# LIGHT-CONE HIGHER-SPIN THEORIES IN FLAT SPACE

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# MOTIVATION

## HIGHER SPIN INTERACTIONS IN FLAT SPACE ARE PROBLEMATIC

Weinberg's soft theorem: couplings of massless higher-spin fields cannot survive in the low-energy limit *[Weinberg'64]*

Aragone-Deser argument: replacement of partial derivatives by covariant ones does not give a consistent minimal coupling *[Aragone, Deser'79]*

Direct obstructions to the Noether procedure

*[Berends, Burgers, van Dam'85; Bengtsson'85]*

*[Bekaert, Boulanger, Leclercq'10; Joung, Taronna'13]*

More recently: *[Roiban, Tseytlin'17; Taronna'17]*

# MOTIVATION

## UNEXPECTED OBSERVATION

Light-cone deformation procedure results into additional local cubic vertices compared to manifestly covariant approaches.

*[Bengtsson, Bengtsson, Brink'83; Bengtsson, Bengtsson, Linden'87]*

*[Bengtsson'14]*

Reasons and a particular mechanism how this happens discussed in

*[Conde, Joung, Mkrtchyan'16; Taronna, Sleight'16]*

In particular, a two-derivative interaction with gravity (minimal coupling) does exist, contrary to covariant approaches (by the Aragone-Deser argument).

# MOTIVATION

## FURTHER ANALYSIS

Deformation procedure was partially solved at the order g^2

This fixes all coupling constants in cubic vertices in terms of a single one

*[Metsaev'91]*

Satisfy Weinberg's equivalence principle (coupling is universal)

Agree with a "flat limit" of cubic vertices found from AdS/CFT

*[Bekaert, Erdmenger, Ponomarev, Sleight'15; Skvortsov'16]*

*[Taronna, Sleight'16]*

(nothing of this can be seen in covariant approaches)

# MOTIVATION

This couple of points suggest that
a consistent higher spin theory may exist in flat space

## GOAL

Revisit higher-spin interactions in flat space focusing on methods that do not require manifest Lorentz covariance (Lorentz tensors).

## PRIMARY TOOL

Light-cone deformation procedure

# MANIFEST LORENTZ INVARIANCE

**FREE THEORIES** UIR's of Poincare group

**INTERACTIONS** Generators are deformed non-linearly. Consistency requirement: still generate the Poincare algebra

## LORENTZ TENSORS

All Poincare symmetry is manifest

Introduces extra d. o. f.
Massless fields = gauge invariance

<u>Fewer local interactions</u>

## DIRECT ANALYSIS

Manual control of Poincare symmetry

Only physical d. o. f.

<u>More local interactions</u>

# HIGHER-SPIN INTERACTIONS FROM LIGHT-CONE

# BASICS OF LIGHT-CONE

Light-cone^2 approach = light-cone gauge x light-cone time

**Light-cone gauge**

$$\phi^{+\cdots} = 0$$

**Light-cone time**

$$x^+ = \frac{1}{\sqrt{2}}(x^3 + x^0)$$

$\partial^-$ is time derivative, $\partial^+$ is not and can be inverted

$\phi^\rho{}_\rho{}^{\cdots} = 0$ and $\partial_\rho \phi^{\rho\cdots} = 0$ are algebraic consequences

This allows to eliminate all unphysical degrees of freedom

**ALTERNATIVELY** Fundamentally define a theory in the light-cone gauge

# BASICS OF LIGHT-CONE: FREE THEORY

The action

($\lambda$ is helicity)

$$S_2 \equiv \int d^4x L_2, \qquad L_2 = -\frac{1}{2}\sum_\lambda \partial_a \Phi^{-\lambda}\partial^a\Phi^\lambda$$

Higher-spin fields look like scalars

Difference: only in spin part of angular momentum

$$S^{+a}\cdot\Phi^\lambda = 0, \qquad S^{ab}\partial_a\cdot\Phi^\lambda = 0 \qquad S^{x\bar{x}}\cdot\Phi^\lambda = -\lambda\Phi^\lambda$$

Noether charges generate associated transformation via the commutator

$$P_2^i = \int d^3x^\perp T^{i,+}, \qquad J_2^{ij} = \int d^3x^\perp L^{ij,+}$$

$$[\Phi^\lambda, P_2^i] = P_2^i\Phi^\lambda, \qquad [\Phi^\lambda, J_2^{ij}] = J_2^{ij}\Phi^\lambda$$

# BASICS OF LIGHT-CONE: INTERACTIONS

Deform dynamical generators

$$D: \qquad H \equiv P^{-}, \qquad J \equiv J^{x-}, \qquad \bar{J} \equiv J^{\bar{x}-}$$

Remaining are not deformed, called kinematical $\qquad K$

## CLASSES OF COMMUTATORS

$[K, K] = K$ immediately satisfied

$$[K, D] = K \quad \Rightarrow \quad [K, \delta D] = 0,$$
$$[K, D] = D \quad \Rightarrow \quad [K, \delta D] = \delta D$$

need to be solved only once

$[D, D] = 0$ main difficulty

# BASICS OF LIGHT-CONE: INTERACTIONS

Deformation

$$H = H_2 + \sum_n H_n$$

$$H_n = \frac{1}{n!}\sum_{\lambda_i}\int d^{3n}q^\perp \delta^3(\sum_{i=1}^n q_i^\perp) h_n^{\lambda_1 \ldots \lambda_n} \prod_{i=1}^n \Phi^{\lambda_i}(q_i^\perp)$$

$$(q^\perp \equiv \{q, \bar{q}, q^+\}, \quad \beta \equiv q^+)$$

Kinematical constraints (solved only once)

+ Fix transverse momentum dependence

$$\bar{\mathbb{P}}_{ij} \equiv \bar{q}_i \beta_j - \bar{q}_j \beta_i, \qquad \mathbb{P}_{ij} \equiv q_i \beta_j - q_j \beta_i$$

+ Impose some homogeneity conditions on h

# BASICS OF LIGHT-CONE: INTERACTIONS

Dynamical constraints (main difficulty)

$$[H, J] = 0 \qquad \Rightarrow \qquad [H_2, J_n] + [H_3, J_{n-1}] + \cdots + [H_{n-1}, J_3] + [H_n, J_2] = 0$$

Reminiscent of the Noether procedure

# CUBIC INTERACTIONS

$$\mathcal{H}^{\{i\}}\left[j_3^{\lambda_1\dots\lambda_3}+\frac{1}{3}(\sum_j\frac{\partial}{\partial\bar{q}_j})h_3^{\lambda_1\dots\lambda_3}\right]+\mathcal{J}^{\{i\}}h_3^{\lambda_1\dots\lambda_3}=0$$

## SOLUTION

$$h_3^{\lambda_1\lambda_2\lambda_3}=C^{\lambda_1\lambda_2\lambda_3}\frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}+\bar{C}^{-\lambda_1-\lambda_2-\lambda_3}\frac{\mathbb{P}_{12}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}$$

where C are arbitrary coupling constants

*[Bengtsson, Bengtsson, Brink'83; Bengtsson, Bengtsson, Linden'87; Metsaev'91]*

## DERIVATIVES

$$N(\partial)=|\lambda_1+\lambda_2+\lambda_3|$$

Individual helicities can be negative. These vertices violate bounds on the number of derivatives in covariant approaches. In particular

$$\{s_1,s_2,s_3\}=\{s,s,2\},\qquad\{\lambda_1,\lambda_2,\lambda_3\}=\{s,-s,2\}\qquad\Rightarrow\qquad N(\partial)=2$$

Light-cone allows to couple minimally higher spins to gravity!

# QUARTIC ORDER ANALYSIS

$$\mathcal{H}^{\{i\}}\Big[j_4^{\lambda_1\dots\lambda_4}+\frac{1}{4}(\sum_j\frac{\partial}{\partial\bar{q}_j})h_4^{\lambda_1\dots\lambda_4}\Big]+\mathcal{J}^{\{i\}}h_4^{\lambda_1\dots\lambda_4}+[H_3,J_3]=0$$

One can adjust: $C^{\lambda_1\lambda_2\lambda_3},\quad \bar{C}^{\lambda_1\lambda_2\lambda_3},\quad h_4^{\lambda_1\dots\lambda_4},\quad j_4^{\lambda_1\dots\lambda_4}.$

A KEY OBSERVATION

only [H3,J3] has a non-vanishing contribution to the q-independent part of the equation. So, this part of [H3,J3] should vanish separately.

*[Metsaev'91]*

# CHIRAL HIGHER-SPIN THEORY

$$[H_3, J_3]|_{q=0} = 0$$

Receives contributions only from antiholomorphic vertices

$$C^{\lambda_1\lambda_2\lambda_3}\frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}$$

## SOLUTION

$$C^{\lambda_1\lambda_2\lambda_3} = \frac{\ell^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}$$

*[Metsaev'91]*

Moreover, if we have only antiholomorphic vertices, the remaining terms in the consistency condition are zero, hence the consistency condition is satisfied (to all orders).
This leads us to a chiral higher spin theory.

# CHIRAL HIGHER-SPIN THEORY

## COMPLETE ACTION

$$S = -\int d^4x \partial_\mu \Phi^{-\lambda} \partial^\mu \Phi^\lambda$$
$$+ g \int d^4x \frac{\ell^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!} \frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{(\partial_1^+)^{\lambda_1}(\partial_2^+)^{\lambda_2}(\partial_3^+)^{\lambda_3}} \Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}$$

1) Consistent to all orders in coupling constant
2) Contains lower derivative couplings, absent in covariant approaches. In particular, minimal coupling to gravity
3) Obeys generalised Weinberg's equivalence principle: coupling to gravity is universal
4) Admits 'truncations'. In particular, a minimal coupling of a single higher spin field to gravity. Classification - ?
5) Has vanishing four-point amplitude. Expected to hold for n-points.
6) Avoids no-go's (in somewhat degenerate manner).

*[Ponomarev, Skvortsov'16]*

# FROM LIGHT-CONE TO SPINOR-HELICITY

# SPINOR-HELICITY REPRESENTATION

In 4d one can factorise null momenta in terms of spinors

$$q_{a\dot{b}} \equiv q_\mu(\sigma^\mu)_{a\dot{b}} = \sqrt{2}\begin{pmatrix} q^- & \bar{q} \\ q & -q^+ \end{pmatrix} \approx \sqrt{2}\begin{pmatrix} -\frac{q\bar{q}}{\beta} & \bar{q} \\ q & -\beta \end{pmatrix} = -|q]_a\langle q|_{\dot{b}},$$

$$|q]_a = \frac{2^{\frac{1}{4}}}{\sqrt{\beta}}\begin{pmatrix} \bar{q} \\ -\beta \end{pmatrix}, \qquad \langle q|_{\dot{b}} = \frac{2^{\frac{1}{4}}}{\sqrt{\beta}}\begin{pmatrix} q & -\beta \end{pmatrix}$$

Then one can construct spinor products

$$[ij] = \frac{2^{\frac{1}{4}}}{\sqrt{\beta_j}}\begin{pmatrix} \bar{q}_j & -\beta_j \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\frac{2^{\frac{1}{4}}}{\sqrt{\beta_i}}\begin{pmatrix} \bar{q}_i \\ -\beta_i \end{pmatrix} = \frac{\sqrt{2}}{\sqrt{\beta_i\beta_j}}\bar{\mathbb{P}}_{ij},$$

$$\langle ij\rangle = \frac{2^{\frac{1}{4}}}{\sqrt{\beta_i}}\begin{pmatrix} q_i & -\beta_i \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\frac{2^{\frac{1}{4}}}{\sqrt{\beta_j}}\begin{pmatrix} q_j \\ -\beta_j \end{pmatrix} = -\frac{\sqrt{2}}{\sqrt{\beta_i\beta_j}}\mathbb{P}_{ij}.$$

# SPINOR-HELICITY REPRESENTATION

## CUBIC VERTICES:

$$\frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} \quad \rightarrow \quad [12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[31]^{\lambda_3+\lambda_1-\lambda_2}$$

*[Ananth'12; Akshay, Ananth'14]*

Provides an off-shell extension of amplitudes found in *[Benincasa, Cachazo'07]*

**BONUS:** The very same expressions can be used off-shell!

# INVARIANCE: HAMILTONIAN VS S-MATRIX

## LORENTZ INVARIANCE

of the Hamiltonian

$$[H, J] = 0$$

of the S-matrix

$$[S, J_2] = 0$$

Our first goal is to rewrite consistency conditions as

$$[H, J] = 0 \qquad \Leftrightarrow \qquad [A, J_2] = 0$$

This will define A. It is an off-shell amplitude unambiguously related to H

SIMPLIFICATION

A effectively treats all generators as kinematical

# TO WARDS IDENTITY

1) Fix "integration by parts" freedom in h

$$j_n^{\lambda_1 \dots \lambda_n} = -\frac{1}{n}\left(\sum_j \frac{\partial}{\partial \bar{q}_j}\right) h_n^{\lambda_1 \dots \lambda_n}$$

2) The first term is already of the right form

$$\mathcal{J} H_n + [H_{n-1}, J_3] + \cdots + [H_3, J_{n-1}] = 0$$

3) Show that

$$[H_k, J_l] + [H_l, J_k] = \mathcal{J}\left(H_k \frac{1}{s} H_l\right) - \frac{1}{s} \mathcal{J} H_k H_l - \frac{1}{s} H_k \mathcal{J} H_l$$

$$\frac{1}{s} = -\frac{2}{(\beta_{k_l} - \beta_{l_k})(\mathcal{H}^{\{k\}} - \mathcal{H}^{\{l\}})}$$

4) Proceeding iteratively one reproduces the Feynman rules for the off-shell amplitude A, which satisfies $\mathcal{J} A = 0$

A related statement for 4-point case was observed in *[Metsaev'91]*

# SOLVING THE WARD IDENTITIES

One has to solve

$$[J, A] = 0, \qquad [\bar{J}, A] = 0, \qquad [K, A] = 0$$

General solution reads

$$A = \chi([ij], \langle ij\rangle),$$

$$(-N_{|i]} + N_{\langle i|} + 2\lambda_i)\chi([ij], \langle ij\rangle) = 0$$

where

$$N_{|i]} = |i]\frac{\partial}{\partial |i]}, \qquad N_{\langle i|} = \langle i|\frac{\partial}{\partial \langle i|}$$

In the 4-point case a general solution in a different form was found in *[Metsaev'91]*

For yet another approach see *[Bengtsson'16]*

# SOLUTION

## OUTCOME

1) This gives a general solution for the amplitude
2) Reversing the Feynman rules we find <u>a general solution for the Hamiltonian</u>
3) Of course, almost all of them are non-local.

Non-local solutions to the deformation procedure are to large extent trivial

*[Barnich, Henneaux'93]*

# COMPARISON WITH THE SPINOR-HELICITY APPROACH

For a gauge-invariant amplitude ambiguity should drop up, which leads to

$$A = \chi([ij], \langle ij\rangle)$$

Correct transformations in the Wigner little group entail

$$(-N_{|i]} + N_{\langle i|} + 2\lambda_i)\chi([ij], \langle ij\rangle) = 0$$

# TOWARDS PARITY INVARIANT THEORY

# TOWARDS PARITY INVARIANT THEORY

## EXISTS

Partial consistency at order g^2

Minimal coupling to gravity, which is universal

“Agreement” with AdS/CFT

## DOES NOT EXIST

Equivalence to spinor-helicity representation
allows to reduce the problem to on-shell methods

BCFW rules out consistent interactions.
Assumption: vanishing of the amplitude at infinity
At least to some extent it can be removed

*[Benincasa, Conde’11; McGady, Rodina’13]*

## WAYS OUT

Amplitudes can be distributions? Summation over spins? Non-localities?

# TOWARDS PARITY INVARIANT THEORY

## MANY KNOWN THEORIES ARE SECRETLY CUBIC

Space-cone gauge: an axial gauge with the axis, depending on the external momenta. Does not require higher vertices

*[Chalmers, Siegel'98]*

BCFW: reconstructs higher amplitudes from analytic properties

*[Britto, Cachazo, Feng, Witten'05]*

Colour-kinematics duality: relies on the representation in terms of cubic diagrams

*[Bern, Carrasco, Johansson'08]*

Holography: n-point functions can be reconstructed from 3-point ones using OPE

*[Pasterski, Shao, Strominger'17]*

This suggests that knowing cubic vertices should be enough!

# SUMMARY

## ON THE CHIRAL THEORY

- One should be careful using Lorentz tensors (AdS, massive fields?)
- There is a chiral higher spin theory, which is consistent to all orders. Avoids no-go's!
- It features local lower-derivative interactions, which are absent in manifestly Lorentz covariant classification. In particular, minimal coupling to gravity
- Generalised equivalence principle holds
- We found a simple way to derive the formula of Metsaev. One can see that there are 'truncations', e g single higher-spin field interacting with gravity
- "Agreement" with AdS/CFT
- Vanishing 4-point function

# SUMMARY

## ON SYSTEMATICS OF LIGHT-CONE DEFORMATION PROCEDURE

- Light-cone deformation procedure can be systematically solved to all orders (up to locality)
- This establishes its relation to the spinor-helicity representation
- Light-cone methods provide a Poincare invariant off-shell extension of spinor-helicity amplitudes

# SUMMARY

## OPEN PROBLEMS

- Parity-invariant completion
- Many other