Renormalization of gauge theories in the background field approach

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"LET'S SEE IF WE COULD PUT A SPIN ON IT AND GET THE PUBLIC INTERESTED."

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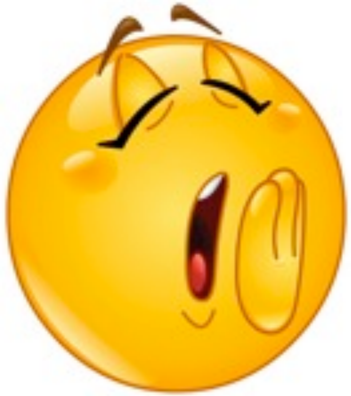

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Task: `Prove that gauge invariance is preserved by renormalization`

40+ year old topic. What news can there be ?

- simplified treatment for standard effective theories (YM with higher dimensional operators, gravity)
- inclusion of Abelian group factors
- extension to a very broad class (e.g. theories without Lorentz invariance)

Renormalization at a glance

generating functional

$$W[J] = -\hbar \log \int d\Phi \exp\left[-\frac{1}{\hbar}(S[\Phi] + J\Phi) \right]$$

mean fields, effective action

$$\langle\Phi\rangle = \frac{\delta W}{\delta J} \; , \qquad \Gamma[\langle\Phi\rangle] = W - J\langle\Phi\rangle$$

subtraction

$$S_{\rm ren} = S_0[\Phi] - \hbar\,\Gamma^{1-{\rm loop}}_{\rm div}[\Phi] - \hbar^2\Gamma^{2-{\rm loop}}_{\rm div}[\Phi] - \ldots$$

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Linearly realized global symmetries of S → symmetries of Γ

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Linearly realized global symmetries of S ➡ symmetries of Γ

But gauge symmetries must be broken by the gauge fixing

Remaining BRST invariance is non-linear:

$\delta\varphi = \varphi\,\omega$, but $\delta\langle\varphi\rangle \neq \langle\varphi\rangle\langle\omega\rangle$ ➡ not a symmetry of Γ

Local BRST cohomology

gauge-fixed action

$$\Sigma_{\rm ren}^{L-{\rm loop}} = S^{L-{\rm loop}}[\varphi] + \boldsymbol{\sigma}\Psi^{L-{\rm loop}}[\varphi, \omega, \ldots]$$

gauge-invariant

BRST operator

gauge fermion

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  $\varphi \mapsto \varphi'(\varphi, \omega, \ldots)\ , \quad \omega \mapsto \omega'(\varphi, \omega, \ldots)$

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for relativistic YM and GR: *Barnich, Brandt, Henneaux (1994)*

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Background gauge

quantum fields (integration variables)	background fields (external)
φ^a	ϕ^a

Gauge-fixing term invariant under simultaneous gauge transformations of φ and ϕ

Very efficient at 1 loop:

$$\Gamma^{1-\text{loop}}\Big|_{\langle\varphi\rangle=\phi,\ \langle\omega\rangle=0} = \Gamma^{1-\text{loop}}[\phi]$$

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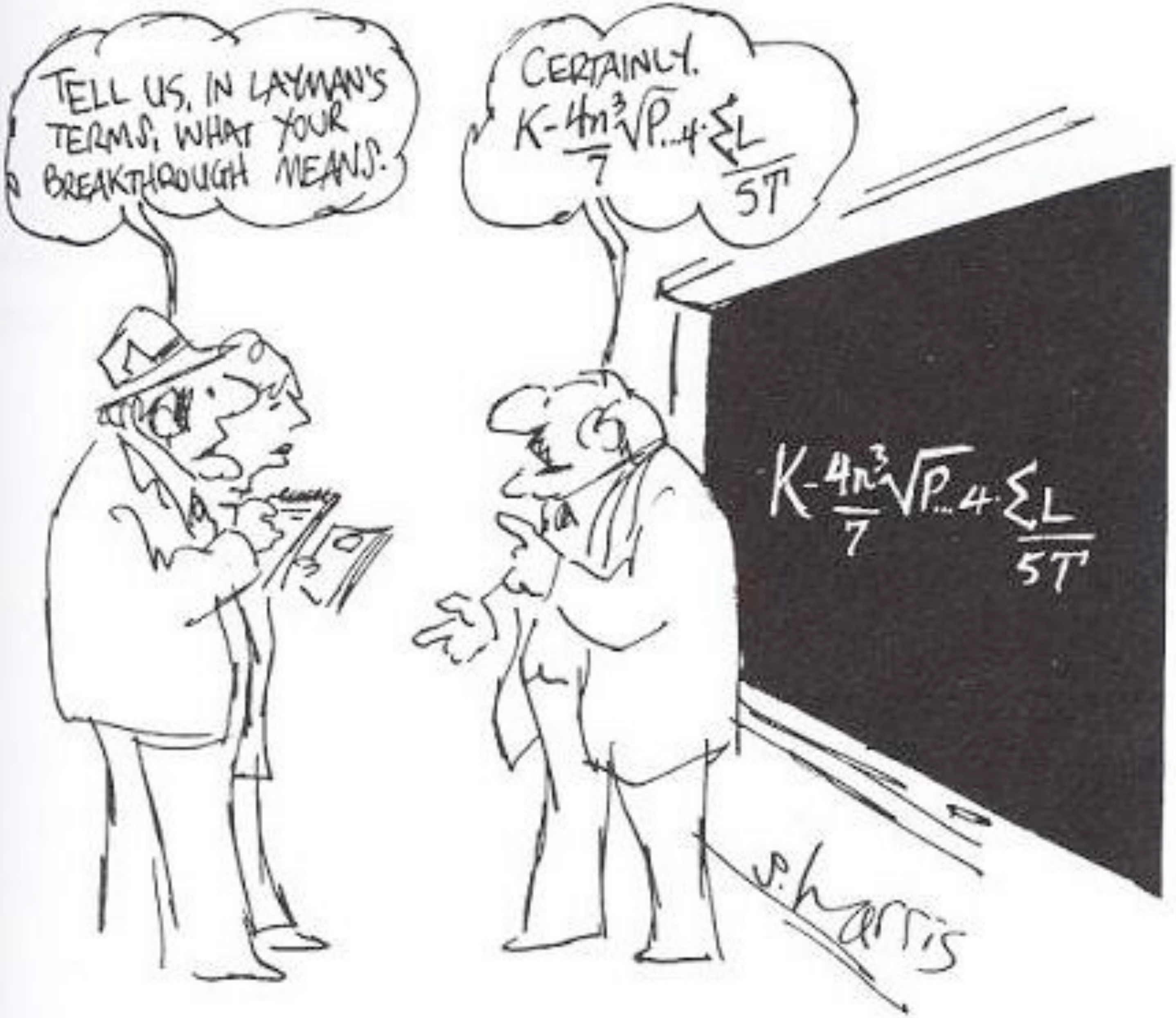
At 2 loop and higher need $\langle\varphi\rangle \neq \phi$ to subtract subdivergences = need to classify the BRST cohomology.

Otherwise non-local divergences at intermediate steps

Kallosh (1974); Arefeva, Faddeev, Slavnov (1974); Abbott (1981); Ichinose, Omote (1982); Barvinsky, Vilkovisky (1987)

Still, presence of background gauge transformations greatly simplifies the task !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S. Harris

Assumptions

i) linear gauge generators: $\delta_\varepsilon \varphi^a = \underbrace{(R^a{}_{b\alpha}\varphi^b + P^a{}_\alpha)}_{R^a{}_\alpha(\varphi)}\varepsilon^\alpha$

ii) off-shell closure: $[\delta_\varepsilon, \delta_\eta]\varphi^a = \delta_\varsigma \varphi^a\,, \quad \varsigma^\alpha = C^\alpha{}_{\beta\gamma}\varepsilon^\beta\eta^\gamma$

iii) local completeness:

$$\frac{\delta S}{\delta \varphi^a} X^a{}_\alpha(\varphi) = 0 \quad \Rightarrow \quad X^a{}_\alpha(\varphi) = R^a{}_\beta Y^\beta_\alpha + \frac{\delta S}{\delta \varphi^b} I^{[ba]}_\alpha$$

iv) irreducibility:

$$R^a{}_\alpha(\varphi_0)\varepsilon^\alpha = 0 \quad \Rightarrow \quad \varepsilon^\alpha = 0$$

v) absence of anomalies

vi) locality of divergences after removal of subdivergences

Examples:

YM: $\delta_\varepsilon A^i_\mu = f^{ijk} A^j_\mu \varepsilon^k + \partial_\mu \varepsilon^i$

GR: $\delta_\varepsilon g_{\mu\nu} = \varepsilon^\lambda \partial_\lambda g_{\mu\nu} + g_{\mu\lambda} \partial_\nu \varepsilon^\lambda + g_{\nu\lambda} \partial_\mu \varepsilon^\lambda$

Also higher-derivative gravity, also non-relativistic (Lifshitz) theories

Counterexample:

Supergravity (the algebra does not close off-shell)

Background gauge-fixing

- choose g.f. function $\chi^\alpha(\varphi,\phi) = \chi^\alpha_a(\phi)(\varphi-\phi)^a$ to be invariant under BGT: $\delta_\varepsilon \varphi^a = R^a{}_\alpha(\varphi)\varepsilon^\alpha$ $\delta_\varepsilon \phi^a = R^a{}_\alpha(\phi)\varepsilon^\alpha$
- promote $\boldsymbol{\sigma} \mapsto \boldsymbol{Q} = \boldsymbol{\sigma} + \Omega^a \frac{\delta}{\delta\phi^a}$

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➡ g.f. term at tree level: $\boldsymbol{Q}\Psi_0$

auxiliary (anti-)fields coupled to $\boldsymbol{\sigma}\varphi^a$, $\boldsymbol{\sigma}\omega^\alpha$

antighost

Lagrange multiplier

$$\Psi_0=-\Big(\underbrace{\gamma_a-\bar\omega_\alpha\chi^\alpha_a(\phi)}_{\hat\gamma_a}\Big)(\varphi-\phi)^a+\zeta_\alpha\omega^\alpha-\frac{1}{2}\bar\omega_\alpha O^{\alpha\beta}(\phi)b_\beta$$

The proposition: BRST structure of renormalized generating functional

$$W_{\rm ren} = -\hbar \log \int d\Phi \exp\left[-\frac{1}{\hbar}\left(\Sigma_{\rm ren} + J_a(\varphi_{\rm ren} - \phi)^a + \bar{\xi}_\alpha \omega^\alpha_{\rm ren} + \xi^\alpha \bar{\omega}_\alpha + y^\alpha b_\alpha\right)\right]$$

$$\Sigma_{\rm ren} = S_{\rm ren}[\varphi] + \boldsymbol{Q}\Psi_{\rm ren}[\varphi, \omega, \bar{\omega}, b, \phi, \gamma, \zeta, \varOmega]$$

gauge-invariant

local

$$\varphi^a_{\rm ren} - \phi^a = -\frac{\delta \Psi_{\rm ren}}{\delta \gamma_a} \qquad \omega^\alpha_{\rm ren} = \frac{\delta \Psi_{\rm ren}}{\delta \zeta_\alpha}$$

$$\Psi_{\rm ren} = \hat{\Psi}_{\rm ren}[\varphi, \omega, \phi, \hat{\gamma}, \zeta, \varOmega] - \frac{1}{2}\bar{\omega} O(\phi) b$$

$$= \underbrace{\Psi_0 + \mathcal{O}(\hbar)}$$

guarantees that the coupling to the source is linear at the leading order

Often $\hat{\Psi}_{\rm ren}$ remains linear in the auxiliary fields:

$$\hat{\Psi}_{\rm ren} = -\hat{\gamma}_a U^a(\varphi, \phi) + \zeta_\alpha \omega^\beta V^\alpha_\beta(\varphi, \phi)$$

➡

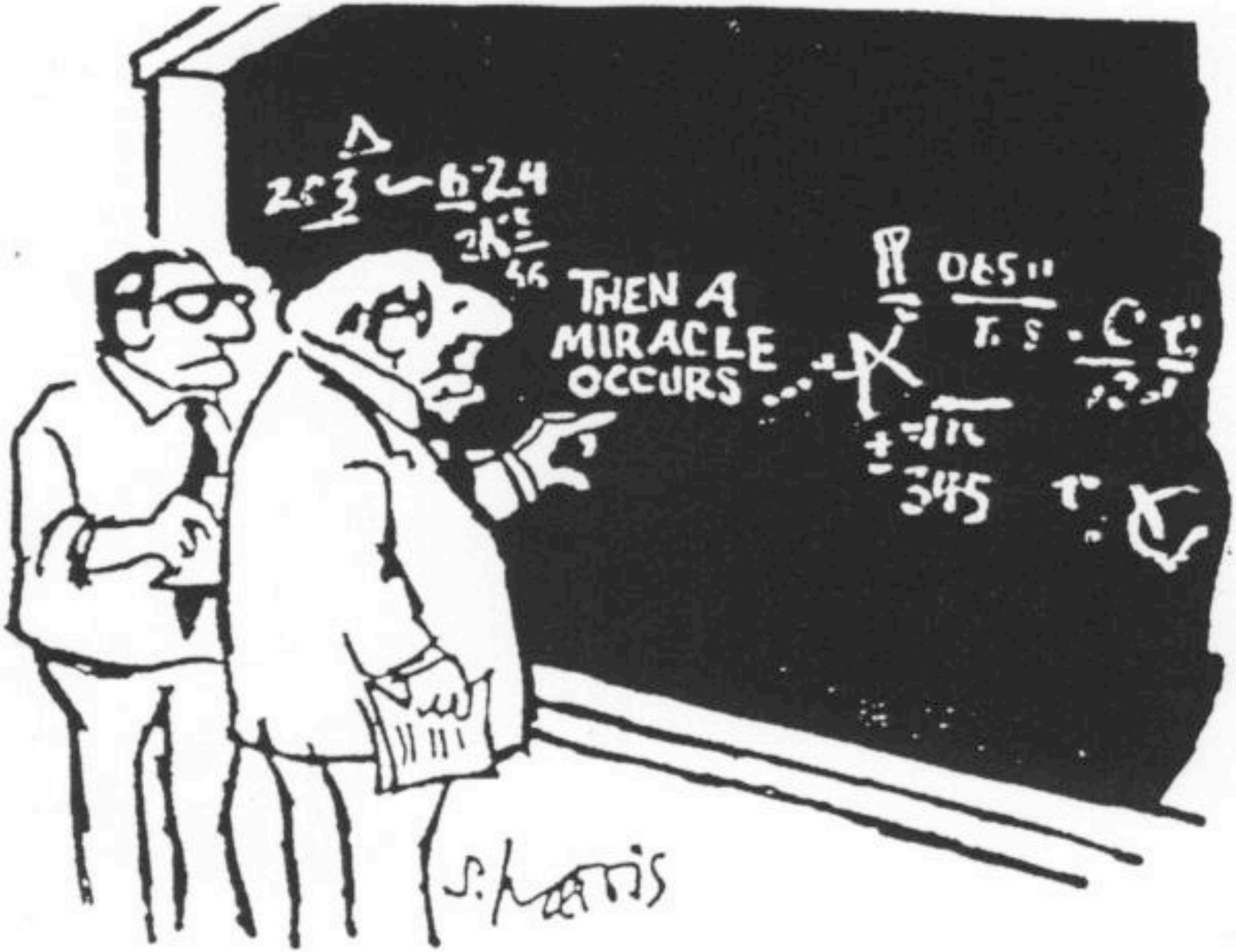
$$\varphi^a_{\rm ren} - \phi^a = U^a(\varphi, \phi)$$

$$\omega^\alpha_{\rm ren} = V^\alpha_\beta(\varphi, \phi)\omega^\beta$$

Typical for renormalizable theories

NB. U , V can still be non-linear if φ, ϕ have zero scaling dimension.

Example: 4d gravity with the Lagrangian $R_{\mu\nu}R^{\mu\nu} + R^2$



I think you should be a little
more specific, here in Step 2

Decoupling of background fields

ST identities: $0 = \boldsymbol{Q}_+ \Gamma_{\rm div} \equiv \left(\boldsymbol{\sigma}_+ + \mathit{\Omega} \frac{\delta}{\delta\phi} \right) \Gamma_{\rm div}$

nilpotent and anticommute

$$\left. \begin{array}{l} \mathit{\Omega} \dfrac{\delta X}{\delta \phi} = 0 \\[2ex] X \big|_{\mathit{\Omega}=0} = 0 \end{array} \right\} \Rightarrow \quad X = \mathit{\Omega} \frac{\delta Y}{\delta \phi}$$

Y is local and inherits all linear symmetries of X.

Is invariant under BGT

$$\Gamma_{\mathrm{div}} = \sum \mathit{\Omega}^k \, \Gamma_k$$

Truncate in number of derivatives + locality + fermionic statistics of $\mathit{\Omega}$ ➡ $0 \leq k \leq K$

$$\mathit{\Omega} \frac{\delta}{\delta\phi} \Gamma_K = 0$$ ➡ $$\Gamma_K = \mathit{\Omega} \frac{\delta}{\delta\phi} \Upsilon_{K-1}$$

$$\mathit{\Omega} \frac{\delta}{\delta\phi} \Gamma_{K-1} + \boldsymbol{\sigma}_+ \Gamma_K = 0$$

➡ $$\mathit{\Omega} \frac{\delta}{\delta\phi} (\Gamma_{K-1} - \boldsymbol{\sigma}_+ \Upsilon_{K-1}) = 0$$

➡ $$\Gamma_{K-1} = \boldsymbol{\sigma}_+ \Upsilon_{K-1} + \mathit{\Omega} \frac{\delta}{\delta\phi} \Upsilon_{K-2}$$

……………………………………

$$\Gamma_0 = \boldsymbol{\sigma}_+ \Upsilon_0 + \mathbb{I}\Gamma[\varphi, \omega, \hat{\gamma}, \zeta]$$

$$\Gamma_{\rm div} = \mathbb{\Gamma}[\varphi, \omega, \hat{\gamma}, \zeta] + \boldsymbol{Q}_+ \Upsilon[\varphi, \omega, \phi, \hat{\gamma}, \zeta, \mathit{\Omega}]$$

$$\mathbb{S}[\varphi] + \Lambda$$

vanishes if $\omega = 0$ or $\hat{\gamma} = \zeta = 0$

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$\mathbb{\Gamma}$ is merely gauge invariant !

NB. Applies to any gauge symmetry satisfying (i)-(iv), including Abelian subgroups

Structure of Λ

ST + Ward identities of BGT

$$\underbrace{\frac{\delta S_0}{\delta \varphi^a}\frac{\delta \Lambda}{\delta \hat{\gamma}_a} - \hat{\gamma}_a R^a{}_\alpha(\varphi)\frac{\delta \Lambda}{\delta \zeta_\alpha}}_{\text{Koszul - Tate differential} \quad \boldsymbol{q}_0} - \underbrace{\frac{1}{2}C^\gamma{}_{\alpha\beta}\omega^\alpha\omega^\beta\frac{\delta \Lambda}{\delta \omega^\gamma}}_{\boldsymbol{q}_1} = 0$$

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Batalin, Vilkovisky (1985)
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ladder equation in powers of $\Rightarrow$ $\Lambda = (\boldsymbol{q}_0 + \boldsymbol{q}_1)\,\Xi$

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+ WI → $\boxed{\Lambda = \boldsymbol{Q}_+\Xi}$

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$$\Gamma_{\rm div} = \mathbb{S}[\varphi] + \boldsymbol{Q}_+ \boldsymbol{\Upsilon}[\varphi, \omega, \phi, \hat{\gamma}, \zeta, \mathit{\Omega}]$$

renormalizes physical couplings in the gauge invariant action

renormalizes the gauge fermion and generates the field redefinition

"ON THE OTHER HAND, MY RESPONSIBILITY
TO SOCIETY MAKES ME WANT TO STOP
RIGHT HERE."

Conclusions and Outlook

- Background field method is not only a convenient calculational tool, but is also efficient for general analysis of the structure of renormalization

cf. Grassi (1996), Anselmi (2014)

- BRST structure (gauge invariance) is preserved by renormalization for non-anomalous theories whose gauge algebra:

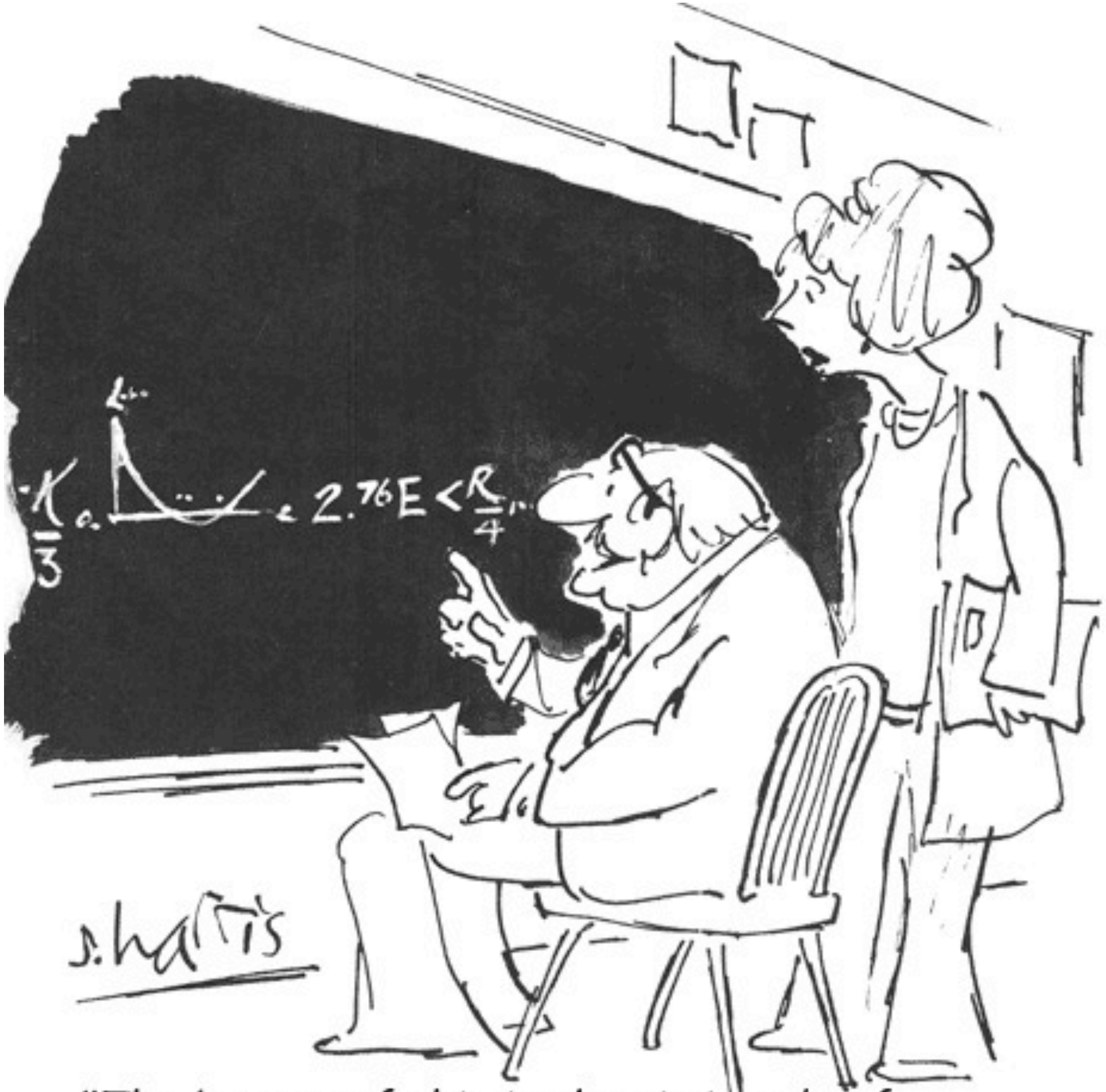
i) has linear generators

ii) closes off-shell *can be relaxed (?)*

iii) is locally complete

iv) is irreducible *can be relaxed*

- Generalizations: open algebras, supersymmetry, composite operators, anomalies



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."