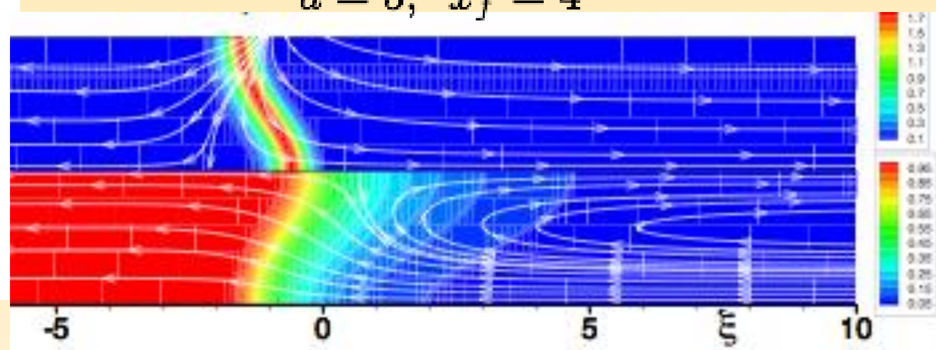
onset of "fast acceleration" occurs earlier when the channel is longer.



$a = 10, \ \dot{x}_f = 14.1$



$a = 5, \ \dot{x}_f = 4$



acceleration - due to frictional forces and gas expansion

compressibility effects

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,$$

$$\rho\Big(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\Big) = -\frac{1}{a^2}\frac{\partial p}{\partial x} + \Pr\Big[\frac{1}{a^2}\frac{\partial^2 u}{\partial y^2} + \frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{1}{3}\frac{\partial^2 v}{\partial x \partial y}\Big]$$

$$\rho\Big(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\Big) = -\frac{1}{a^4}\frac{\partial p}{\partial y} + \Pr\Big[\frac{1}{a^2}\Big(\frac{4}{3}\frac{\partial^2 v}{\partial y^2} + \frac{1}{3}\frac{\partial^2 u}{\partial x \partial y}\Big) + \frac{\partial^2 v}{\partial x^2}\Big]$$

$$\rho\Big(\frac{\partial Y}{\partial t} + u\frac{\partial Y}{\partial x} + v\frac{\partial Y}{\partial y}\Big) - \frac{1}{Le}\Big(\frac{\partial^2 Y}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 Y}{\partial y^2}\Big) = -\omega,$$

$$\rho\Big(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\Big) - \Big(\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 \theta}{\partial y^2}\Big) = \underline{\Lambda\frac{\gamma - 1}{q}\Big(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + \Pr\Phi\Big)} + \omega$$

$$1 + \underline{\gamma\Lambda p} = \rho(1 + q\theta),$$

where $\Lambda = \mathrm{Ma}^2/a^2$ with $\mathrm{Ma} = S_L/c$ a representative Mach number to highlight compressibility effects and minimize the influences of frictional forces and gas expansion we consider $a \ll 1$

$$\partial p/\partial y = 0$$

$$Pr \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$$

$$\Rightarrow$$

$$\boxed{\begin{gathered} p = p(x,t) \\ u = 6Uy(1-y) \\ U(x,t) = \int_0^1 u\,dy = -\frac{1}{12Pr}\frac{\partial p}{\partial x} \end{gathered}}$$

mean velocity U determined by solving

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U)}{\partial x} = 0,$$

$$\rho\frac{\partial Y}{\partial t} + U\frac{\partial Y}{\partial x} - \frac{1}{Le}\frac{\partial^2 Y}{\partial x^2} = -\omega,$$

$$\rho\left(\frac{\partial \theta}{\partial t} + U\frac{\partial \theta}{\partial x}\right) - \frac{\partial^2 \theta}{\partial x^2} = \underline{\Lambda\frac{\gamma-1}{q}\frac{\partial p}{\partial t}} + \omega$$

$$1 + \underline{\gamma\Lambda p} = \rho(1+q\theta),$$

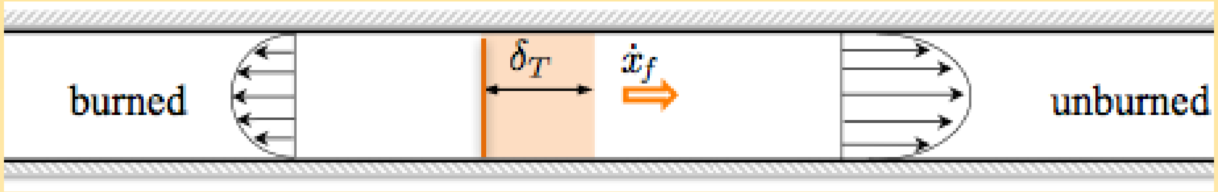
relevant parameter

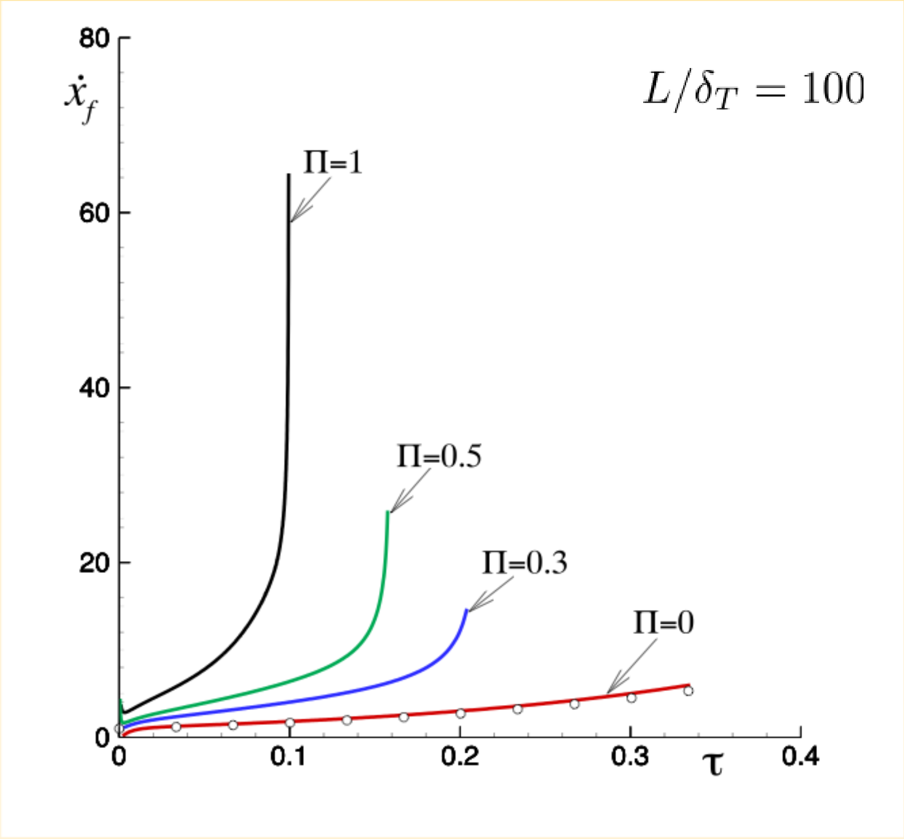
$$\Pi = 12Pr\Lambda\ell$$

$$\boxed{\Pi = \frac{12\mu}{\lambda/c_p}\frac{L\delta_T}{h^2}\mathrm{Ma}^2}$$

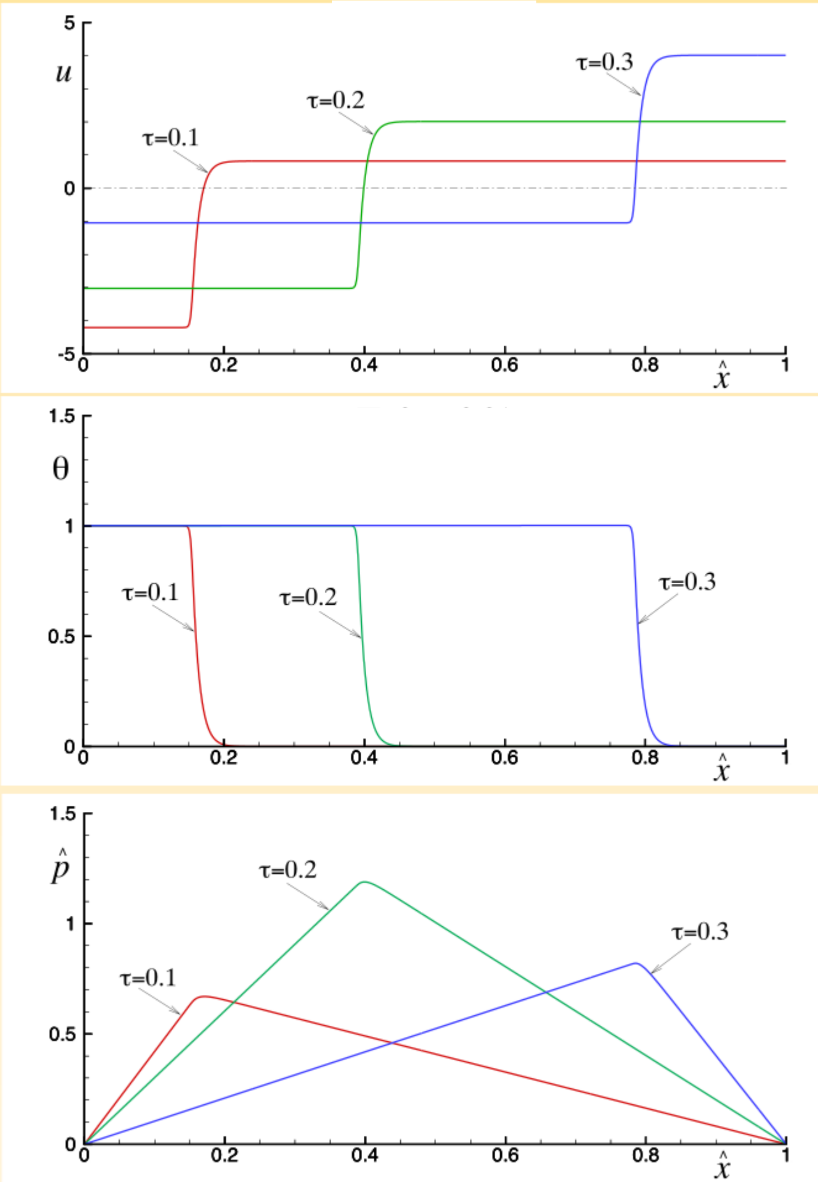
scaled Mach number

Propagation in a channel open at both ends



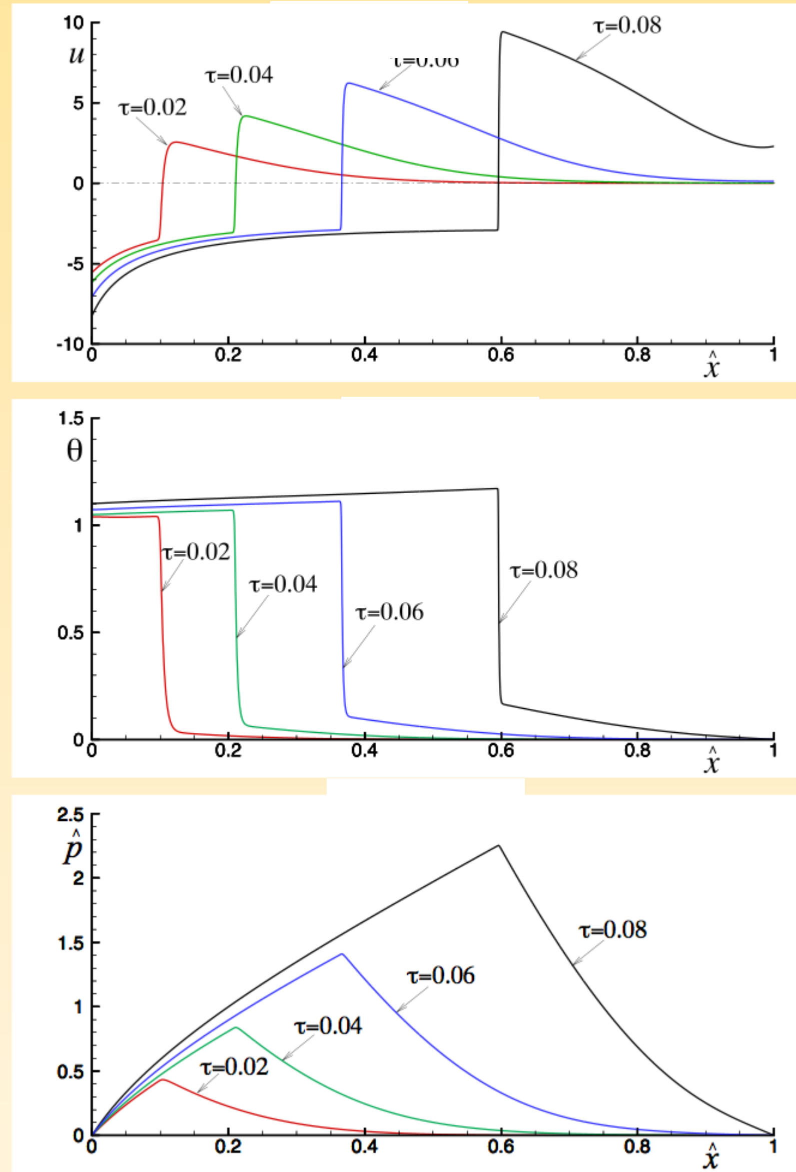


$\Pi = 0$



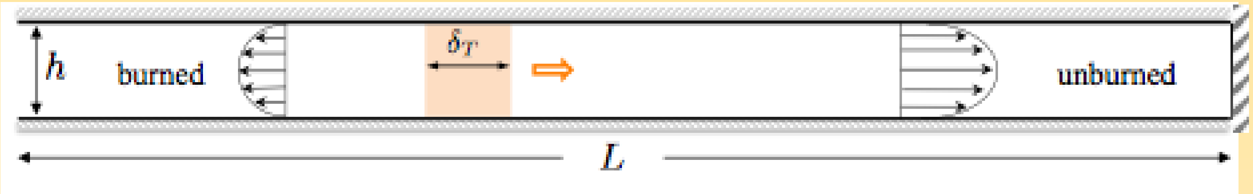
u
τ=0.1
τ=0.2
τ=0.3
θ
τ=0.1
τ=0.2
τ=0.3
p̂
τ=0.1
τ=0.2
τ=0.3
x̂

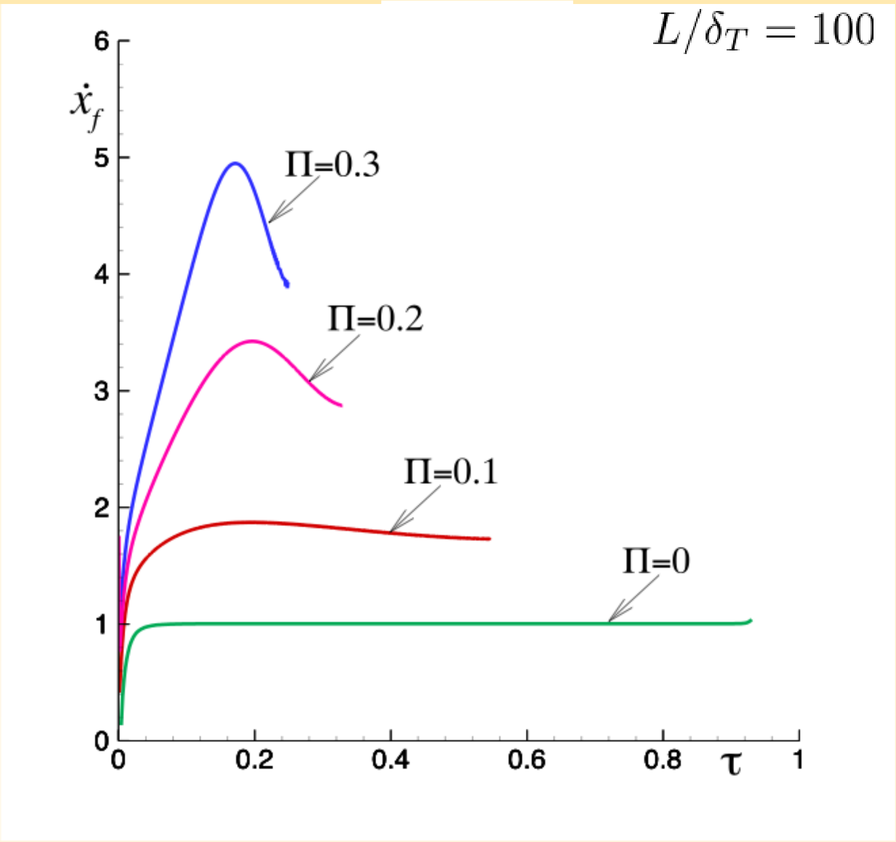
$\Pi = 0.3$



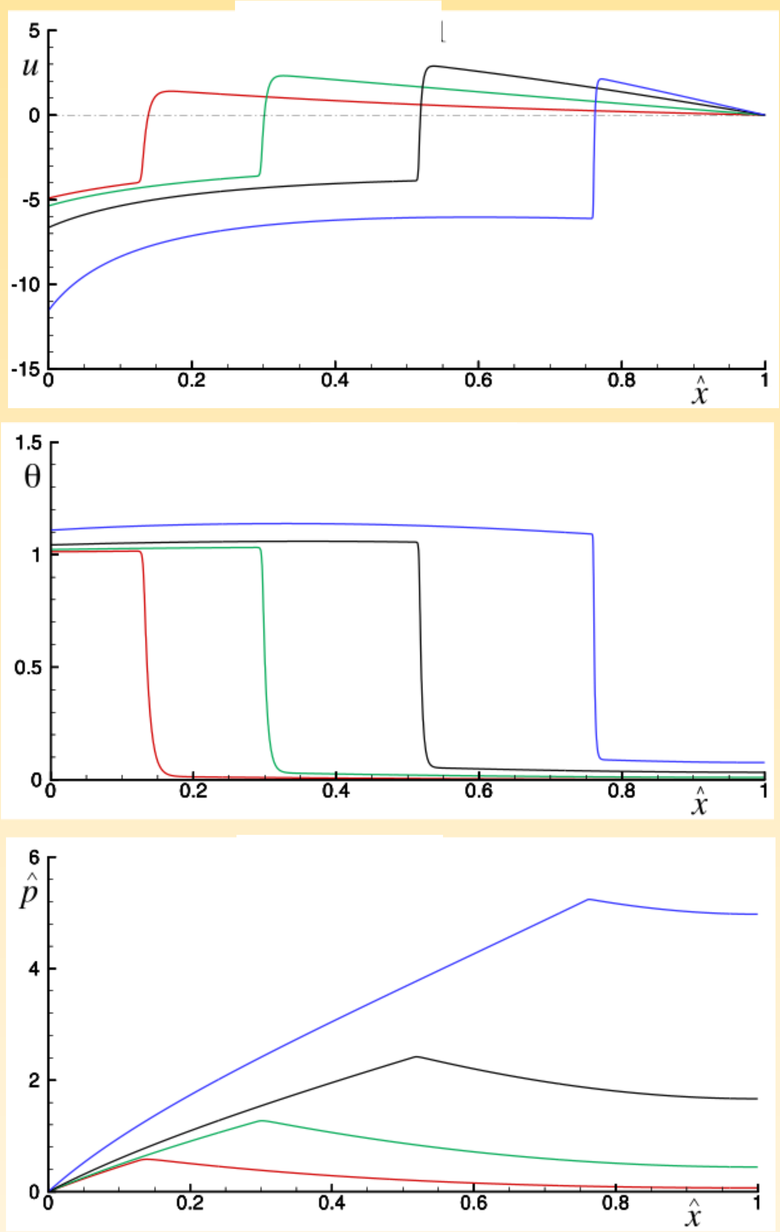
u
τ=0.02
τ=0.04
τ=0.08
θ
τ=0.02
τ=0.04
τ=0.06
τ=0.08
p̂
τ=0.02
τ=0.04
τ=0.06
τ=0.08
x̂

propagation towards a closed end

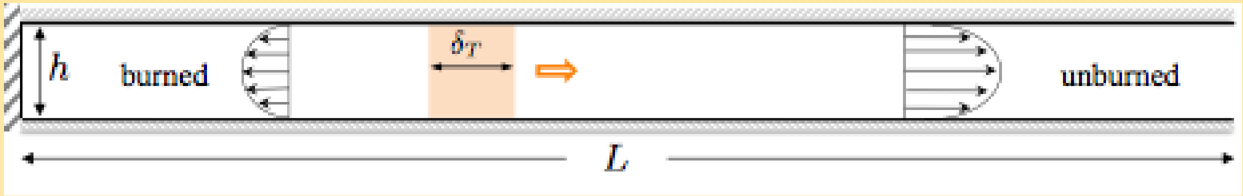


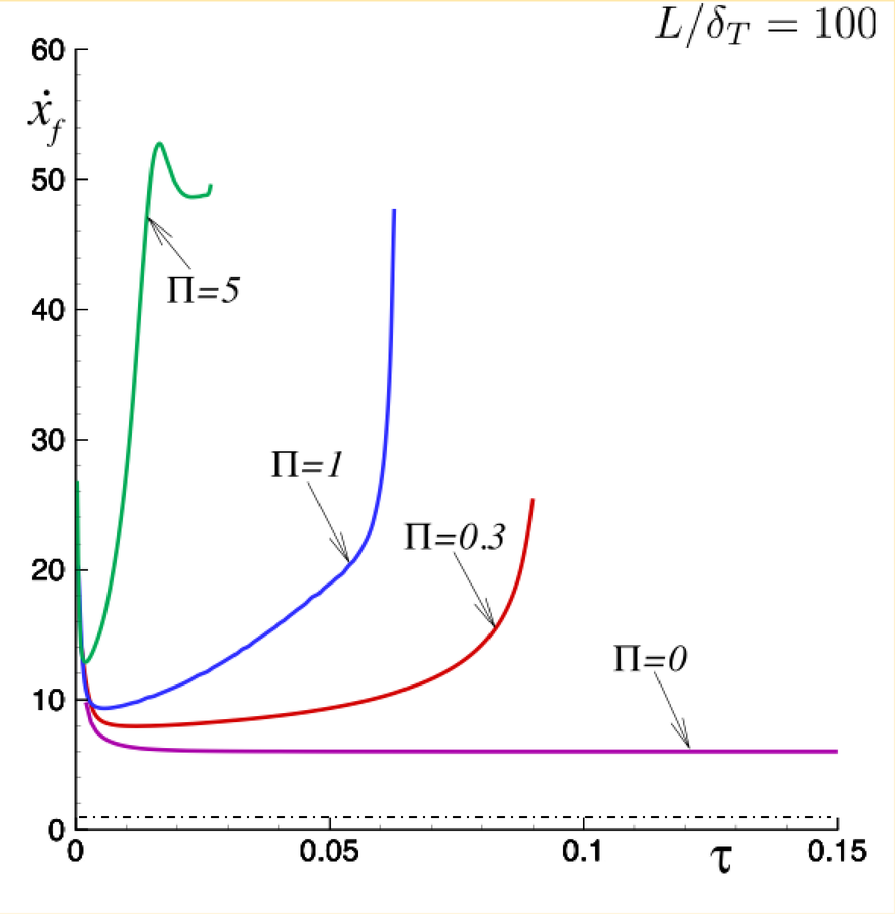


$$\Pi = 0.3$$

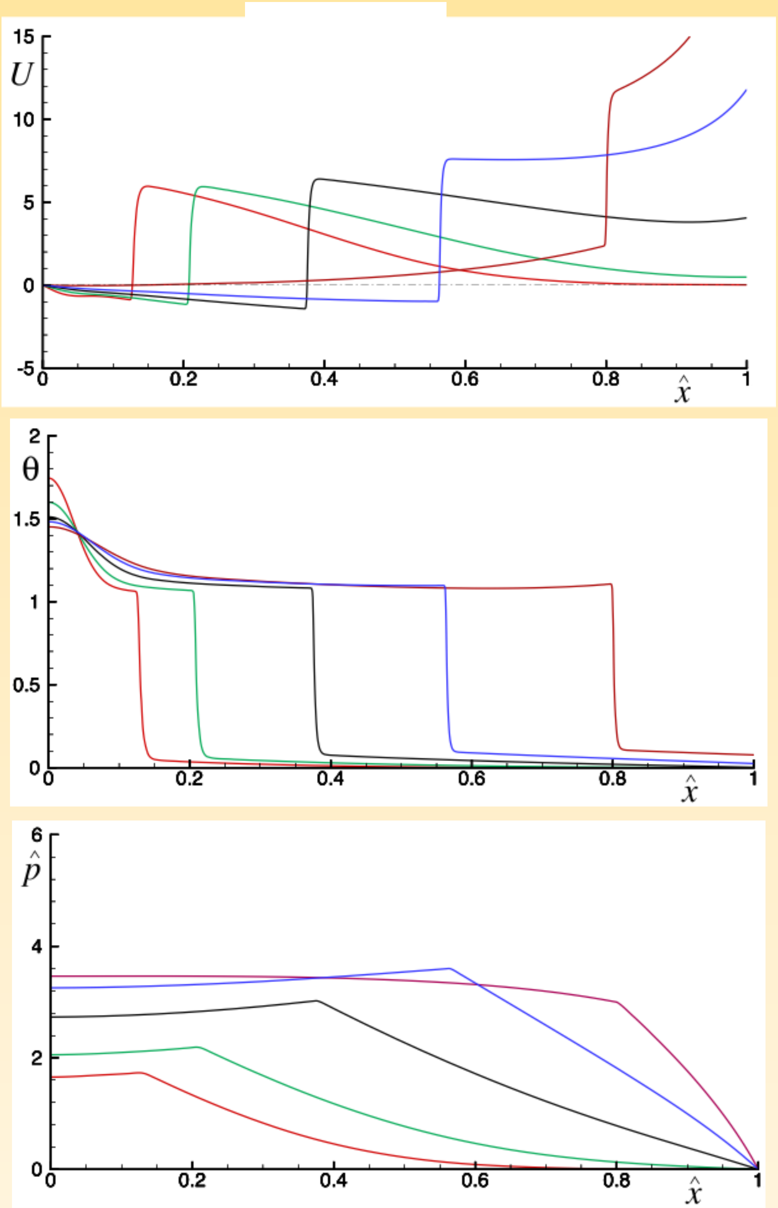


propagation from a closed end





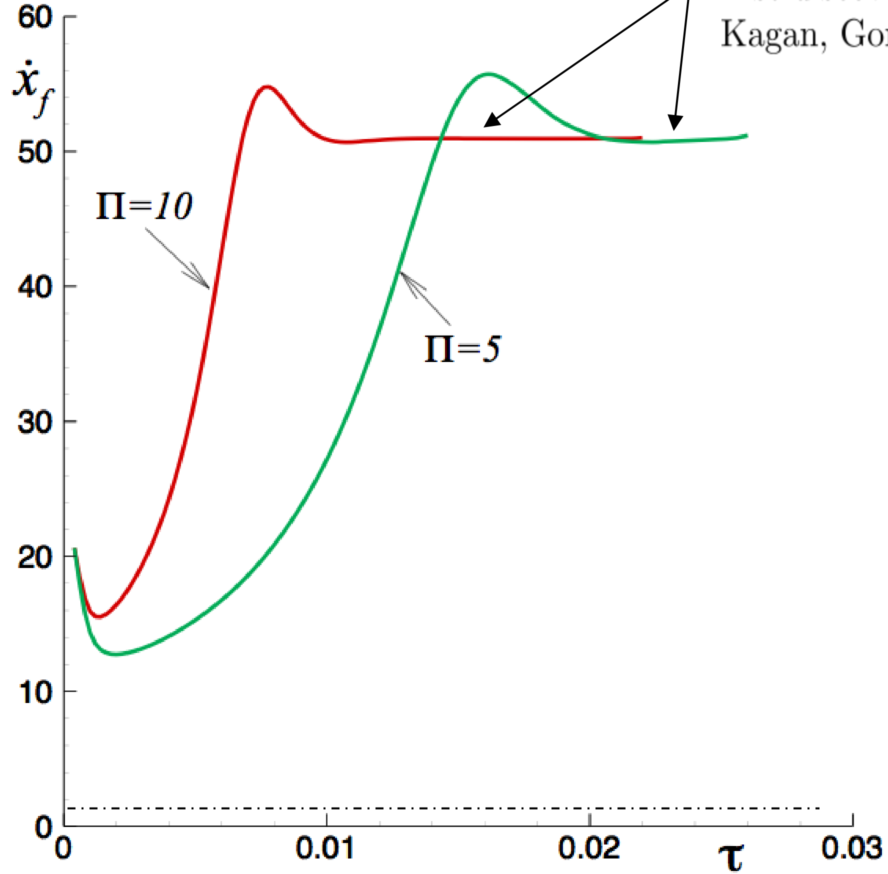
$\Pi = 0.3$



15
U
10
5
0
-5
0
0.2
0.4
0.6
0.8
$\hat{x}$
1
2
θ
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
$\hat{x}$
1
6
$\hat{p}$
4
2
0
0
0.2
0.4
0.6
0.8
$\hat{x}$
1

steady propagation

first discovered by
Kagan, Gordon & Sivashinsky (2015)



steady propagation of compression-driven flames

$$\xi = x - S_c t$$

$$\frac{d}{d\xi}\left[\rho(U - S_c)\right] = 0$$

$$-S_c \frac{dY}{d\xi} - \frac{1}{Le}\frac{d^2Y}{d\xi^2} = -\omega$$

$$-S_c \frac{d\theta}{d\xi} - \frac{d^2\theta}{d\xi^2} = -S_c\Lambda\frac{\gamma-1}{q}\frac{dp}{d\xi} + \omega$$

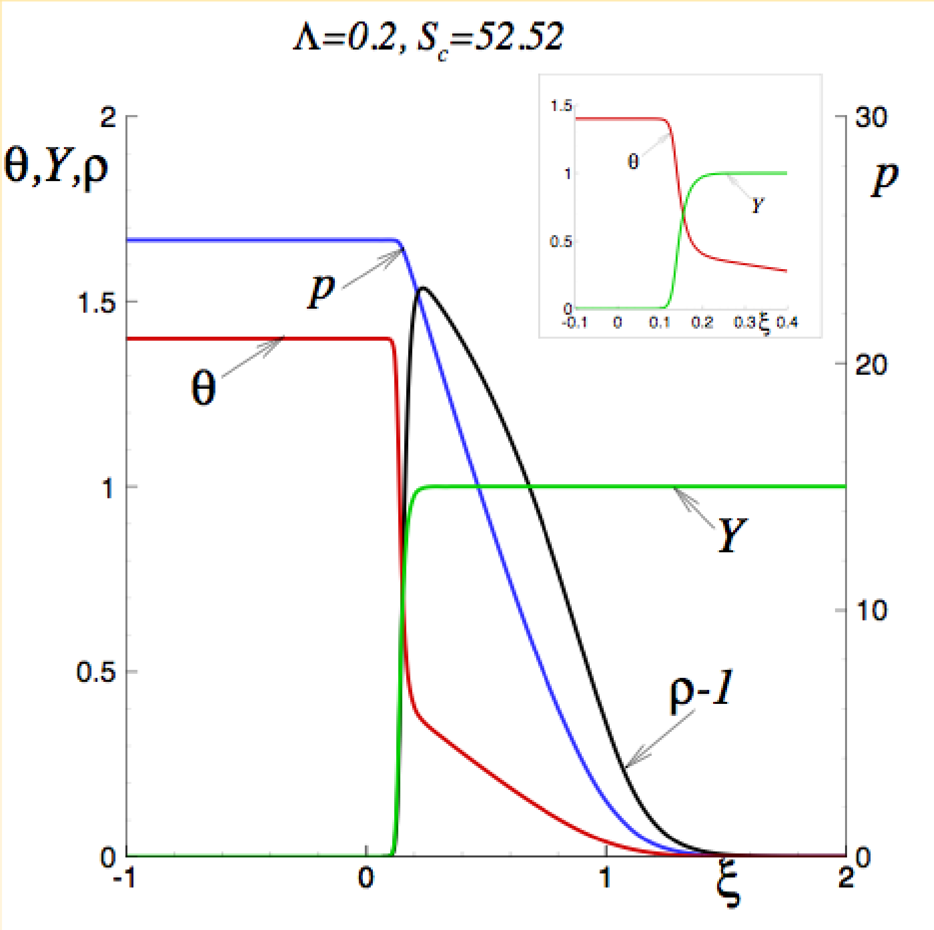
$$1 + \gamma\Lambda p = \rho(1 + q\theta)$$

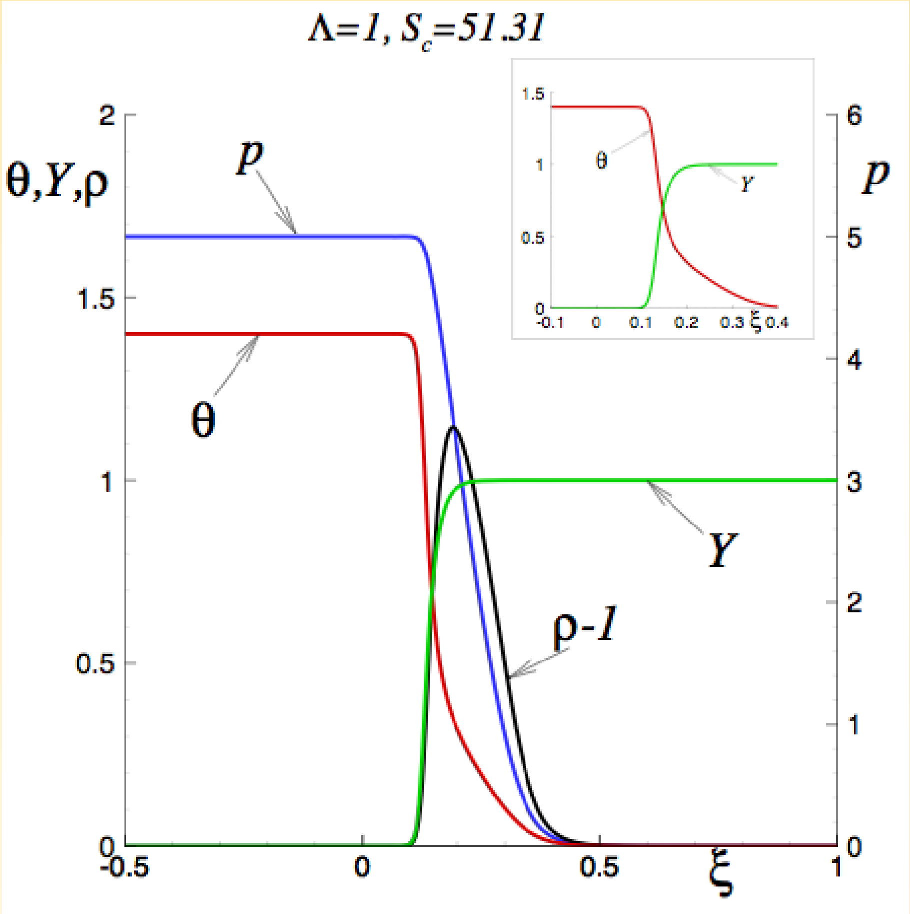
$$p \sim q/\Lambda\,, \quad \theta \sim \gamma, \quad \rho \sim 1 \qquad \text{as } \xi \to -\infty$$

$$p = \theta = 0, \quad \rho = Y = 1, \qquad \text{as } \xi \to +\infty$$

nonlinear eigenvalue problem for the determination of the propagation speed S_c

structure of compression-driven flames



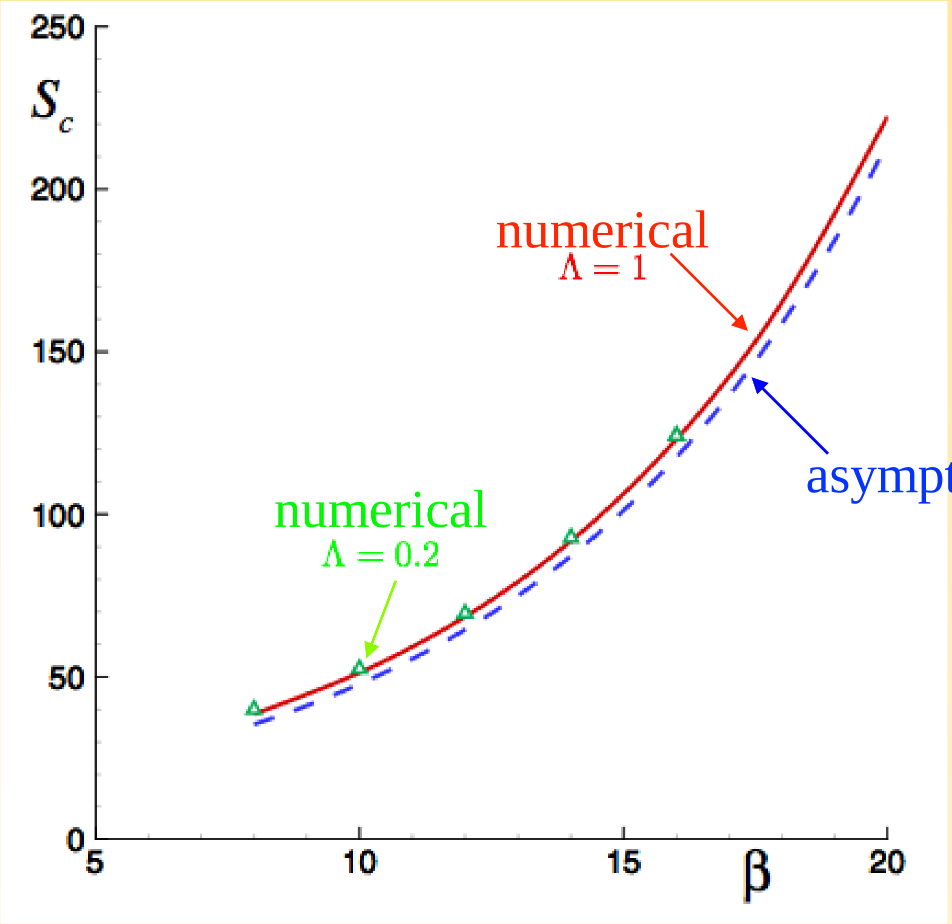


the density increases in the compression region but drops in the flame zone $\rho \to 1$

the pressure increases significantly throughout the flame zone, $p \to q/\Lambda$

the flame temperature is significantly larger than the adiabatic flame temperature, $\theta \to \gamma$

propagation speed of compression-driven flames



$$S_c^{\rm asp} = \sqrt{\frac{2(\lambda/c_p)\mathcal{B}}{Le^{-1}\beta^2}}\left(\frac{T_e}{T_a}\right)^2 \mathrm{e}^{-E/2\mathcal{R}T_e}$$

$$T_e = T_u + QY_u/c_v$$

constant pressure laminar flame speed

$$S_L^{\rm asp} = \sqrt{\frac{2(\lambda/c_p)\mathcal{B}}{Le^{-1}\beta^2}}\,\frac{T_u}{T_a}\,\mathrm{e}^{-E/2\mathcal{R}T_a}\,,$$

$$T_a = T_u + QY_u/c_p$$

CONCLUSIONS

- Dependence on the **boundary conditions** - open vs closed

- Acceleration - combined effects of **wall friction** and **thermal expansion**, further enhanced by effects due to **compressibility**

- Acceleration in sufficiently long channels - in a near-explosion fashion **beyond a critical distance**

- **Compression waves** tend to heat up the unburned gas ahead of the flame leading to significant increase in propagation speed

- The propagation is **dampened** when the far end is closed, and a evolves into a **steady-propagation** when the ignition end is closed with the burned gas trapped behind the flame.

- The flame propagation speed of **compression-derived flames** is significantly higher than the propagation speed of isobaric flames (up to 50 times S_L).

Thank You