Sound in anisotropic hydrodynamics

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Anisotropic hydrodynamics: origins

- Initial state created in high energy nuclear collisions is highly anisotropic: negative longitudinal pressure and positive resulting in fast, mainly longitudinal, expansion.
- Experimental data shows that collective effects play important role in this expansion.
- At the tree level and leading gradient approximation the simplest idea is to construct anisotropic hydrodynamics.
- In this talk we discuss equations describing propagation of sound in non relativistic and relativistic anisotropic hydrodynamics.

Non-relativistic hydro: generic equations

- Continuity equation, equations for velocity and temperature:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} &= \frac{\rho}{m} \mathbf{F} - \nabla \overset{\leftrightarrow}{p} \\
\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \Theta &= -\frac{2}{3} \nabla \cdot \mathbf{q} - \frac{2}{3} \overset{\leftrightarrow}{p} \cdot \overset{\leftrightarrow}{\Lambda}
\end{aligned}$$

- In the RHS of the above equations

$$\left(\overset{\leftrightarrow}{p} \right)_{ij} = p_{ij}, \quad \left(\nabla \overset{\leftrightarrow}{p} \right)_i = \partial_i p_{ij}, \quad \overset{\leftrightarrow}{p} \cdot \overset{\leftrightarrow}{\Lambda} = p_{ij} \Lambda_{ij}$$

- Fileds:

$$\begin{aligned}
\rho(\mathbf{r},t) &= m\int d^3v\, f(\mathbf{r},\mathbf{v},\mathbf{t}) \\
\mathbf{u}(\mathbf{r},t) &= \langle \mathbf{v}(\mathbf{r},t)\rangle \\
\Theta(\mathbf{r},t) &= \frac{1}{3}m\langle|\mathbf{v}-\mathbf{u}|^2\rangle \\
\mathbf{q}(\mathbf{r},t) &= \frac{1}{2}m\rho\langle(\mathbf{v}-\mathbf{u})|\mathbf{v}-\mathbf{u}|^2\rangle \\
p_{ij} &= \rho\langle(v_i-u_i)(v_j-u_j)\rangle \\
\Lambda_{ij} &= \frac{1}{2}m\left(\partial_j u_i+\partial_i u_j\right)
\end{aligned}$$

- Averaging:

$$\langle A\rangle = \frac{\int d^3vAf}{\int d^3v}$$

Non-relativistic hydro: isotropic case

- Distribution function:

$$f^{(0)}(\mathbf{r}, \mathbf{v}, \mathbf{t}) = n(\mathbf{r}, t) \left(\frac{m}{2\pi\theta}\right)^{3/2} \exp\left[-\frac{m}{2\theta}(\mathbf{v} - \mathbf{u})^2\right]$$

- Fields:

$$\begin{aligned} \rho(\mathbf{r}, t) &= mn(\mathbf{r}, t) \\ \Theta(\mathbf{r}, t) &= \theta(\mathbf{r}, t) \\ \mathbf{q}^{(0)}(\mathbf{r}, t) &= 0 \\ p_{ij}^{(0)}(\mathbf{r}, t) &= \delta_{ij} n(\mathbf{r}, t)\theta(\mathbf{r}, t) \end{aligned}$$

Non-relativistic hydro: isotropic case

- Hydrodynamic equations:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} &= \frac{\rho}{m} \mathbf{F} - \nabla \overset{\leftrightarrow}{p} \\
\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \Theta &= -\frac{2}{3} \nabla \cdot \mathbf{q} - \frac{2}{3} \overset{\leftrightarrow}{p} \cdot \overset{\leftrightarrow}{\Lambda}
\end{aligned}$$

$$\left(\overset{\leftrightarrow}{p} \right)_{ij} = p_{ij}, \quad \left(\nabla \overset{\leftrightarrow}{p} \right)_i = \partial_i p_{ij}, \quad \overset{\leftrightarrow}{p} \cdot \overset{\leftrightarrow}{\Lambda} = p_{ij} \Lambda_{ij}$$

Non-relativistic hydro: anisotropic case

- Distribution function:

$$f^{(0)}(\mathbf{r},\mathbf{v},\mathbf{t}) = n\left(\frac{m}{2\pi\theta}\right)^{3/2}\exp\left\{-\frac{m}{2\theta}\left[(\mathbf{v}-\mathbf{u})^2+\xi(\mathbf{r},t)(v_z-u_z)^2\right]\right\}$$

- Fields:

$$\begin{aligned}
\rho^{(an)}(\mathbf{r},t) &= \frac{mn(\mathbf{r},t)}{\sqrt{1+\xi}} \\
\Theta^{(an)}(\mathbf{r},t) &= \theta(\mathbf{r},t)\frac{1}{3}\left[\frac{3+2\xi}{1+\xi}\right] \\
\mathbf{q}^{(an)}(\mathbf{r},t) &= 0 \\
p_{11}^{(an)}(\mathbf{r},t) &= p_{22}^{(an)}(\mathbf{r},t) = n\theta, \quad p_{33}^{(an)}(\mathbf{r},t) = \frac{n\theta}{1+\xi}
\end{aligned}$$

Non-relativistic hydro: anisotropic case

- Hydrodynamic equations:

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= \frac{1}{2}\frac{\rho}{1+\xi}\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\xi \\
\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} &= \frac{\mathbf{F}}{m} - \frac{1}{m}\frac{\sqrt{1+\xi}}{\rho}\nabla(\rho\theta) \\
&- \frac{1}{m}\frac{\sqrt{1+\xi}}{\rho}\partial_3\left(\frac{\xi}{1+\xi}\rho\theta\right)\mathbf{e}_3 \\
\frac{1}{3}\frac{3+2\xi}{1+\xi}\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\Theta &= -\frac{2}{3}\sqrt{1+\xi}(\nabla\cdot\mathbf{u})\theta + \frac{\xi\theta}{\sqrt{1+\xi}}\partial_3 u_3 \\
&+ \frac{\theta}{(1+\xi)^2}\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\xi
\end{aligned}
$$

Non-relativistic anisotropic hydro: sound

- Generic equations:

$$
\begin{aligned}
\partial_t \rho + \rho \nabla \cdot \mathbf{u} &= 0 \\
(\partial_t + \mathbf{u} \cdot \nabla)\, \xi &= 0 \\
\partial_t \mathbf{u} + \frac{\sqrt{1+\xi}}{\rho} \nabla \cdot (\rho\theta) &= \frac{\sqrt{1+\xi}}{\rho} \partial_3 \left(\frac{\xi}{1+\xi} \rho\theta \right) \mathbf{e}_3 \\
\phi(\xi)\partial_t \theta - \frac{2}{3}\sqrt{1+\xi}\frac{1}{\rho}\partial_t \rho\theta &= \frac{\xi\theta}{\sqrt{1+\xi}} \partial_3 u_3
\end{aligned}
$$

- Leading order in spatial gradients of ξ:

$$
\begin{aligned}
-\partial_t^2 \rho + \sqrt{1+\xi}\,\Delta(\rho\theta) &= \frac{\xi}{1+\xi}\partial_3^2(\rho\theta) + \sqrt{1+\xi}\left(\partial_3^2 \frac{\xi}{1+\xi}\right)(\rho\theta) \\
\theta^{\chi(\xi)} &= \rho^{\phi(\xi)} \exp\left[\int_{t_0}^{t} dt \frac{\xi}{1+\xi}\partial_3 u_3\right]
\end{aligned}
$$

Relativistic anisotropic hydrodynamics: basics

- Distribution function

$$
\begin{aligned}
f(x,p) &= f_{iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\Lambda(x)}\right) \\
p^\mu \Xi_{\mu\nu} p^\nu &= p^2 + \xi(x) p_L^2
\end{aligned}
$$

- Energy-momentum tensor:

$$
T^{\mu\nu} = (\varepsilon + P_T) u^\mu u^\nu - P_T g^{\mu\nu} + (P_L - P_T) z^\mu z^\nu
$$

- Fields (energy, density, pressure)

$$
\begin{aligned}
\varepsilon &= R(\xi)\varepsilon_{iso}(\Lambda, \tilde{\mu}) \\
n &= R_0(\xi) n_{iso}(\Lambda, \tilde{\mu}) \\
P_{T,L} &= R_{T,L}(\xi) P_{iso}(\Lambda, \tilde{\mu})
\end{aligned}
$$

Relativistic anisotropic hydrodynamics: basics and sound

- Basic equations:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu j^\mu = 0$$

- Equation for sound waves:

$$c_T \Delta n^{(1)} + c_\delta \partial_z^2 n^{(1)} - c_\varepsilon \partial_t^2 n^{(1)} = \left[\ddot{\xi} \frac{\partial \varepsilon^{(0)}}{\partial \xi} + (\dot{\xi})^2 \frac{\partial^2 \varepsilon^{(0)}}{\partial \xi^2} \right]$$