Cosmological bounce beyond Horndeski

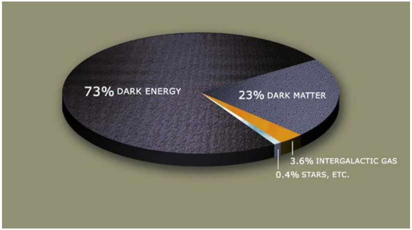
1705.06626

R. Kolevatov, SM, N. Suchov, V. Volkova

INR RAS

Moscow, Lebedev Inst., May 30, 2017

There are several "dark" questions: matter and energy.



In fact, we know nothing about 95% of the Universe.

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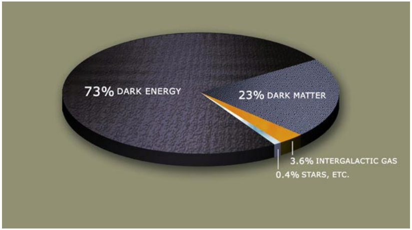

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But generally Big Bang theory is accepted and widely known.

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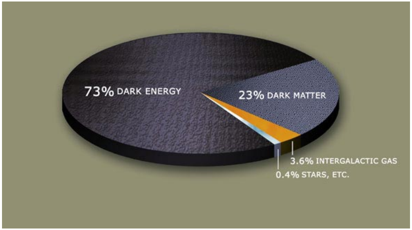

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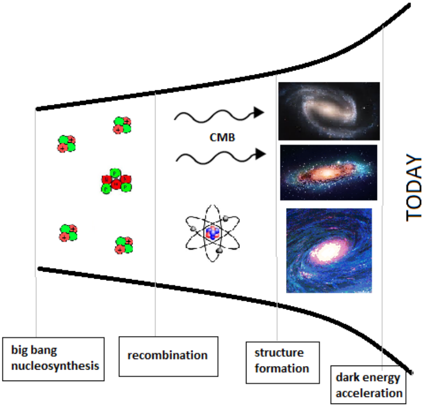
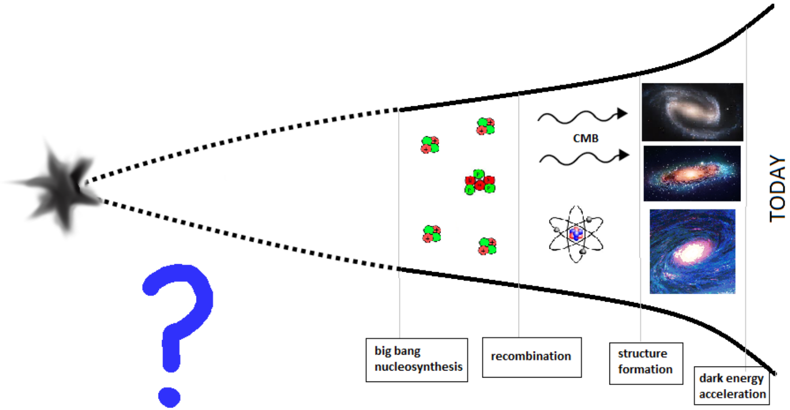
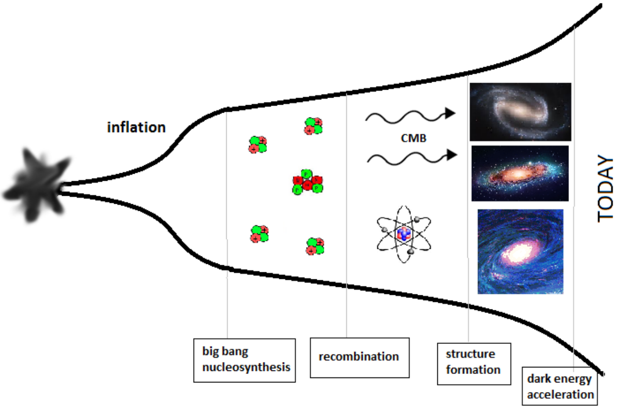


We know a lot about the Universe and its evolution from at least hundreds KeV (BBN) till 3K (today).



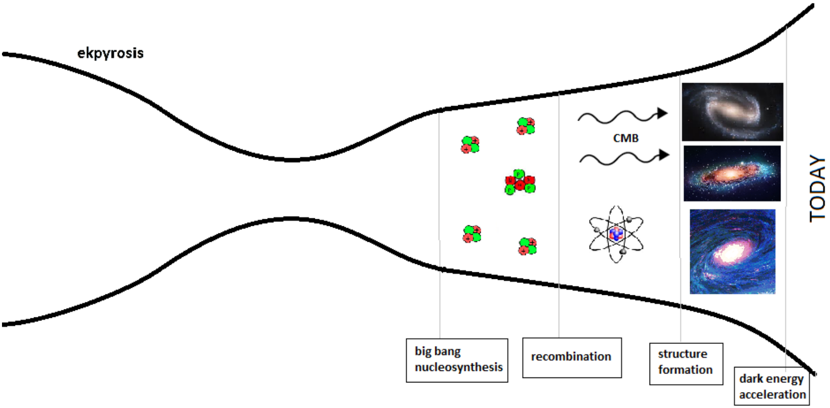
We realize that before the hot stage there should have been some other period of evouliton



Inflation

rapid expansion

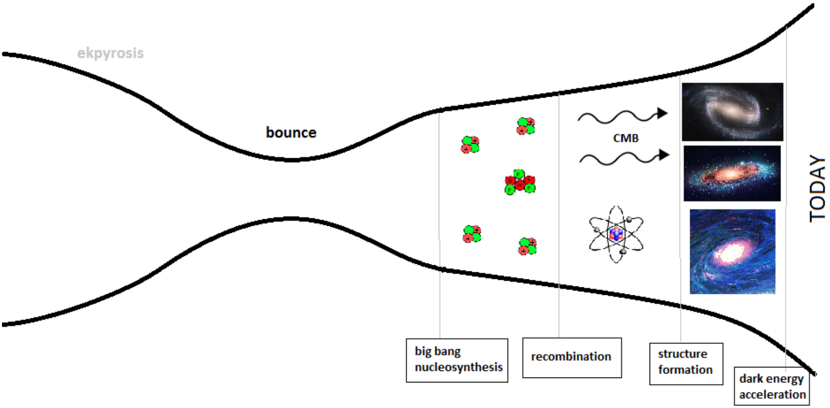
energy decreases



Ekpyrosis

slow contraction $p \gg \rho$

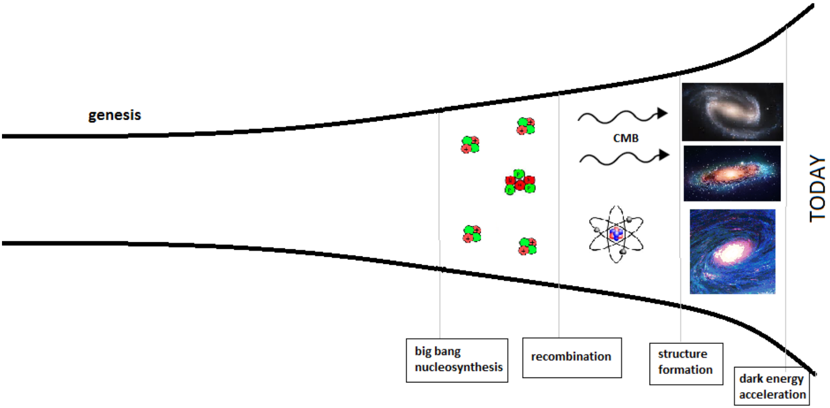
energy decreases



Bounce

contraction $\rightarrow$ expansion

energy INCREASES!



Genesis

static, empty → expansion

energy INCREASES!

The first solution was Inflation

It elegantly solves BB problems:

- flatness
- homogenity
- **flat spectrum**
- ...

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Several Inflation models at first

Chaotic Inflation

Novel Inflation

Hybrid Inflation

But now..

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Higgs Inflation (HI) Radiatively Corrected Higgs Inflation (RCHI) Large Field Inflation (LFI) Mixed Large Field Inflation (MLFI) Radiatively Corrected Massive Inflation (RCMI) Radiatively Corrected Quartic Inflation (RCQI) Natural Inflation (NI) Exponential SUSY Inflation (ESI) Power Law Inflation (PLI) Kahler Moduli Inflation (KMII) Horizon Flow Inflation at first order (HF1I) Coleman Weinberg Inflation (CWI) Loop Inflation (LI) $R + R^{2p}$ Inflation (RpI) Double Well Inflation (DWI) Mutated Hilltop Inflation (MHI) Radion Gauge Inflation (RGI) MSSM Inflation (MSSMI) Renormalisable Inflection Point Inflation (RIPI) Arctan Inflation (AI) Constant ns A Inflation (CNAI) Constant ns B Inflation (CNBI) Open String Tachyonic Inflation (OSTI) Witten-O'Raifeartaigh Inflation (WRI) Small Field Inflation (SFI) Intermediate Inflation (II)

Inflation faces several problems

THE LATEST ASTROPHYSICAL MEASUREMENTS,
COMBINED WITH THEORETICAL PROBLEMS, CAST DOUBT
ON THE LONG-CHERISHED INFLATIONARY THEORY
OF THE EARLY COSMOS AND SUGGEST WE NEED NEW IDEAS

By Anna Ijjas, Paul J. Steinhardt and Abraham Loeb

Inflation faces several problems

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Initial singularity

Initial conditions

Multiverse

It stimulates the search for alternatives.

Ekpyrosis

Bounce

Cyclic

Genesis

Pre-Big-Bang

Anamorphic Universe

String gas

...

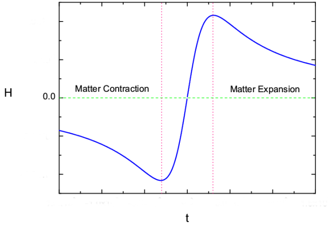
Some of them require energy growth

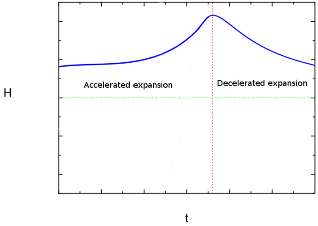
Null Energy Condition

$T_{\mu\nu} k^\mu k^\nu \geq 0$

Friedmann equations

$\dot{H} = -4\pi G(p + \rho) + \frac{\kappa}{a^2}$





Bounce and genesis require NEC-violation

Penrose theorem

Absence of singularity requires NEC-violation

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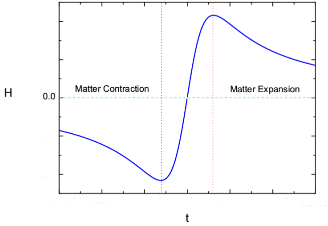

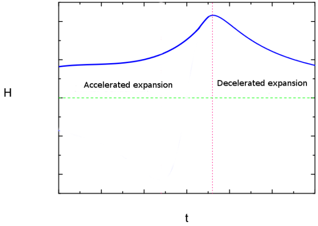

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$T_{\mu\nu} k^\mu k^\nu \geq 0$

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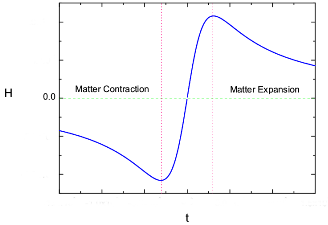
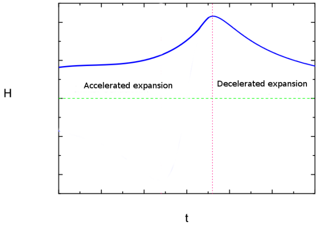
$\dot{H} = -4\pi G(p + \rho) \quad \leq 0$





Bounce and genesis require NEC-violation

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Absence of singularity requires NEC-violation

NEC-violation

Unhealthy?

NEC-violation

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Not necessarily

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Lagrangians with first derivatives $\Rightarrow$ NEC-violation $=$ ghosts and/or gradient instabilities

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Hence we need to consider Lagrangians with second derivatives:

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Hence we need to consider Lagrangians with second derivatives:

- Deal with higher derivative equations

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Not necessarily

Lagrangians with first derivatives $\Rightarrow$ NEC-violation = ghosts and/or gradient instabilities

Hence we need to consider Lagrangians with second derivatives:

- Deal with higher derivative equations
- Get 2 derivatives equations only

$$\mathcal{L} = F(\pi, X) + K(\pi, X)\Box\pi$$

here $X = \partial_\mu \pi \partial^\mu \pi$

$$\delta\mathcal{L} = F_\pi \delta\pi + F_X \delta X + K_\pi \Box\pi\delta\pi + \underline{K_X \Box\pi\delta X + K\Box\delta\pi} =$$

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$$= ... + K_X \Box\pi\delta\partial_\mu\pi\partial^\mu\pi + K\partial_\mu\partial^\mu\delta\pi$$

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$$= ... + K_X \Box\pi\delta\partial_\mu\pi\partial^\mu\pi + K\partial_\mu\partial^\mu\delta\pi$$

$$= ... + 2K_X \Box\pi\partial_\mu\pi\partial^\mu\delta\pi + \partial_\mu\partial^\mu K\delta\pi$$

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$$= ... + 2K_X \Box\pi\partial_\mu\pi\partial^\mu\delta\pi + \partial_\mu\partial^\mu K\delta\pi$$

$$= ... - \underline{2K_X\partial^\mu\Box\pi\partial_\mu\pi\delta\pi} + \partial_\mu(K_\pi\partial^\mu\pi + \underline{2K_X\partial^\mu\partial_\nu\pi\partial^\nu\pi})\delta\pi$$

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$$... - 2K_X\partial^\mu\partial_\nu\partial^\nu\pi\partial_\mu\pi\delta\pi + 2K_X\partial_\mu\partial^\mu\partial_\nu\pi\partial^\nu\pi\delta\pi$$

$$= ...\text{only second derivatives}$$

Horndeski

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X) \Box \pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X) R + 2 G_{4X}(\pi, X) \left[(\Box \pi)^2 - \pi_{;\mu\nu} \pi^{;\mu\nu} \right],$$

$$\mathcal{L}_5 = G_5(\pi, X) G^{\mu\nu} \pi_{;\mu\nu} + \frac{1}{3} G_{5X} \left[(\Box \pi)^3 - 3 \Box \pi \pi_{;\mu\nu} \pi^{;\mu\nu} + 2 \pi_{;\mu\nu} \pi^{;\mu\rho} \pi_{;\rho}^{\ \ \nu} \right]$$

where π is the Galileon field, $X = g^{\mu\nu} \pi_{,\mu} \pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu \pi$, $\pi_{;\mu\nu} = \nabla_\nu \nabla_\mu \pi$, $\Box \pi = g^{\mu\nu} \nabla_\nu \nabla_\mu \pi$, $G_{4X} = \partial G_4 / \partial X$

NEC-violation without pathologies (ghost or gradient instabilities)

T. Qiu, J. Evslin, Y. F. Cai, M. Li and X. Zhang, 1108.0593
D. A. Easson, I. Sawicki and A. Vikman, 1109.1047
M. Osipov and V. Rubakov, 1303.1221
T. Qiu, X. Gao and E. N. Saridakis, 1303.2372

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But situation is not so bright for complete cosmological models

If there are no pathologies during or near the NEC-violation phase, they will appear somewhere else:

Y. F. Cai, D. A. Easson and R. Brandenberger, 1206.2382
M. Koehn, J. L. Lehners and B. A. Ovrut, 1310.7577
L. Battarra, M. Koehn, J. L. Lehners and B. A. Ovrut, 1404.5067
T. Qiu and Y. T. Wang, 1501.03568
T. Kobayashi, M. Yamaguchi and J. Yokoyama, 1504.05710
Y. Wan, T. Qiu, F. P. Huang, Y. F. Cai, H. Li and X. Zhang, 1509.08772
A. Ijjas and P. J. Steinhardt, 1606.08880

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A. Ijjas and P. J. Steinhardt, 1606.08880

Theorem: there is no healthy bounce in Horndeski theory

M. Libanov, S. M and V. Rubakov, 1605.05992
R. Kolevatov and S. M, 1607.04099
T. Kobayashi, 1606.05831
S. Akama and T. Kobayashi, 1701.02926

$$S = \int \mathrm{d}t\mathrm{d}^3 x a^3 \Big[\frac{\mathcal{G_T}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F_T}}{8a^2}\left(\partial_i h^T_{kl}\right)^2 + \mathcal{G_S}\dot{\zeta}^2 - \mathcal{F_S}\frac{(\nabla\zeta)^2}{a^2}\Big]$$

where the coefficients are related:

$$\mathcal{G_S} = \frac{\Sigma \mathcal{G_T}^2}{\Theta^2} + 3\mathcal{G_T},$$
$$\mathcal{F_S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F_T},$$
$$\xi = \frac{a\mathcal{G_T}^2}{\Theta}.$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F_T}}{\mathcal{G_T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F_S}}{\mathcal{G_S}}$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\mathcal{G_T} > \mathcal{F_T} > 0, \quad \mathcal{G_S} > \mathcal{F_S} > 0$$

No-go in Horndeski

$$\xi(t_2) - \xi(t_1) = \int_{t_1}^{t_2} a(t)\,(\mathcal{F}_{\mathcal{T}} + \mathcal{F}_{\mathcal{S}})\,\mathrm{d}t$$

Suppose that $\xi(t_2) > 0$. As we have

$$\xi(t_1) = \xi(t_2) - \int_{t_1}^{t_2} a(t)\,(\mathcal{F}_{\mathcal{T}} + \mathcal{F}_{\mathcal{S}})\,\mathrm{d}t,$$

taking a long enough period of time (for instance, $t_1 \to -\infty$) results in $\xi(t_1) < 0$.
Another possibility is that $\xi(t_1) < 0$:

$$\xi(t_2) = -|\xi(t_1)| + \int_{t_1}^{t_2} a(t)\,(\mathcal{F}_{\mathcal{T}} + \mathcal{F}_{\mathcal{S}})\,\mathrm{d}t,$$

Then taking $t_2 \to \infty$ gives $\xi(t_2) > 0$.
Hence, there must be a moment of time when $\xi(t)$ changes sign, i.e, it crosses zero, $\xi(t_0) = 0$. This requires $\Theta \to \infty$ or $\mathcal{G}_{\mathcal{T}} \to 0$

Beyond Horndeski

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

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$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\ \ \nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$

$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

$$S = \int \mathrm{d}t\mathrm{d}^3 x a^3 \Big[\frac{\hat{\mathcal{G}}_{\mathcal{T}}}{8} \left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_{\mathcal{T}}}{8a^2} \left(\partial_i h^T_{kl}\right)^2 + \mathcal{G}_{\mathcal{S}} \dot{\zeta}^2 - \mathcal{F}_{\mathcal{S}} \frac{(\nabla\zeta)^2}{a^2} \Big]$$

where the modified coefficients are

$$\mathcal{G}_{\mathcal{S}} = \frac{\Sigma \hat{\mathcal{G}}^2_{\mathcal{T}}}{\Theta^2} + 3\hat{\mathcal{G}}_{\mathcal{T}},$$

$$\mathcal{F}_{\mathcal{S}} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_{\mathcal{T}},$$

$$\xi = \frac{a\mathcal{G}_{\mathcal{T}}\hat{\mathcal{G}}_{\mathcal{T}}}{\Theta} = \frac{a\left(\hat{\mathcal{G}}_{\mathcal{T}} - \mathcal{D}\dot{\pi}\right)\hat{\mathcal{G}}_{\mathcal{T}}}{\Theta}.$$

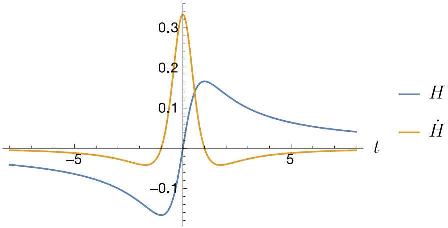
The speeds of sound for tensor and scalar perturbations are, again, respectively,

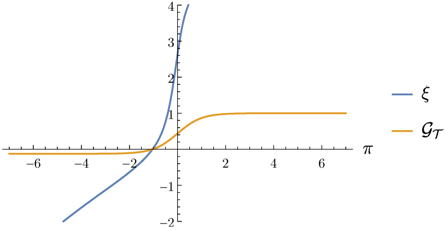
$$c^2_{\mathcal{T}} = \frac{\mathcal{F}_{\mathcal{T}}}{\hat{\mathcal{G}}_{\mathcal{T}}}, \qquad c^2_{\mathcal{S}} = \frac{\mathcal{F}_{\mathcal{S}}}{\mathcal{G}_{\mathcal{S}}}$$

Again, we require correct signs for kinetic and gradient terms as well as subluminal propagation:

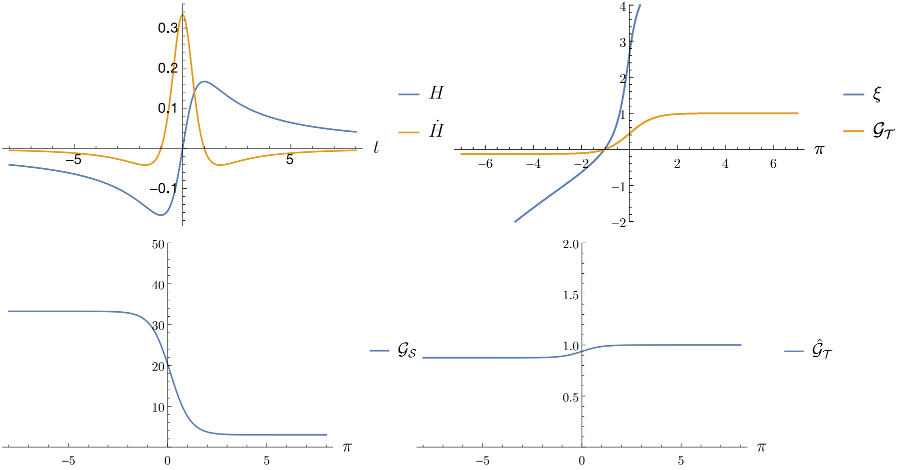
$$\hat{\mathcal{G}}_{\mathcal{T}} > \mathcal{F}_{\mathcal{T}} > 0, \quad \mathcal{G}_{\mathcal{S}} > \mathcal{F}_{\mathcal{S}} > 0$$

Bounce: example

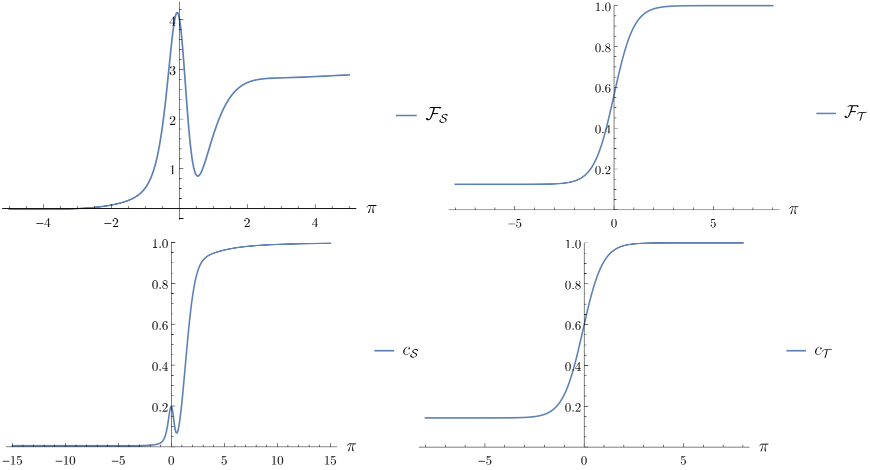




Bounce: example



Bounce: example



Conclusion

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There are

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spartially-flat

Conclusion

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solutions

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Y. Cai and Y. S. Piao, 1705.03401

R. Kolevatov, SM, N. Suchov, V. Volkova, 1705.06626

EFT:

Y. Cai, Y. Wan, H. Li, T. Qiu and Y. Piao, 1610.03400

P. Creminelli, D. Pirtskhalava, L. Santoni and E. Trincherini, 1610.04207

THANK YOU FOR YOUR ATTENTION!

$$\mathcal{G}_\mathcal{T} = 2G_4 - 4G_{4X}X + G_{5\pi}X - 2HG_{5X}X\dot{\pi},$$

$$\mathcal{F}_\mathcal{T} = 2G_4 - 2G_{5X}X\ddot{\pi} - G_{5\pi}X,$$

$$\mathcal{D} = 2F_4X\dot{\pi} + 6HF_5X^2,$$

$$\hat{\mathcal{G}}_\mathcal{T} = \mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi},$$

$$\Theta = -K_XX\dot{\pi} + 2G_4H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi} -$$
$$- 5H^2G_{5X}X\dot{\pi} - 2H^2G_{5XX}X^2\dot{\pi} + 3HG_{5\pi}X + 2HG_{5\pi X}X^2 +$$
$$+ 10HF_4X^2 + 4HF_{4X}X^3 + 21H^2F_5X^2\dot{\pi} + 6H^2F_{5X}X^3\dot{\pi},$$

$$\Sigma = F_XX + 2F_{XX}X^2 + 12HK_XX\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2 -$$
$$- 6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 -$$
$$- 6HG_{4\pi}\dot{\pi} - 30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 30H^3G_{5X}X\dot{\pi} +$$
$$+ 26H^3G_{5XX}X^2\dot{\pi} + 4H^3G_{5XXX}X^3\dot{\pi} - 18H^2G_{5\pi}X - 27H^2G_{5\pi X}X^2 -$$
$$- 6H^2G_{5\pi XX}X^3 - 90H^2F_4X^2 - 78H^2F_{4X}X^3 - 12H^2F_{4XX}X^4 -$$
$$- 168H^3F_5X^2\dot{\pi} - 102H^3F_{5X}X^3\dot{\pi} - 12H^3F_{5XX}X^4\dot{\pi}.$$