P.N.Lebedev Physical Institute of the Russian Academy of Sciences

Физический институт имени П.Н.Лебедева Российской академии наук

ФИАН

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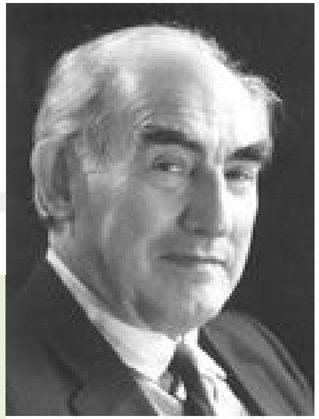

Investigation of Self-Organization Mechanisms In Non-Equilibrium Systems Using Block Model Approach

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GC100

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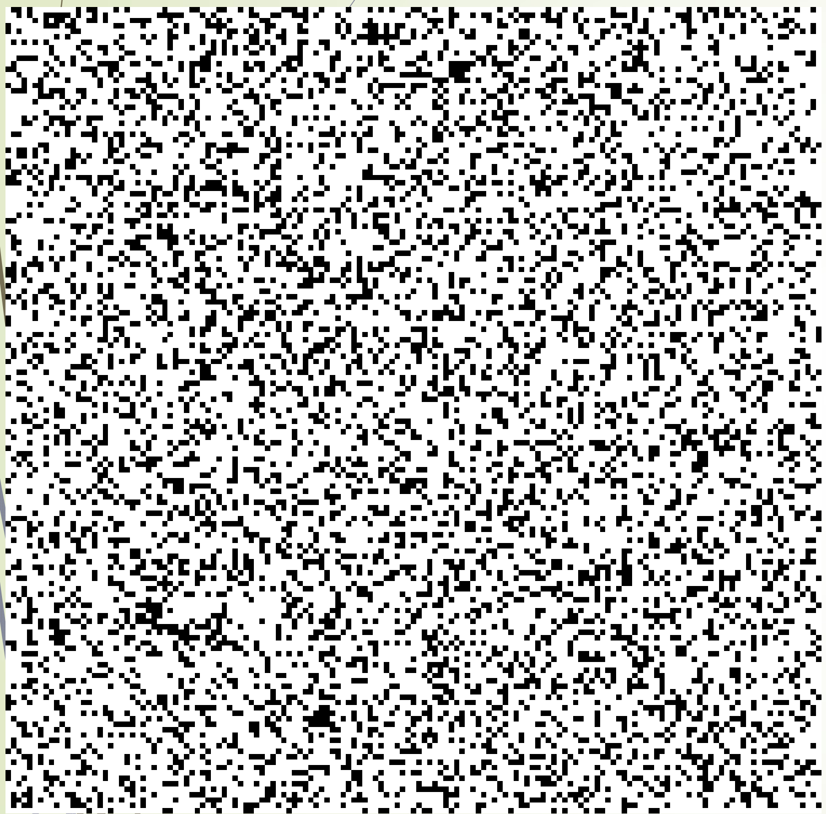
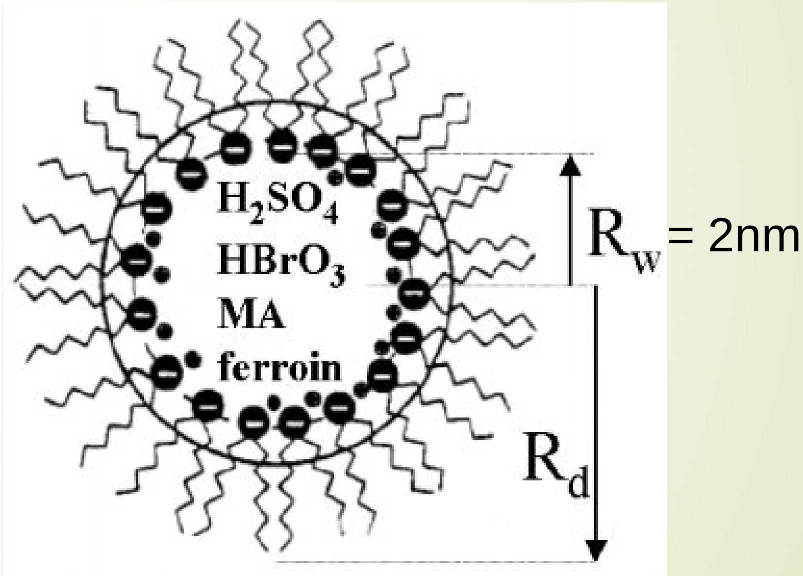
Spatio-temporal patterns are widely spread in nature

Belousov-Zhabotinsky reaction dispersed in a water-in-oil aerosol OT microemulsion

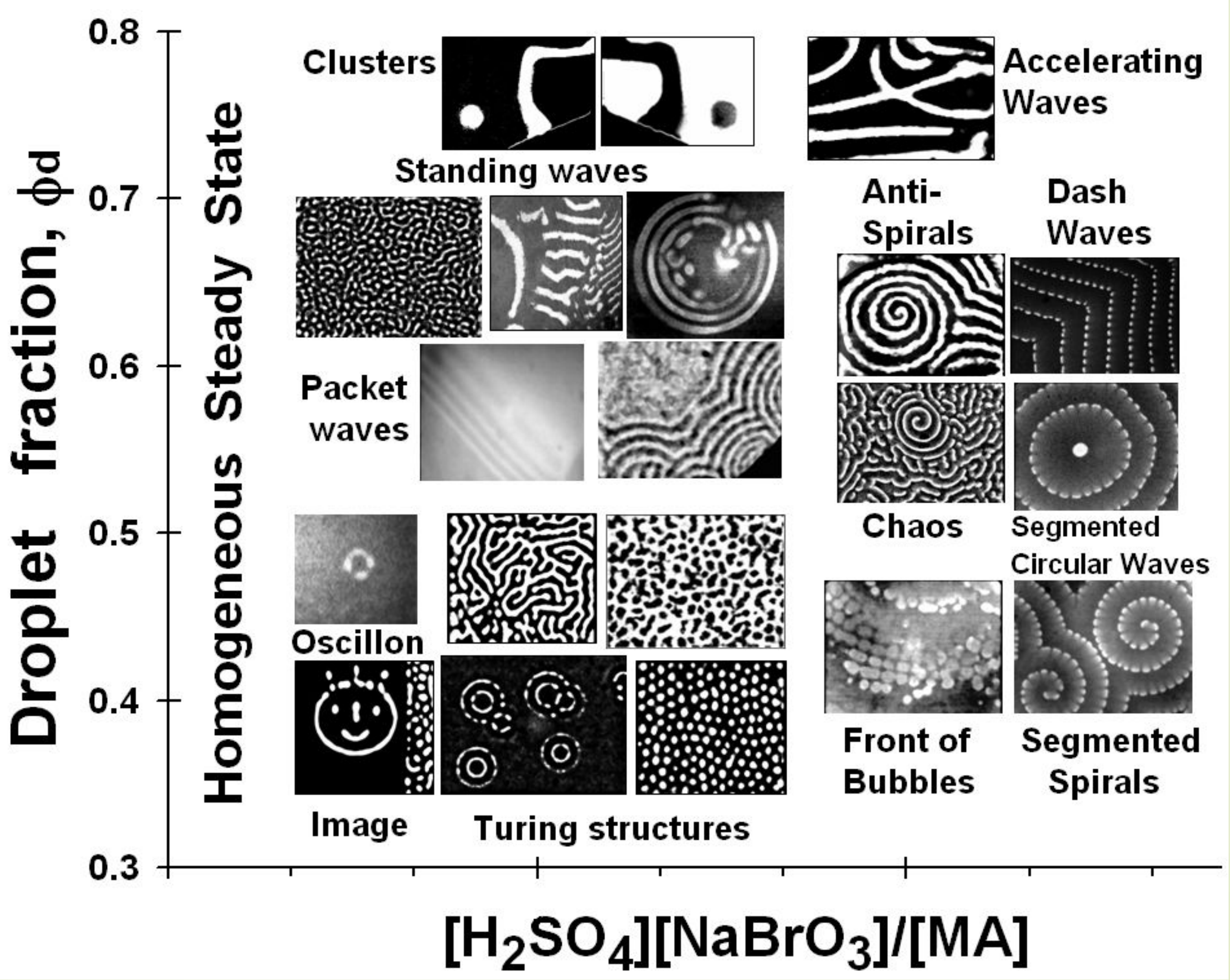


1 μm × 1 μm

Volume fraction of droplets $\varphi_d = 0.3$

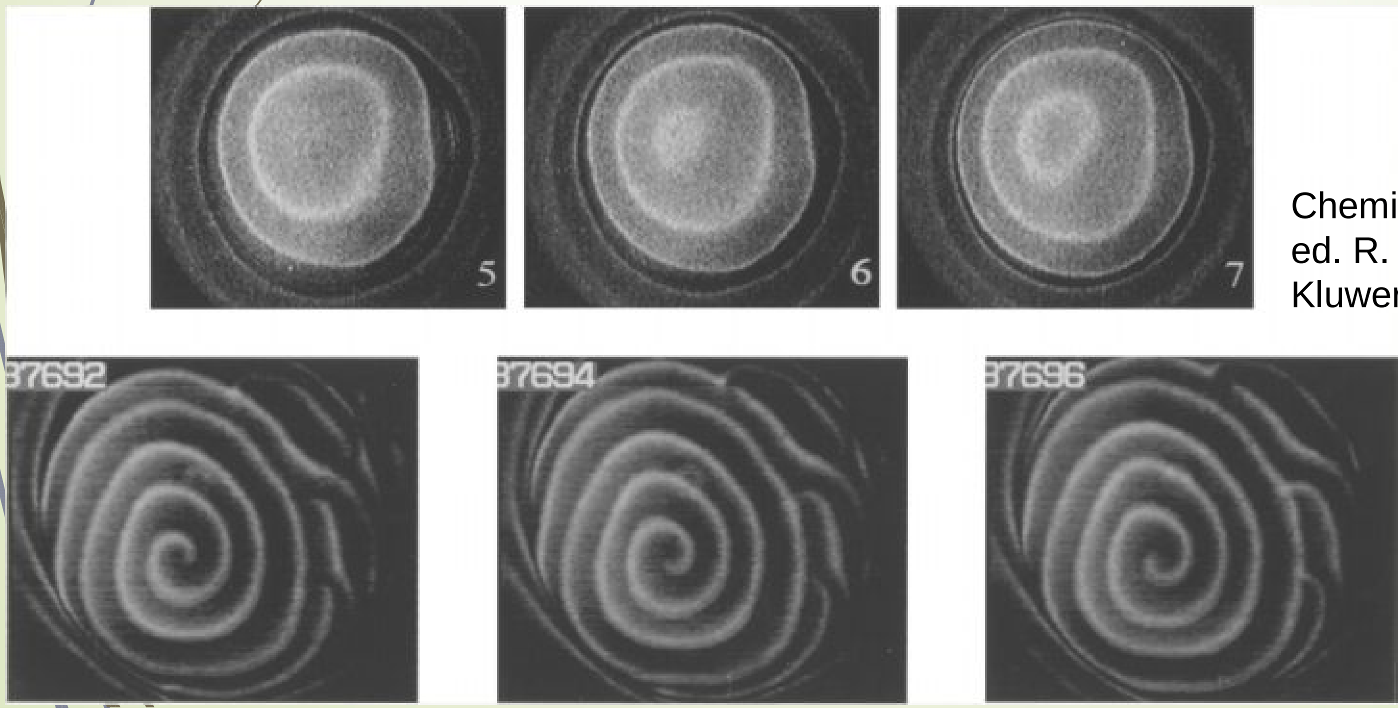


Overview of pattern formation in the BZ-AOT system



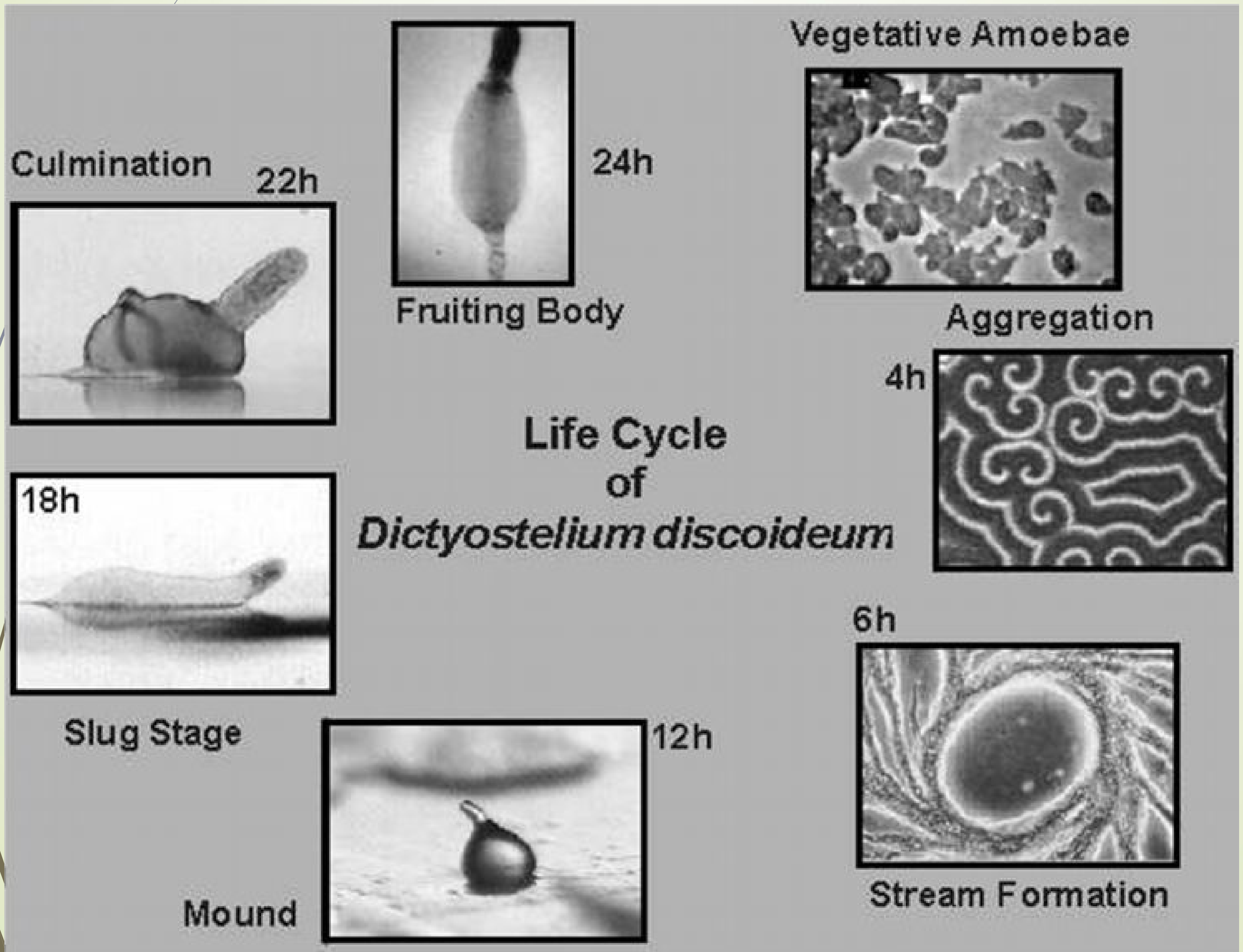
Epstein B.R., Berenstein I.B., Dolnik M., Vanag V.K., Yang L., Zhabotinsky A.M. Coupled and forced patterns in reaction–diffusion systems// Phil. Trans. R. Soc. A, 2008, v. 366, p. 397–408.

Patterns on combustion fronts

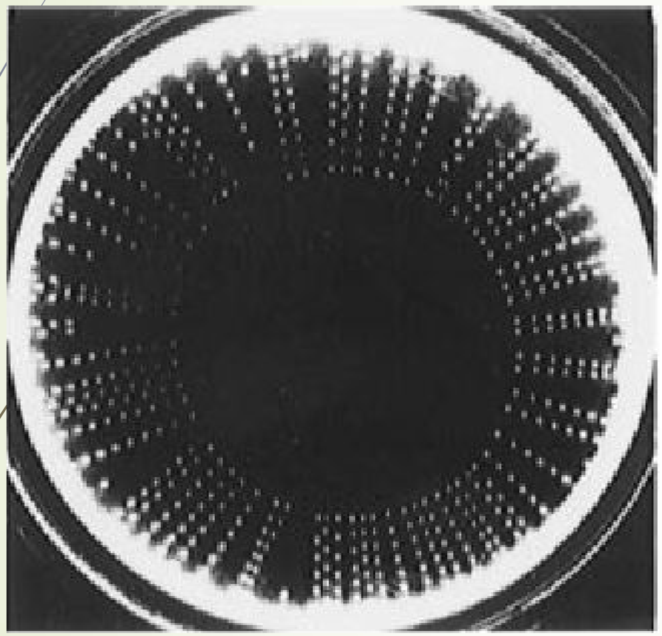


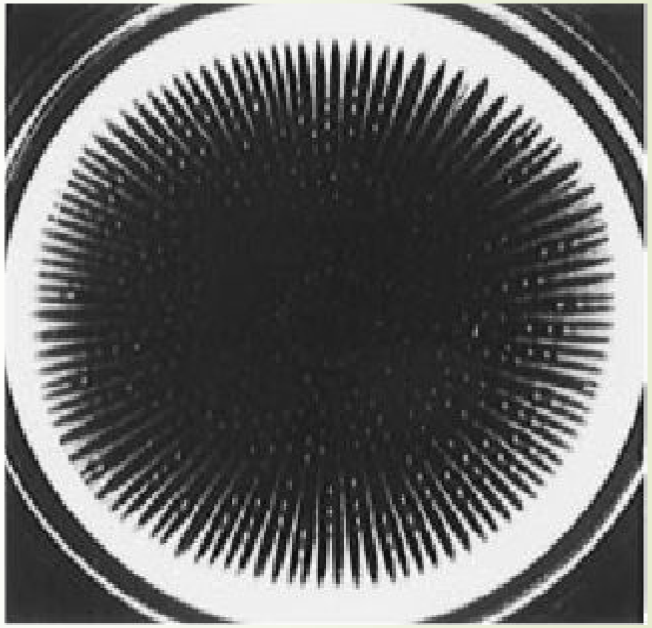
Chemical Waves and Patterns, ed. R. Kapral and K. Showalter, Kluwer, Dordrecht, 1995

Aggregation of *Dictyostelium discoideum*



Pattern formation in bacterial colonies

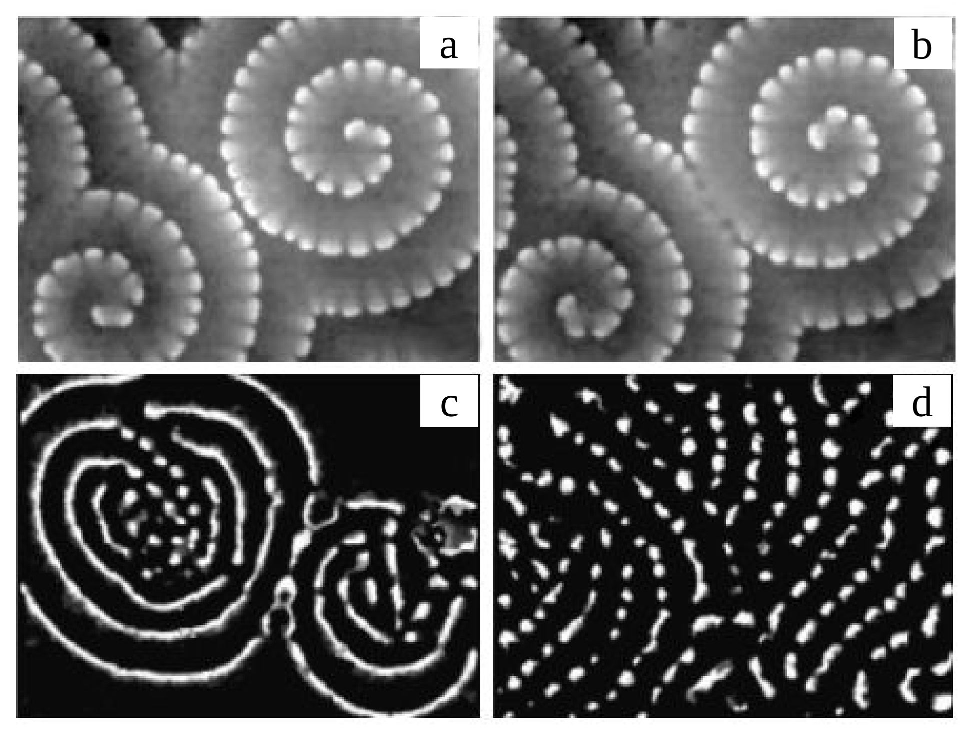




Patterns formed by *E.coli* colonies

E. Budrene, H. Berg. Complex patterns formed by motile cells of Escherichia coli, Nature, 1991, v.345, p.630-633

Segmented waves in the BZ-AOT system



(a, b) Segmented spirals[1], (c, d) segmented waves[2] (dash waves) in the BZ-AOT system. Frame size (mm×mm) for (a, b) 3.72 ×4.82; for (c, d) 2.54×1.88. Time between (a) and (b) is 66 s; (c) and (d) is 1800 s.

[1] *Vanag V.K., Epstein I.R.* Segmented spiral waves in a reaction-diffusion system. //Proc. Natl. Acad. Sci. USA, 2003, v. 100, p. 14635

[2] *Vanag V.K., Epstein I.R.* Dash waves in a reaction-diffusion system. //Phys. Rev. Lett., 2003, v.90, p. 098301

Interaction of two coupled subsystems, one of which is excitable, and the other one has Turing instability

FitzHugh-Nagumo + Brusselator

$$
\begin{cases}
\dfrac{dU}{dt} = U - \frac{U^3}{3} - V + D_F \nabla^2 U, \\
\dfrac{dV}{dt} = (U - \gamma V + \delta)\varepsilon, \\
\dfrac{dX}{dt} = a - (b(U) + 1)X + X^2 Y + \nabla^2 X, \\
\dfrac{dY}{dt} = b(U)X - X^2 Y + D_B \nabla^2 Y,
\end{cases}
$$

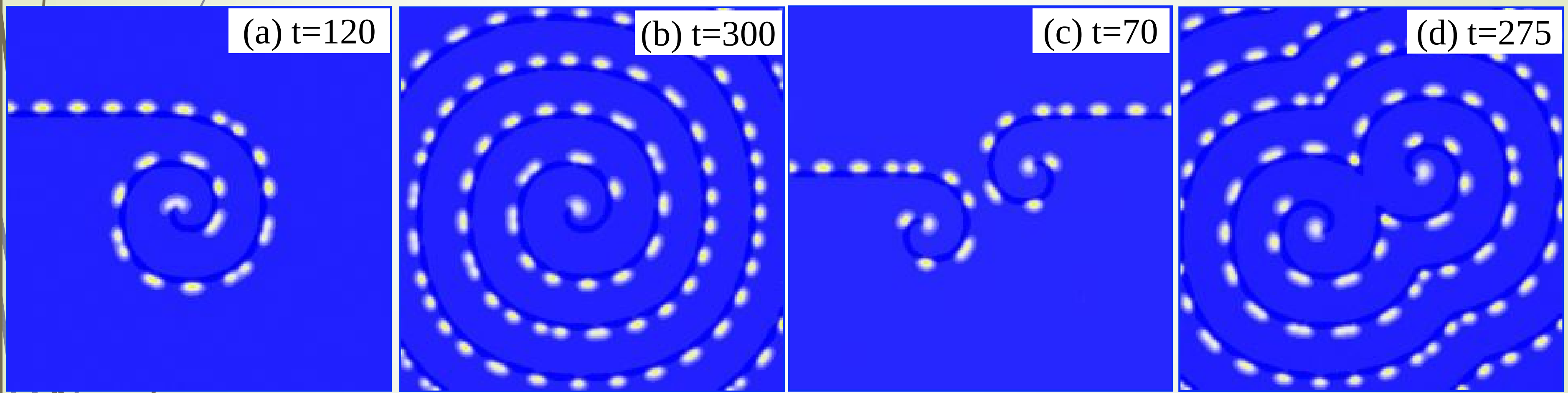
$$
b(u) = \begin{cases} b_c + \Delta \cdot U, & U \geq 0; \\ b_c, & U < 0. \end{cases}
$$

b_c – bifurcation value of the control parameter b

(Borina, Polezhaev, 2013)

FitzHugh-Nagumo + Brusselator

ФитцХью-Нагумо и Брюсселятор



(a, b) Formation of a single segmented spiral and (c, d) interaction of two segmented spirals in two-dimensional 200×200 system with zero flux boundary conditions.
Model parameters: ε=0.09, γ=0.5, δ=0.7, D_F=0.1, a=2, D_B=100, b_c=1.25, Δ=2.

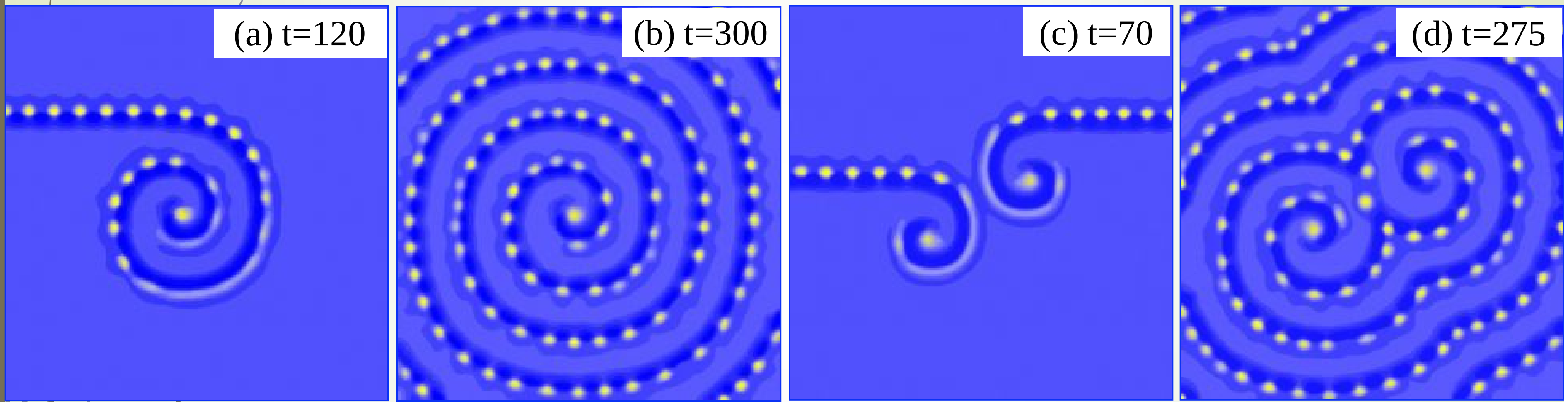
FitzHugh-Nagumo + Brusselator

$$\begin{cases} \dfrac{dU}{dt} = U - \frac{U^3}{3} - V + D_F \nabla^2 U, \\ \dfrac{dV}{dt} = \left(U - \gamma V + \delta\right)\varepsilon, \\ \dfrac{dX}{dt} = a(U) - (b+1)X + X^2 Y + \nabla^2 X, \\ \dfrac{dY}{dt} = bX - X^2 Y + D_B \nabla^2 Y. \end{cases}$$

$$a(u) = \begin{cases} a_c + \Delta \cdot U, & U \geq 0; \\ a_c, & U < 0. \end{cases}$$

a_c – bifurcation value of the control parameter a

FitzHugh-Nagumo + Brusselator



(a, b) Formation of a segmented spiral and (c, d) interaction of two segmented spirals in two-dimensional 200×200 system with zero flux boundary conditions.
Model parameters: ε=0.09, γ=0.5, δ=0.7, D_F=0.1, b=2, D_B=100, a_c=4.2, Δ=-1.

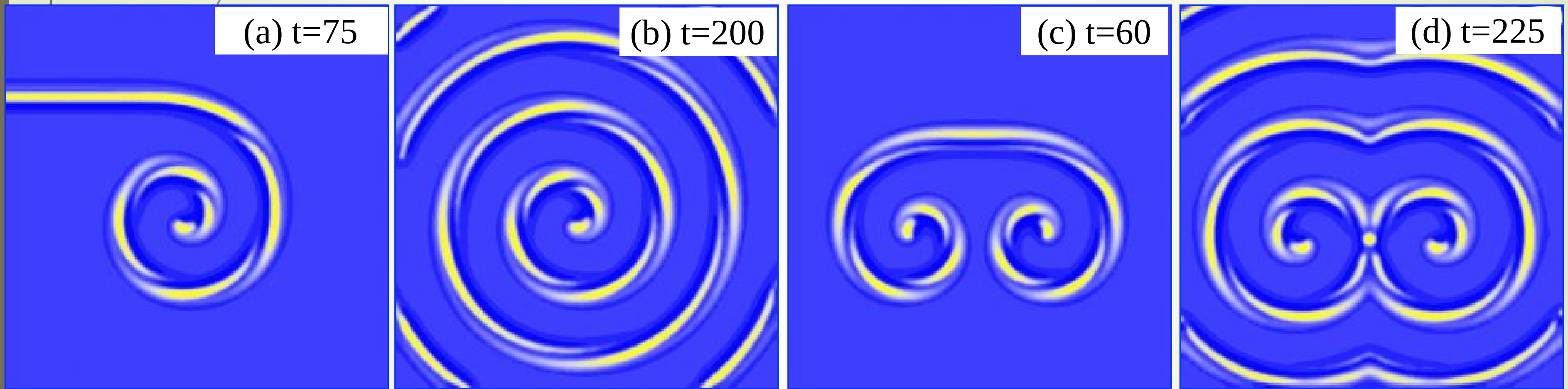
FitzHugh-Nagumo + FitzHugh-Nagumo

$$\begin{cases} \dfrac{dU}{dt} = U - \frac{U^3}{3} - V + D_1 \nabla^2 U, \\ \dfrac{dV}{dt} = \left(U - \gamma_1 V + \delta_1\right)\varepsilon, \\ \dfrac{d\tilde{U}}{dt} = \left(\tilde{U} - \frac{\tilde{U}^3}{3} - \tilde{V} + I(U) + D_2 \nabla^2 \tilde{U}\right)\alpha, \\ \dfrac{d\tilde{V}}{dt} = \left(\tilde{U} - \gamma_2 \tilde{V} + \delta_2 + \nabla^2 \tilde{V}\right)\alpha. \end{cases}$$

$$I(u) = \begin{cases} I_c + \Delta \cdot U, & U \geq 0; \\ I_c, & U < 0. \end{cases}$$

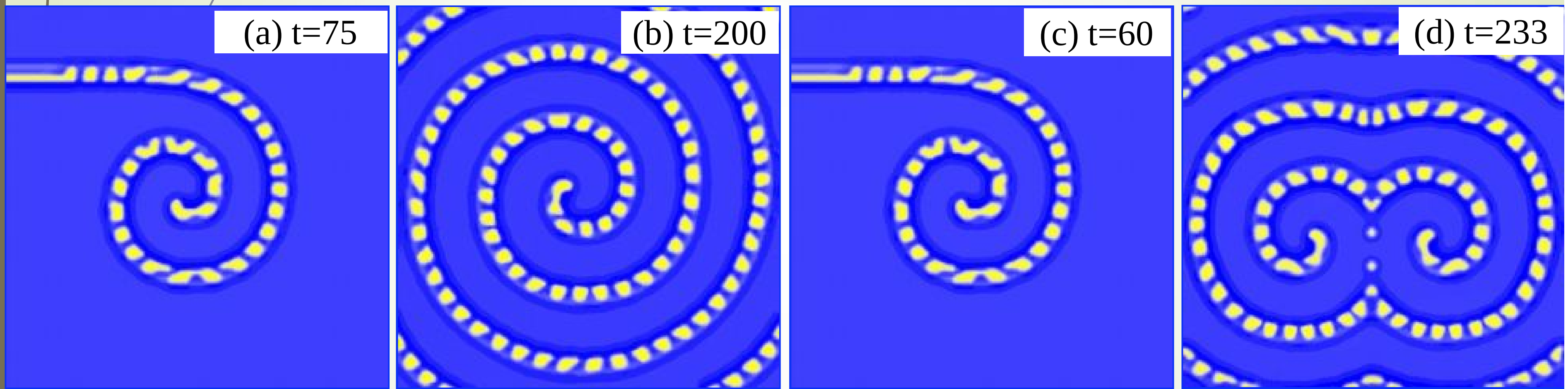
I_c – bifurcation value of the control parameter I

FitzHugh-Nagumo + FitzHugh-Nagumo



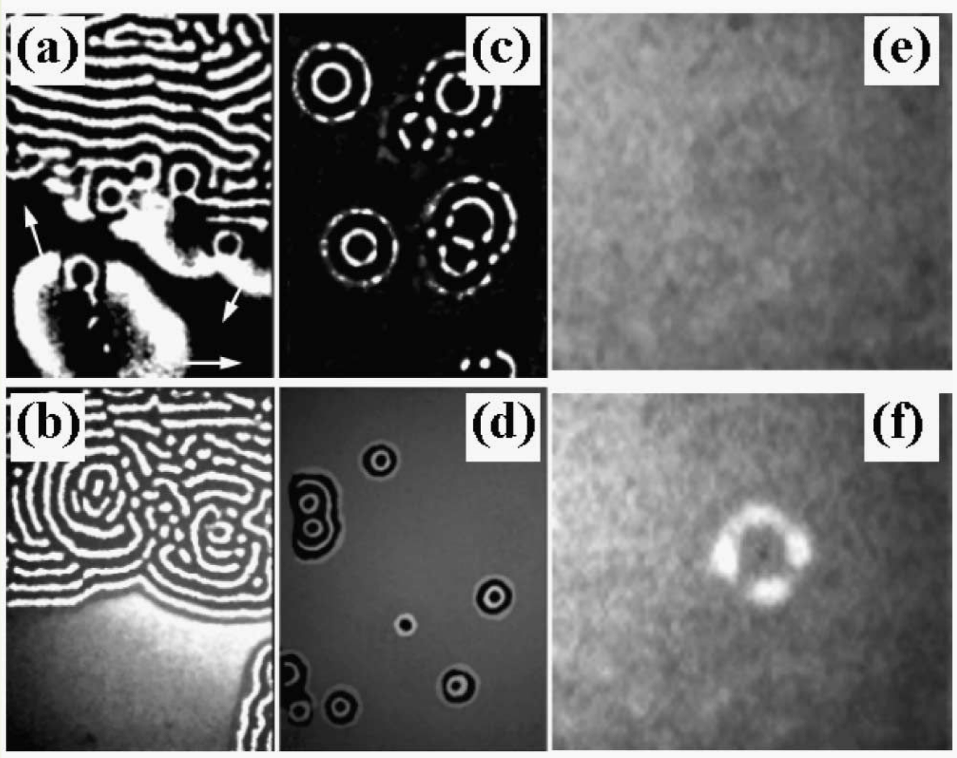
(a, b) Formation of a segmented spiral and (c, d) interaction of two segmented spirals in two-dimensional 100×100 system with zero flux boundary conditions. Model parameters: ε=0.09, γ_1=0.5, δ_1=0.7, D_1=0.1, γ_2=0.5, δ_2=0.5, D_2=0.05, α=1, I_c=0.1, Δ=0.03.

FitzHugh-Nagumo + FitzHugh-Nagumo



(a, b) Formation of a segmented spiral and (c, d) interaction of two segmented spirals in two-dimensional 100×100 system with zero flux boundary conditions. Model parameters: ε=0.09, γ_1=0.5, δ_1=0.7, D_1=0.1, γ_2=0.5, δ_2=0.5, D_2=0.05, α=5, I_c=0.1, Δ=0.03.

Oscillons in the BZ-AOT system



Turing patterns (a),(b), localized Turing patterns (c), and oscillons (d) –(f) in the Ru(bpy) - (a),(c),(e),(f) and ferroin - (b),(d) catalyzed BZ-AOT systems.
Period of the oscillon is 47 s for (e),(f) and about 250 s for (d).

Vanag V. K., *Epstein I. R.* Stationary and Oscillatory Localized Patterns, and Subcritical Bifurcations. // Phys. Rev. Lett. — vol. 92(12), 2004. —128301.

Interaction of two coupled subsystems, one of which forms localized Turing patterns, and the other one is potentially oscillatory

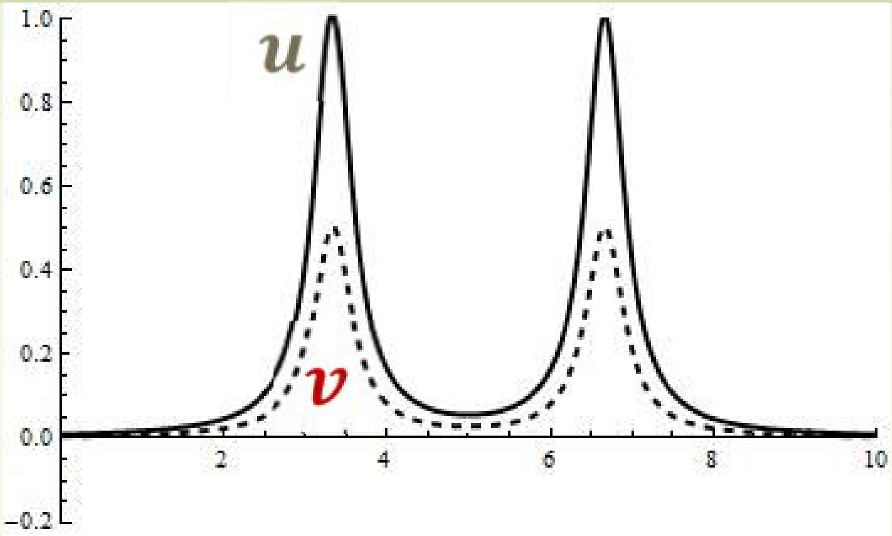
$$\begin{cases} \dfrac{dU}{dt} = -U(U+\alpha)(U-1) - V + D\nabla^2 U, \\ \dfrac{dV}{dt} = U - V + \nabla^2 V, \\ \dfrac{dX}{dt} = A - (B(U)+1)X + X^2 Y + D\nabla^2 X, \\ \dfrac{dY}{dt} = B(U)X - X^2 Y + D\nabla^2 Y, \end{cases}$$

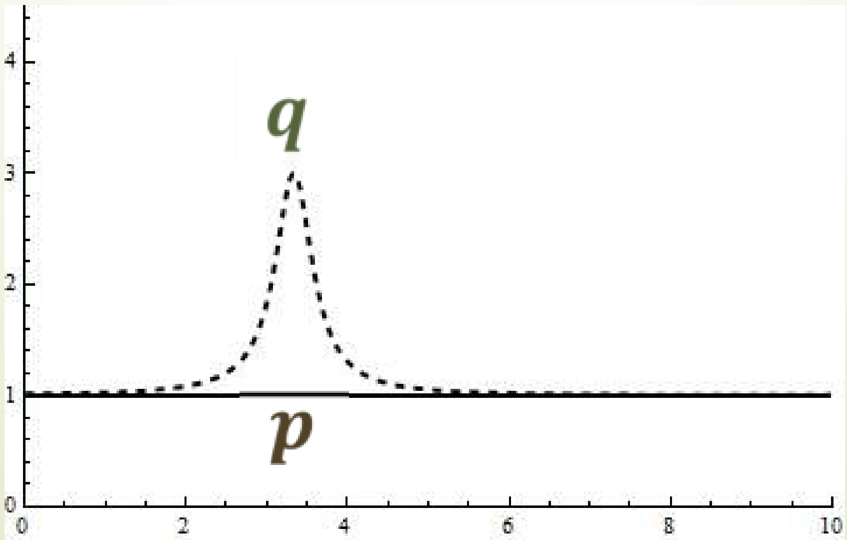
$$B(U) = 1 + 2U.$$

Subcritical Turing bifurcation in the first subsystem takes place for sufficiently small diffusion coefficient D.
If D=0.001 then the bifurcation value of the parameter α is 0.0622.

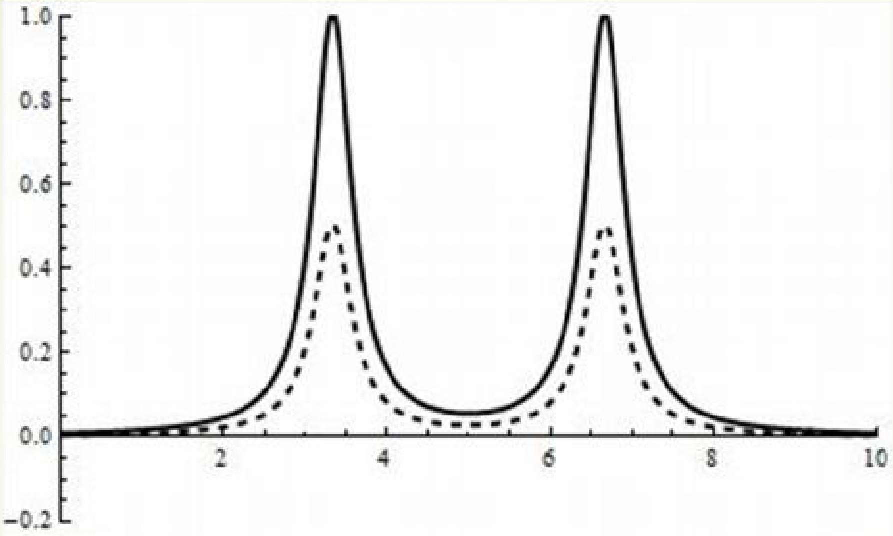
(Kuznetsov, Polezhaev, 2015)

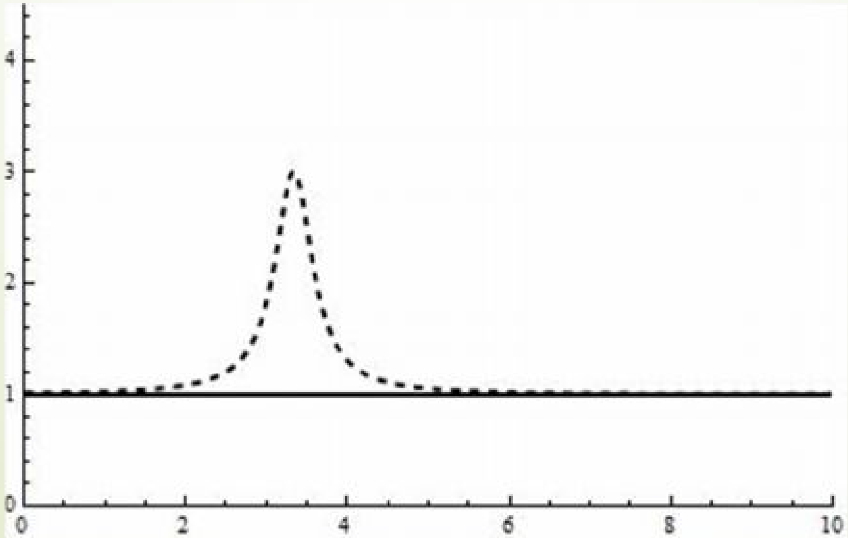
2D simulation (initial distribution)

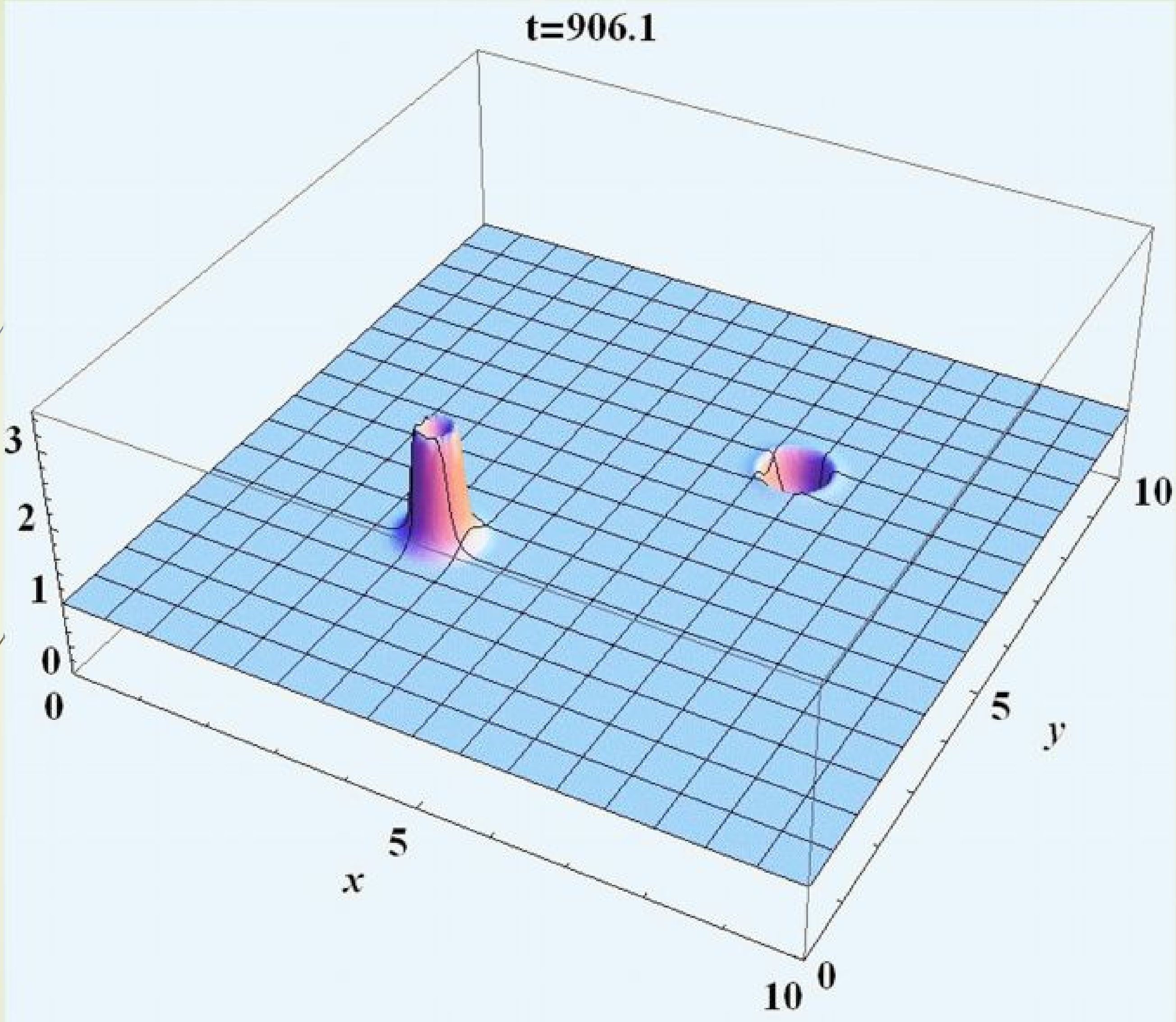




2D simulation







Oscillations of the variable X.
$D = 0.001$, $\alpha = -0.1$, $A = 1$, $B = 1 + 2U$.

Aggregation of *Dictyostelium discoideum*

Oscillations in cell suspension of *D.discoideum*

(Martiel, Goldbeter, 1987)

- cAMP is secreted by cells into the external environment in response to a cAMP signal;
- cAMP is hydrolyzed by extracellular phosphodiesterase;
- cAMP binds to receptors on the outer surface of the cell membrane;
- Synthesis of cAMP inside the cell is induced by binding of cAMP to receptors on the surface;
- Sensitivity of membrane receptors decreases with prolonged stimulation of intercellular cAMP.

Mathematical model

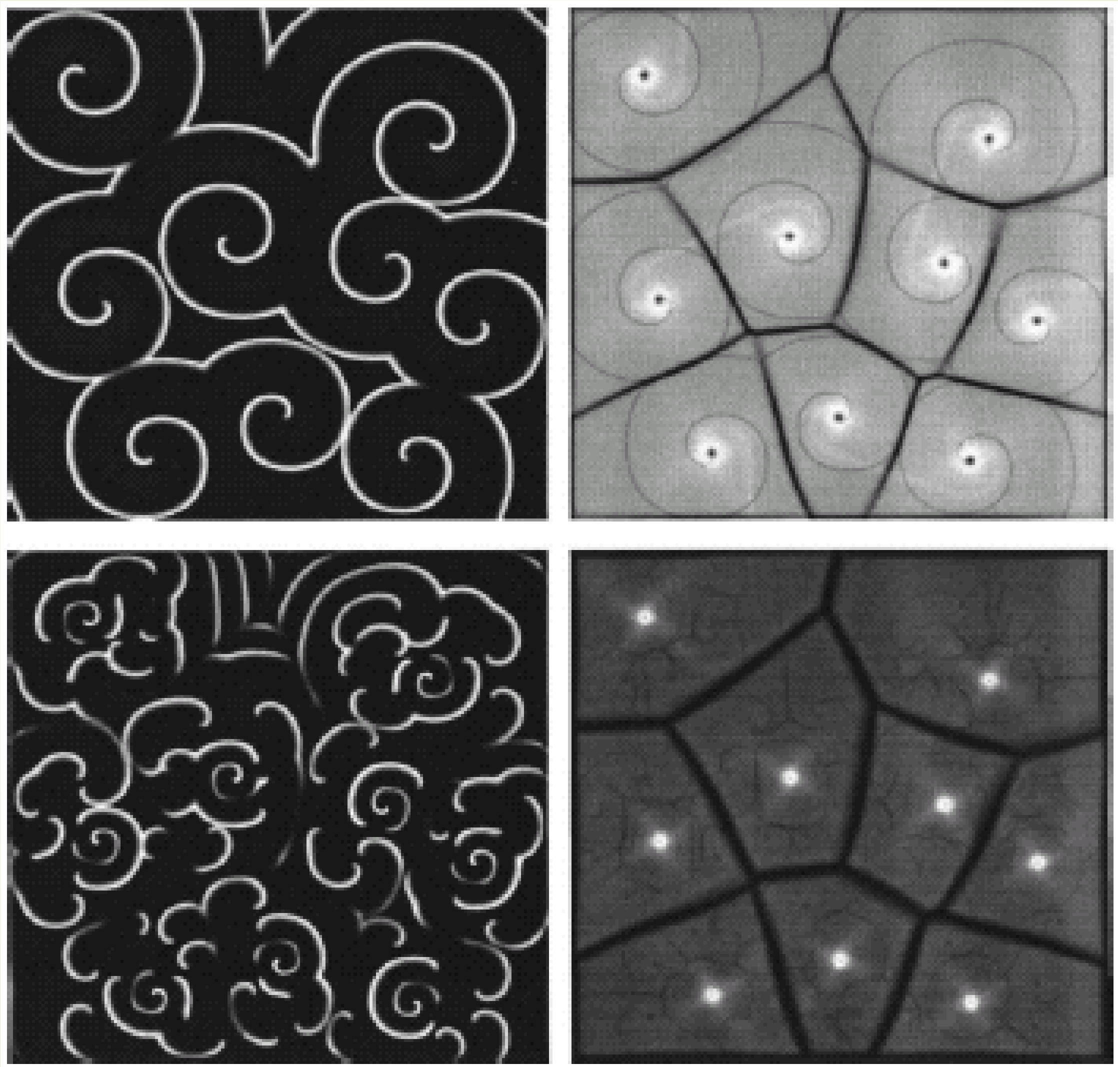
$$\frac{\partial v}{\partial t} = \gamma u \left(g \frac{v^2 + A^2}{v^2 + 1} - \delta v \right) + D_v \Delta v,$$

$$\frac{\partial g}{\partial t} = B - (1 + Hv) g,$$

$$\frac{\partial u}{\partial t} = D_u \Delta u - \nabla \left(\chi (g - g_0)^4 u \nabla v \right).$$

(Polezhaev et al., 1998, 2005)

Simulation



Thank you for attention!