Field theretical approach to gravity

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Abstract

The metric of spherically symmetric ball of ideal liquid is considered in G^2- approximation with the help of theory of sources. Using the integral equations of this theory gives the exterior metric depending upon the radius of the ball of matter in some terms proportional to G^2.I argue that according to this metric from measurement outside the ball one can obtain the radius b of the ball of matter

Interior solution The following notation is used

$$g_{ik}=\eta_{ik}+h^{(1)}_{ik}+h^{(2)}_{ik}+\cdots, \quad h^{(n)}_{ik}\propto G^n, \quad \eta_{ik}=\mathrm{diag}(-1,1,1,1), \quad \frac{\partial h}{\partial x^\alpha}=h_{,\alpha},$$

$$\bar h^{(n)}_{ik}=h^{(n)}_{ik}-\frac{1}{2}\eta_{ik}h^{(n)}, \quad i,k=0,1,2,3, \quad h^{(n)}=h^{(n)}_{,\alpha\alpha}-h^{(n)}_{00}, \quad \alpha,\beta=1,2,3. \qquad (1)$$

In linear approximation in Hilbert gauge $\bar h^{(1)}_{\alpha\beta.\alpha}(m',r)=0$ we have

$$h^{(1)}_{ik}(m',r)=-2\phi(m',r)\delta_{ik}, \quad \bar h^{(1)}_{ik}=-4\phi(m',r)\delta_{0i}\delta_{0k}, \quad \phi(m',r)=\frac{m'G}{2b}(\frac{r^2}{b^2}-3), \quad m'=\frac{4}{3}\pi b^3\mu. \qquad (2)$$

We note that according to the theory of sources the Hilbert gauge $\bar h^{ik}{}_{,k}=0$ means that gauge degree of freedom are excluded. This is true for any $\bar h^n_{ik}$. In (2) μ is the density of liquid. The function $\bar h^{(2)}_{ik}(m',r)$ in this gauge satisfies the differential equation

$$\nabla^2\bar h^{(2)}_{ik}(m',r)=-16\pi G(T^{(1)}_{ik}+t_{ik}), \qquad (3)$$

This is the differential form of eq. (17.6) in Ch.3, §17 in J.Schwinger. Particles, sources. and fields. Addison- Wesley, 1970.

Here $T^{(1)}_{ik}$ is the tensor of matter, t_{ik}-tensor due to 3-graviton interaction In this paper we assume that t_{ik} is given by general relativity, see Ch. 7, §6 in S. Weinberg. Gravitation and Cosmology. New York 1972.

In functions proportional to G^2, such as $\bar h^{(2)}_{ik}(m',r)$, $h^{(2)}_{ik}(m',r)$ and so on m' indicate only that these functions are obtained directly from equations of perturbation theory. In terms proportional to G^2 in the considered approximation it does not matter whether it stands there m' or the dressed mass $m=m'(1+\frac{3mG}{b})$ In the region inside the ball we have, see A.I.Nikishov. arXiv: 1605.06305 v.I [physics gen -ph] 16 May 2016]

$$t_{00}=-\frac{3}{8\pi G}(\nabla\phi)^2-6\mu\phi=\frac{m^2G^2}{8\pi Gb^4}(54-\frac{21r^2}{b^2}), \quad T^{(1)}_{00}=2\mu\phi=\frac{m^2G^2}{4\pi Gb^4}(\frac{3r^2}{b^2}-9), \qquad (4)$$

$$T^{(1)}_{00}+t_{00}=\frac{m^2G^2}{8\pi Gb^4}(36-\frac{15r^2}{b^2}),$$

$$T^{(1)}_{\alpha\beta}+t_{\alpha\beta}=\frac{m^2G^2}{8\pi Gb^4}[\delta_{\alpha\beta}\left(-9+\frac{4r^2}{b^2}\right)-\frac{2x_\alpha x_\beta}{b^2}]. \tag{5}$$

Now using the integral equation of theory of sources (see Schwinger book)

$$\bar{h}^{(2)}_{ik}(m',r)=16\pi G\int d^4x'D_+(x-x')[T^{(1)}_{ik}(x')+t_{ik}(x')], \tag{6}$$

we find

$$\bar{h}^{(2)}_{\alpha\beta}(m',r)=\frac{m^2G^2}{b^2}[\delta_{\alpha\beta}(-5+\frac{18}{5}\frac{r^2}{b^2}-\frac{3}{7}\frac{r^4}{b^4})-\frac{9}{5}\frac{x_\alpha x_\beta}{b^2}+\frac{2}{7}\frac{r^2x_\alpha x_\beta}{b^4}]=\bar{h}^{(2)}_{\alpha\beta}(m,r), \tag{7}$$

$$\bar{h}^{(2)}_{00}(m',r)=\frac{m^2G^2}{b^2}(\frac{51}{2}-12\frac{r^2}{b^2}+\frac{3}{2}\frac{r^4}{b^4}). \tag{8}$$

The expression (7) remains valid when m' is replaced by m because $\bar{h}^{(1)}_{\alpha\beta}(m',r)=0$ in (2) for $\alpha,\beta=1,2,3$, see the text below eq. (13). From (7) we have

$$\bar{h}^{(2)}_{\alpha\alpha}(m',r)=\bar{h}^{(2)}_{\alpha\alpha}(m,r)=\frac{m^2G^2}{b^2}[-15+9\frac{r^2}{b^2}-\frac{r^4}{b^4}]. \tag{9}$$

Next from solutions $\bar{h}^{(2)}_{ik}(m',r)$ we should obtain the solution $h^{(2)}_{ik}(m',r)$:

$$h^{(2)}_{ik}(m',r)=\bar{h}^{(2)}_{ik}(m',r)-\frac{1}{2}\eta_{ik}\bar{h}(m',r),\quad \bar{h}^{(2)}==\bar{h}^{(2)}_{\alpha\alpha}-\bar{h}^{(2)}_{00}=-h^{(2)}. \tag{10}$$

Using the eqs. (9) and (8), we get from the second eq. in (10)

$$\bar{h}^{(2)}(m',r)=\frac{m^2G^2}{b^2}(-\frac{81}{2}+21\frac{r^2}{b^2}-\frac{5}{2}\frac{r^4}{b^4}). \tag{11}$$

From the first eq. in (10) (in which i,k are replaced by α,β) we obtain

$$h^{(2)}_{\alpha\beta}(m',r)=\frac{m^2G^2}{b^2}[\delta_{\alpha\beta}(\frac{61}{4}-\frac{69}{10}\frac{r^2}{b^2}+\frac{23}{28}\frac{r^4}{b^4})-\frac{9}{5}\frac{x_\alpha x_\beta}{b^2}+\frac{2}{7}\frac{r^2x_\alpha x_\beta}{b^4}]. \tag{12}$$

Similarly from (10) when $i=k=0$ we find

$$h^{(2)}_{00}(m',r)=\frac{m^2G^2}{b^2}(\frac{21}{4}-\frac{3}{2}\frac{r^2}{b^2}+\frac{1}{4}\frac{r^2}{b^2}). \tag{13}$$

Besides solutions $h^{(2)}_{ik}(m',r)$ we need solutions $h^{(2)}_{ik}(m,r)$. The latter solutions can be obtained from the former ones by adding to them G^2-terms from $h^{(1)}_{ik}(m',r)$ when in them m' is replaced by $m(1-3\frac{mG}{b})$; m is the dressed mass. Indeed we have

$$h^{(1)}_{ik}(m',r)=\frac{m'G}{b}\delta_{ik}(3-\frac{r^2}{b^2})=h^{(1)}_{ik}(m,r)+\frac{m^2G^2}{b^2}\delta_{ik}(3\frac{r^2}{b^2}-9),\quad m'=m(1-3\frac{mG}{b}). \tag{14}$$

Similarly from (2)

$$\bar{h}^{(1)}_{00}(m',r)=-4\phi(m',r)=\bar{h}^{(2)}_{00}(m,r)+\frac{m^2G^2}{b^2}(6\frac{r^2}{b^2}-18). \tag{15}$$

Equations (10) remain valid also when m' is replaced by m. So taking into account (9) and equation (19) below from the second equation in (10) we find

$$\bar{h}^{(2)}(m,r) = \frac{m^2G^2}{b^2}(-\frac{45}{2} + 15\frac{r^2}{b^2} - \frac{5}{2}\frac{r^4}{b^4}), \tag{16}$$

Similarly from the first equation in (10) with the help of (7) and (16) we get

$$h^{(2)}_{\alpha\beta}(m,r) = \frac{m^2G^2}{b^2}[\delta_{\alpha\beta}(\frac{25}{4} - \frac{39}{10}\frac{r^2}{b^2} + \frac{23}{28}\frac{r^4}{b^4}) - \frac{9}{5}\frac{x_\alpha x_\beta}{b^2} + \frac{2}{7}\frac{r^2 x_\alpha x_\beta}{b^4}], \tag{17}$$

In the same manner with the help of (19) and (15) we obtain

$$h^{(2)}_{00}(m,r) = \frac{m^2G^2}{b^2}(-\frac{15}{4} + \frac{3}{2}\frac{r^2}{b^2} + \frac{1}{4}\frac{r^2}{b^2}). \tag{18}$$

Similarly from (8) and (15) we find

$$\bar{h}^{(2)}_{00}(m,r) = \frac{m^2G^2}{b^2}(\frac{15}{2} - 6\frac{r^2}{b^2} + \frac{3}{2}\frac{r^4}{b^4}). \tag{19}$$

We note that $h^{(2)}_{\alpha\beta}(m,r)$ differs from harmonic $h^{(2)har}_{\alpha\beta}(m,r)$ and isotropic$h^{(2)iso}_{\alpha\beta}(m,r)$ only by gauge terms, in more detail see A.I.Nikishov. arXiv: 1605.06305 v.I [physics gen -ph] 16 May 2016. Equation (18) holds in all these systems.

Exterior solution. With the help of (5) from (6) we obtain the contribution to $\bar{h}^{(2)}_{\alpha\beta}(m',r)$ from $r' < b$

$$16\pi G\int_{r'<b<r}\frac{d^3x'}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}(T^{(1)}_{\alpha\beta} + t_{\alpha\beta}) = (mG)^2[-\frac{14}{3}\frac{\delta_{\alpha\beta}}{rb} - \frac{4}{5\cdot 7}b(\frac{x_\alpha x_\beta}{r^5} - \frac{\delta_{\alpha\beta}}{3r^3})]. \tag{20}$$

Here were used the equations (4), (5) and (16) in A.I.Nikishov. arXiv: 1605.06305 v.I [physics gen -ph] 16 May 2016). With the help of relation (see for example equation (70) in A.I.Nikishov. Part. Nucl. v.32, No 1, p.5 (2001))

$$16\pi G t_{\alpha\beta} = (mG)^2[\frac{14\delta_{\alpha\beta}}{r^4} - \frac{28x_\alpha x_\beta}{r^6}] \tag{21}$$

we find the contribution to $\bar{h}^{(2)}_{\alpha\beta}(m',r)$ from $b < r'$

$$16\pi G\int_{b<r',r}\frac{d^3x'}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}t_{\alpha\beta}(x') = (mG)^2[\frac{14}{3}\frac{\delta_{\alpha\beta}}{rb} - \frac{7x_\alpha x_\beta}{r^4} + \frac{28}{5}b(\frac{x_\alpha x_\beta}{r^5} - \frac{\delta_{\alpha\beta}}{3r^3})]. \tag{22}$$

Here were used the equations (A18) and (15) in arXiv: 1605.06305 v.I [physics gen -ph] 16 May 2016. The sum of these two contribution (20) and (22) gives

$$\bar{h}^{(2)}_{\alpha\beta}(m',r) = (mG)^2[-7\frac{x_\alpha x_\beta}{r^4} + \frac{192}{35}b(\frac{x_\alpha x_\beta}{r^5} - \frac{\delta_{\alpha\beta}}{3r^3})].. \tag{23}$$

We remind here that $\bar{h}^{(2)}_{\alpha\beta}(m',r) = \bar{h}^{(2)}_{\alpha\beta}(m,r)$ because $\bar{h}^{(2)}_{\alpha\beta}(m',r) = 0$, see the second equation in (2). It is usefull to note that the region $b < r'$ contributes to (23) essentially more then the region $r' < b$.

From the first equation in (10) and the relation

$$h^{(2)}(m',r) = (mG)^2(\frac{12}{rb} + \frac{10}{r^2}) = -\bar{h}(m',r) \tag{24}$$

it follows

$$h^{(2)}_{\alpha\beta}(m',r) = m^2G^2[\frac{6\delta_{\alpha\beta}}{rb} + (\frac{5\delta_{\alpha\beta}}{r^2} - \frac{7x_\alpha x_\beta}{r^4}) + \frac{192b}{35}(\frac{x_\alpha x_\beta}{r^5} - \frac{\delta_{\alpha\beta}}{3r^3})]. \tag{25}$$

The expression for $h^{(2)}_{\alpha\beta}(m,r)$ is obtained from here by dropping the term $\frac{6\delta_{\alpha\beta}}{rb}$ because it is cancelled by the term in $h^{(1)}_{\alpha\beta}(m',r)$:

$$h^{(1)}_{\alpha\beta}(m',r) = h^{(1)}_{\alpha\beta}(m,r) - m^2G^2\frac{6\delta_{\alpha\beta}}{rb}, \quad h^{(1)}_{\alpha\beta}(m,r) = \delta_{\alpha\beta}\frac{2mG}{r}. \tag{26}$$

Thus

$$h^{(2)}_{\alpha\beta}(m,r) = m^2G^2[(\frac{5\delta_{\alpha\beta}}{r^2} - \frac{7x_\alpha x_\beta}{r^4}) + \frac{64b}{35}(3\frac{x_\alpha x_\beta}{r^5} - \frac{\delta_{\alpha\beta}}{r^3})]. \tag{25a}$$

It is easy to check that the interior solution (18) continuously goes over to exterior one (25a):

$$h^{(2)}_{\alpha\beta}(m,r)\Big|_{r\to b} = \left(\frac{mG}{b}\right)^2[\frac{111}{35}\delta_{\alpha\beta} - \frac{53}{35}\frac{x_\alpha x_\beta}{b^2}] = h^{(2)}_{\alpha\beta}(m,r)\Big|_{b\leftarrow r}. \tag{27}$$

We note that the linear approximation (2) holds in harmonic, isotropic and considered here coordinates systems. Our system, given in G^2 approximation by (17) and (25a) we will call the preferred one, because the gauge degrees of freedom do not contribute to it.

Scattering of a particle by gravitational field

1. Scattering of scalar particle in Born approximation

$$d\sigma = \frac{\pi}{2}r_g^2(1 + \frac{1}{\beta^2})\frac{\cos\frac{\theta}{2}}{\sin^3\frac{\theta}{2}}d\theta = \frac{\pi}{2}r_g^2(1 + \frac{1}{\beta^2})\frac{1}{y^3}(1 - \frac{1}{15}y^3 + \cdots)d\theta. \tag{28}$$

Here β is velocity at infinity. For a massless particle $\beta = 1$.

2.Scattering of a classical zero mass particle

$$d\sigma = \frac{\pi}{2}r_g^2\frac{1}{y^3}(1 + a_1y + a_3y^3 + a_4y^4 + \cdots)d\theta, \quad y = \frac{\theta}{2} \tag{29}$$

Here $a_i, i = 1,3,4.$ are numbers of order unity. The quantum formula (28) does not contain $\hbar$. So the difference with (29) is surprising.

3.For graviton scattering in Born approximation we have

$$d\sigma = \frac{\pi}{4}r_g^2\frac{\cos^8\frac{\theta}{2} + \sin^8\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}d\Omega. \tag{30}$$

The first (second) term in the numerator are contribution from nonspin flip (from spin flip) scattering.

Modified 3-graviton vertex.

The 3-graviton vertex has the form

$$L = \frac{1}{2}h^{ik}t_{ik}. \tag{31}$$

In general relativity $t_{ik} = \overset{GR}{t}_{ik}$ is not the gravitational energy-momentum tensor In field-theoretical approach we can calculate t_{ik} either by Belifante or Rosenfeld method. Both method give the same result for space without matter. Where there is a matter we have to add interaction terms. In approximation of point-like particles we can choose these interaction terms in such a way that particles move along geodesics. Then for t_{ik} I pick up a specific tensor which I call $\overset{MTW}{t}_{ik}$ For slowly moving particles we have

$$T_{00} + \overset{MTW}{t}_{00} = T_{00}^{(0)} + \mu\phi + \frac{8\pi G}{(\nabla\phi)^2}, \quad T_{00}^{(0)} = T_{00}(G = 0). \tag{32}$$

This formula gives the positive gravitational field energy density as it should be. It agrees with Newtonian interaction energy $\frac{1}{2}\mu\phi$.It follows from here that in Newtonian $\frac{Gm_1m_2}{R}$ masses m_1, m_2 are dressed ones (together with gravitational energy). This is impotent for interaction of heavy compact object (neutron stars) as the essential part of their energy resides in the gravitation field outsside the matter.

As Schwinger shown the Newtonian interaction $\frac{1}{2}\mu\phi$ leads to modification of the potential $\phi \to \phi + \frac{1}{2}\phi^2$. In our case this means that $phi \to \phi + \phi^2$. This is necessary for the correct perihelion precession of a planet.

For small angle graviton scattering $\overset{MTW}{t}_{ik}$ gives the same result as $t_{ik} = \overset{GR}{t}_{ik}$.

Conclusion.

We may expect the modification of GR results for the movement of a relativistic particle in strong gravitational field.