Charges in Nonlinear Higher-Spin Theory

(V.E. Didenko, N.M., M.A. Vasiliev, JHEP 03 (2017) 164)

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Ginzburg Conference, Moscow, 23/05/17

Outline

- Nonlinear HS theory
- AdS_4 charges
- Black Holes in HS Theory
- HS BH Charges

Nonlinear HS Equations

- Nonlinear HS theory is described by Vasiliev equations [Vasiliev' 92] – generating equations on HS fields, described by spacetime 1-forms $\omega(Y|x)$ and 0-forms $C(Y|x)$, polynomial in $sp(4)$-spinors Y^A;
- HS equations have schematic form

$$\mathcal{R} := \mathrm{d}\omega + \omega * \omega = \Upsilon(\omega, \omega, C) + \Upsilon(\omega, \omega, C, C) + \ldots$$
$$\mathrm{d}C + [\omega, C]_* = \Upsilon(\omega, C, C) + \ldots$$

where vertices $\Upsilon(\ldots)$ have to be determined from Vasiliev equations.

- $sp(4)$-spinors Y^A form HS algebra with following product:

$$f(Y) * g(Y) = \int d^4U d^4V \exp\left\{iU_A V^A\right\} f(Y+U)\, g(y+V)$$

$$\left[Y^A, Y^B\right]_* = 2i\epsilon^{AB},$$

$$\mathrm{tr} f(Y) := f(0); \quad \mathrm{tr}(f * g) = \mathrm{tr}(g * f).$$

AdS_4 vacuum

- Vacuum:

$$C = 0, \quad \mathrm{d}\Omega + \Omega * \Omega = 0.$$

- Symmetries:

$$\delta\Omega = \mathrm{d}\epsilon + [\Omega, \epsilon]_* \,;$$
$$D_0\epsilon := \mathrm{d}\epsilon + [\Omega_0, \epsilon]_* = 0$$

– leftover global symmetries for fixed Ω_0.

- Natural choice is AdS_4-connection $(so\,(3,2) \approx sp\,(4))$

$$\Omega_0 = \Omega_0^{AB} Y_A Y_B := \frac{i}{4}\left(\omega_L^{\alpha\beta} y_\alpha y_\beta + \bar{\omega}_L^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}} + 2\lambda e^{\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}\right),$$

$$D_0\left(K^{AB} Y_A Y_B\right) = 0;$$

$K^{AB}(x)$ – usual Killing vector along with its covariant derivative.

Physical and topological HS fields

- Perturbative expansion over AdS_4 background results in two different representations for HS fields:
  - adjoint representation

$$D_0 f(Y|x) = \mathrm{d}f + [\omega_L + e, f]_* \, ;$$

  - twisted-adjoint representation

$$\widetilde{D}_0 f(Y|x) = \mathrm{d}f + [\omega_L, f]_* + \{e, f\}_* \, ;$$

- This leads at the free level to the splitting into topological and physical sectors:
  - physical

$$\widetilde{D}_0 C^{phys} = 0;$$

  - topological

$$D_0 C^{top} = 0;$$

- At the interaction level sectors become entangled.

AdS_4 asymptotic charges

- GR asympt. charge for *AdS*-space [Ashtekar, Das '99]:

$$Q = const \cdot \oint_C \mathcal{E}_{ab} \xi^a dS^b,$$

ξ^a – asympt. symm. parameter, $\mathcal{E}_{ab}$ – 'electric' part of Weyl tensor.

- Analogous construction can be built in HS theory. From $\mathcal{R} := \mathrm{d}\omega + \omega * \omega$ it follows

$$D\mathcal{R} := \mathrm{d}\mathcal{R} + [\omega, \mathcal{R}]_* = 0.$$

- If asymptotically $(x_z \to 0)$

$$\omega\,(Y|x) \to \Omega_0, \quad C\,(Y|x) \to 0, \quad D\epsilon \to D_0\epsilon = 0,$$

$$J_\epsilon := \mathtt{tr}\,(\epsilon * \mathcal{R})\,, \quad \mathrm{d}J_\epsilon \to 0.$$

$$Q_\epsilon = \int_{\Sigma^2} J_\epsilon$$

– asymptotically conserved HS charge.

AdS$_4$ HS charges

- Nonlinear HS theory allows a completion of the asympt. charge to the bulk:

$$D_0 \epsilon = 0$$

  holds at the free level for topological fields $\epsilon = C^{top}$.

- In higher orders C^{top} get nonlinear corrections and source phys. fields:

$$DC^{top} = \Upsilon\left(\omega, C^{top}, C^{top}\right) + \Upsilon\left(\omega, C, C\right) + \dots$$
$$\mathcal{R} := \mathrm{d}\omega + \omega * \omega = \Upsilon\left(\omega, \omega, C, C^{top}\right) + \dots$$

- This permits to define

$$J := \mathrm{tr}\left(\mathcal{R}\right), \quad Q := \int_{\Sigma^2} J,$$

  and to treat

$$\mathcal{Z} := \exp\left\{-\int_{\Sigma^2} J\left(C^{top}\right)\right\}$$

  as a partition and C^{top} as chemical potentials conjugated to HS charges.

HS AdS_4 Black Holes

- BH metric in Kerr-Schild form [Carter '68]

$$g_{mn} = g^{AdS}_{mn} + \frac{2M}{r} k_m k_n .$$

- Kerr-Schild vectors k_m:

$$k_m k^m = 0, \quad k_n D_0^n k_m = 0, \quad \frac{1}{r} = -\frac{1}{2} D_0^m k_m .$$

- HS generalisation [Didenko, Matveev, Vasiliev '08]:

$$\phi_{m_1 \ldots m_s} = \frac{2M}{r} k_{m_1} \ldots k_{m_s}$$

obeys free spin-s equation in AdS.

HS AdS_4 Black Holes

- $k_m = k_m(K_{AB})$ – generic AdS_4 BH is completely determined by global symmetry K_{AB} of empty AdS_4 [Didenko, Matveev, Vasiliev '08, '09]
- HS BH Weyl tensors [Didenko, Vasiliev '09]:

$$C_{\alpha(2s)} = \frac{m_s \lambda^{-2s}}{s! 2^s q^{2s+1}} (K_{\alpha\alpha})^s \qquad \bar{C}_{\dot{\alpha}(2s)} = \frac{\bar{m}_s \lambda^{-2s}}{s! 2^s \bar{q}^{2s+1}} \left(\bar{K}_{\dot{\alpha}\dot{\alpha}}\right)^s ,$$

$$q = 2^{-3/2} \lambda^{-2} \sqrt{-K_{\alpha\beta} K^{\alpha\beta}}.$$

- Kerr-like HS BH is generated by $\frac{\partial}{\partial t} = (1, 0, 0, 0)$ in AdS_4 Boyer-Lindquist metric with rotation parameter a.

Charge for Kerr-like HS BH

- Free equations generate linear vacuum partition:

$$D_0\omega\left(y,\bar{y}|x\right)=-\frac{i\eta}{4}e_{\beta}{}^{\dot{\alpha}}e^{\beta\dot{\alpha}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\alpha}}}C\left(0,\bar{y}|x\right)-\frac{i\bar{\eta}}{4}e^{\alpha}{}_{\dot{\beta}}e^{\alpha\dot{\beta}}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\alpha}}C\left(y,0|x\right)$$

- To the vacuum partition at the free level only spin-1 contributes:

$$J^0=-\frac{i\eta}{4}e_{\beta}{}^{\dot{\alpha}}e^{\beta\dot{\alpha}}C_{\dot{\alpha}\dot{\alpha}}\left(x\right)-\frac{i\bar{\eta}}{4}e^{\alpha}{}_{\dot{\beta}}e^{\alpha\dot{\beta}}C_{\alpha\alpha}\left(x\right)$$

$$Q^0=4\pi\frac{m_1\bar{\eta}-\bar{m}_1\eta}{1+\Lambda a^2}.$$

First order topological contribution

- First order corrections:

$$D_0\omega\left(y,\bar{y}|x\right) = -\frac{i}{4}\left(\mu e_{\beta}{}^{\dot{\alpha}}e^{\beta\dot{\alpha}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\alpha}}}\left\{C, C^{top}\right\}_*(0,\bar{y}|x) + h.c.\right)$$

- Spin-2 sector

$$C^{top} = K_{AB}\left(x\right)Y^AY^B, \quad D_0C^{top} = 0,$$

contribution to spin-2 charge is

$$J^1_{s=2} = -\frac{i\mu}{2}e_{\beta}{}^{\dot{\alpha}}e^{\beta\dot{\alpha}}\bar{K}^{\dot{\alpha}\dot{\alpha}}C_{\dot{\alpha}(4)}\left(x\right) + h.c.$$

$$Q^1_{s=2} = 3\pi\Lambda\frac{m_2\bar{\mu} - \bar{m}_2\mu}{1+\Lambda a^2}.$$

Conclusion

- A generalisation of Ashtekar-Das asymptotic charge for HS theory is proposed, allowing a nonlinear bulk completion;
- From thermodynamical perspective, topological fields play a role of chemical potentials in HS partition;
- Free vacuum contribution to the partition is calculated, agreed with ADM-like behaviour;
- Possible relations to the BH entropy and information paradox?