Brane SUSY Breaking: Old Puzzles and New Results

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Ten-Dimensional Superstrings

(Briefly)

Ten-Dimensional (Closed) Superstrings

- **Building principles:** spin-statistics *(GSO projections)* and modular invariance

- **Building Blocks:** SO(8) level-one characters ($q = e^{2\pi i \tau}$):

Even(odd) # Fermi osc

$$O_8(V_8) = \frac{\theta^4\begin{bmatrix}0\\0\end{bmatrix}(0|\tau) \pm \theta^4\begin{bmatrix}0\\1/2\end{bmatrix}(0|\tau)}{2\,\eta^4(\tau)} \qquad S_8(C_8) = \frac{\theta^4\begin{bmatrix}1/2\\0\end{bmatrix}(0|\tau) \pm \theta^4\begin{bmatrix}1/2\\1/2\end{bmatrix}(0|\tau)}{2\,\eta^4(\tau)}$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n) \qquad \theta\begin{bmatrix}\alpha\\\beta\end{bmatrix}(z|\tau) = \sum_{n\in Z} q^{\frac{1}{2}(n+\alpha)^2}\, e^{i2\pi(n+\alpha)(z-\beta)}$$

Opposite Weyl proj's

- **Behavior under Modular Transformations:**

$$\tau \to -\frac{1}{\tau} \;:\; S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \qquad \tau \to \tau + 1 \;:\; T = e^{-i\frac{\pi}{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tau \to -\frac{1}{\tau} \;:\; \eta(-1/\tau) = (-i\tau)^{\frac{1}{2}}\eta(\tau) \qquad \tau \to \tau + 1 \;:\; \eta(\tau+1) = e^{i\frac{\pi}{12}}\,\eta(\tau)$$

- IIA and IIB:

$$\mathcal{T}_{IIA} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{(V_8 - S_8)(\bar{V}_8 - \bar{C}_8)}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}$$

- 0A and 0B:

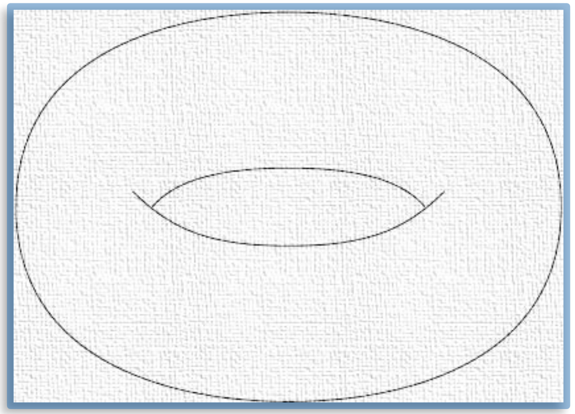
$$\mathcal{T}_{0A} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|O_8|^2 + |V_8|^2 + S_8\bar{C}_8 + C_8\bar{S}_8}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{0B} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}$$

- HO and HE:

$$\mathcal{T}_{HE} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2}\,\frac{(V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})^2}{Im\tau^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{HO} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2}\,\frac{(V_8 - S_8)(\bar{O}_{32} + \bar{S}_{32})}{Im\tau^4\,\eta^8\,\bar{\eta}^8}$$

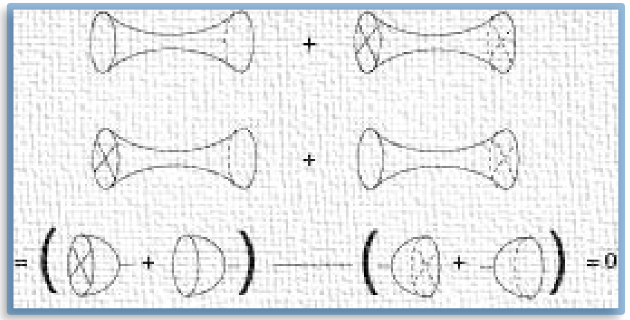
- $H_{SO(16)\times SO(16)}$:

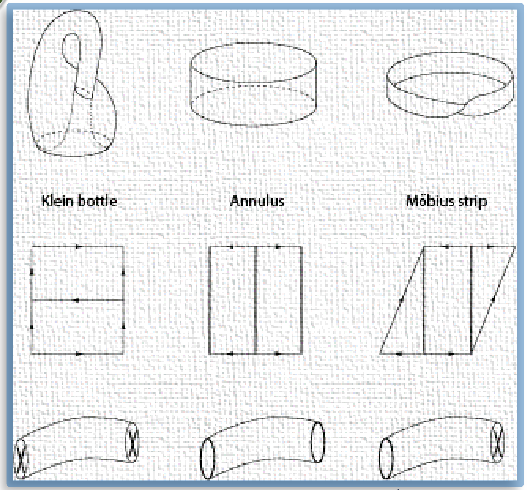
$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2}\,\frac{1}{Im\tau^4\,\eta^8\,\bar{\eta}^8}\,\big[O_8(\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}) + V_8(\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}) - S_8(\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16}) - C_8(\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16})\big]$$



Ten-Dimensional Orientifolds

- Follow from the IIB, 0A and 0B models: **orientifold projection** *(AS, 1987; + Bianchi, Pradisi, 1988-92)* which fills vacuum with D-branes and O-planes *(Polchinski, 1995)*





- **RR tadpole condition(s):** neutrality conditions

- **Three 10D NON-TACHYONIC orientifold models:**

  - **Type-I SO(32) superstring** *(Green and Schwarz, 1984)*: descends from IIB, vacuum filled with **BPS** combinations of **O-orientifold** (T<0,Q<0) and **D-branes** (T>0,Q>0). Massless Weyl fermions in the adjoint of SO(32)

  - **U(32) O'B model** *(AS, 1995)*: non-BPS D and O; → Applications to large-N QCD: *Armoni et al*

  - **USp(32) Sugimoto model** (simplest example with **"Brane SUSY Breaking"**): descends from IIB, vacuum filled with **NON BPS** combination of **O+ orientifold** (T>0,Q>0) and **anti D-branes** (T>0,Q<0). Massless fermions in the (reducible) two-fold antisymmetric of USp(32) → **Goldstino**

    (Sugimoto; Antoniadis, Dudas, AS; Angelantonj, Aldazabal and Uranga, 1999)

- *We had long spotted (with M. Bianchi and G. Pradisi) «tadpole conditions» yielding NEGATIVE solutions!*

  (See AS, ``Anomaly cancellations and open string theories'', hep-th/9302099)

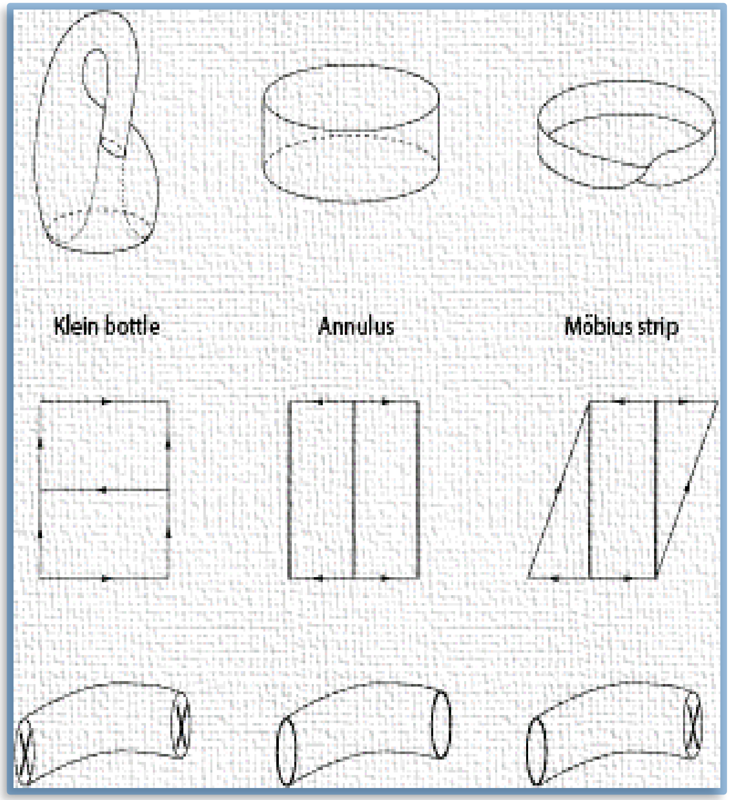
10D Tachyon-Free Orientifolds

1. USp(32) type-I: $(e^a_\mu, B_{\mu\nu}, \phi, \psi_\mu, \psi) \oplus (A^{(ab)}_\mu, \lambda^{[ab]})$, USp(32) gauge group [non-linear SUSY, T > 0]
   (Sugimoto, 1999)

$$\mathcal{T}_{Usp(32)} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar\eta^8}[\tau,\bar\tau] \qquad \mathcal{K}_{USp(32)} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{V_8 - S_8}{(\tau_2)^4\,\eta^8}[2i\tau_2]$$

$$\mathcal{A}_{Usp(32)} = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{V_8 - S_8}{(\tau_2)^4\,\eta^8}[i\tau_2/2] \qquad \mathcal{M}_{USp(32)} = \frac{\mathcal{N}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{(\hat V_8 + \hat S_8)}{(\tau_2)^4\,\hat\eta^8}[i\tau_2/2 + 1/2]$$

$$\mathcal{N} = 32$$



2. U(32) type-0'b: $(e^a_\mu, B_{\mu\nu}, D^+_{\mu\nu\rho\sigma}, \phi, \phi') \oplus (A_{\mu\, a}{}^b, \lambda^{[ab]}, \lambda_{[ab]})$, U(32) gauge group [NO SUSY, T > 0]
   (AS, 1995)

$$\mathcal{T}_{0'B} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\,\eta^8\,\bar\eta^8}[\tau,\bar\tau] \qquad \mathcal{K}_{0'B} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{-O_8 + V_8 + S_8 - C_8}{(\tau_2)^4\,\eta^8}[2i\tau_2]$$

$$\mathcal{A}_{0'B} = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{\mathcal{N}\bar{\mathcal{N}}\,V_8 - \frac{\mathcal{N}^2+\bar{\mathcal{N}}^2}{2}\,C_8}{(\tau_2)^4\,\eta^8}[i\tau_2/2] \qquad \mathcal{M}_{0'B} = -\frac{\mathcal{N}+\bar{\mathcal{N}}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{\hat C_8}{(\tau_2)^4\,\hat\eta^8}[i\tau_2/2 + 1/2]$$

$$\mathcal{N} = \bar{\mathcal{N}} = 32$$

3. SO(16) x SO(16) heterotic model: $(e^a_\mu, B_{\mu\nu}, \phi) \oplus (A^{[ab]}_\mu, A^{[a'b']}_\mu, \lambda^{aa'}_L, \lambda^A_R, \lambda^{A'}_R)$, SO(16) × SO(16) gauge group [NO SUSY, $\Lambda_{Torus} > 0$]
   (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1986; Dixon and Harvey, 1986)

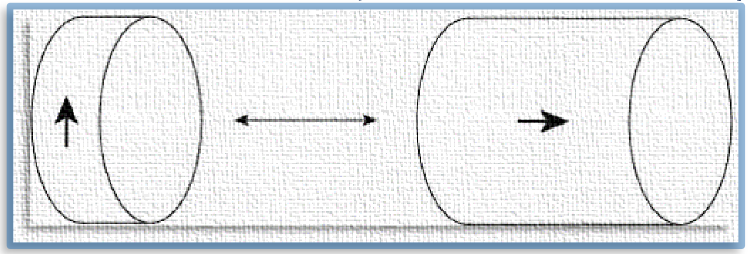
Brane SUSY Breaking

The Problem

- String Theory can be efficiently described around supersymmetric vacua (especially with extended SUSY);

- On the other hand, **broken supersymmetry** is accompanied by **vacuum redefinitions**:

  - In models of **oriented closed strings** these originate from "loop" diagrams (torus and beyond)
  - In **orientifolds** the distinction between tree and loop levels is blurred by **open-closed duality**



  - **"Brane SUSY Breaking":** results from insertion of non-BPS combinations of BPS branes and/or orientifolds
    (Sugimoto; Antoniadis, Dudas, AS; Angelantonj, Aldazabal and Uranga, 1999)

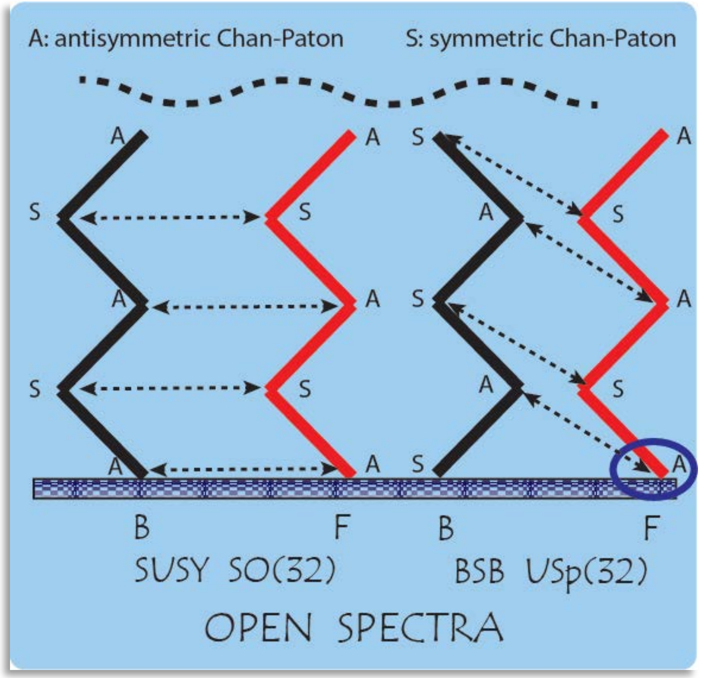
  - **Runaway exponential potential** for dilaton ($\sim e^{-\varphi}$ in string frame, or $\sim e^{\frac{3}{2}\varphi}$ in 10D in Einstein frame)

  - [**Torus (genus-one)** correction in 10D closed-string modes: ~ 1 in string frame, or $\sim e^{\frac{5}{2}\varphi}$ in Einstein frame]

- **In Field Theory** → one can shift fields. **BUT in String Theory** → must start around "wrong vacua"

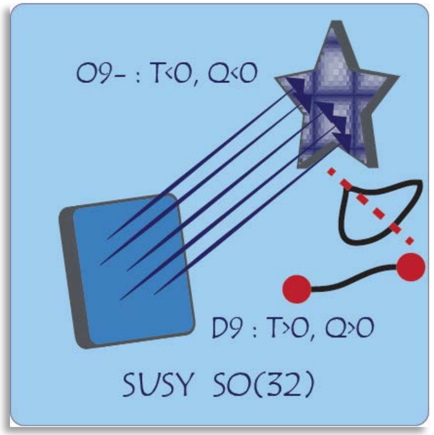
Brane SUSY Breaking (BSB)

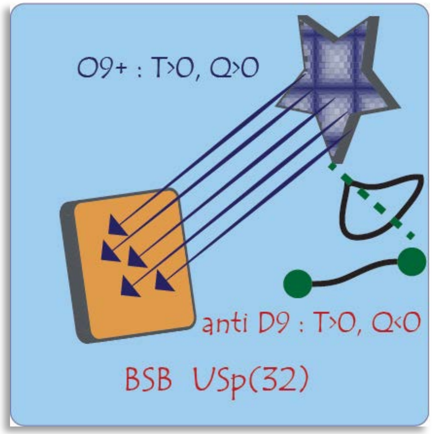
(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)



Tree – level

- SUSY broken at string scale in open sector, exact in closed sector
- Stable vacuum (classically)
- Goldstino in open sector





BSB: Tension unbalance → exponential potential

$$S_{10} = \frac{1}{2k_{10}^2} \int d^{10}x\sqrt{-g}\,\{e^{-2\phi}\left(-R + 4(\partial\phi)^2\right) - Te^{-\phi} + \ldots\}$$

BSB: The Low-Energy Supergravity

(Dudas, and Mourad, 2000, 2001)

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} \right\}$$

1. **Dudas and Mourad** solved the field equations for a 9D profile that (in string frame) describes a 9D S_1/Z_2 compactification. It leads to **finite 9D Planck mass and gauge coupling**, but there are **singularities** at the ends

$$ds^2 = |\alpha\,y^2|^{\frac{2}{9}}\, e^{\frac{1}{2}\phi_0}\, e^{\frac{1}{4}\alpha\,y^2}\, \eta_{\mu\nu}\,dx^\mu\,dx^\nu + |\alpha\,y^2|^{-\frac{1}{3}}\, e^{-\phi_0}\, e^{-\frac{3}{4}\alpha\,y^2}\, dy^2$$
$$e^\phi = e^{\phi_0}\,|\alpha\,y^2|^{\frac{1}{3}}\, e^{\frac{3}{4}\alpha\,y^2}$$

[similar, albeit more complicated looking, results also obtain for SO(16) x SO(16)]

2. There is an amusing **cosmological counterpart**. The dilaton tadpole lies precisely at the onset of the **"climbing phenomenon"**. The scalar is bound to emerge from the initial singularity **"climbing up" the potential**.

$$ds^2 = |\alpha\,t^2|^{\frac{2}{9}}\, e^{\frac{1}{2}\phi_0}\, e^{-\frac{1}{4}\alpha\,t^2}\, \delta_{ij}\,dx^i\,dx^j - |\alpha\,t^2|^{-\frac{1}{3}}\, e^{-\phi_0}\, e^{\frac{3}{4}\alpha\,t^2}\, dt^2$$
$$e^\phi = e^{\phi_0}\,|\alpha\,t^2|^{\frac{1}{3}}\, e^{-\frac{3}{4}\alpha\,t^2}$$

Halliwell , 1987)
…………
(Dudas, Mourad, 2000)
(Russo, 2004)
(Dudas, Kitazawa, AS, 2010)
…………

AdS Vacua and Branes

from

Dilaton Tadpoles and Fluxes

Low-Energy Lagrangians

In string frame (T→ Λ for heterotic):

$$\mathcal{S} \;=\; \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{2(p+2)!}\,e^{-2\,\beta_S\,\phi}\,\mathcal{H}_{p+2}^2-\frac{1}{4}\,e^{-\,2\,\alpha_S\,\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{\gamma_S\,\phi}\right\}$$

In Einstein frame (T→ Λ for heterotic):

$$\mathcal{S} \;=\; \frac{1}{2\,k_{10}^2}\int d^{10}x\sqrt{-g}\left\{-R-\frac{1}{2}\,(\partial\phi)^2-\frac{1}{2\,(p+2)!}\,e^{-2\,\beta_E^{(p)}\,\phi}\,\mathcal{H}_{p+2}^2-\frac{1}{4}\,e^{-2\,\alpha_E\,\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{\gamma_E\,\phi}\right\}$$

Here:

Orientifolds:

$$\gamma_E=\frac{3}{2}\;\;(\gamma_S=-1)\;,\;\alpha_E=-\,\frac{1}{4}\;\left(\alpha_S=\frac{1}{2}\right)\;,\;\beta_E^{(1)}=-\,\frac{1}{2}\;,\;\beta_E^{(5)}=\frac{1}{2}\;\;(\beta_S=0)$$

Heterotic:

$$\gamma_E=\frac{5}{2}\;\;(\gamma_S=0)\;,\;\alpha_E=\frac{1}{4}\;\;(\alpha_S=1)\;,\;\beta_E^{(1)}=\frac{1}{2}\;,\;\beta_E^{(5)}=-\frac{1}{2}\;\;\left(\beta_S^{(1)}=1\right)$$

Brane Profiles

1. The starting point is a **class of metrics of the type** (whose structure is familiar from the SUSY case):

$$\boxed{ds^2 = e^{2A(r)}\, g(\mathrm{L_k}) + e^{2B(r)} dr^2 + e^{2C(r)}\, g(\mathrm{E_{k'}})}$$

2. There exists a **CFT analysis** of (charged and uncharged) brane configurations for this type of orientifold systems. *(Dudas, Mourad, AS, 2001)*

How will the CFT analysis, which is set up around the flat space vacuum, and thus ignoring the dilaton tadpole, **connect to the actual deformed backgrounds**?

Radial Dynamical System

Second-order equations: $\left(F \;=\; (p+1)A \;-\; B \;+\; (D-p-2)C \right)$

$$\begin{aligned}
\mathcal{E}_A \;&\equiv\; A'' \;+\; A'\,F' \;=\; -\;\frac{T}{D-2}\;e^{\,2\,B+\gamma\,\phi} \;+\; k\;p\;e^{2(B-A)} \\
&+\;\frac{(D-p-3)}{2\,(D-2)}\;e^{\,-\,2\,\beta_E\,\phi\,-\,2(p+1)A}\,(b\,')^2 \;+\;4\;\frac{(D-p-2)}{(D-2)}\;e^{\,2\,C\,-\,2\,\alpha_E\,\phi}\;(a')^2 \;+\;8\;\frac{(D-p-2)(D-p-3)}{(D-2)}\;e^{\,2\,B-4\,C\,-\,2\,\alpha_E\,\phi}\,a^2\;(1\;-\;a)^2 \\
\mathcal{E}_C \;&\equiv\; C'' \;+\; C'\,F' \;=\; -\;\frac{T}{D-2}\;e^{\,2\,B+\gamma\,\phi} \;+\; k'(D-p-3)\;e^{2(B-C)} \\
&-\;\frac{(p+1)}{2\,(D-2)}\;e^{\,-\,2\,\beta_E\,\phi\,-\,2(p+1)A}\,(b\,')^2 \;-\;\frac{4\,p}{D-2}\;e^{\,-\,2\,C\,-\,2\,\alpha_E\,\phi}\;(a')^2 \;-\;8\;\frac{(D+p-2)(D-p-3)}{(D-2)}\;e^{\,2\,B-4\,C\,-\,2\,\alpha_E\,\phi}\,a^2\;(1\;-\;a)^2 \\
\mathcal{E}_\phi \;&\equiv\; \phi'' \;+\; \phi'\,F' \;=\; \frac{T\,\gamma\,(D-2)}{8}\;e^{\,2\,B+\gamma\,\phi} \;+\;\frac{\beta_E\,(D-2)}{8}\;e^{\,-\,2\,\beta_E\,\phi\,-\,2(p+1)A}\,(b\,')^2 \\
&-\;\alpha_E\,(D-2)\,(D-p-2)\,(a')^2\;e^{-2C-2\,\alpha_E\,\phi}\;-\;2\,\alpha_E\,(D-2)\;(D-p-2)\,(D-p-3)\,a^2\,(1-a)^2\;e^{2B-4C-2\,\alpha_E\,\phi} \\
\mathcal{E}_a \;&\equiv\; a'' \;+\; a'\,(F' \;-\;2\,C' \;-\;2\,\alpha_E\;\phi') \;-\;2\;e^{\,2(B\,-\,C)}\;(D-p-3)\,a\,(1\;-\;a)\,(1\;-\;2\,a) \;=\; 0 \\
\mathcal{E}_b \;&\equiv\; \left(e^{\,-\,2\,\beta_E\,\phi\,-\,(p+1)A\,-\,B\,+\,(D-p-2)C}\;b\,'\right)' \;=\; 0
\end{aligned}$$

First-order constraint:

$$\begin{aligned}
&(p+1)A'[p\,A' \;+\;(D-p-2)C'] \;+\;(D-p-2)C'[(D-p-3)C' + (p+1)A'] \\
&-\frac{4\,(\phi')^2}{D-2} \;+\;T\,e^{\,2\,B+\gamma\,\phi} \;-\;k\,p\,(p+1)\;e^{2(B-A)} \;-\;k'(D-p-3)(D-p-2)\;e^{2(B-C)} \\
&+\tfrac{1}{2}\,e^{\,-\,2\,\beta_E\,\phi\,-\,2\,(p+1)\,A}\,(b\,')^2 \;-\;4\,e^{\,-\,2\,C\,-\,2\,\alpha_E\,\phi}\,(D-p-2)\;(a')^2 \\
&+8\,e^{\,2\,B\,-\,4\,C\,-\,2\,\alpha_E\,\phi}\,(D-p-2)(D-p-3)\,a^2\,(1-a)^2 \;=\; 0
\end{aligned}$$

Example: Supersymmetric Branes

Gauge choice: (k=0,k′=1)

$$C \;=\; B \;+\; \log r$$

$$ds^2 \;=\; e^{2A(r)}\, g(\mathrm{L_k}) \;+\; e^{2B(r)} dr^2 \;+\; e^{2C(r)}\, g(\mathrm{E_{k'}})$$

$A \sim B \sim \phi$:

$$(p+1)A \;+\; (D-p-3)B \;=\; 0$$

$$\frac{\beta_E\,(D-2)^2}{4}\; B \;+(p+1)\,\phi \;=\; 0$$

If $\beta_E \neq 0$:

$$\lambda \;=\; \beta_E \;+\; \frac{4\,(p+1)(D-p-3)}{\beta_E\,(D-2)^2}$$

$$e^{-\lambda\,\phi} \;=\; e^{-\lambda\,\phi_0}\left[\left(\frac{r_0}{r}\right)^{D-p-3} \;+\; 1\right]$$

$$e^{2\,A} \;=\; e^{\frac{8\,\phi_0\,(D-p-3)}{\beta_E\,(D-2)^2}}\left[\left(\frac{r_0}{r}\right)^{D-p-3} \;+\; 1\right]^{-\,\frac{8(D-p-3)}{\beta_E\,\lambda\,(D-2)^2}}$$

$$e^{2\,B} \;=\; e^{-\,\frac{8\,\phi_0\,(p+1)}{\beta_E\,(D-2)^2}}\left[\left(\frac{r_0}{r}\right)^{D-p-3} \;+\; 1\right]^{\frac{8(p+1)}{\beta_E\,\lambda\,(D-2)^2}}$$

Non-Singular Vacuum Configurations

1. The class of **metrics**: $$\boxed{ds^2 = e^{2A(r)}\, g(\mathrm{L_k}) + dr^2 + e^{2C(r)}\, g(\mathrm{E_{k'}})}$$

2. **Constant dilaton profiles :** aim at "fixing" the dilaton, despite the runaway potentials

3. Vacuum configurations for $\mathcal{H}_{p+2}$: $$\boxed{\mathcal{H}_{p+2} = h\, e^{(p+1)\, A(r) + 2\,\beta_E^{(p)}\,\phi - (8-p)\, C}\ \epsilon(p+1)\, dr}$$

4. Allow also for **non-trivial internal gauge fields**, identifying subgroups of the gauge groups with the tangent space to internal spheres ($\tilde{y}^T\,\tilde{y} = 1$) : the simplest option is for SU(2): *(Wu and Yang, 1969)*

$$\boxed{\mathcal{A} = i\, a(r)\left(\tilde{y}\, d\tilde{y}^T - d\tilde{y}\,\tilde{y}^T\right) \longrightarrow \mathcal{F} = i\,\xi\, d\tilde{y}\, d\tilde{y}^T \qquad \left(\text{or } a = \frac{\xi}{2}\right)}$$

Orientifold Vacuum Configurations

In this fashion the field equations reduce to

$$
\begin{aligned}
(\star):\ T\, e^{\gamma_E \phi} &= \frac{\xi\,\alpha_E}{\gamma_E}\,(8-p)\,(7-p)\,e^{-4C-2\alpha_E\phi} - \frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2(8-p)C+2\beta_E^{(p)}\phi} \\
16\,k'\,e^{-2C} &= \xi\left[8+p+\frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right]e^{-4C-2\alpha_E\phi} + \frac{h^2\left(p+1-\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}}{(7-p)} \\
(A')^2 &= k\,e^{-2A} + \xi\,\frac{(8-p)(7-p)}{16(p+1)}\left(1-\frac{2\alpha_E}{\gamma_E}\right)e^{-4C-2\alpha_E\phi} + \frac{h^2}{16(p+1)}\left(7-p+\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}
\end{aligned}
$$

(*): dilaton eq: strong constraints due to **positivity of *l.h.s.*** (α_E , β_E < 0 for orientifolds & T>0, NEED H_3 fluxes)

First two eqs: determine k′=1 (internal sphere), and (implicitly) its radius e^C and ϕ in terms of h and ξ

Third eq: determines for k=0 A ~r, and thus AdS in Poincaré coordinates (or in other slicings for k ≠ 0)

AdS Slicings

Let us take a closer look at the last equation: $$(A')^2 \;=\; k\, e^{-2A} \;+\; \frac{1}{R^2} \qquad (k=\pm 1, 0)$$

It is solved by:

$$\begin{aligned} ds^2 &= R^2 \sinh^2\left(\frac{r}{R}\right) \left(ds^2\right)_{k=1} + dr^2 \\ ds^2 &= R^2 \cosh^2\left(\frac{r}{R}\right) \left(ds^2\right)_{k=-1} + dr^2 \\ ds^2 &= R^2\, e^{2\frac{r}{R}} \left(ds^2\right)_{k=0} + dr^2 \end{aligned}$$

These metrics emerge from **three different slicing of the same AdS space**, for which

$$\begin{aligned} X_{-1}^2 + X_0^2 - \sum_{i=1}^{p+1} X_i^2 &= R^2 \\ ds^2 = -\,dX_{-1}^2 - dX_0^2 &+ \sum_{i=1}^{p+1} dX_i^2 \end{aligned}$$

1. **K = 1 slicing:** $$-\eta_{\mu\nu}\, X^\mu\, X^\nu - X_{p+1}^2 = R^2\,, \quad X^\mu = R\, y^\mu \cosh\xi\,, X_{p+1} = R \sinh\xi\,, \quad -\eta_{\mu\nu}\, y^\mu\, y^\nu = 1$$

2. **K=-1 slicing:** $$X_{-1}^2 - \eta_{\mu\nu}\, X^\mu\, X^\nu = R^2\,, \quad X^\mu = R\, y^\mu \sinh\xi\,, X_{-1} = R \cosh\xi\,, \quad \eta_{\mu\nu}\, y^\mu\, y^\nu = 1$$

3. **K=0 slicing:** $$X_{-1}^2 - X_{p+1}^2 - \eta_{\mu\nu}\, X^\mu\, X^\nu = R^2\,, \; U, V = X_{-1} \pm X_{p+1}\,, \quad V = \frac{R^2}{U} + \eta_{\mu\nu}\, \frac{U}{R^2}\, y^\mu\, y^\nu$$

$AdS_3 \times S_7$ Orientifold Vacua $\left(T = \frac{16}{\pi^2}\,(BSB),\ \frac{8}{\pi^2}\,(0'B) \right)$

Concentrate on the **first two equations** [Here $\alpha_E < 0$, $\beta_E < 0$, and therefore we **NEED** a 3-form flux, with p=1]:

$$(\star):\ T\, e^{\gamma_E\,\phi} = \frac{\xi\,\alpha_E}{\gamma_E}\,(8-p)\,(7-p)\,e^{-4C-2\alpha_E\,\phi} - \frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2(8-p)C+2\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{-2C} = \xi\left[8+p+\frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right]e^{-4C-2\alpha_E\,\phi} + \frac{h^2\left(p+1-\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\,\phi}}{(7-p)}$$

Combining them leads to **two branches** of solutions :

$$e^{2C} = \frac{2\,\xi\,e^{\frac{\phi}{2}}}{1 \pm \sqrt{1-\frac{\xi T}{3}\,e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{\xi^7\,e^{4\phi}}{\left(1 \pm \sqrt{1-\frac{\xi T}{3}\,e^{2\phi}}\right)^7}\left[\frac{42}{\xi}\left(1 \pm \sqrt{1-\frac{\xi T}{3}\,e^{2\phi}}\right) + 5\,T\,e^{2\phi}\right]$$

The **lower sign** yields an asymptotically weak-coupling solution (small coupling for large sizes)

$$g_s \equiv e^{\phi} \sim \frac{12}{(2\,h\,T^3)^{\frac{1}{4}}}\,,\qquad R^4\,g_s^3 \sim \frac{144}{T^2}\,,\qquad (A')^2 \sim k\,e^{-2A} + \frac{6}{R^2}$$

$AdS_3 \times S_7$ Orientifold Vacua $\left(T = \frac{16}{\pi^2}\ (BSB), \frac{8}{\pi^2}\ (0'B) \right)$

$$
\begin{aligned}
e^{2C} &= \frac{2\xi\, e^{\frac{\phi}{2}}}{1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}} \\
\frac{h^2}{32} &= \frac{\xi^7\, e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}} \right) + 5\, T\, e^{2\phi} \right]
\end{aligned}
$$

- The solution corresponding to the **lower sign** exists for $\xi=0$ (with residual unbroken gauge group Usp(32) or U(32)), and for $\xi=1$ (with residual unbroken gauge group Usp(24) or U(24))

- The solution corresponding to the **upper sign** only exists for $\xi=1$, associates **large radii** to **strong couplings**, and **survives in the limit of vanishing T**, so that it is a solution also for the type-I superstring. It lies outside the perturbative reach, but it has an interesting heterotic counterpart, as we shall see shortly.

Heterotic Vacuum Configurations

In this fashion the field equations reduce to

$$(\star):\ \Lambda\, e^{\gamma_E \phi} = \frac{\xi\, \alpha_E}{\gamma_E}\,(8-p)\,(7-p)\, e^{-4C-2\alpha_E \phi} - \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \xi\left[8+p+\frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right] e^{-4C-2\alpha_E\phi} + \frac{h^2\left(p+1-\frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C+2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \xi\,\frac{(8-p)(7-p)}{16(p+1)}\left(1-\frac{2\alpha_E}{\gamma_E}\right) e^{-4C-2\alpha_E\phi} + \frac{h^2}{16(p+1)}\left(7-p+\frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

(*): dilaton eq: now $\alpha_E > 0$ for heterotic & $\Lambda > 0$, but we can allow, surprisingly, unbounded H_3 or H_7 fluxes ($\beta_E >$ &< 0). Actually, one can also eliminate these fluxes altogether, and then the solutions, supported by the gauge fields alone, exist for all p. Only for p=3,7, however, can they have unbounded radii and small string coupling.

First two eqs: determine again k′=1 (internal sphere), and (implicitly) its radius e^C and ϕ in terms of h and ξ

Third eq: determines again for k=0 A ~r, and thus AdS in Poincaré coordinates (or in other slicings for $k \neq 0$)

$AdS_3 \times S_7$ Heterotic Vacua ($\Lambda \approx \frac{4\pi^2}{25}$)

Let us concentrate on the first two equations (here $\alpha_E > 0$, but we want to allow for large h fluxes, with p=1):

$$(\star):\; T\,e^{\gamma_E\,\phi} = \frac{\xi\,\alpha_E}{\gamma_E}\,(8-p)\,(7-p)\,e^{-4C-2\alpha_E\phi} - \frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

$$16\,k'\,e^{-2C} = \xi\left[8+p+\frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right]e^{-4C-2\alpha_E\phi} + \frac{h^2\left(p+1-\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}}{(7-p)}$$

Combining them leads to one branch of solutions :

$$e^{2C} = \frac{2\,\xi\,e^{-\frac{\phi}{2}}}{1+\sqrt{1+\frac{\xi\Lambda}{3}\,e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3}\,e^{2\phi}}\right]^7}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3}\,e^{2\phi}}\right) - 13\,\Lambda\,e^{2\phi}\right]$$

This solution affords again a weak-coupling limit, and asymptotically

$$g_s \equiv e^{\phi} \sim \left(\frac{21}{h^2}\right)^{\frac{1}{4}}, \qquad g_s\,R^4 \sim 1\,, \qquad (A')^2 \sim k\,e^{-2A} + \frac{21}{4\,R^2}$$

BUT: it continues to exist even if $\Lambda = 0$ (strong-weak coupling dual of second orientifold one in the limit)

$AdS_7 \times S_3$ Heterotic Vacua $\left(\Lambda \approx \frac{4\pi^2}{25}\right)$

Let us concentrate on the first two equations (here $\alpha_E > 0$, but we want to allow for large h fluxes, with p=5):

$$(\star):\ T\, e^{\gamma_E \phi} = \frac{\xi\, \alpha_E}{\gamma_E}\,(8-p)\,(7-p)\, e^{-4C - 2\alpha_E \phi} - \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \xi \left[8 + p + \frac{2\alpha_E}{\gamma_E}(8-p)\right] e^{-4C - 2\alpha_E \phi} + \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

Combining them leads to one branch of solutions :

$$e^{2C} = \frac{\xi}{2}\, \frac{e^{-\frac{\phi}{2}}}{1 - \sqrt{1 - \xi\, \Lambda\, e^{2\phi}}}\,,$$

$$\frac{h^2}{3} = \xi^3\, \frac{\left[\frac{17\Lambda}{24}\, e^{2\phi} - \frac{1}{\xi}\left(1 - \sqrt{1 - \xi\, \Lambda\, e^{2\phi}}\right)\right]}{\left(1 - \sqrt{1 - \xi\, \Lambda\, e^{2\phi}}\right)^3}\,;$$

This solution also affords again a weak-coupling limit, and asymptotically

$$g_s \equiv e^{\phi} \sim \left(\frac{5}{h^2\, \Lambda^2}\right)^{\frac{1}{4}}, \qquad g_s^5\, R^4 \sim \frac{1}{\Lambda^2}\,, \qquad (A')^2 \sim k\, e^{-2A} + \frac{1}{4\, R^2}$$

BUT: this branch exists only if $\Lambda \neq 0$

Thank You