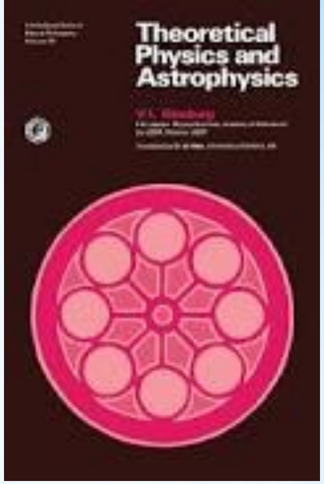
GINZBURG CENTENNIAL CONFERENCE on PHYSICS

May 29 - June 3, 2017
Lebedev Institute,
Moscow, Russia



“The local and integrate dispersion relations and the analytic structure of hadron elastic scattering amplitude at LHC energies

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Contents

* Introduction
* Elastic hadron scattering – new data LHC
* Comparing the data with High Energy Generalized structure model (HEGS)
* Integral and Derivative dispersion relations
* The real part of the scattering amplitude from the data
* Results and Summery

Total cross sections

TOTEM **ATLAS**

$$\sigma_{tot} = (98.3 \pm 2.8)\, mb;$$

$$\sqrt{s} = 7000\, GeV;$$

$$(98.6 \pm 2.2)\, mb;$$

$$(99.1 \pm 4.3)\, mb;$$

$$(98.0 \pm 2.5)\, mb;$$

$$\sigma_{tot} = (98.5 \pm 2.9)\, mb$$

$$\sigma_{tot} = (95.35 \pm 2.0)\, mb$$

$$\Delta = 3.15\ mb;$$

$$\sqrt{s} = 8000\, GeV;$$

$$\sigma_{tot} = (101.7 \pm 2.9)\, mb$$

$$\sigma_{tot} = (102.0 \pm 2.2)\, mb$$

$$\sigma_{tot} = (102.3 \pm 2.3)\, mb$$

$$\sigma_{tot} = (96.07 \pm 1.34)\, mb$$

$$\Delta = 5.93\ mb;$$

$$F_1^{em}(t) = \alpha f_1^2(t)\frac{s-2m^2}{t}; \quad F_3^{em}(t) = F_1^{em};$$

and for spin-flip amplitudes:

$$F_2^{em}(t) = \alpha\frac{f_2^2(t)}{4m^2}s; \quad F_4^{em}(t) = -F_2^{em}(t),$$

$$F_5^{em}(t) = \alpha\frac{s}{2m\sqrt{|t|}}f_1(t)\ f_2(t),$$

where the form factors are:

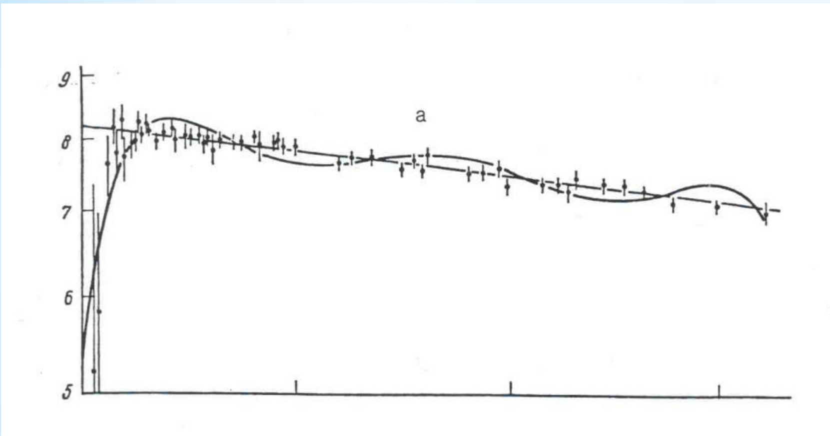
$$f_1(t) = \frac{4m_p^2 - (1+k)\ t}{4m_p^2 - \ t}G_d(t);$$

$$f_2(t) = \frac{4m_p^2\ k}{4m_p^2 - \ t}\ G_d(t);$$

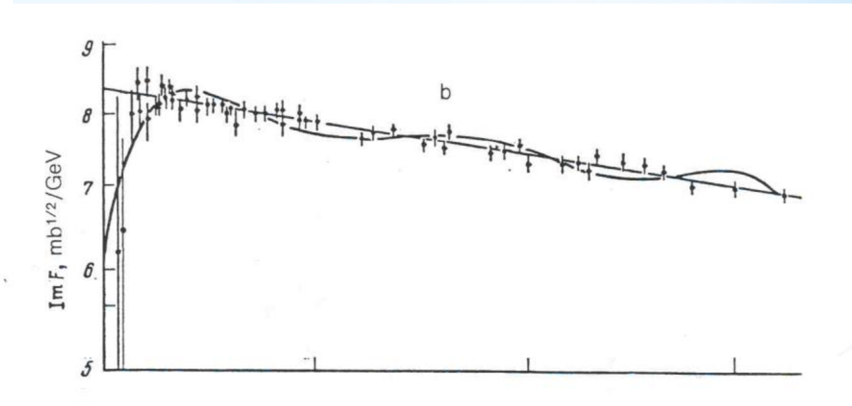
$$\frac{dN}{dt} = \mathcal{L}\left[\frac{4\pi\alpha^2}{|t|^2}G^4(t) - \frac{2\alpha\left(\rho(s,t) + \phi_{CN}(s,t)\right)\sigma_{tot}G^2(t)e^{-\frac{B(s,t)|t|}{2}}}{|t|} + \frac{\sigma_{tot}^2(1+\rho(s,t)^2)e^{-B(s,t)|t|}}{16\pi}\right] \quad (\text{

Proton-proton $\sqrt{s} = 27.4\,GeV;$

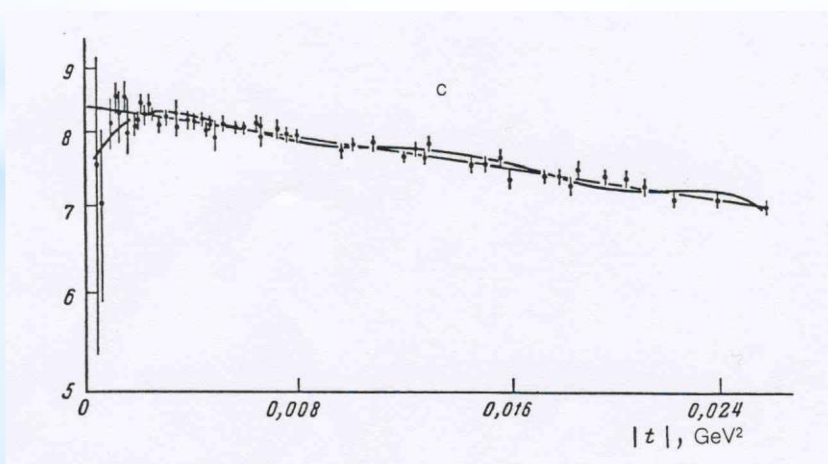
$Exp.: \rho = 0.012,\ B = 11.95\,GeV^{-2},\ \sigma_{tot} = 40.65\,mb;$



$fit: \rho = 0.033,\ B = 14.5\,GeV^{-2},\ \sigma_{tot} = 40.9\,mb;$

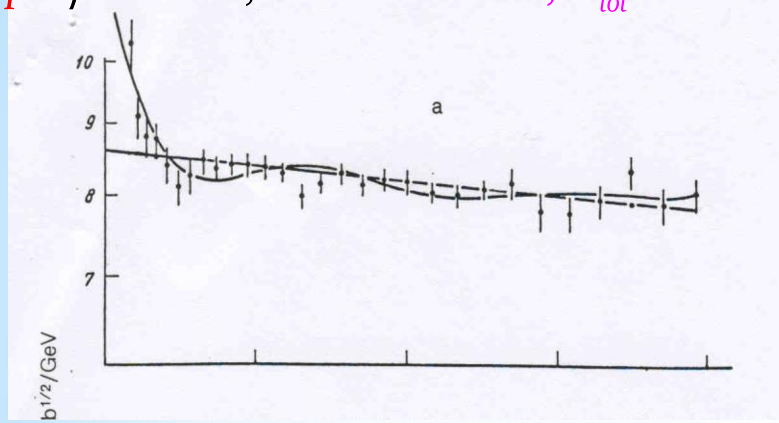


$fit: \rho = 0.02,\ B = 13.8\,GeV^{-2},\ \sigma_{tot} = 41.34\,mb,\ n = 1.02;$

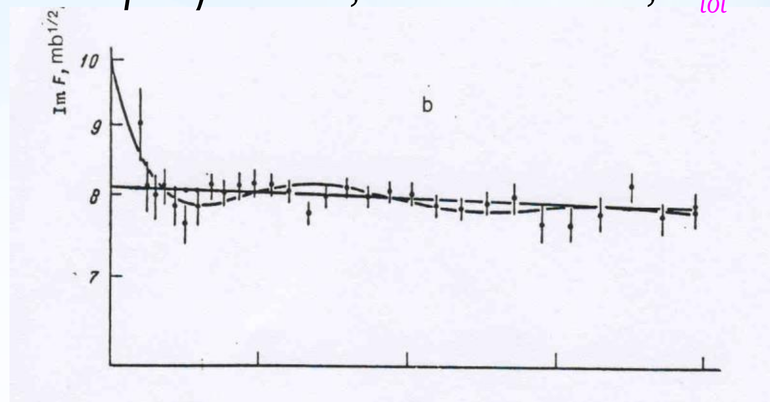


Proton-antiproton $\sqrt{s} = 30.4\,GeV;$

$Exp.: \rho = 0.055,\ B = 12.7\,GeV^{-2},\ \sigma_{tot} = 42.13\,mb;$



$fit: \rho = 0.12,\ B = 14.1\,GeV^{-2},\ \sigma_{tot} = 39.65\,mb;$



$fit: \rho = -0.01,\ B = 11.75\,GeV^{-2},\ \sigma_{tot} = 40.22\,mb,\ n = 0.9;$

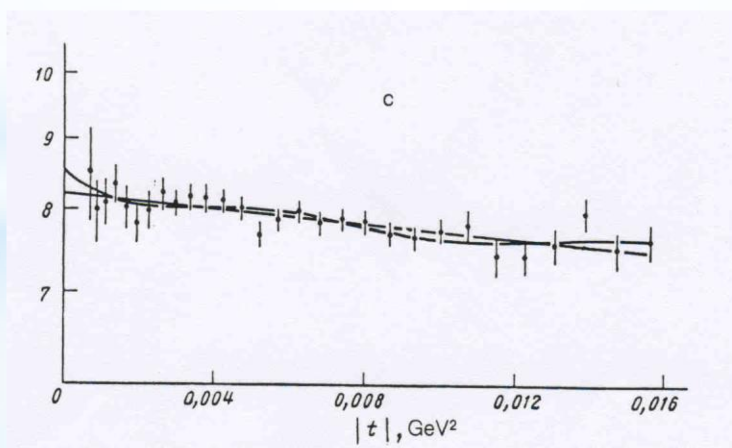


Table 4: Fit results with $N_b = 1$. Each column corresponds to a fit with different interference formula and/or nuclear phase.

	SWY, constant	KL, constant	KL, peripheral
step 1: χ^2/ndf	$48.0/27 = 1.78$	$48.1/27 = 1.78$	$27.7/27 = 1.03$
step 2: χ^2/ndf	$180.8/58 = 3.12$	$181.2/58 = 3.12$	$64.3/58 = 1.11$
a [mb/GeV^2]	533 ± 23	533 ± 23	551 ± 23
b_1 [GeV^{-2}]	19.42 ± 0.05	19.42 ± 0.05	19.74 ± 0.05
ρ	0.05 ± 0.02	0.05 ± 0.02	0.10 ± 0.02
ζ_1			800
κ			2.311
ν [GeV^{-2}]			8.161
σ_{tot} [mb]	102.0 ± 2.2	102.0 ± 2.2	103.4 ± 2.3

In each case, the fit results are used to calculate the total cross-section via the optical theorem:

$$\sigma_{\mathrm{tot}}^2 = \frac{16\pi\,(\hbar c)^2}{1+\rho^2}\,a\,. \qquad (27)$$

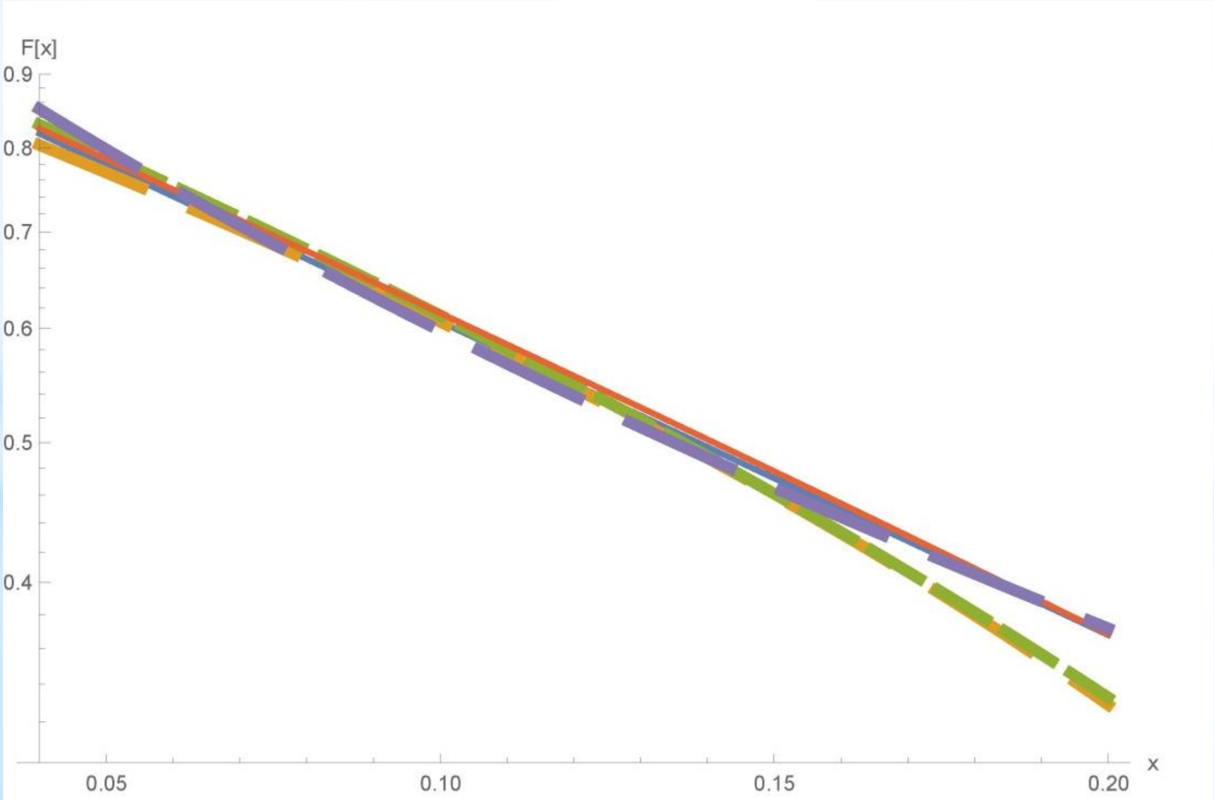
$$\Gamma(b) = e^{-b^2/R^2}; \quad \rightarrow \quad F_1(t) \sim e^{5t};$$

$$\Gamma(b) = \delta(R_0); \quad \rightarrow \quad F_1(t) \sim J_0(R\sqrt{|t|});$$

$$\Gamma(b) = e^{-(b-R_0)^\wedge 2/R_0^2}; \rightarrow \quad F_1(t) \sim e^{2.8t} J_0(2.8\sqrt{|t|});$$

$$\Gamma(b) = C(0 - R_0); \quad \rightarrow \quad F_1(t) \sim J_1(R_0\sqrt{|t|}) / R_0\sqrt{|t|};$$

$$\Gamma(b) = e^{-\mu(\sqrt{4R_o^2 - t})}; \quad \rightarrow \quad F_1(t) \sim e^{-5R_0(\sqrt{\mu^2 - t})};$$



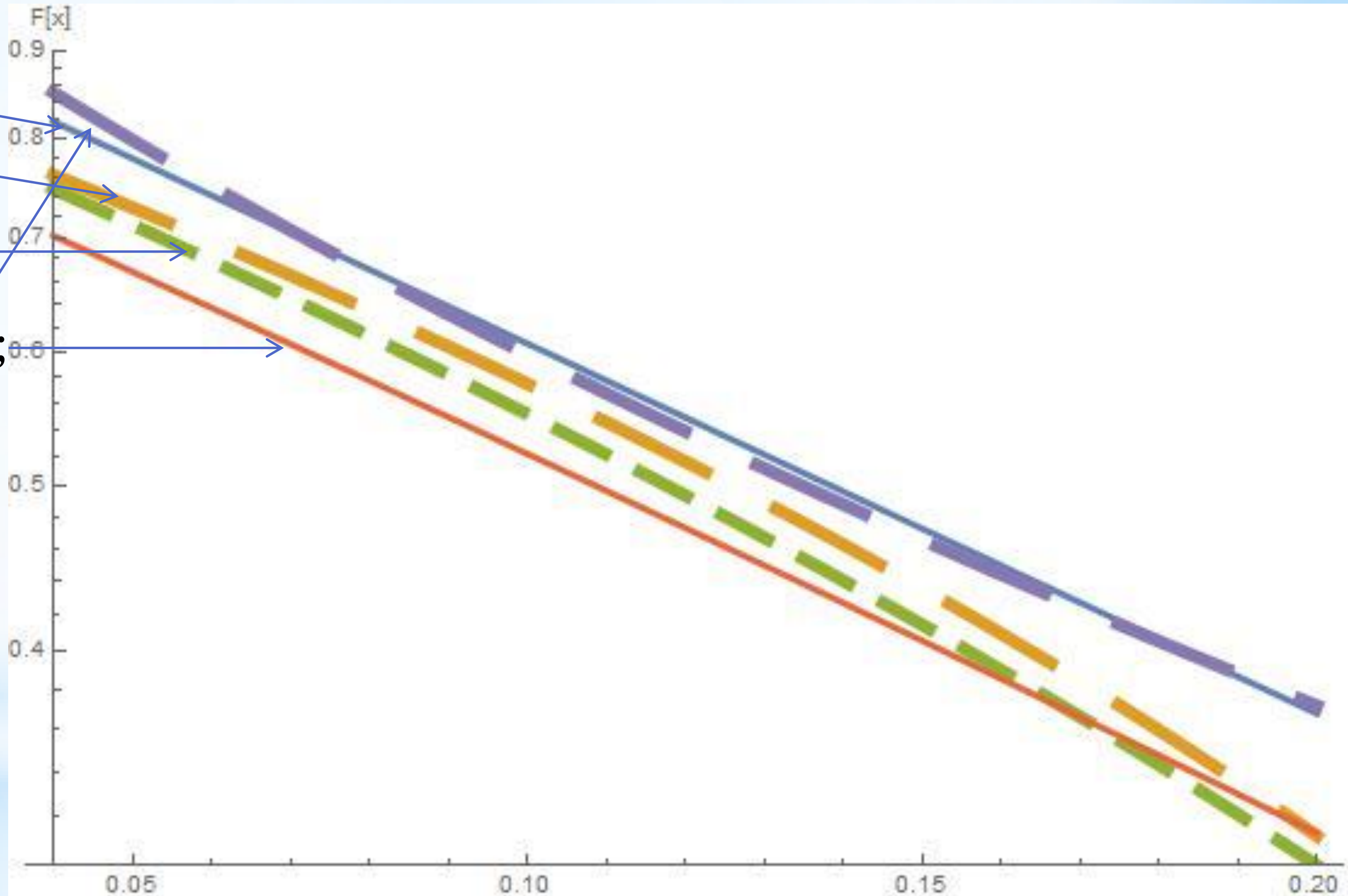
e^{5t};

$J_0(4\sqrt{|t|})$;

$e^{2.8t} J_0(2.8\sqrt{|t|})$;

$J_1(6\sqrt{|t|}) / 6\sqrt{|t|}$;

$e^{4.5 R_0(\sqrt{\mu^2 - t})}$;



High Energy General Structure

(HEGS0) model. O.V.S - Eur. Phys. J. C (2012) 72:2073

Extending of model (HEGS1) – O.V. S. Phys.Rev. D 91, (2015) 113003

	BSW_1	BSW_2	AGN	MN	HESG0	HESG1
N_exp.	369	955	1728+238	2600+300	980	3416
n_par.	7+Regge	11	36	36+7	3+2	6+3
$\sqrt{s}\ GeV$	23.4-630	13.4 - 1800	9.3-1800	5-1800	52-1800	9-8000
$\Delta t\ GeV^2$		0.1 - 5	0,1-2.6	0.1-16	0.000875-10	0.00037-15
$\sum \chi_i^2 / N$	4.45	1.95	1.16	1.23	2.	1.28

$$n = 980 \rightarrow 3416; \quad 9 \le \sqrt{s} \le 8000\, GeV;\ 0.00037 < |t| < 15\ GeV^2;$$

GPDs

$$\lim it \; Q_{\gamma}^{2} = 0, \; and \; \xi = 0$$

X.Ji Sum Rules (1997)

$$\mathcal{F}_{\xi=0}(x;t) = \mathcal{F}(x;t)$$

$$F_1^q(t) = \int_{-1}^{1} dx \; H^q(x,\xi,t);$$

$$F_2^q(t) = \int_{-1}^{1} dx \; E^q(x,\xi,t);$$

$$\mathcal{H}^q(x;t) = H^q(x,0,t) + H^q(-x,0,t)$$

$$\mathcal{E}^q(x;t) = E^q(x,0,t) + E^q(-x,0,t)$$

$$F_1^q(t) = \int_{0}^{1} dx \; \mathcal{H}^q(x,\xi,t);$$

$$F_2^q(t) = \int_{0}^{1} dx \; \boldsymbol{\mathcal{E}}^q(x,\xi,t);$$

$$\int_{-1}^{1} dx \; x[H^q(x,\xi,t) + E^q(x,\xi,t)] = A_q(\Delta^2) + B_q(\Delta^2);$$

High Energy General Structure (HEGS) model O.V.S. - Eur. Phys. J. C (2012) 72:2073

Extending of model (HEGS1) – O.V. S. Phys.Rev. D 91, (2015) 113003

$$\hat{s} = s / s_0 \, e^{-i\pi/2}; \qquad s_0 = 4m_p^2.$$

$$F_1^B(s,t) = h_2 G_{em}(t) \, (\hat{s})^{\Delta_1} e^{\alpha_1 t \ln(\hat{s})}; \; F_3^B(s,t) = h_3 G_A(t)^2 \, (\hat{s})^{\Delta_1} e^{\alpha_1/4 t \ln(\hat{s})};$$

$$F^B(\hat{s},t) = F_2^B(\hat{s},t)(1 + R_1/\sqrt{\hat{s}} \,)] + F_3^B(\hat{s},t)(1 + R_2/\sqrt{\hat{s}} \,)] + F_{odd}^B(s,t);$$

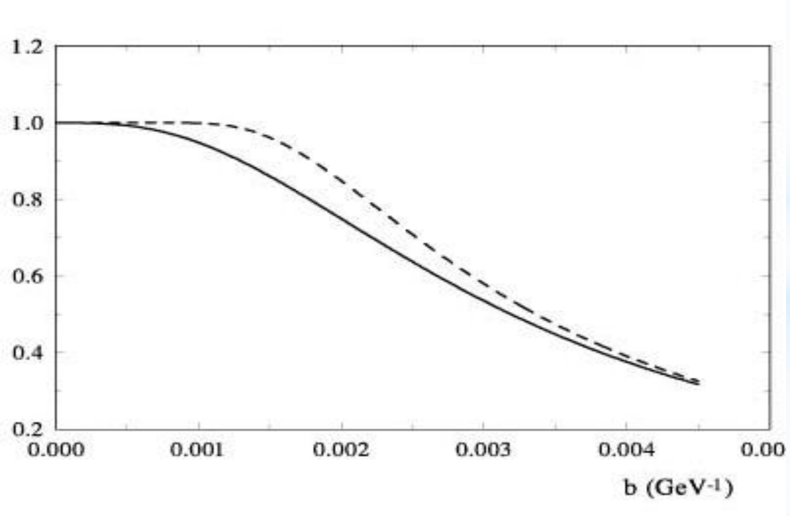
$$F_{Odd}^B(s,t) = h_{Odd} \; G_A(t)^2 \; (\hat{s})^{\Delta_1} \; \frac{t}{1 - r_o^2 t} e^{\alpha_1/4 t \ln(\hat{s})};$$

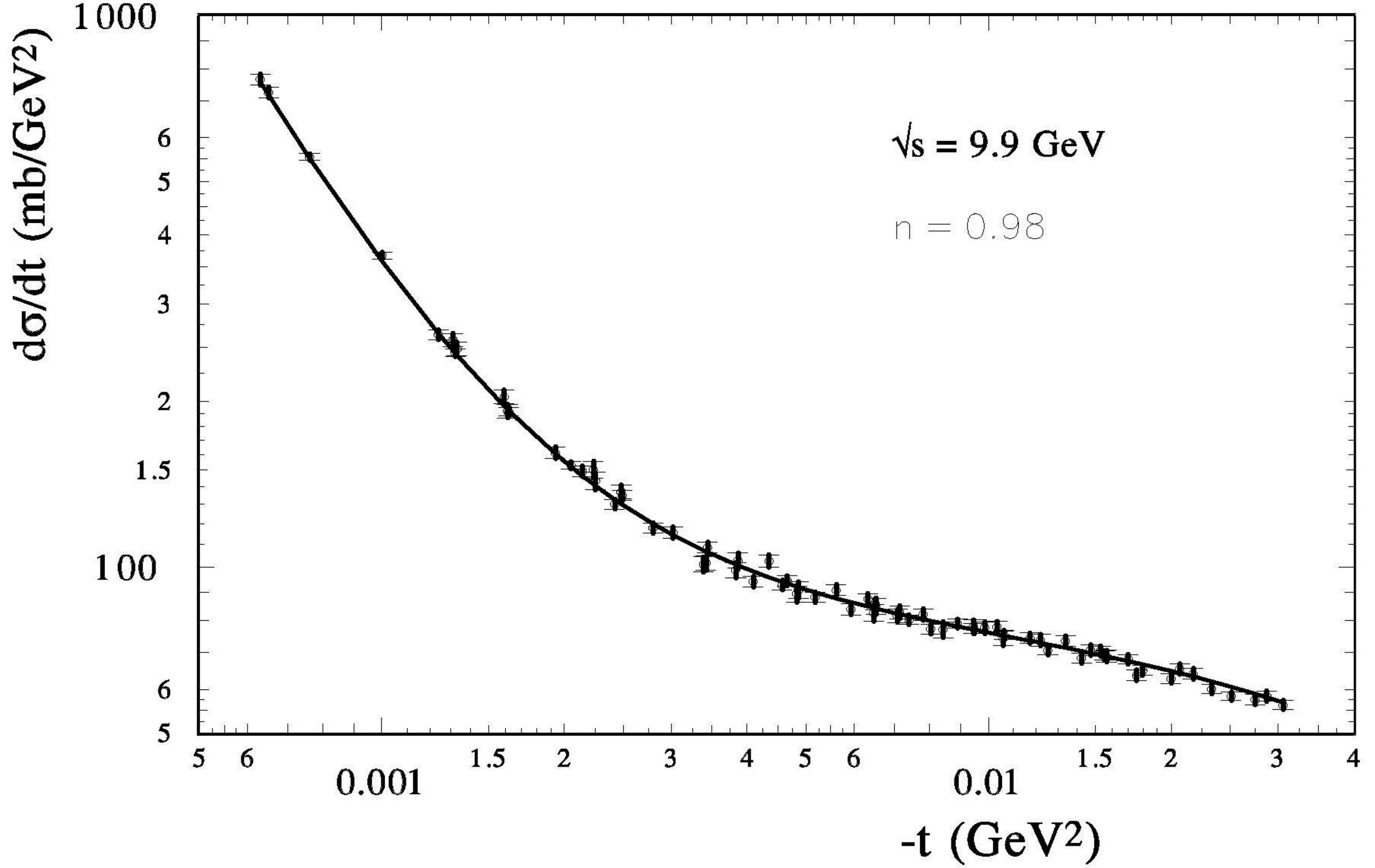
UNITARIZATION → eikonal representation

$$\chi(s,b) = 2\pi \int_0^\infty q\ J_0(bq)\ F_B^h(s,q)\,dq$$

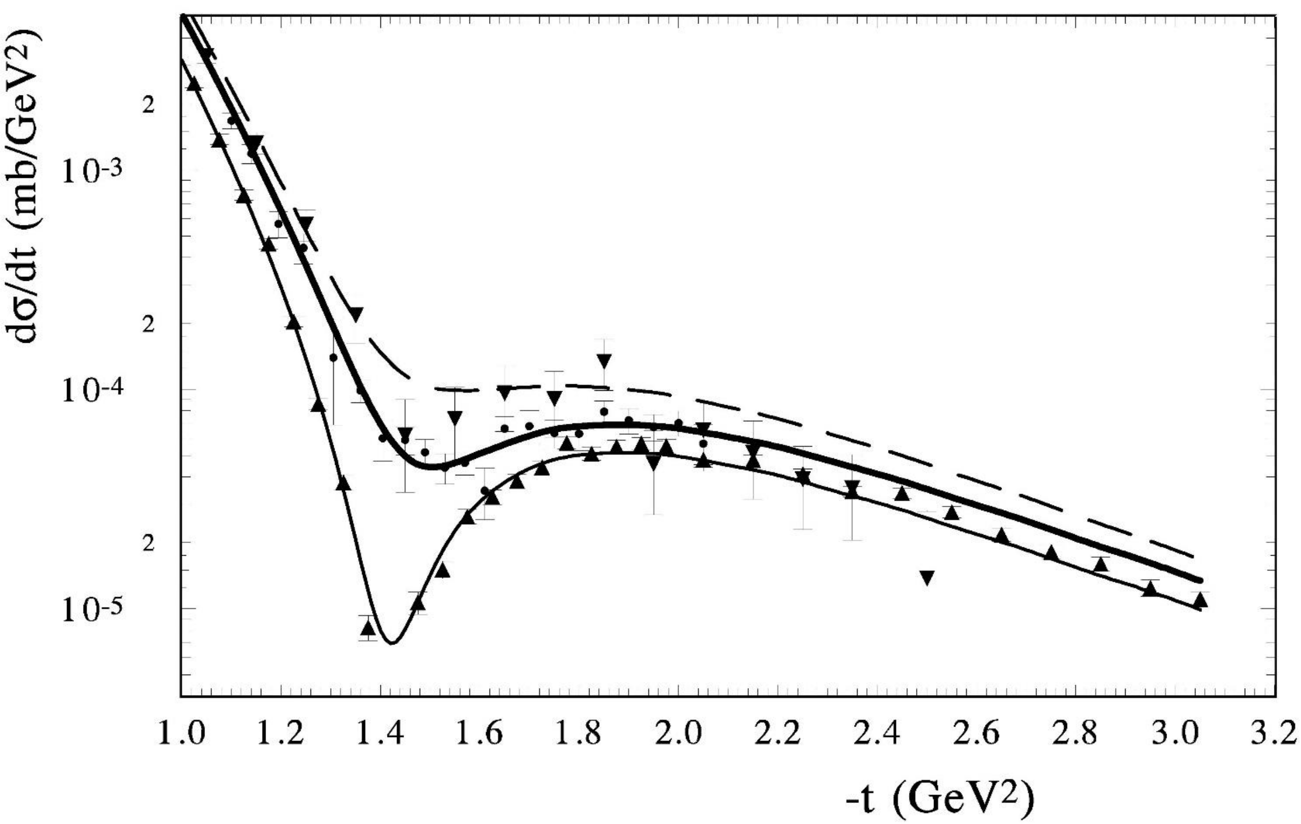
$$\chi(s,b) = -\frac{1}{2k}\int_{-\infty}^{\infty} dz V[\sqrt{z^2+b^2}]$$

$$F^h(s,t) = \frac{1}{2\pi}\int_0^\infty b\ J_0(bq)\ [1-e^{-\chi(s,b)}]\,db$$



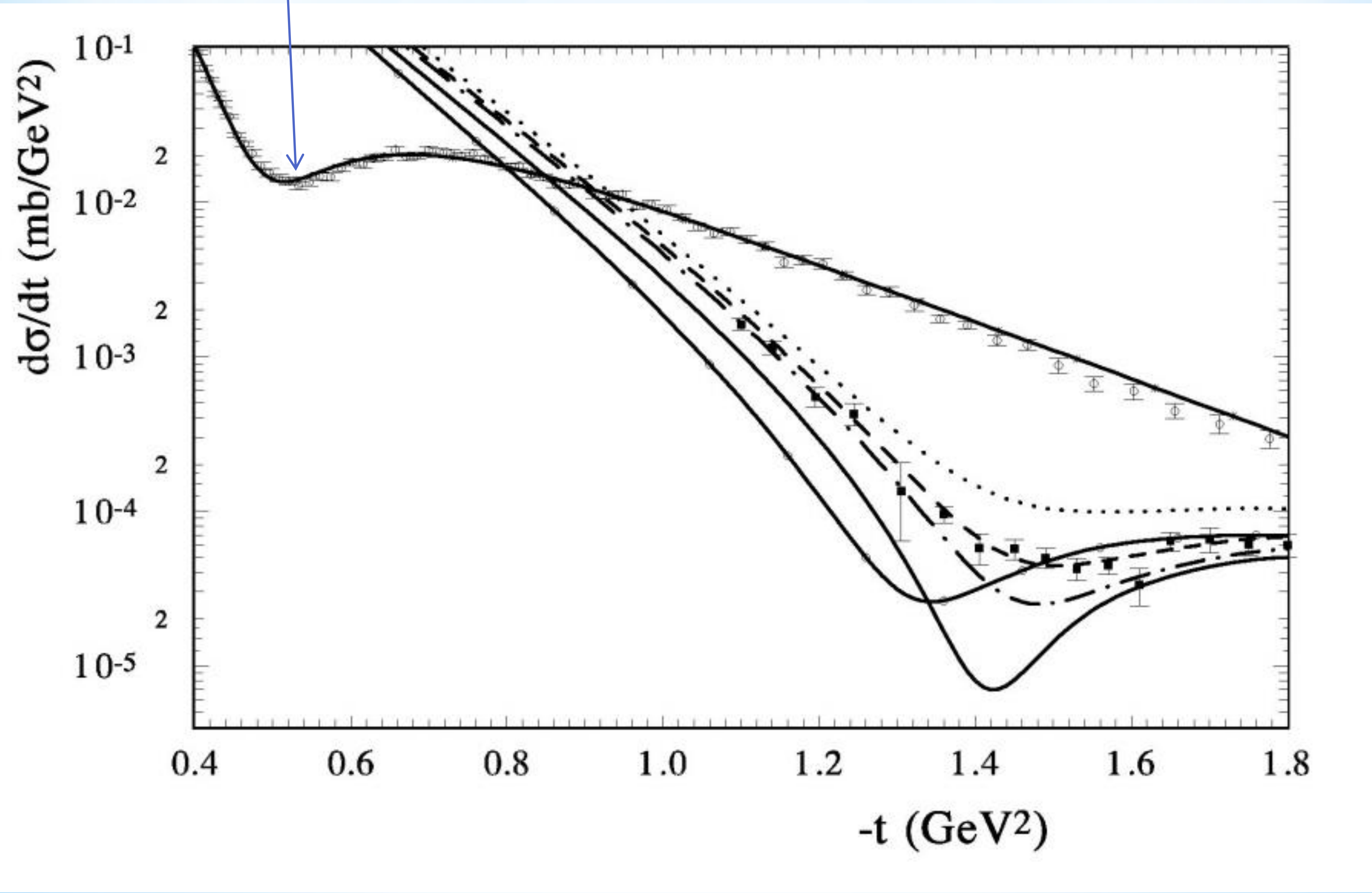


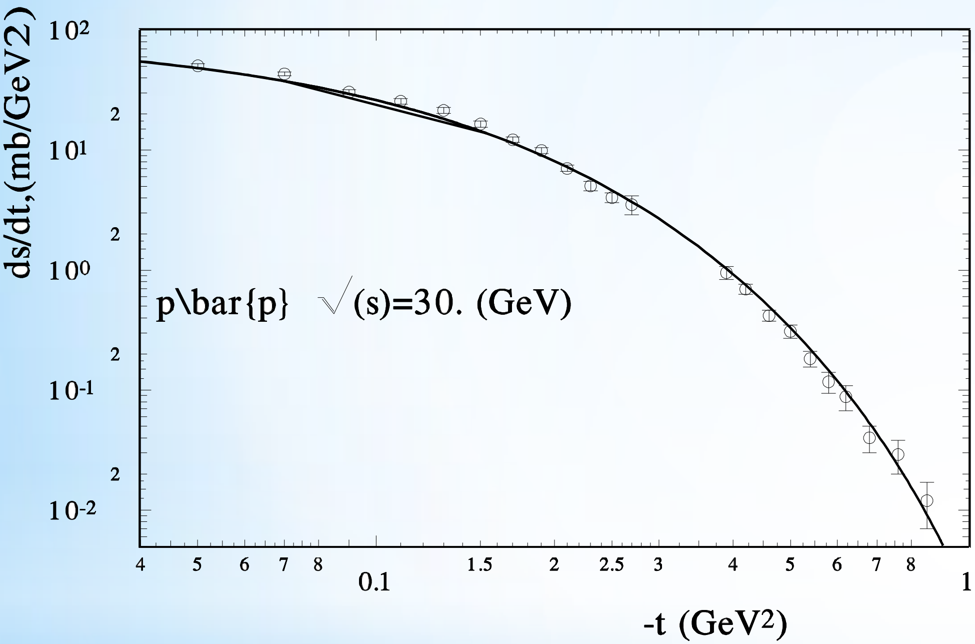
1000
100
dσ/dt (mb/GeV2)
√s = 9.9 GeV
n = 0.98
0.001
0.01
-t (GeV2)

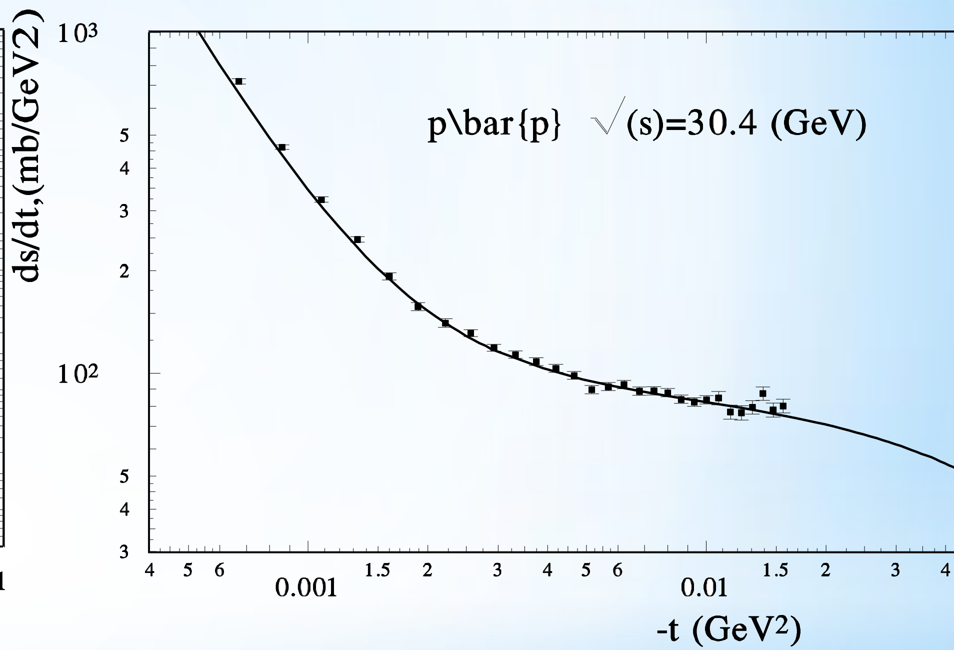


dσ/dt (mb/GeV2)
2
10-3
2
10-4
2
10-5
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
-t (GeV2)

TOTEM data – 7 TeV

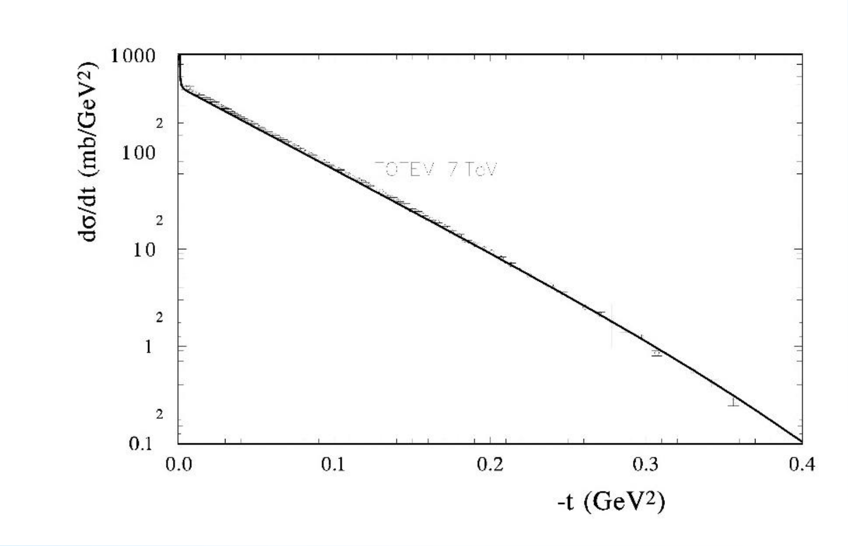



ds/dt,(mb/GeV2)
p\bar{p} √(s)=30. (GeV)
-t (GeV2)


ds/dt,(mb/GeV2)
p\bar{p} √(s)=30.4 (GeV)
-t (GeV2)

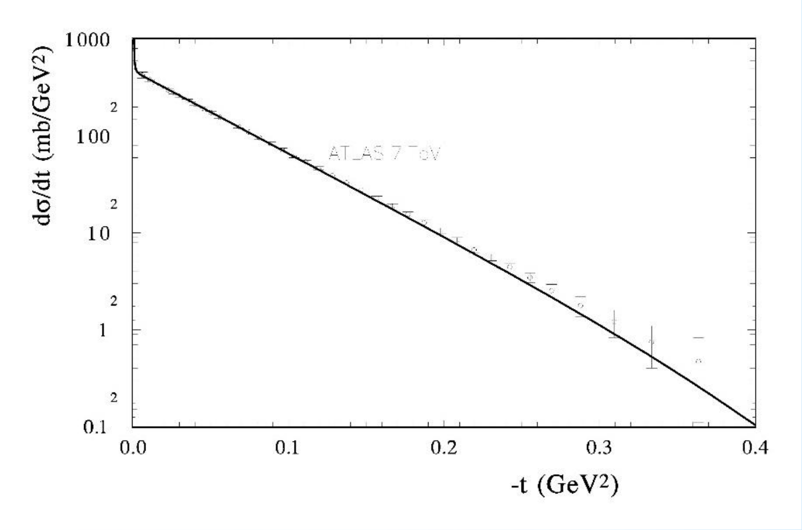
7 TeV (TOTEM) - t [0.00515 – 0.371]

n = 0.94

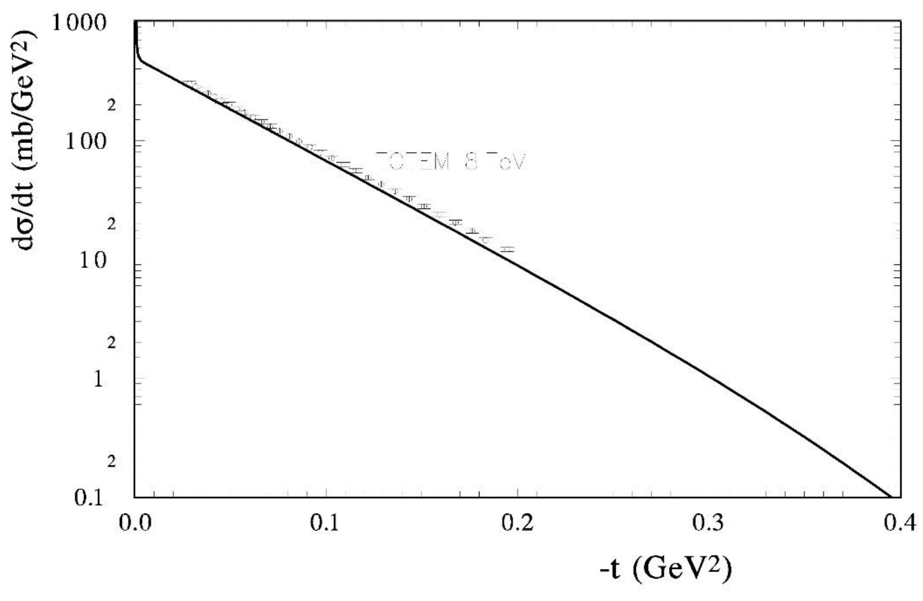


7 TeV (ATLAS) - t [0.0062 – 0.3636]

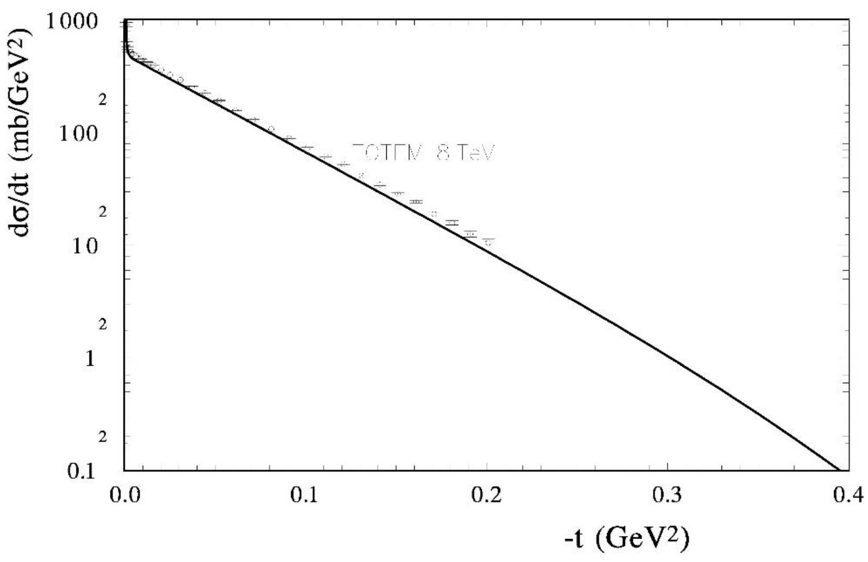
n=1.0



8 TeV (TOTEM) - t [0.0285 – 0.1947]
n=0.9

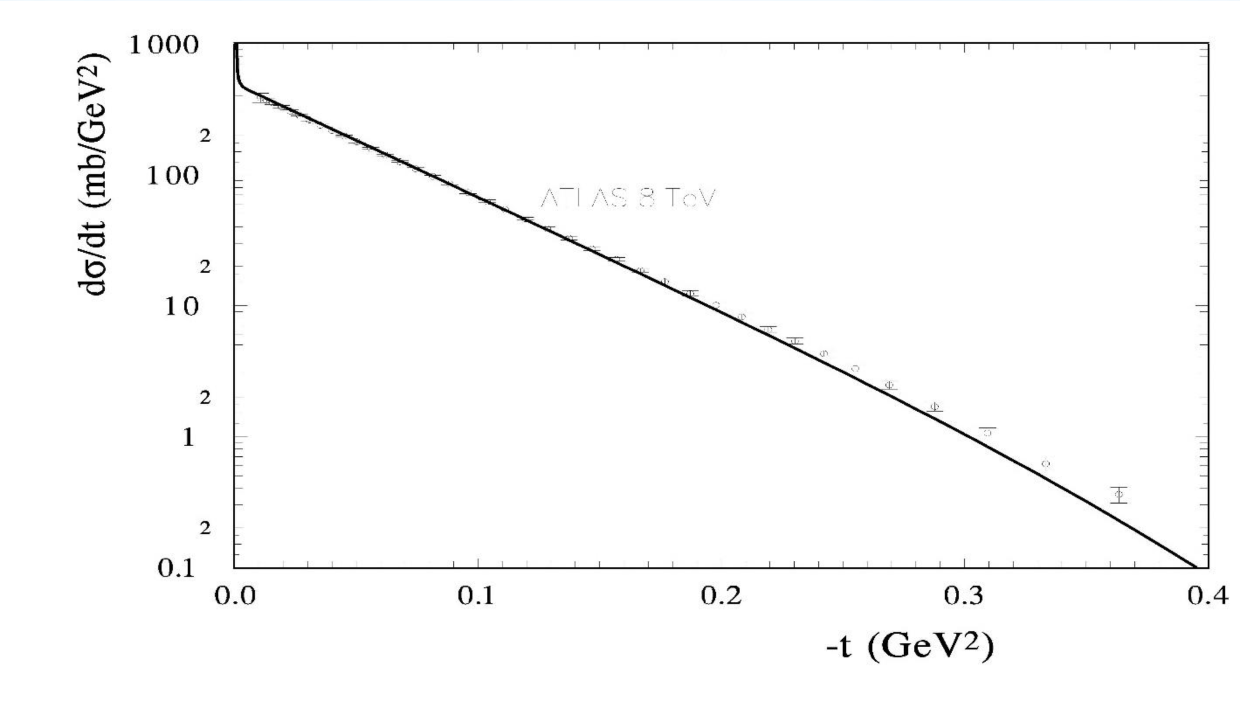


8 TeV (TOTEM) - t [0.000741 – 0.201]
n=0.9

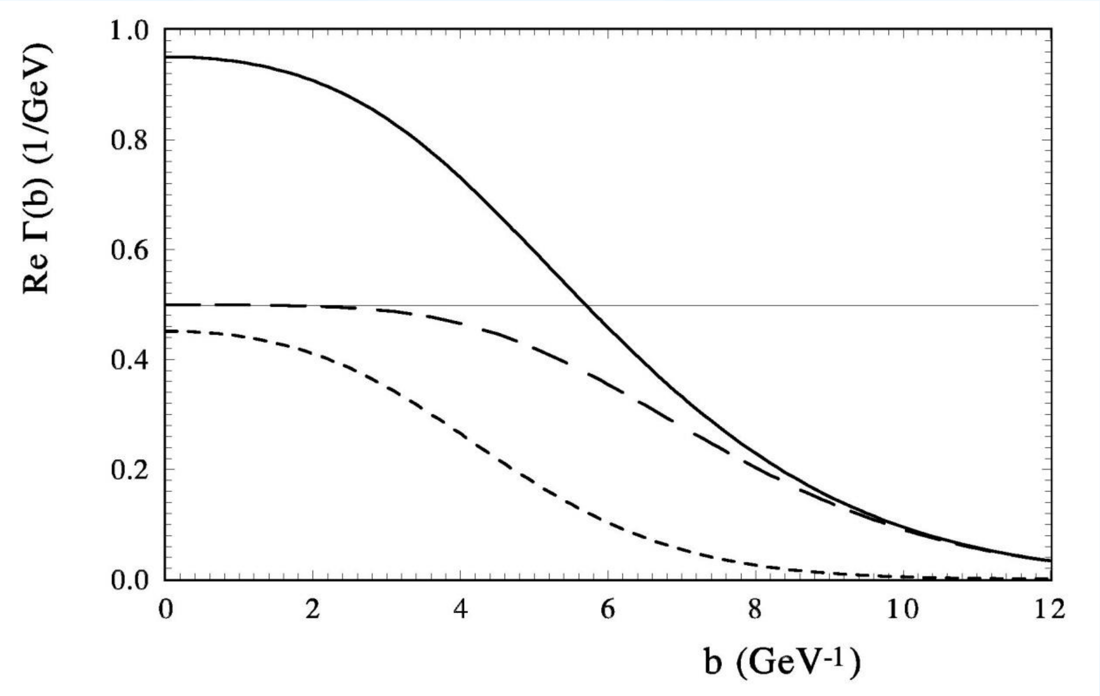


8 TeV (ATLAS) - t [0.01050 – 0.3635]

n=1.0



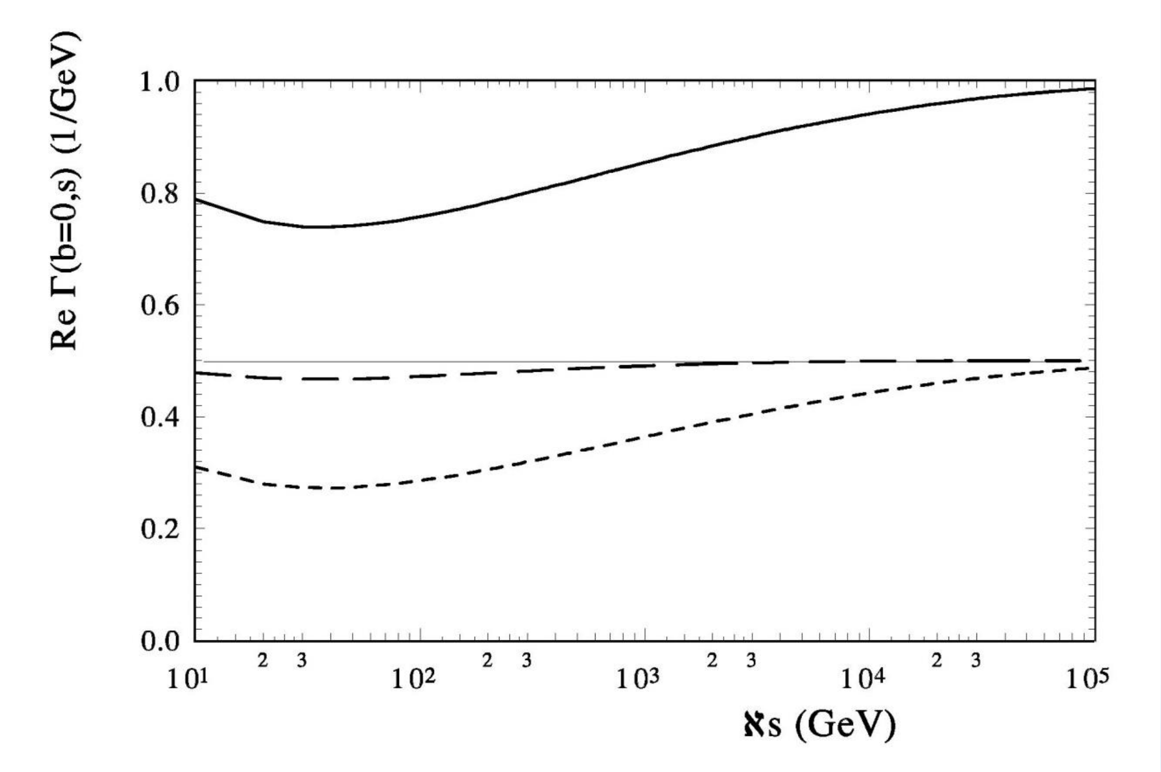
$$\sqrt{s} = 14 TeV$$



$\Gamma(s,b)_{tot}$ (hard line)

$\Gamma(s,b)_{elast.}$ (dashed line); $\Gamma(s,b)_{inel.}$ (long dashed line)

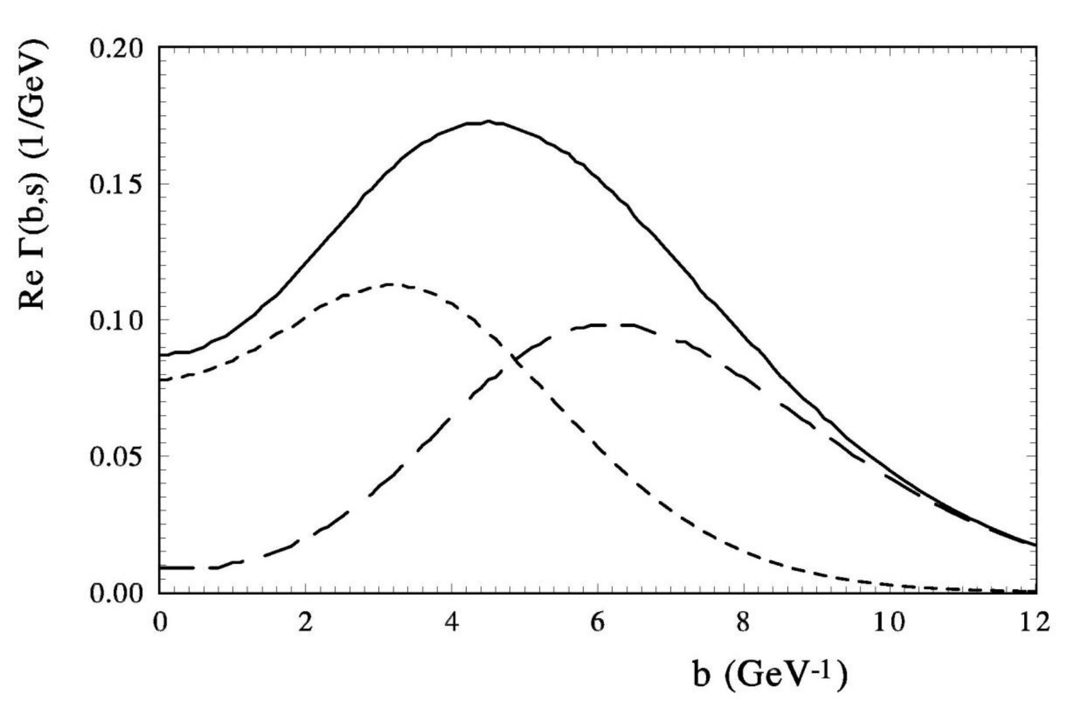
The energy dependence of



$$\Gamma(s,b=0)_{tot}$$ (hard line)

$\Gamma(s,b=0)_{elast.}$ (dashed line); $\Gamma(s,b=0)_{inel.}$ (long dashed line)

The difference between $\sqrt{s} = 14 TeV$ and $\sqrt{s} = 7 TeV$



$\Gamma(s,b)_{tot}$ (hard line)

$\Gamma(s,b)_{elast.}$(dashed line); $\Gamma(s,b)_{inel.}$ (long dashed line)

Integral dispersion relations $\qquad S \longrightarrow \infty$

N.M. Queen, G. Violint "Dispersion theory in high energy physics" (1974)

$$\rho_{\pm}(E)\,\sigma_{\pm}(E) = \frac{C}{P} + \frac{E}{\pi P}\int_{m}^{\infty} dE'\,P'[\frac{\sigma_{\pm}(E')}{E'(E'-E)} - \frac{\sigma_{\mp}(E')}{E'(E'+E)}].$$

Local DDR

COMPETE Collaboration

$$\mathrm{Re}\,F_{+}(E,0)=(\frac{E}{m_p})^{\alpha}\tan[\frac{\pi}{2}(\alpha-1+E\frac{d}{dE})]\,\mathrm{Im}\,F_{+}(E,0)/(\frac{E}{m_p})^{\alpha}$$

Roy (2016)

$$\mathrm{Re}\,F_{+}(s,t)=(\frac{\pi}{\ln(s)})\,\frac{d}{dt}[t\,\mathrm{Im}\,F_{+}(s,t)\,/\,\mathrm{Im}\,F_{+}(s,t=0)], \quad as \quad s\rightarrow\infty.$$

$$\mathrm{Re}\,F_{+}(s,t) \sim \rho(s,t)(1.+d\ f(t)).$$

E. Ferreira, A.K. Kohara – arXiv: 1704.08866

$$\mathrm{Re}_+(E,t) = K + \frac{2E^2}{\pi} P\int_m^\infty dE' \, \frac{\mathrm{Im}\, F_+(E')}{E'(E'^2 - E^2)} \; . \qquad x = E/m; \;\; \frac{s}{2m^2} = x+1;$$

$$\mathrm{Re}_+(E,t) = K + \frac{2m^2 x}{\pi} P\int_1^\infty dx' \, \frac{1}{x'^2 - x^2}[\sigma_{\bar{p}p}(x')\, e^{B_I^{\bar{p}p}(x')t/2} - \sigma_{pp}(x')\, e^{B_I^{pp}(x')t/2}].$$

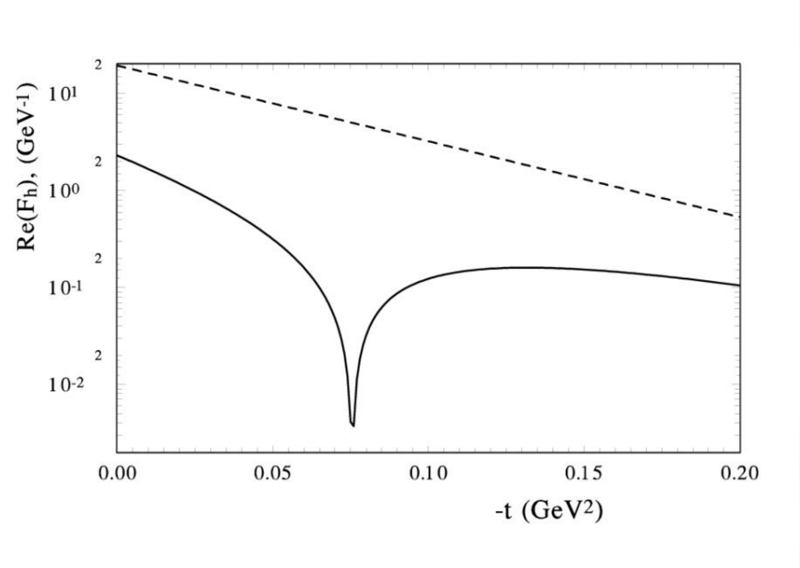
$$I(\nu,\lambda,x) = P\int_1^\infty dx' \, \frac{x'^\lambda (\log x')^\nu}{x'^2 - x^2} = \frac{x^{-2}}{2^{\nu+1}} P\int_1^\infty dt \, \frac{t^\lambda e^{(1+\lambda)t/2}}{x^{-2} e^t - 1}$$

By using the series representation of the Lerch’s transcendent,

$$I(n,\lambda,x) = \frac{\pi}{2x} \frac{\partial^n}{\partial \lambda^n}(x^\lambda \tan(\pi\lambda/2)) + (-1)^n n! \sum_{m=1}^\infty \frac{x^{-2m}}{(2m-1+\lambda)^{n+1}}.$$

$$\int_m^\infty dE'\,P'\left[\frac{\operatorname{Im}F_\pm(E',t)}{E'(E'-E)}-\frac{\operatorname{Im}F_\mp(E',t)}{E'(E'+E)}\right]=\int_m^\infty dE'\,P'\left[\frac{2\operatorname{Im}F(E',t)}{E'(E'^2-E^2)}\right]$$

$$=\int_m^{E-\varepsilon} dE'\,P'\left[\frac{2\operatorname{Im}F(E',t)}{E'(E'^2-E^2)}\right]+\int_{E-\varepsilon}^{E+\varepsilon} dE'\,P'\left[\frac{2\operatorname{Im}F(E',t)}{E'(E'^2-E^2)}\right]+\int_{E+\varepsilon}^{\infty} dE'\,P'\left[\frac{2\operatorname{Im}F(E',t)}{E'(E'^2-E^2)}\right].$$



$$\int_m^\infty dE' P'[\frac{\operatorname{Im} F_\pm (E',t)}{E'(E'-E)} - \frac{\operatorname{Im} F_\mp (E',t)}{E'(E'+E)}];$$

The term for which a principal part is needed is the first term of the integral. Split it into 3 part

$$I_1 = \int_m^{E-\varepsilon} P' \frac{\operatorname{Im} F(E',t)}{E'(E'-E)} dE';\ I_2 = \int_{E+\varepsilon}^{2E-m} P' \frac{\operatorname{Im} F(E',t)}{E'(E'-E)} dE';\ I_3 = \int_{2E-m}^{\infty} P' \frac{\operatorname{Im} F(E',t)}{E'(E'-E)} dE';$$

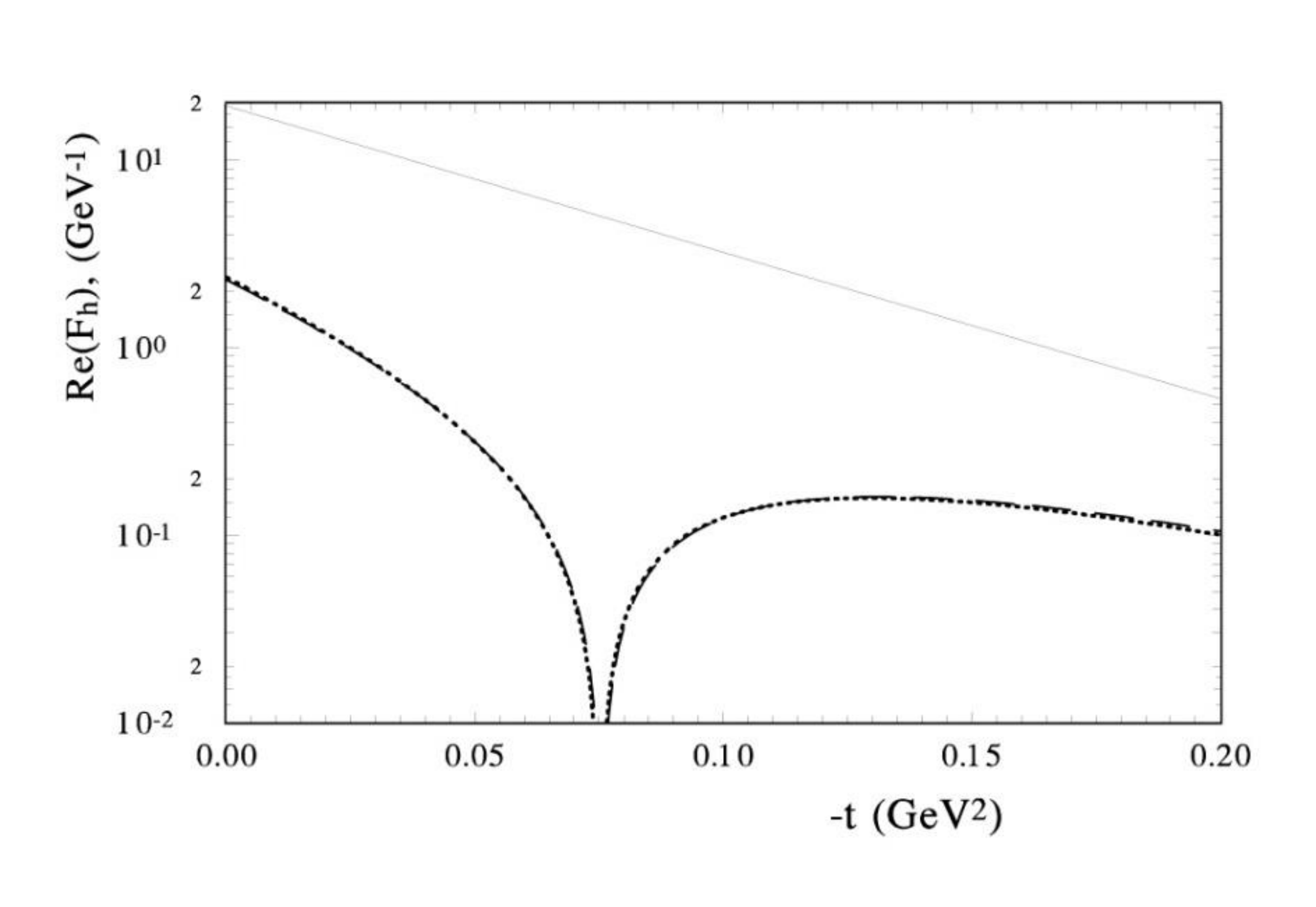
Let us perform a change of variable on I_2, so $E'' = 2E - E';\ E''_{\min} = m;\ E''_{\max} = E - \varepsilon;$ the bound are as in the first

$$I_2 = \int_{E+\varepsilon}^{2E-m} \frac{\sqrt{E'^2 - m^2} \operatorname{Im} F(E',t)}{E'(E'-E)} dE' = \int_m^{E-\varepsilon} \frac{\sqrt{(2E-E'')^2 - m^2} \operatorname{Im} F(E',t)}{(2E-E'')(E-E'')} dE';$$

We can sum I_1 and I_2, and call the integration variable x

$$I_2 + I_2 = \int_m^{E-\varepsilon} [\frac{\sqrt{(2E-x)^2 - m^2} \operatorname{Im} F(x,t)}{(2E-x)} - \frac{\sqrt{x^2 - m^2} \operatorname{Im} F(x,t)}{x}] \frac{dx}{E-x};$$

$$\operatorname{Im} F(s,t) = (C + a\ln(s)^2)\, e^{-Bt\ln(s)};$$



Non-exponential behavior (origins)

1. Non-linear Regge trajectory

   a) Contributions of the meson cloud (J.Pumplin, G.L. Kane; O.V.S.)

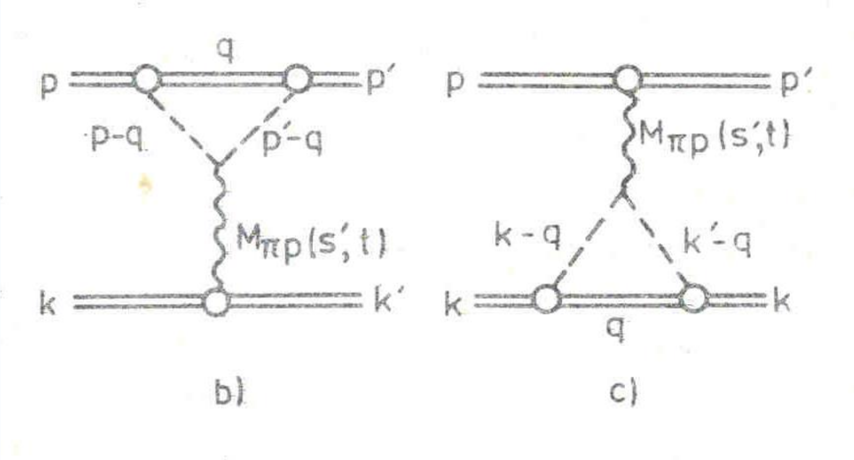
   b) Pion loops (Jenkovski ,Anselm, Gribov, Khoze, Martin et al.)

2. Different slopes of the other contributions
   (real part, odderon, spin-flip amplitude)

3. Unitarization

Pi-meson cloud

J.Pumplin, G.L. Kane, 1975, Phys.Rev. D11; pi-mezon scattering
O.V.S., S. Goloskokov, 1980, Yad.Fiz. 31, pp-scattering



$$i\chi(b) = -h\, e^{-\mu(s)\sqrt{b^2-b_0^2}}; \quad V(r) = \frac{2is\mu h}{\pi} K_0(\mu\sqrt{b_0^2 - r^2});$$

$$T(s,t) = -is\sum_{n=1}^{\infty} \frac{(-h)^n}{(n-1)!} \frac{\mu}{(n^2\mu^2 - t))^{3/2}} (1 - b\sqrt{n^2\mu^2 - t})\, e^{-b\sqrt{n^2\mu^2 - t}});$$

The additional term in the slope like $h(t)=\sqrt{t_0-t};$

was obtained in old time which can be produce a set of canceling Regge cuts

H. Cheng, T.Wu (1970) ; J.L. Cardy (1971)

Fnd leads to the Schwarz type trajectories (J.M. Schwarz (1968)

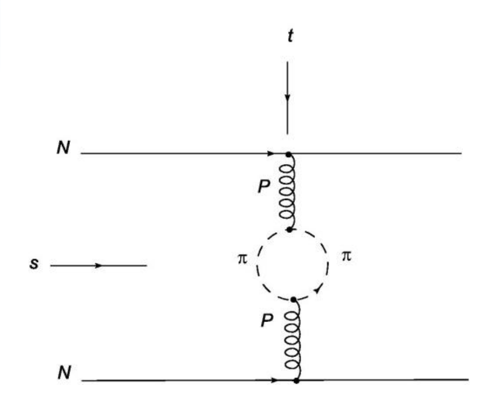
$$\alpha(t)_{\pm}=1\pm\gamma t^{1/2}+2\rho(1/2\gamma^{2}t)^{3/2}(-\ln(t))^{1/2};$$

The appearing complex trajectory heavely complicated picture.
Based on works O. Barrut, D.E. Zwanziger (1962, 1963)
by G. Cohen-Tannoudji, V.V. Ilyin, L.L. Jenkovszky (1972)
was proposed the simplest form

$$\alpha(t)=1.041-0.15\sqrt{t_0-t};$$

The analytic calculation of mezon loop

S. Barshay, Y.-A. Chao (1972)



$$D_R^{NN}(t)=n[A-(1+\frac{1}{a_1})^{3/2}\ Ln\,(\frac{\sqrt{1+a_1}+1}{\sqrt{1+a_1}-1})+Ln\,(\frac{m^2}{\mu^2})]^{-1};$$

$$D_R^{NN}(t)\sim[\frac{1}{5\mu^{-2}-t}+C].$$

A.A. Anselm, V.N. Gribov, Phys.Lett. 40B, (1972).

$$\alpha_P(q^2)=1-C_P q^2-(\sigma_{\pi\pi}/32\pi^2)h_1(q^2);$$

$$h_1(q^2)=\frac{q^2}{\pi}[\frac{8\mu^2}{q^2}-(1+\frac{4\mu^2}{q^2})^{3/2}\ Ln\,(\frac{\sqrt{1+(q^2/4\mu^2)}+1}{\sqrt{1+(q^2/4\mu^2)}-1})+Ln\,(\frac{m^2}{\mu^2})];$$

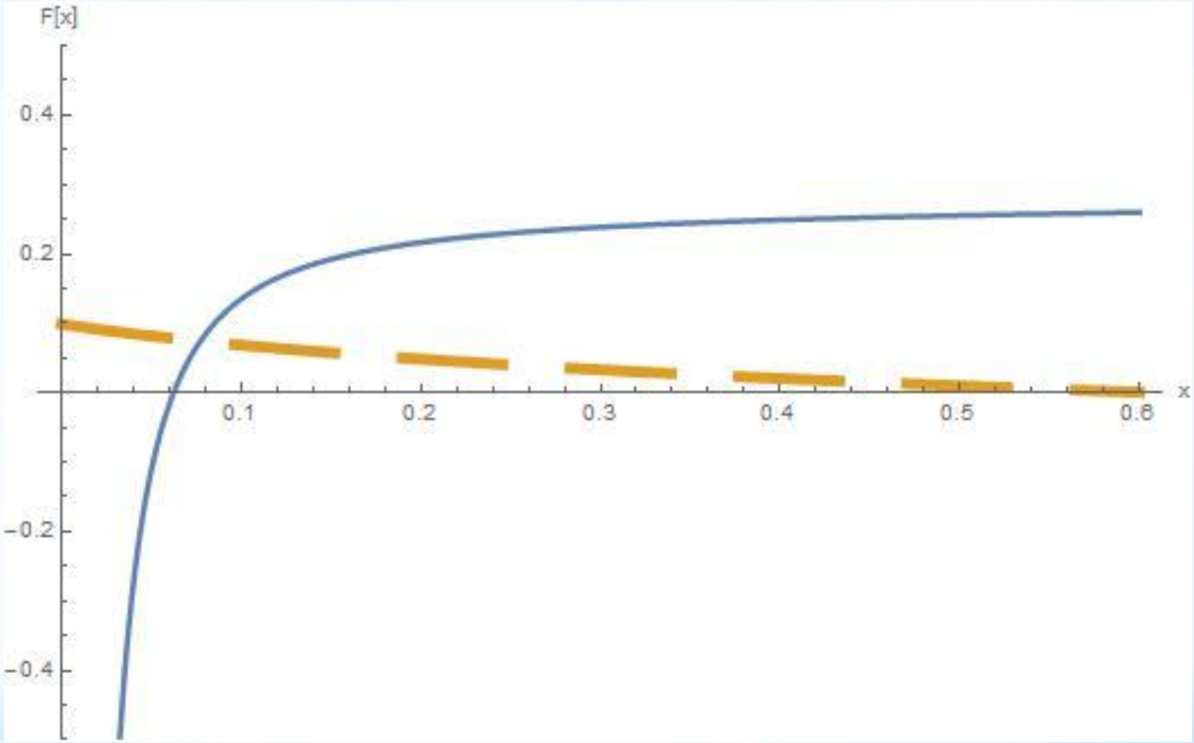
$$q^2<<4\mu^2;$$

$$\frac{q^2}{\pi}[Ln\,(\frac{m^2}{\mu^2})-\frac{8}{3}];$$

V.A. Khoze, A.D. Martin and M.G. Ryskin, J.Phys.G (2014)

$$h_1(q^2)=\frac{q^2}{\pi}[\frac{8\mu^2}{q^2}-(1+\frac{4\mu^2}{q^2})^{3/2}\ Ln\,(\frac{\sqrt{1+(4\mu^2/q^2)}+1}{\sqrt{1+(4\mu^2/q^2)}-1})+Ln\,(\frac{m^2}{\mu^2})];$$

* 0.139^2
* (2.* tau − (1. +tau)^(1.5) * Log[(Sqrt[(1. + 1./tau)] + 1.)/(Sqrt[(1. + 1./tau)] − 1.)]
+ Log[1./0.139^2]);



4.* 0.139^2
* (2.* tau − (1. +tau)^(1.5) * Log[(Sqrt[(1. +tau)] + 1.)/(Sqrt[(1. +tau)] − 1.)]
+ Log[1./0.139^2]);

LHC

The elastic scattering reflects the generalized structure of the hadron

And the scattering amplitude is satisfied the basic analitical properties.

* The new data bounded essentially the limits of the models.

GPDs open the new way to connections of the elastic and inelastic intaractions

5. The standard eikonal approximation works perfectly
   from Sqrt(s}=9 GeV up to 8 TeV.

But: where is the hard Pomeron?; t and s dependence of the Odderon?
Unitarization – eikonal?

* The problems of the determination of $\rho(s,t)$ and $\sigma_{tot}(s)$

- The thin structure of the slope - B(s,t),
- (non-exponential, oscillations)

Asymptotic of the scattering amplitude - wide region

Wait the high precision of the new data at small t and 13 TeV

TOTEM; ATLAS

Thanks for your attention