Hypernetwork as an upper level of describing dynamics and evolution of a neuron network

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O.V. Maslennikov, D.S. Shchapin, V.I. Nekorkin, Transient sequences in a hypernetwork generated by an adaptive network of spiking neurons. // Phil. Trans. R. Soc. A. 2017. Vol. 375 (2096), 20160288
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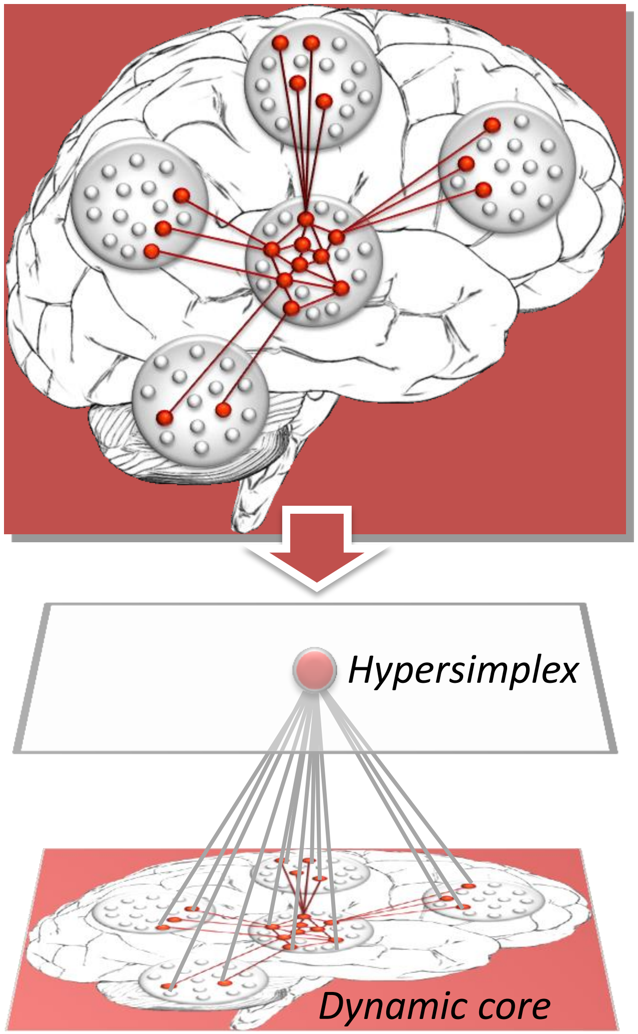
Some definitions

Hypersimplices are ordered, or structured, sets of nodes with an explicit relation; in other words, they exist at a higher level of representation than network's nodes.

A structured set of hypercimplices determines a *hypernetwork* where the connections between hypercimplices describe their functional relations.

Modern neurobiological view on cognitive function

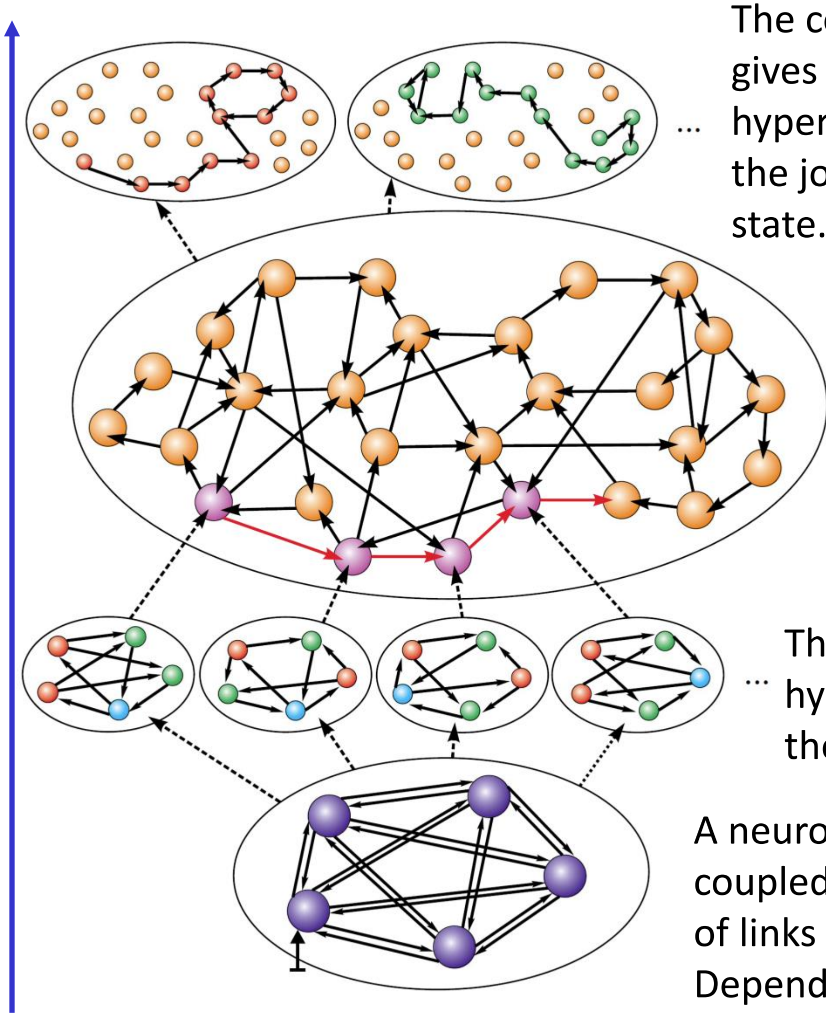


Each perception or cognitive act there emerges an assembly of spatially distributed neurons—the dynamic core—that operates in a coordinated manner. The neurons within the assembly preferentially communicate with each other giving rise to some integration activity.

This process is highly transient which enables the neuronal network to rapidly change the activation pattern reflecting the multitasking property of brain and functional segregation.

We relate this distributed neuronal group specified by the common task to a corresponding hypersimplex. A structured set of hypersimplices forms a hypernetwork.

A hypernetwork generated by an adaptive network



The coevolution of the structure and dynamics at the network gives rise to a kind of traffic at the upper level – in the hypernetwork. An actual path in the hypernetwork depends on the joint action of external inputs and the network's internal state.

A structured set of hypersimplices determines a hypernetwork where the connections between hypersimplices describe their functional relations. In our model, we assume that two hypersimplices are connected if the network can change its state from one hypersimplex to another due to the evolutionary rules governing its structure and dynamics.

The network generates a family of relational simplices (or hypersimplices). In our model hypersimplices are specified by the coupling topology that leads to a certain cluster activity.

A neuronal population as a network of interacting nodes-oscillators coupled by directed links. The nodal dynamics as well as the activity of links are governed by some deterministic evolutionary operators. Depending on nodal dynamics the links become active or inactive thus forming different structural patterns in the network.

Different perception stimuli or cognitive tasks give rise to the activation of different distributed neuronal groups. We relate this distributed neuronal group specified by the common task to a corresponding hypersimplex. Hypersimplices are ordered sets of nodes with an explicit relation. They exist at a higher level of representation than network's nodes.

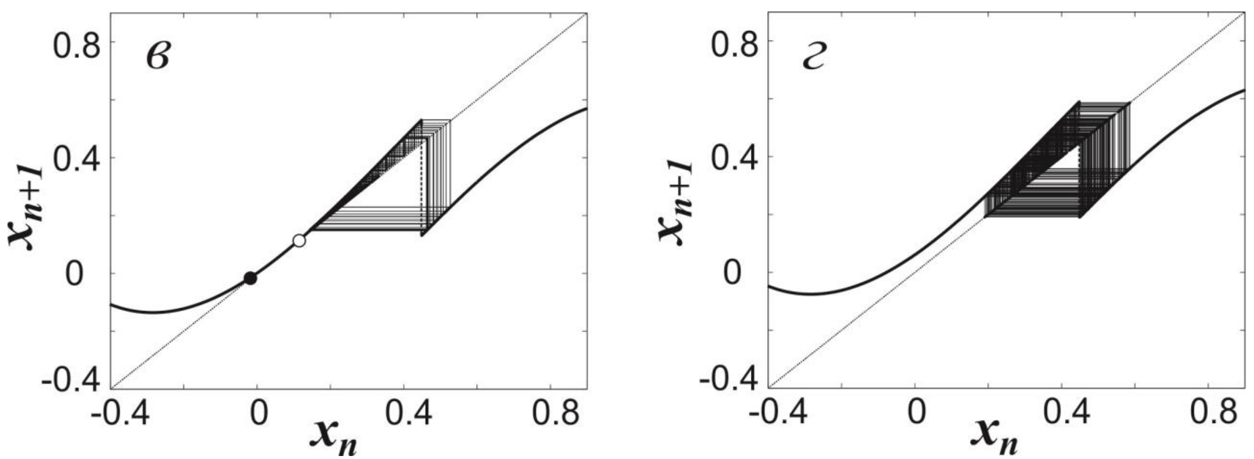
Dynamics of a node

$$\begin{cases} x_{n+1} = x_n + F(x_n) - \beta H(x_n - d) - y_n + I, \\ y_{n+1} = y_n + \varepsilon(x_n - J), \end{cases}$$

where $F_H(x) = x(x-a)(1-x) - \beta H(x-d)$

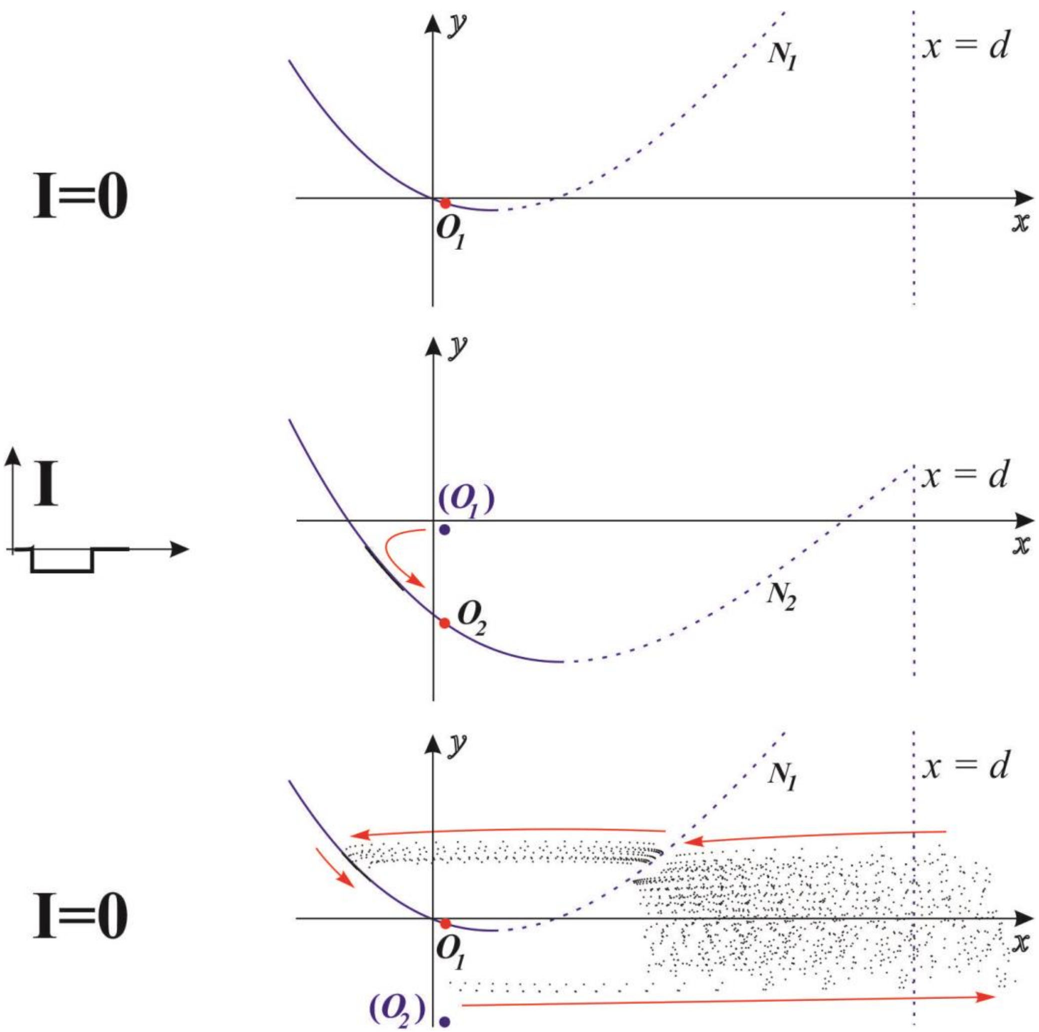
***n*=0,1,2… is discrete time; *H*(*x*) is the Heaviside function; *a*, *β*, *d* are positive parameters which control a dynamic regime; *ε* determines the rate for *y*; *J* characterizes excitatory properties of a node; *I* is an input current.**

For fixed *y*, there are two different behaviors depending on *y*:



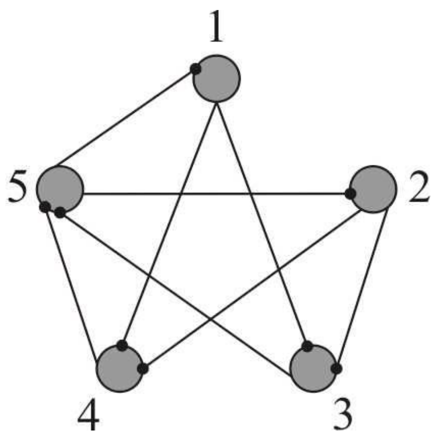
Dynamics of a node

$$
\begin{cases}
x_{n+1} = x_n + F(x_n) - \beta H(x_n - d) - y_n + I, \\
y_{n+1} = y_n + \varepsilon (x_n - J),
\end{cases}
$$

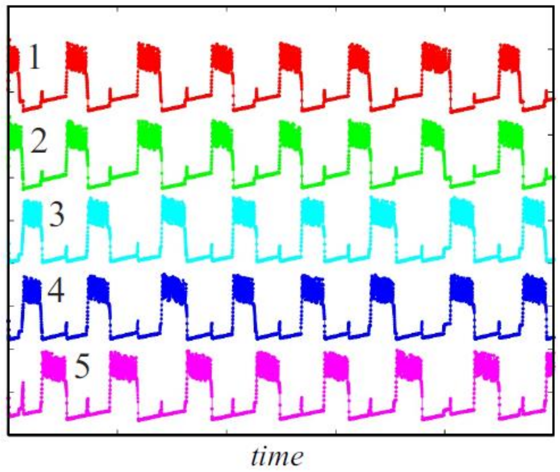


When stimulated by a rectangular impulse, the system exhibits a burst.

Example: 5-node oscillatory network with inhibitory connections



Corresponding dynamics: the cluster state in the form of the cyclic sequence
1,2 → 3,4 → 5



$$x_{i,n+1} = x_{i,n} + F_H(x_{i,n}) - y_{i,n} + I_{i,n},$$
$$y_{i,n+1} = y_{i,n} + \varepsilon(x_{i,n} - J_i), i = 1, ..., 5,$$

System of nodal dynamics

$$F_H(x) = x(x-a)(1-x) - \beta H(x-d)$$

Nonlinearity

$$I_{i,n} = -g \sum_{j=1, j\neq i}^{j=N} a_{ij,n}(x_{i,n} - \nu)H(x_{j,n} - \theta),$$

Internodal coupling

5-node oscillatory network with inhibitory connections

$$q_{n+1} = q_n + \mu X_n, \quad X_n = \frac{1}{5}\sum_{i=1}^{5} x_{i,n},$$

$$\text{if } q_n > 1 \text{ then } q_n := 0 \text{ and } n^* := n.$$

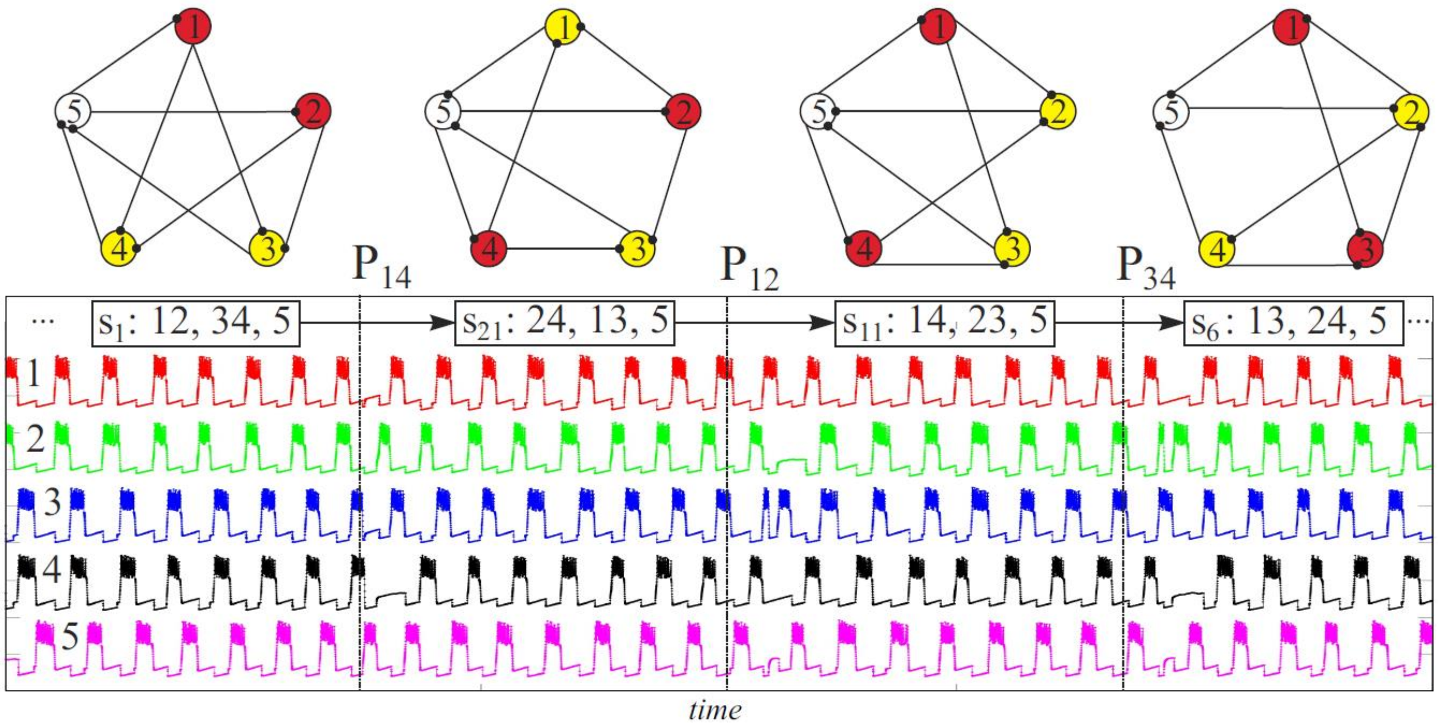
System for the auxiliary variable *q* which defines the moment $n=n*$ of topology switching

$$A_{n+1} = T_{kl} A_n T_{kl}$$

Operator for topology switching

Which numbers *k* and *l* are chosen depends on the following algorithm. Suppose at the moment of rewiring $n= n*$ the nodes with indexes (*i*3*; i*4) are active, and before that the nodes (*i*1*; i*2) were active. In the clockwise ordered set starting from i5 there is a pair of nodes ik$*\in$ (i3; i4) and il$*\in$ (i1; i2) that have a minimum index distance determined from the clockwise ordering of the nodes. The indexes *ik* and *il* specify the numbers $k = k*$ and $l = l*$. The matrix *Tkl* is obtained by swapping row *k* and row *l* of the identity 5 × 5 matrix with 1's on the main diagonal and 0's elsewhere.

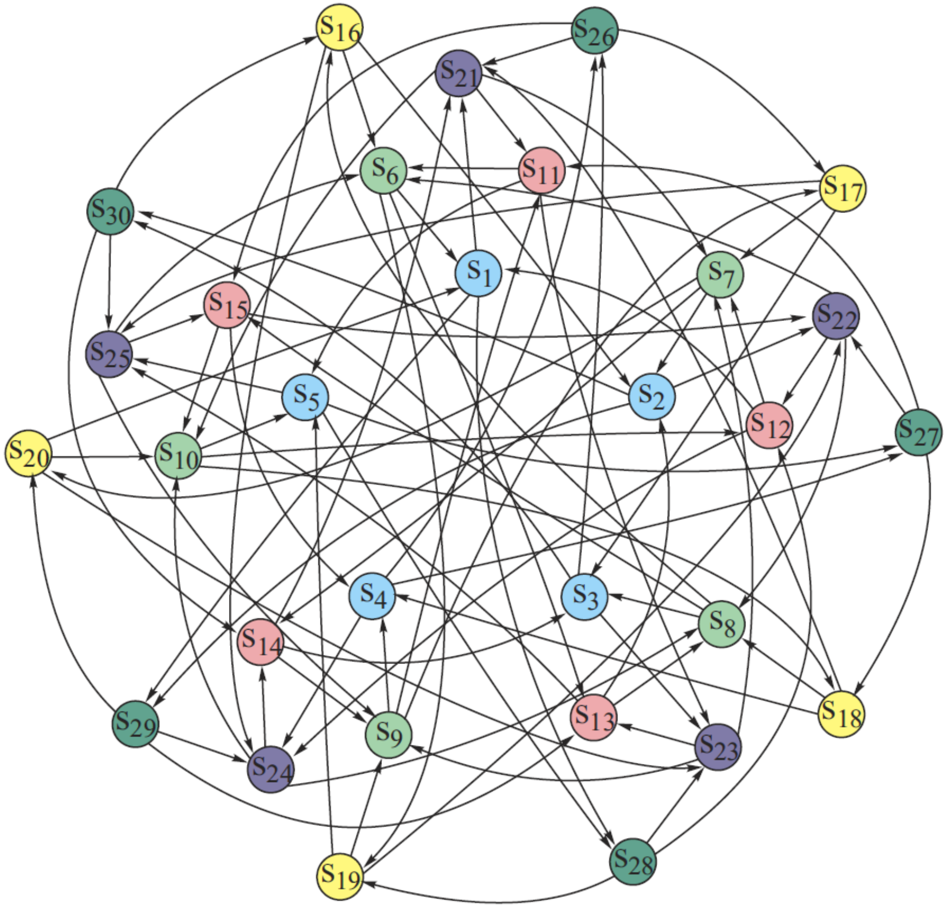
Hypersimplices and a transient sequence of cluster states



There are 30 different 3-cluster states and corresponding hypersimplices in the network of 5 nodes where two clusters consists of two nodes, and the third cluster contains only one:

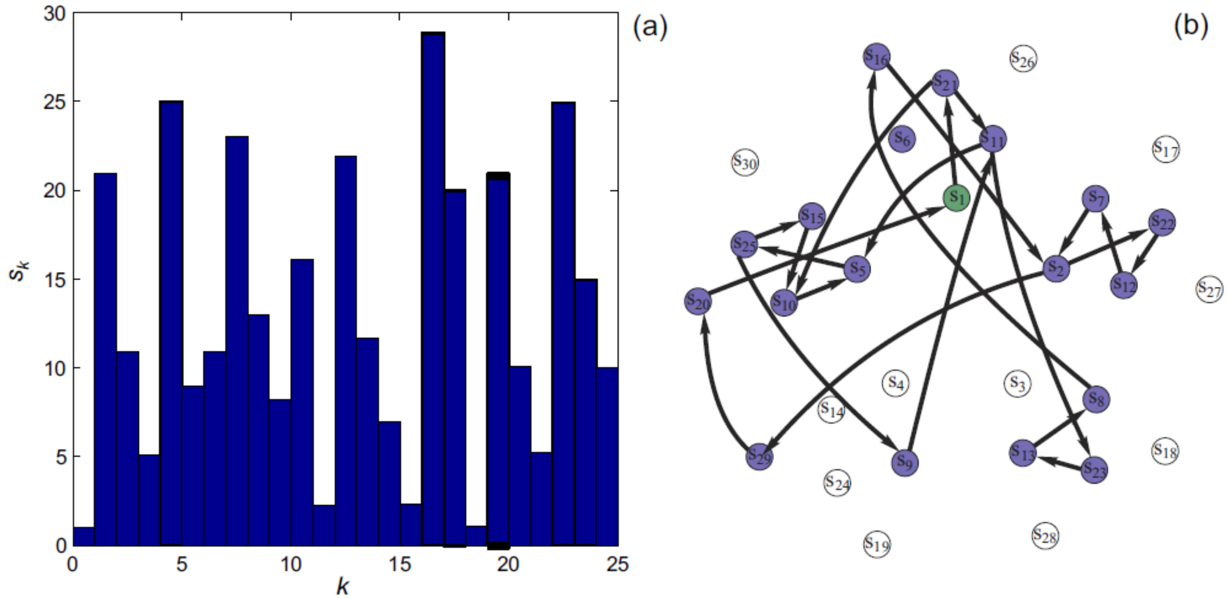
s_1 : 12, 34, 5	s_2 : 23, 45, 1	s_3 : 34, 51, 2	s_4 : 45, 12, 3	s_5 : 51, 23, 4
s_6 : 13, 24, 5	s_7 : 24, 35, 1	s_8 : 35, 14, 2	s_9 : 14, 25, 3	s_{10} : 25, 13, 4
s_{11} : 14, 23, 5	s_{12} : 25, 34, 1	s_{13} : 13, 45, 2	s_{14} : 24, 15, 3	s_{15} : 35, 12, 4
s_{16} : 23, 14, 5	s_{17} : 34, 25, 1	s_{18} : 45, 13, 2	s_{19} : 15, 24, 3	s_{20} : 12, 35, 4
s_{21} : 24, 13, 5	s_{22} : 35, 24, 1	s_{23} : 14, 35, 2	s_{24} : 25, 14, 3	s_{25} : 13, 25, 4
s_{26} : 34, 12, 5	s_{27} : 45, 23, 1	s_{28} : 51, 34, 2	s_{29} : 12, 45, 3	s_{30} : 23, 51, 4

Resulting hypernetwork

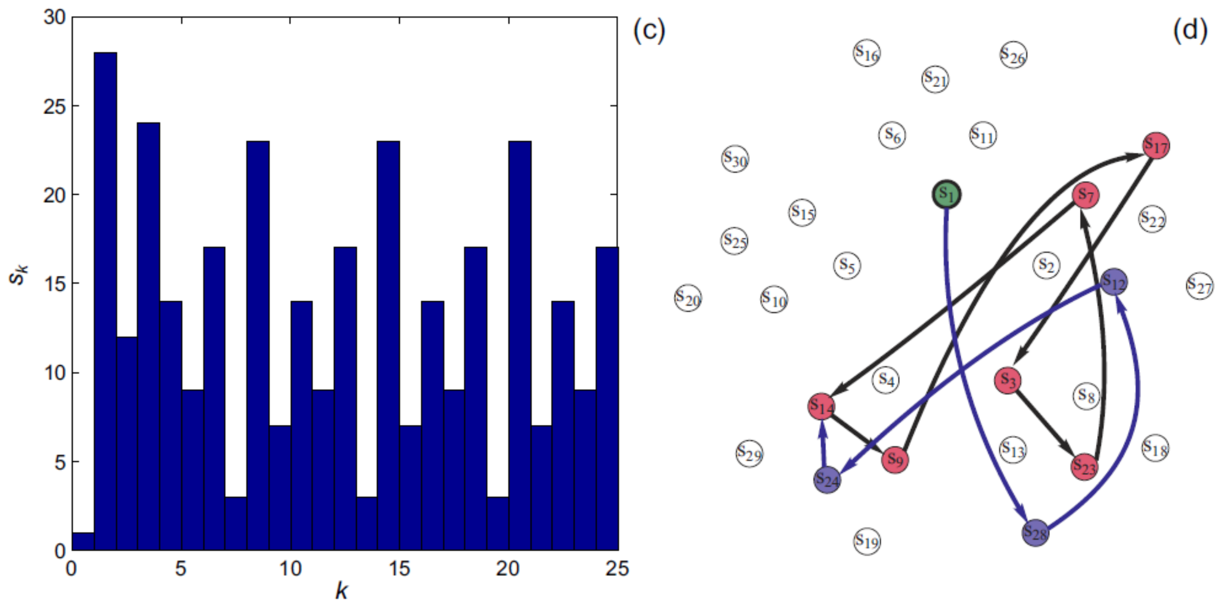


All these 30 hypersimplices and transitions between them constitute a hypernetwork. Note that up to three different transitions are possible from one hypersimplex s_i to others. Starting from some initial conditions one cannot predict a sequence of states s_i because the moments n^* of switching are unknown a priori. Applying an input to the network, the path becomes stimulus-specified.

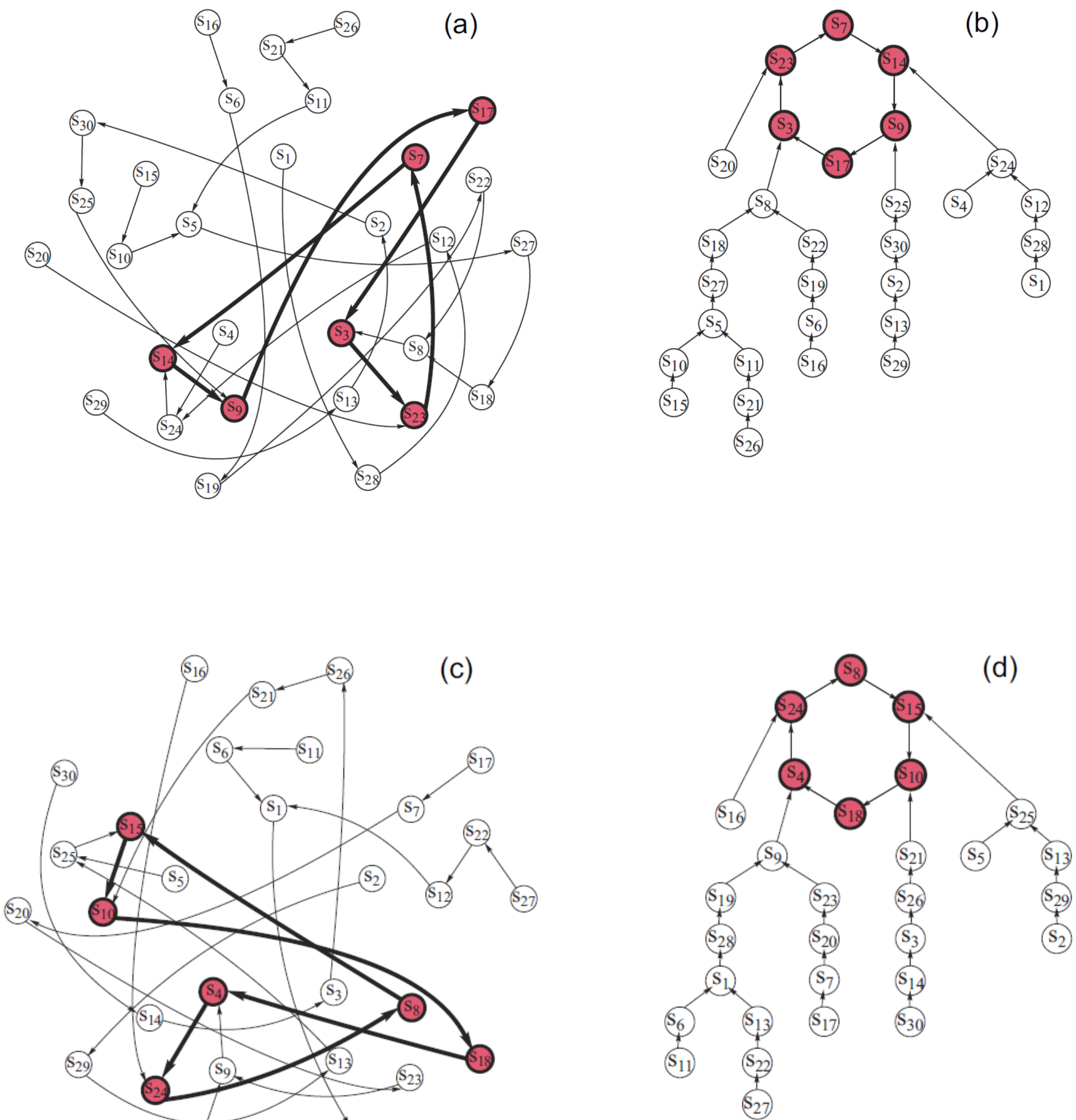
Sequences of clusters states and the corresponding paths in the hypernetwork in the autonomous case and under constant inputs



Without inputs: Sequence of different cluster states and the path in the corresponding hypernetwork is a random walk.

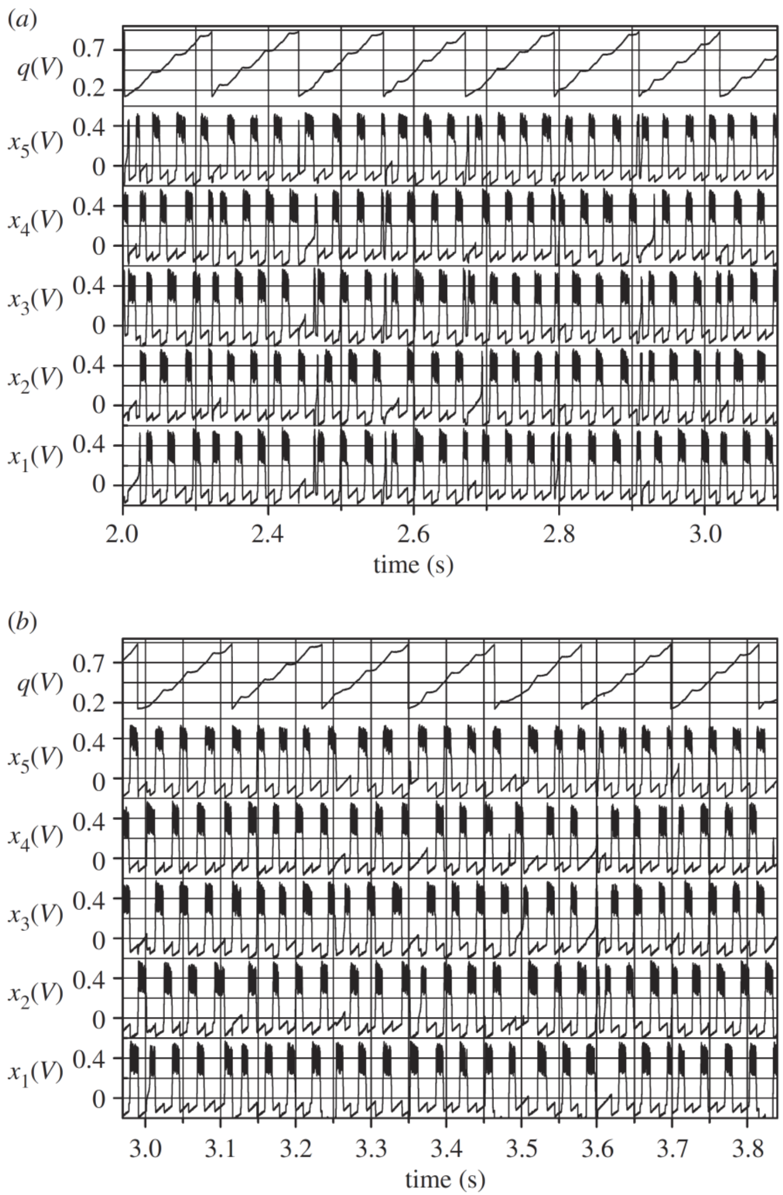


A constant input applied to node 1 while the network initially is in the state $s1$: The trajectory in the hypernetwork goes through the sequence $s1; s28; s12; s24$ and comes to the cycle $s14; s9; s17; s3; s23; s7$.



The application of a constant stimulus to node 1 (a) and node 2 (c) specifies active links in the hypernetwork; thus this reduced hypernetwork governs transitions until node 1 (respectively, node 2) remains under the stimulus. In both cases all the trajectories come to cycles of 6 cluster states shown by red circles. The same reduced hypernetworks can be presented in an untangled way (b,d).

Experimental setup



To create a real-time functioning 5-node dynamical network with inhibitory connections, we realized its electronic model on FPGA Xilinx Artix-7. The FPGA electronic model allows one to process external (sensory, informational) signals in this network in real time. The analysis shows a full agreement of theoretical and numerical predictions with experimental data. Without external inputs the experimental set-up displays random transitions between different cluster states (a). In the case of an input application, the network demonstrates transitions relating to the corresponding reduced hypernetwork where each hypersimplex has only one outward link and all paths are converged to a six-cycle. For example, when an input is applied to node 1 the network activity gives rise to a transition in the hypernetwork towards the cyclic sequence s23, s7, s14, s9, s17, s3, s23, etc (b).

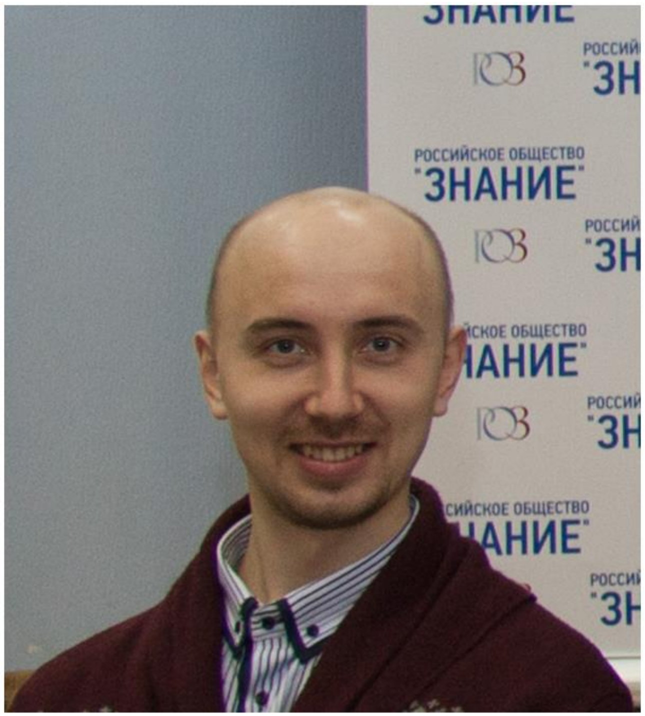
Conclusions

- We proposed a paradigmatic model of how a network of spiking neurons can create complex responses at the higher level or representation – in the corresponding hypernetwork.

- Despite the simplicity of the coupling structure, the neuron model, and the evolutionary operator we show the basic idea: how dynamics of the adaptive oscillatory network of spiking neurons leads to the emergence of various transient behaviors in the hypernetwork.

Acknowledgments

Oleg Maslennikov and Dmitry Shchapin



Thank you for attention