Towards an accurate and efficient description of cosmic Large-Scale Structure

Sergey Sibiryakov

Ginzburg Conference, 2017

In the past century, and even nowadays, one could encounter the opinion that in physics nearly everything had been done. (...) I consider these views as some kind of blindness. The entire history of physics, as well as the state of present-day physics and, in particular, astrophysics, testifies to the opposite. In my view we are facing a boundless sea of unresolved problems.

V. L. Ginzburg, Nobel lecture

In the past century, and even nowadays, one could encounter the opinion that in physics nearly everything had been done. (...) I consider these views as some kind of blindness. The entire history of physics, as well as the state of present-day physics and, in particular, astrophysics, *testifies to the opposite. In my view we are facing a boundless sea of unresolved problems.*

V. L. Ginzburg, Nobel lecture

Current cosmological paradigm

Timeline:

Composition:

Current cosmological paradigm

Timeline:

inflation

Composition:

Current cosmological paradigm

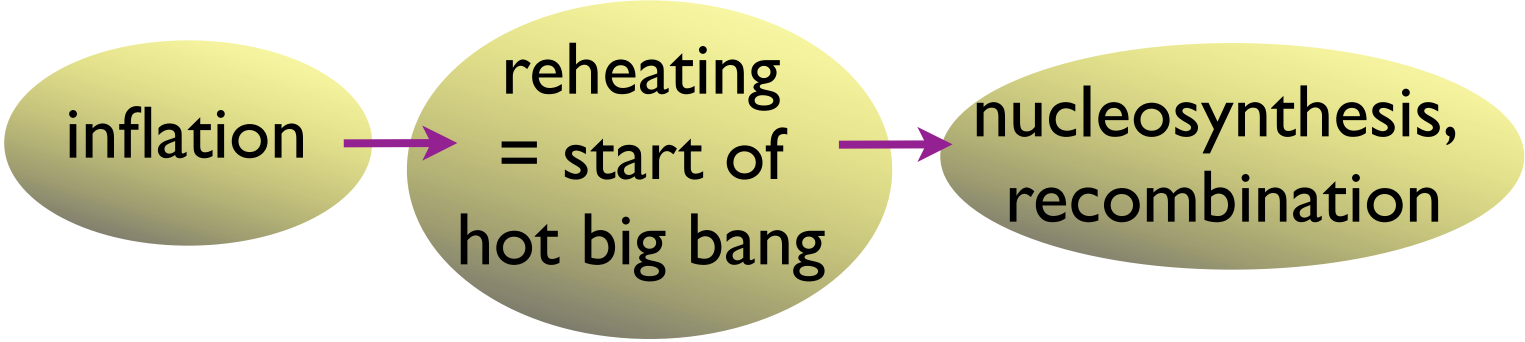
Timeline:

inflation → reheating
= start of
hot big bang

Composition:

Current cosmological paradigm

Timeline:



inflation → reheating = start of hot big bang → nucleosynthesis, recombination

Composition:

Current cosmological paradigm

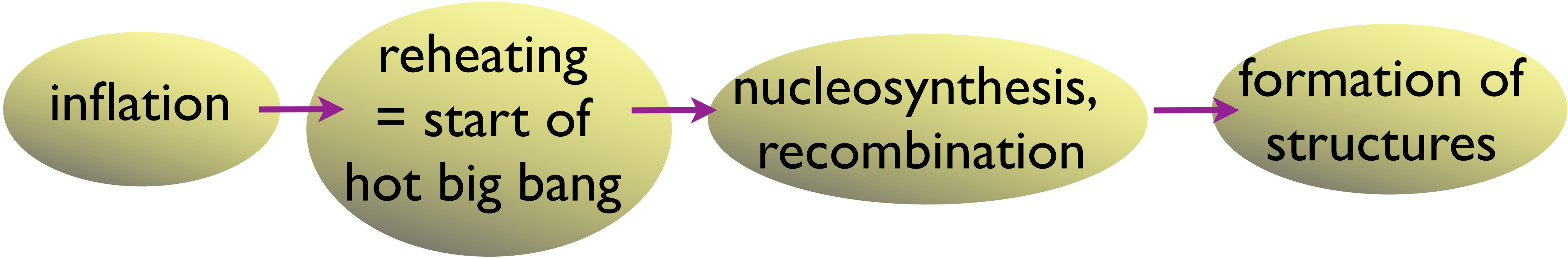
Timeline:

inflation → reheating = start of hot big bang → nucleosynthesis, recombination → formation of structures

Composition:

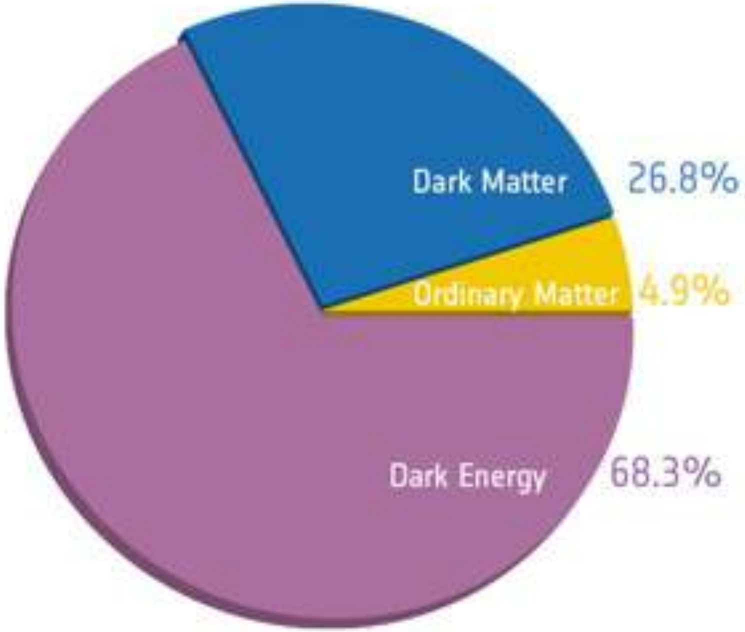
Current cosmological paradigm

Timeline:



inflation → reheating = start of hot big bang → nucleosynthesis, recombination → formation of structures

Composition:



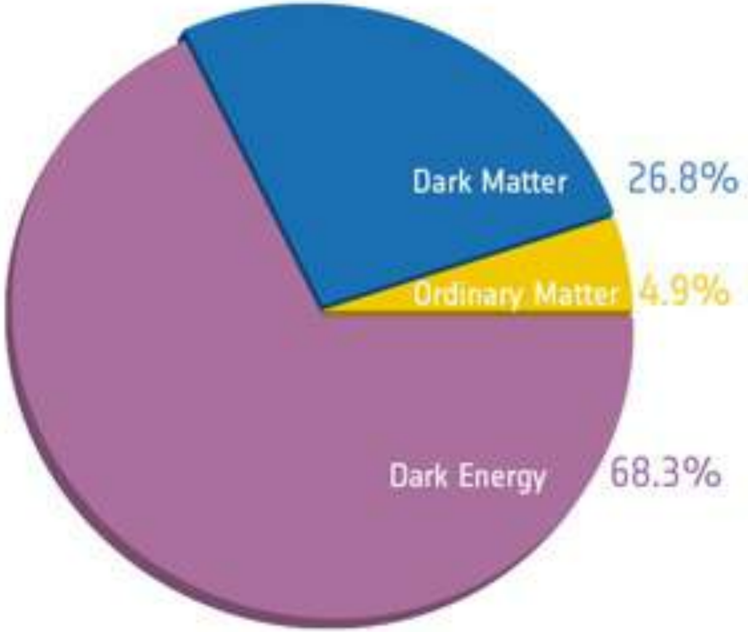
Current cosmological paradigm

Timeline:

inflation → reheating = start of hot big bang → nucleosynthesis, recombination → formation of structures

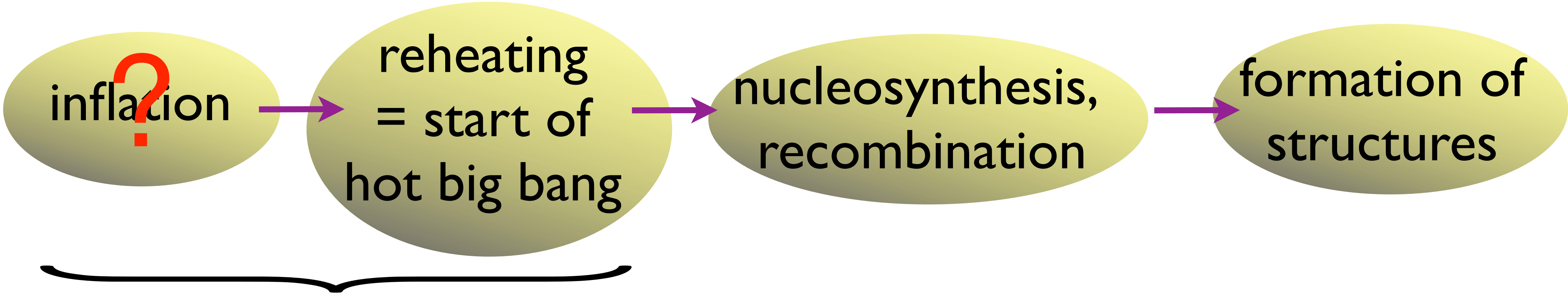
no direct information

Composition:



Current cosmological paradigm

Timeline:



inflation ?

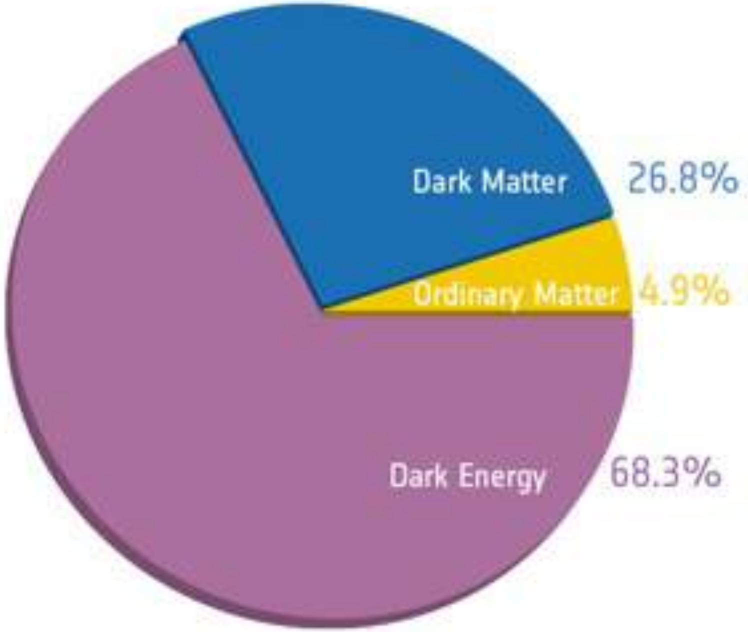
→ reheating = start of hot big bang

→ nucleosynthesis, recombination

→ formation of structures

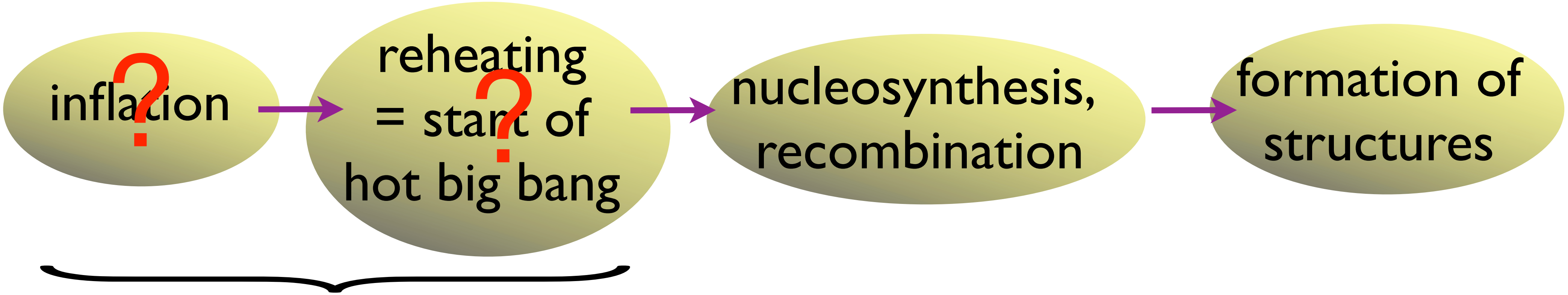
no direct information

Composition:



Current cosmological paradigm

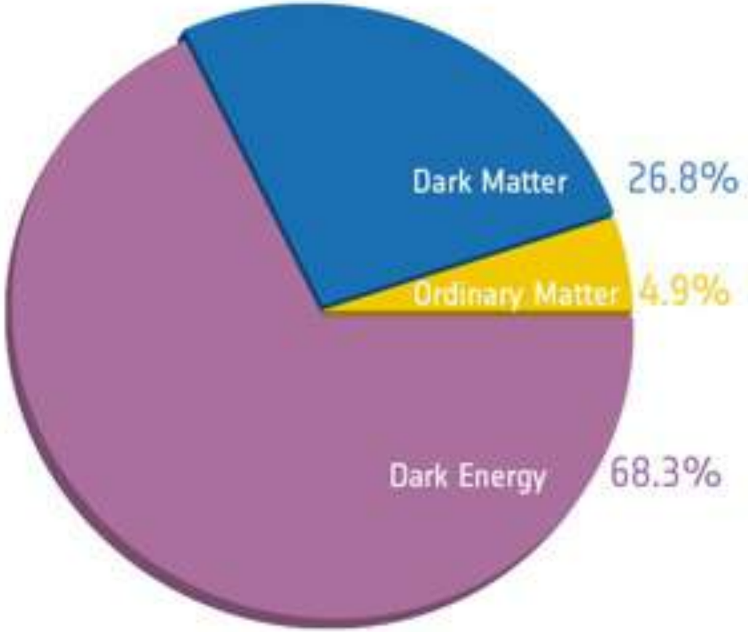
Timeline:



inflation ? → reheating = start of hot big bang ? → nucleosynthesis, recombination → formation of structures

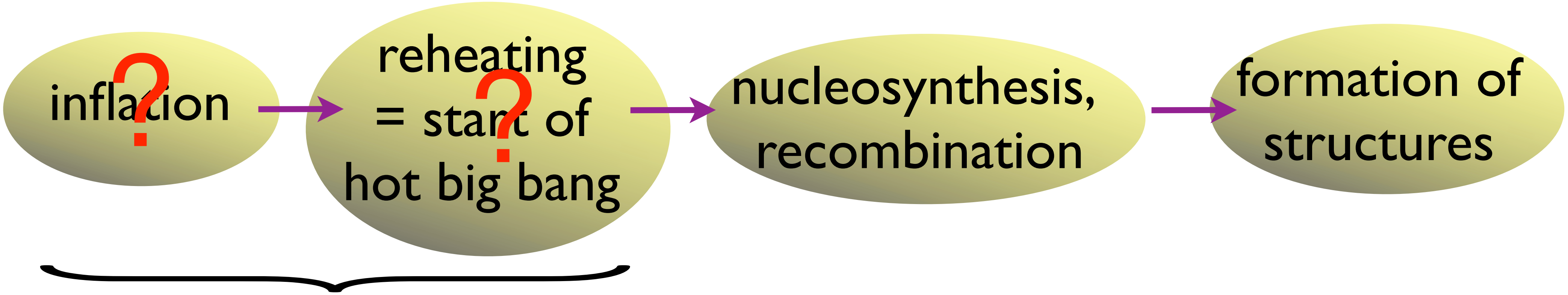
no direct information

Composition:



Current cosmological paradigm

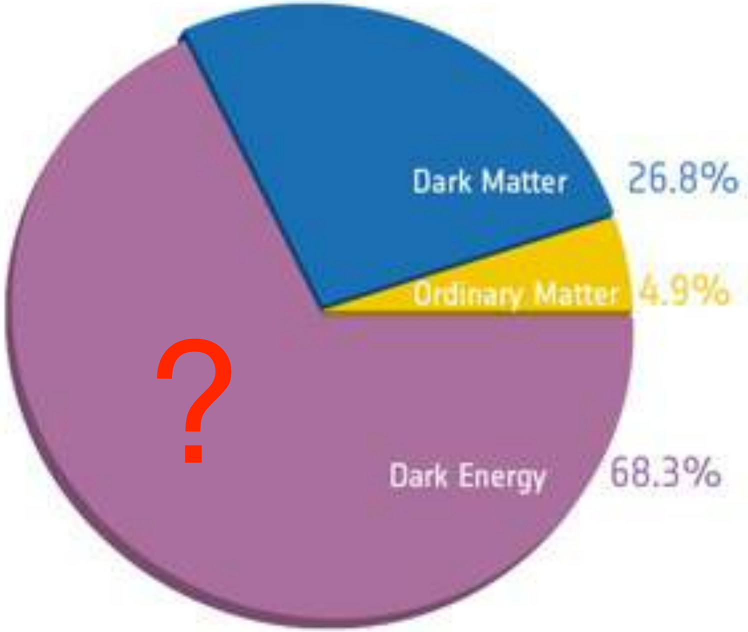
Timeline:



inflation ? → reheating = start of hot big bang ? → nucleosynthesis, recombination → formation of structures

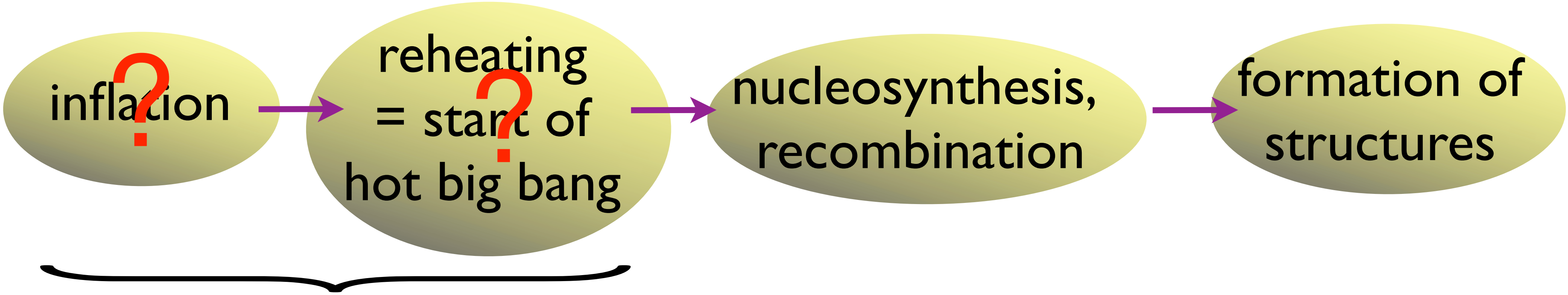
no direct information

Composition:



Current cosmological paradigm

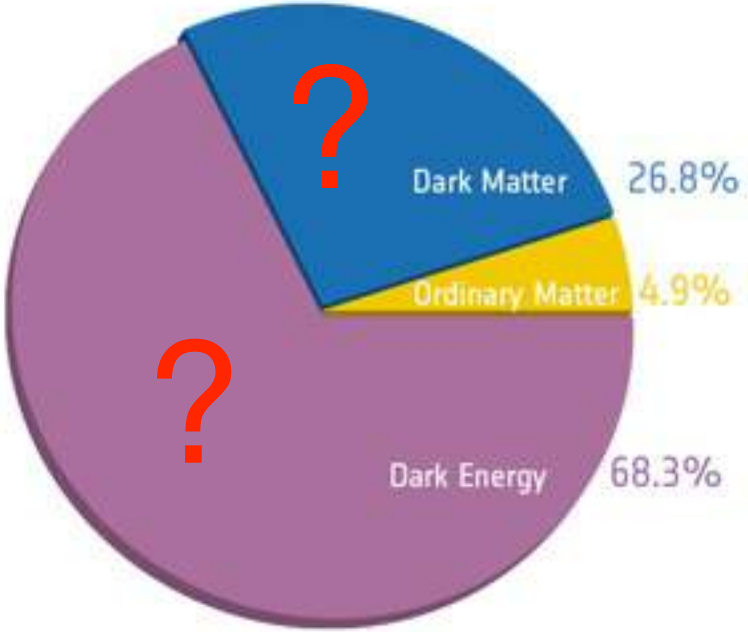
Timeline:



inflation ? → reheating = start of hot big bang ? → nucleosynthesis, recombination → formation of structures

no direct information

Composition:



Current cosmological paradigm

Timeline:

inflation ?

reheating = start of hot big bang ?

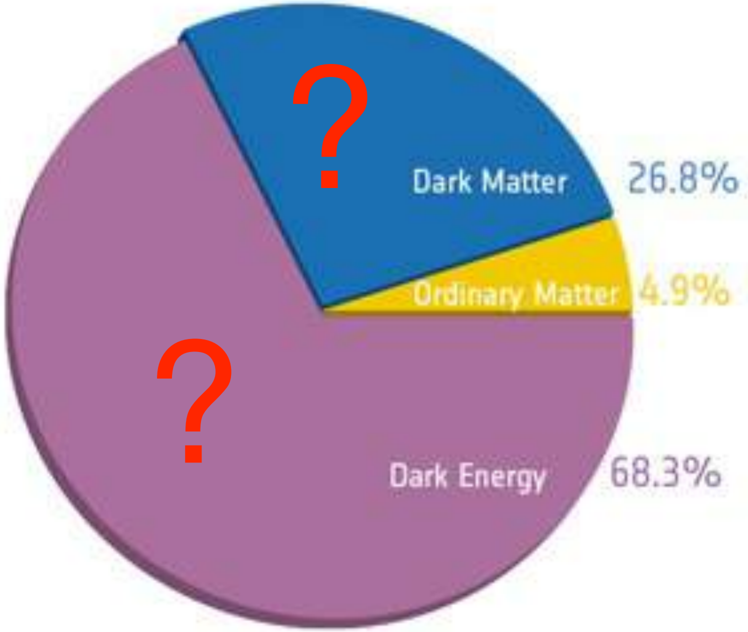
nucleosynthesis, recombination

formation of structures

no direct information

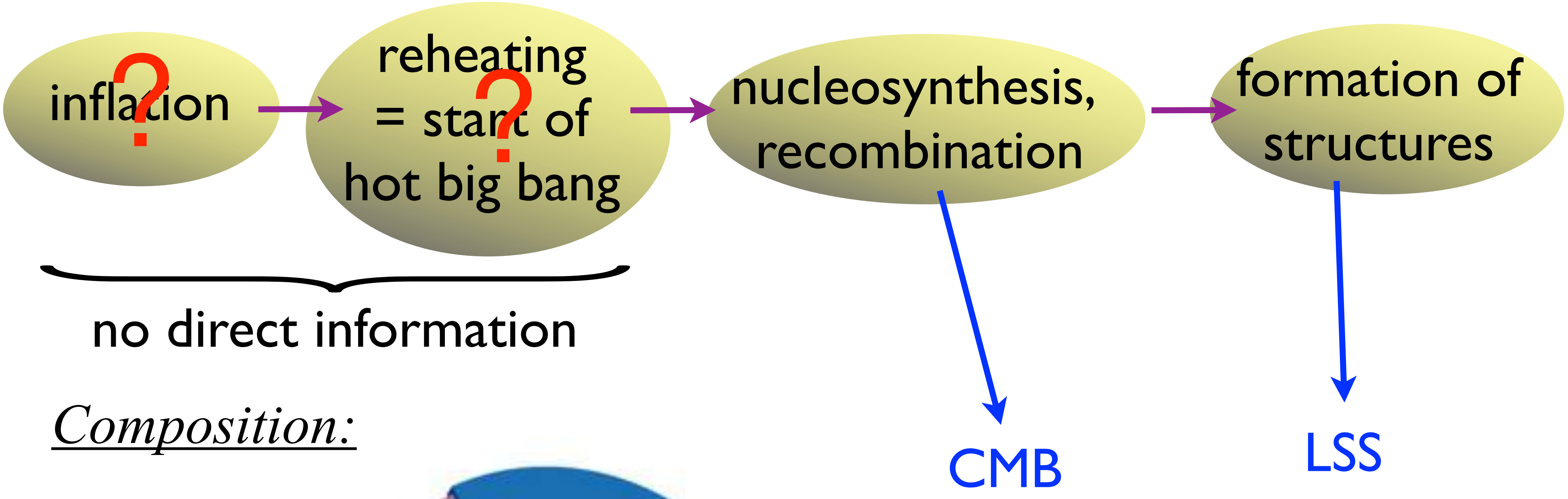
CMB

Composition:



Current cosmological paradigm

Timeline:



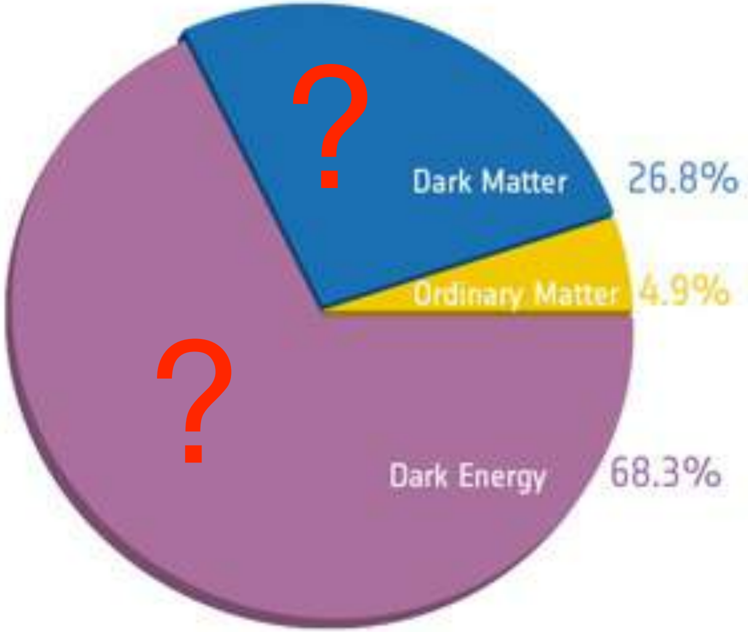
inflation ? → reheating ? = start of hot big bang → nucleosynthesis, recombination → formation of structures

no direct information

CMB

LSS

Composition:



Existing galaxy surveys:

SDSS

DARK ENERGY SURVEY

2MASS
2 MICRON ALL SKY SURVEY

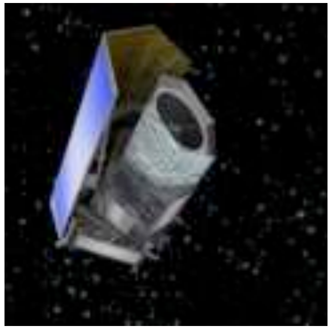
PAU

Future surveys:

LSST

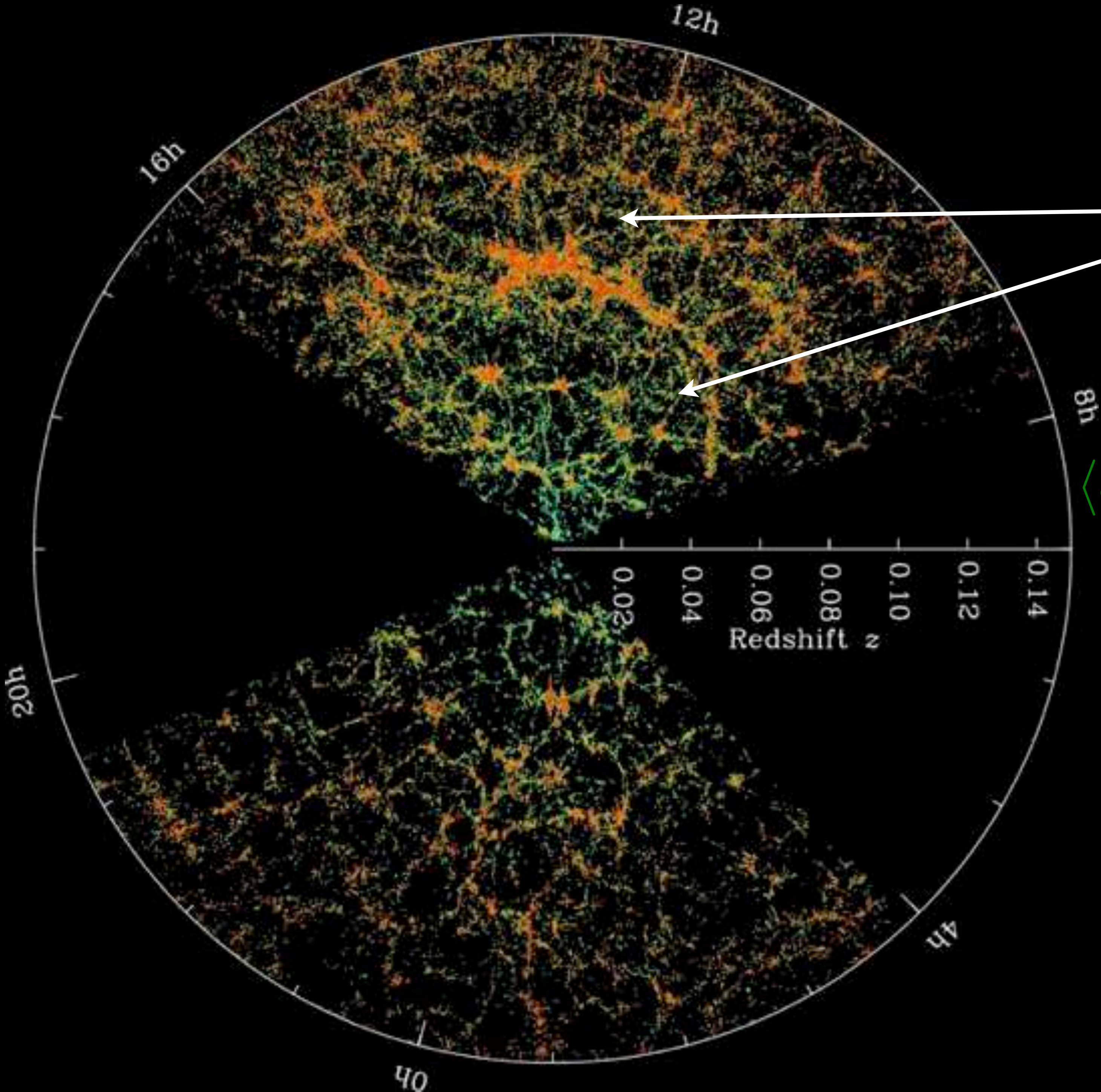
SKA
SQUARE KILOMETRE ARRAY

Euclid



DARK ENERGY SPECTROSCOPIC INSTRUMENT

The beautiful Universe of SDSS



$$\delta_\rho \equiv \frac{\delta\rho}{\rho}$$

$$\langle \delta_\rho(x_1)\delta_\rho(x_2)\rangle$$

$$\langle \delta_\rho(x_1)\delta_\rho(x_2)\delta_\rho(x_3)$$

$$\vdots$$

Physics with LSS

- primordial non-gaussianity
  - ➡ interactions in the inflationary sector
- baryon acoustic oscillations = standard ruler in the Universe
  - ➡ dark energy equation of state
- evolution of perturbations
  - ➡ neutrino mass

    properties of dark matter (e.g. fifth force, WDM)
    and dark energy (e.g. clustering)

Primordial non-gaussianity

gaussian random field: $\langle \delta_\rho(k_1)\delta_\rho(k_2)\rangle = P(k_1)\delta(k_1 + k_2)$

$$\langle \delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = 0$$

$$\delta_\rho \equiv \frac{\delta\rho}{\rho}$$

Primordial non-gaussianity

gaussian random field: $\langle \delta_\rho(k_1)\delta_\rho(k_2)\rangle = P(k_1)\delta(k_1+k_2)$

$$\langle \delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = 0$$

$$\delta_\rho \equiv \frac{\delta\rho}{\rho}$$

non-gaussianity:

$$\langle \delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = B(k_1,k_2)\delta(k_1+k_2+k_3) \neq 0$$

quantified by $f_{NL} \propto \frac{B}{P^2}$

Primordial non-gaussianity

gaussian random field: $\langle \delta_\rho(k_1)\delta_\rho(k_2)\rangle = P(k_1)\delta(k_1+k_2)$

$$\langle \delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = 0$$

$$\delta_\rho \equiv \frac{\delta\rho}{\rho}$$

non-gaussianity:

$$\langle \delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = B(k_1,k_2)\delta(k_1+k_2+k_3) \neq 0$$

quantified by $f_{NL} \propto \dfrac{B}{P^2}$

Planck 2015 constraints: $f_{NL} = \begin{cases} 0.8 \pm 5.0 & \text{local} \\ -4 \pm 43 & \text{equilateral} \end{cases}$

Primordial non-gaussianity

gaussian random field: $\langle\delta_\rho(k_1)\delta_\rho(k_2)\rangle = P(k_1)\delta(k_1+k_2)$

$$\langle\delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = 0$$

$$\delta_\rho \equiv \frac{\delta\rho}{\rho}$$

non-gaussianity:

$$\langle\delta_\rho(k_1)\delta_\rho(k_2)\delta_\rho(k_3)\rangle = B(k_1,k_2)\delta(k_1+k_2+k_3)\neq 0$$

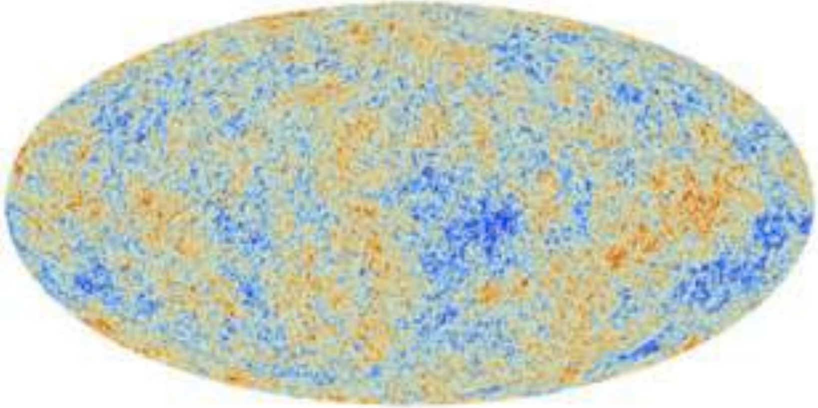
quantified by $f_{NL} \propto \dfrac{B}{P^2}$

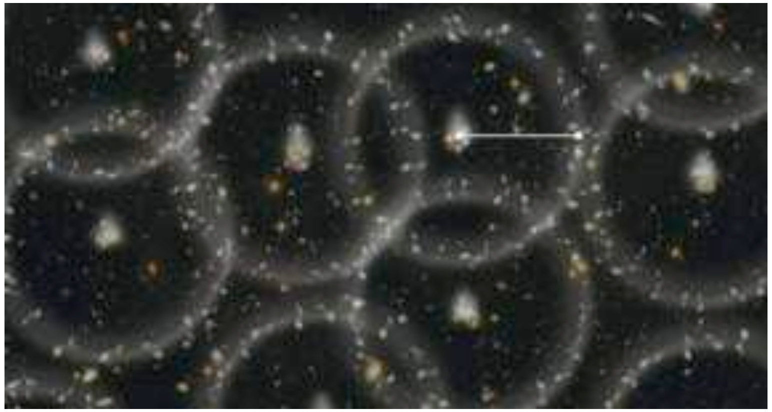
Planck 2015 constraints: $f_{NL} = \begin{cases} 0.8 \pm 5.0 & \text{local} \\ -4 \pm 43 & \text{equilateral} \end{cases}$

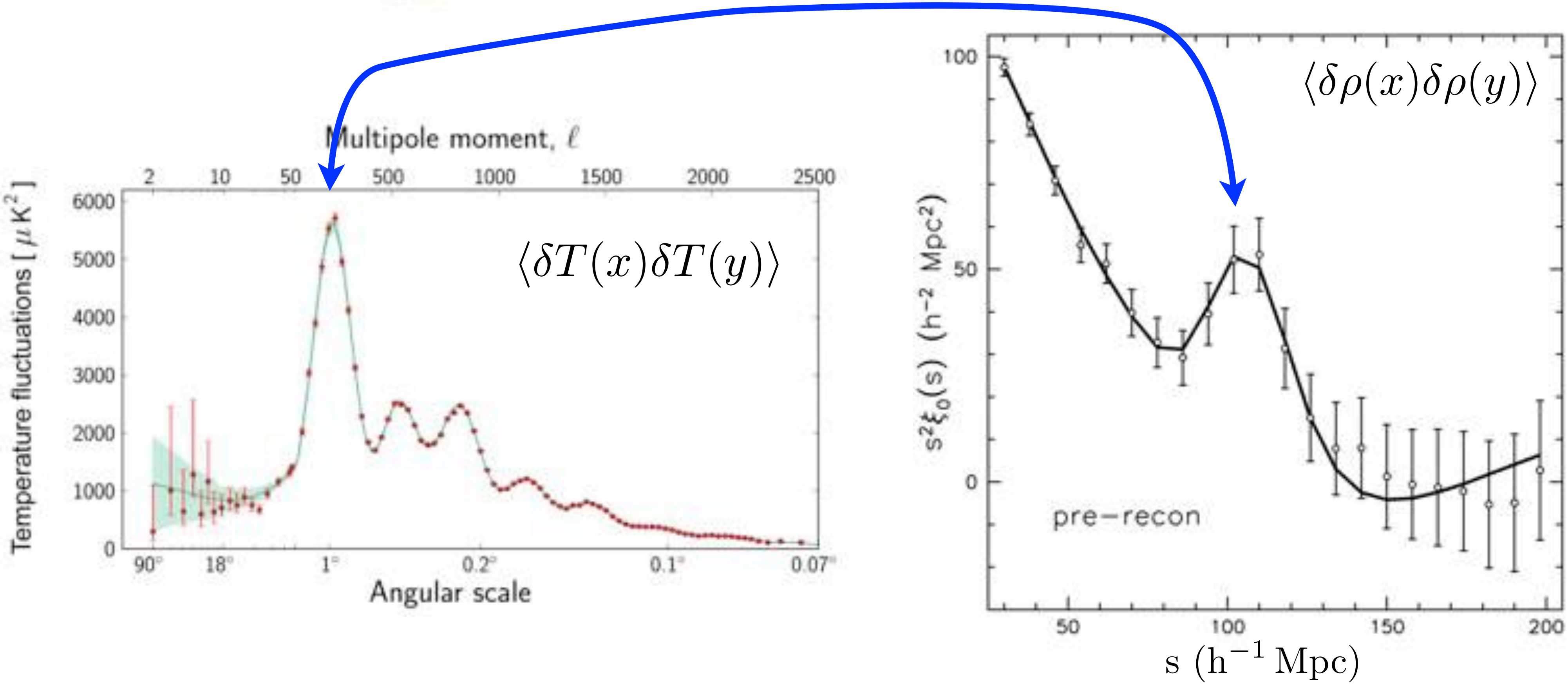
cf. predictions of the minimal inflation: $f_{NL} \sim \epsilon, \eta \sim 10^{-2}$

$f_{NL} \sim 1$ naturally appears in extended inflationary models (multiple fields, extended kinetic action, ...)

Baryon acoustic oscillations







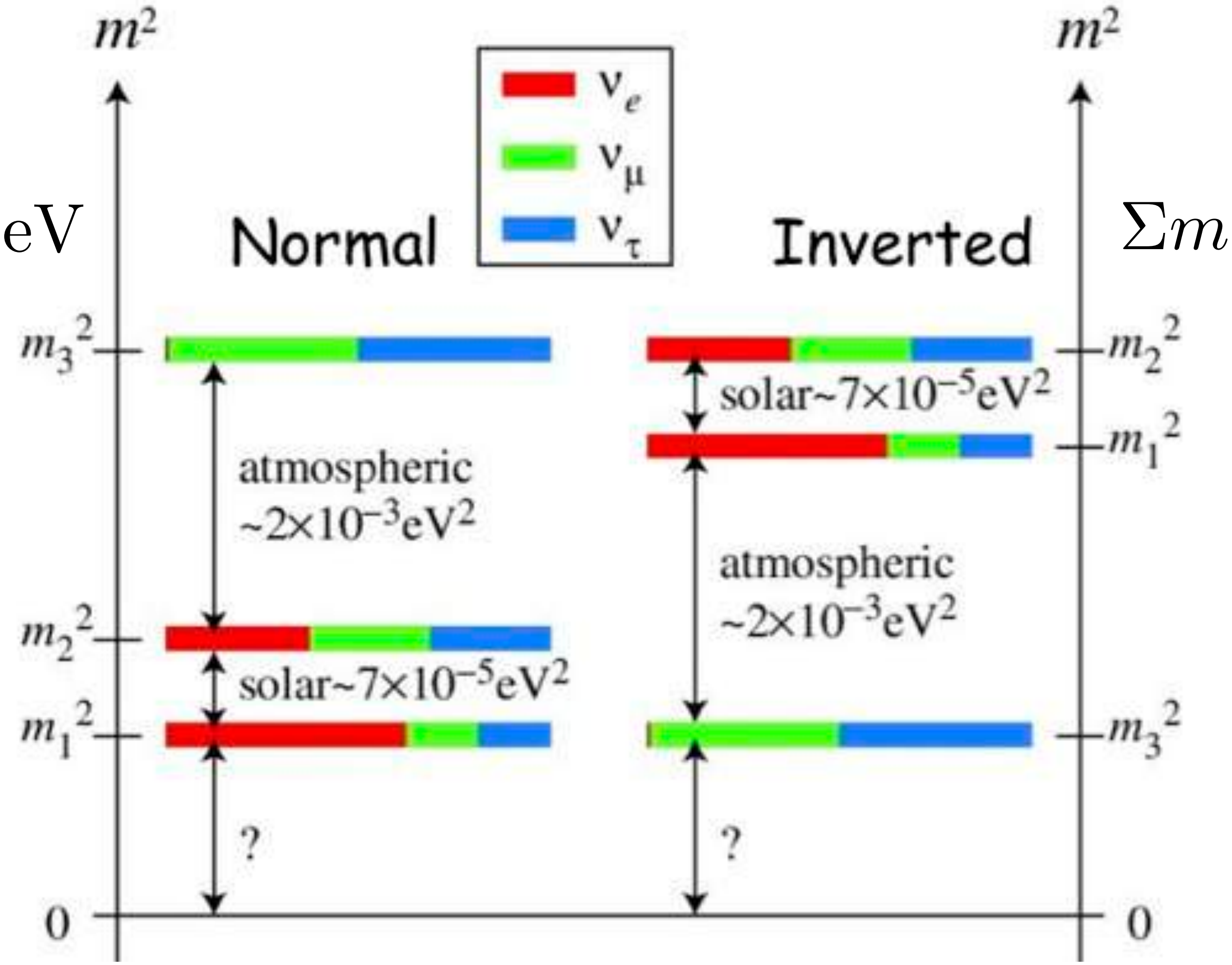
Planck collaboration

Anderson et al. (BOSS collaboration)

Neutrino mass: current status

$\Sigma m_\nu > 0.05$ eV

$\Sigma m_\nu > 0.1$ eV

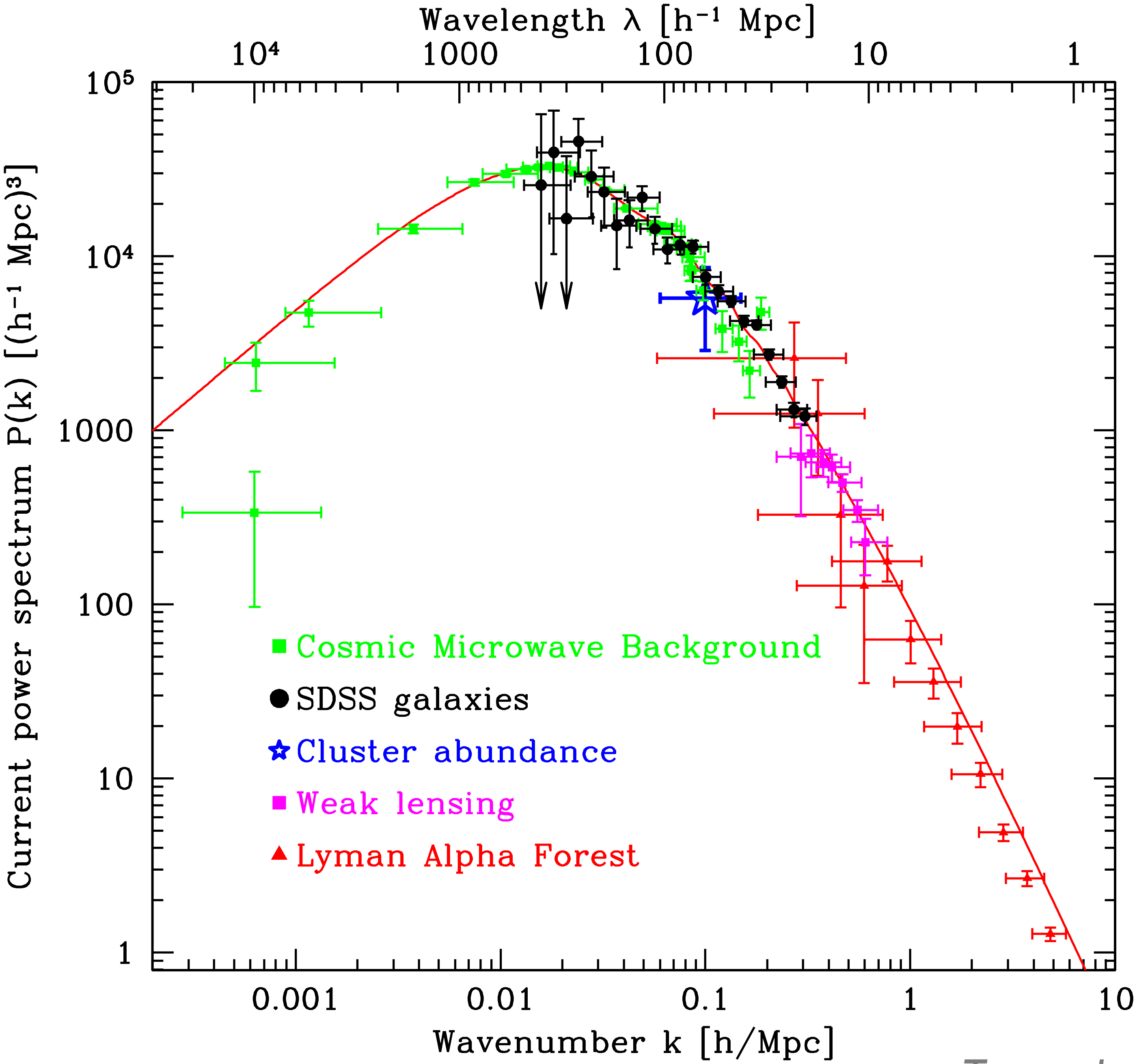


$m_{\nu_e} < 2$ eV (tritium decay)

$\Sigma m_\nu < 0.15$ eV (CMB + BAO) *Vagnozzy et al. (2017)*

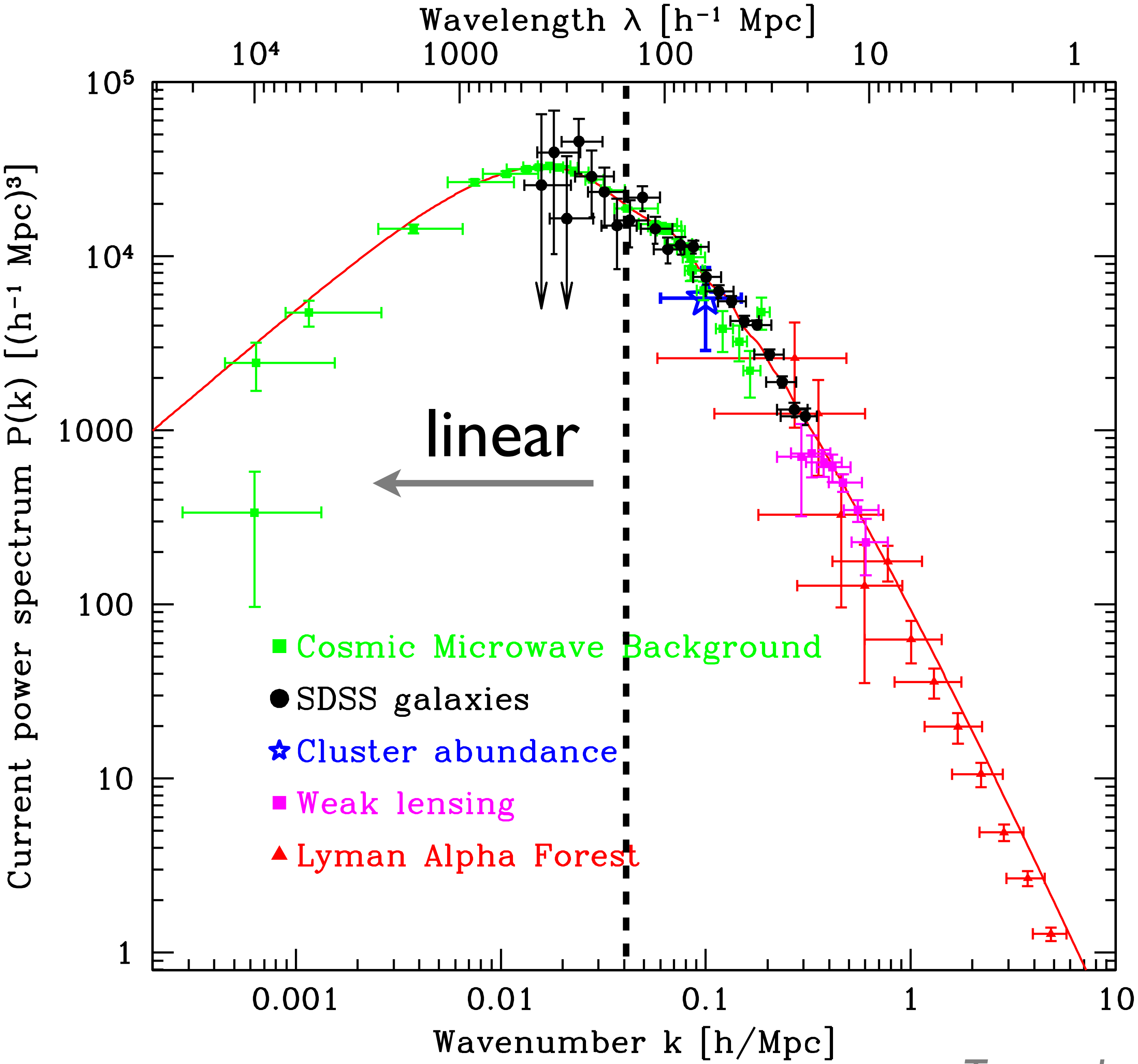
< 0.12 eV (CMB + Lyα) *Palanque-Delabrouile et al. (2015)*

About scales



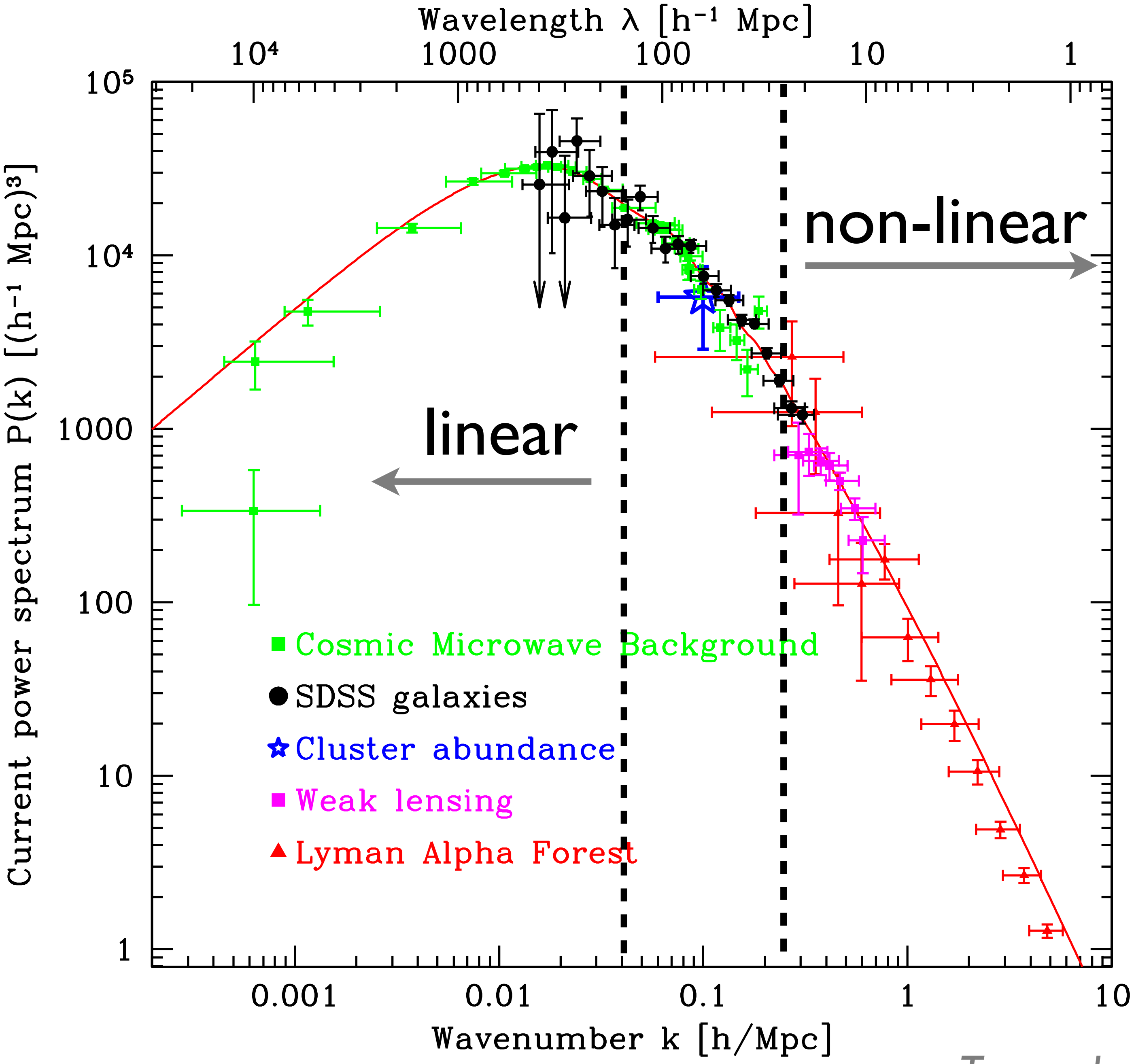
Tegmark et al. (2004)

About scales



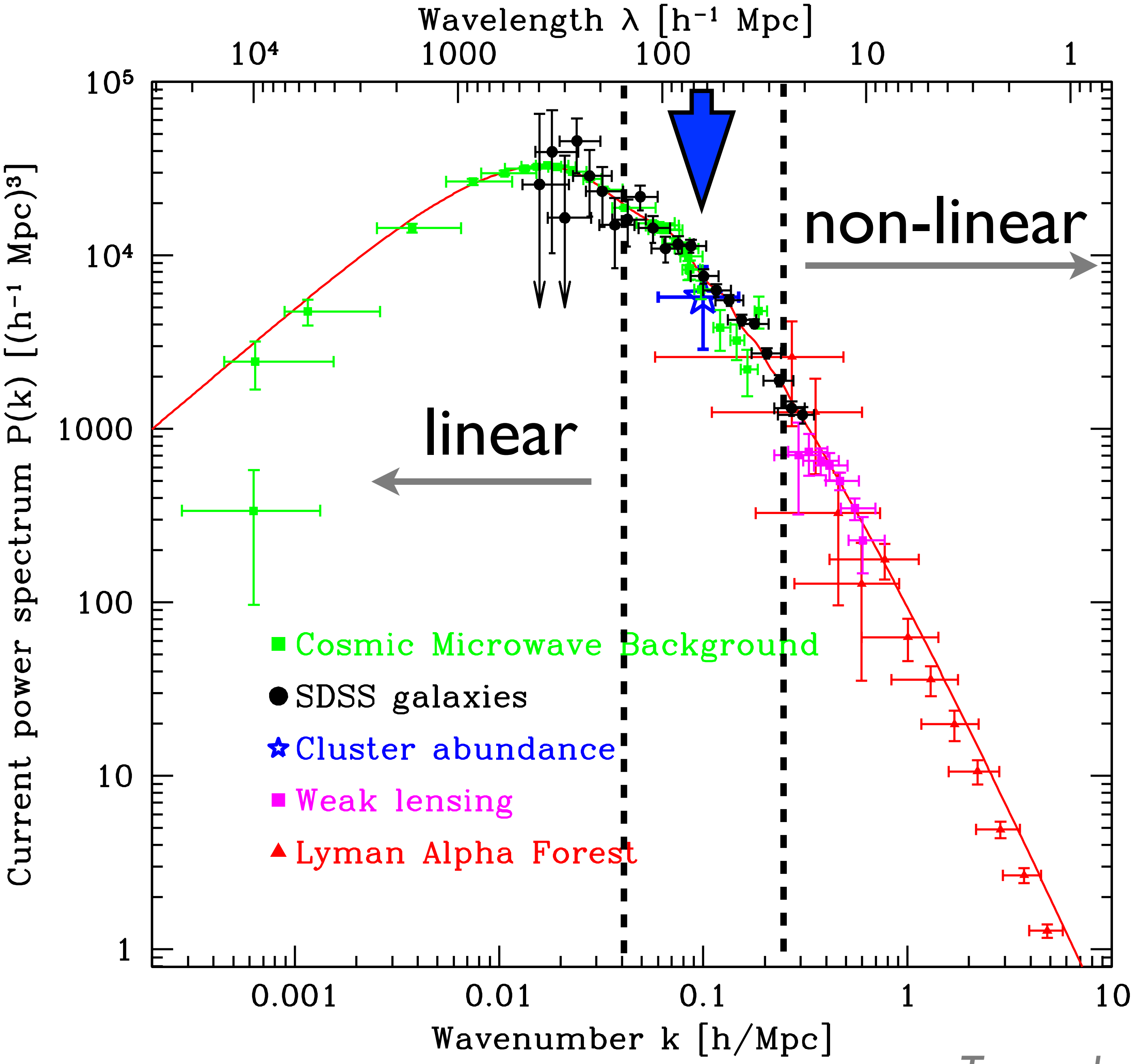
Tegmark et al. (2004)

About scales



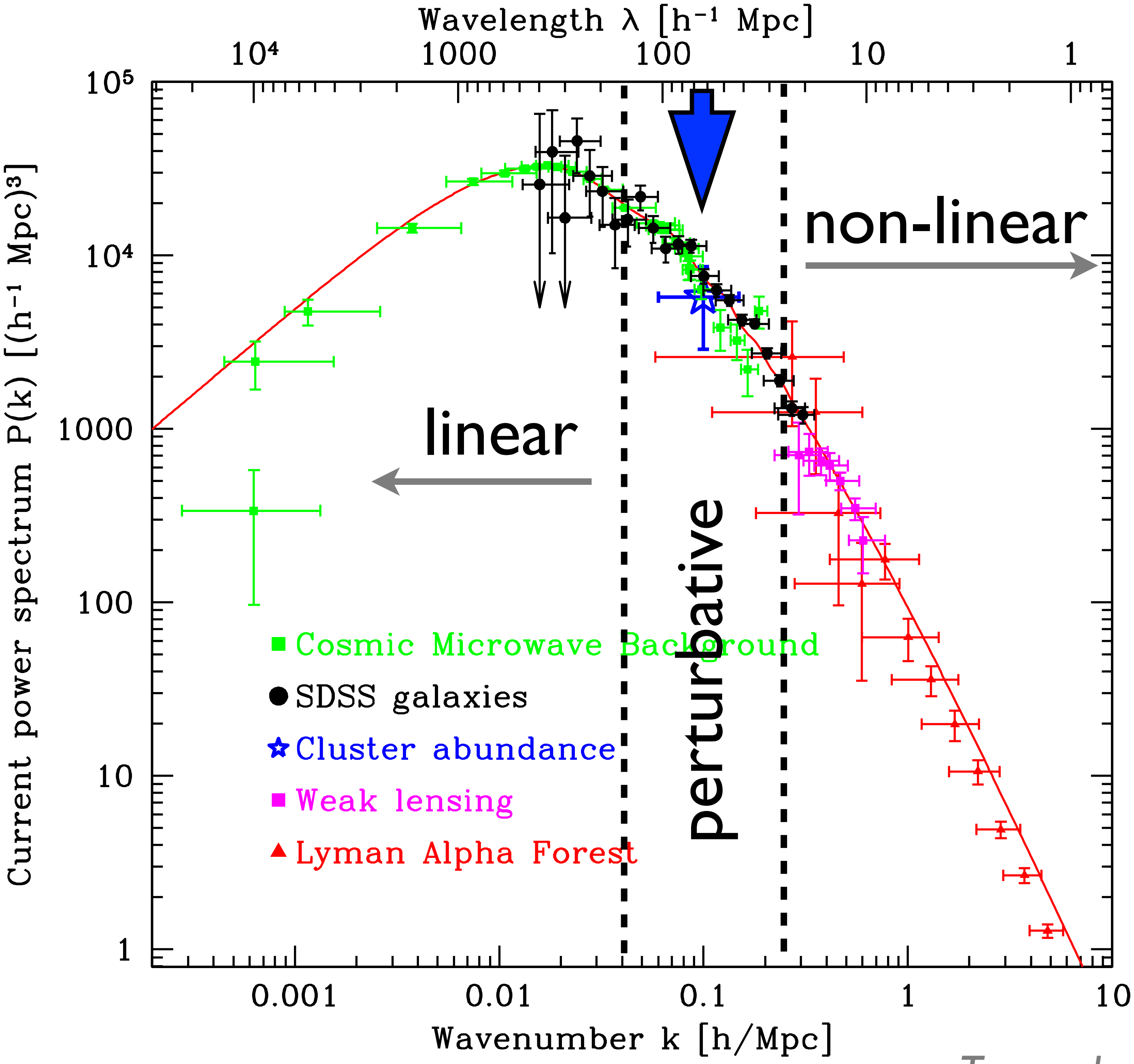
Tegmark et al. (2004)

About scales



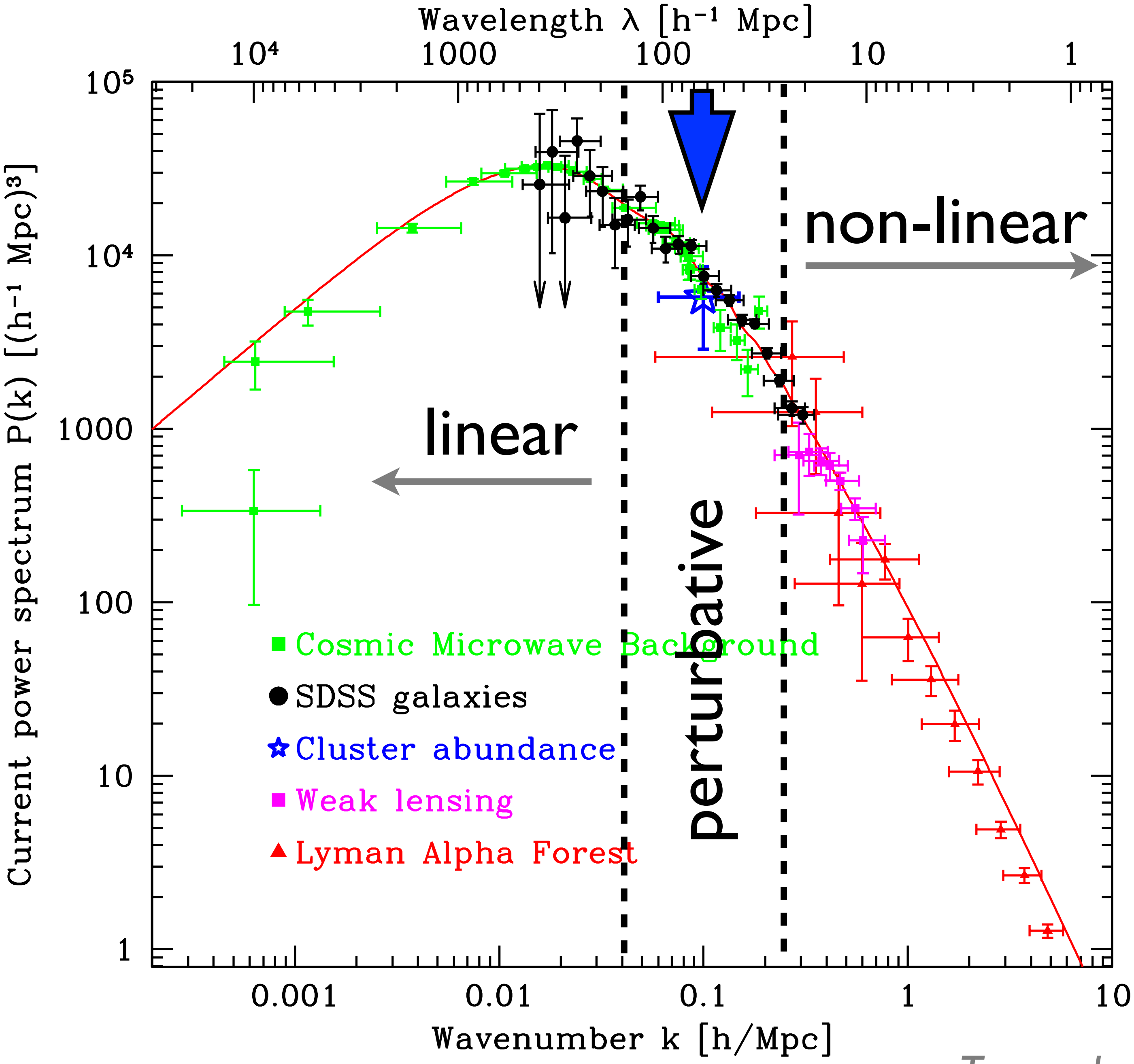
Tegmark et al. (2004)

About scales



Tegmark et al. (2004)

About scales

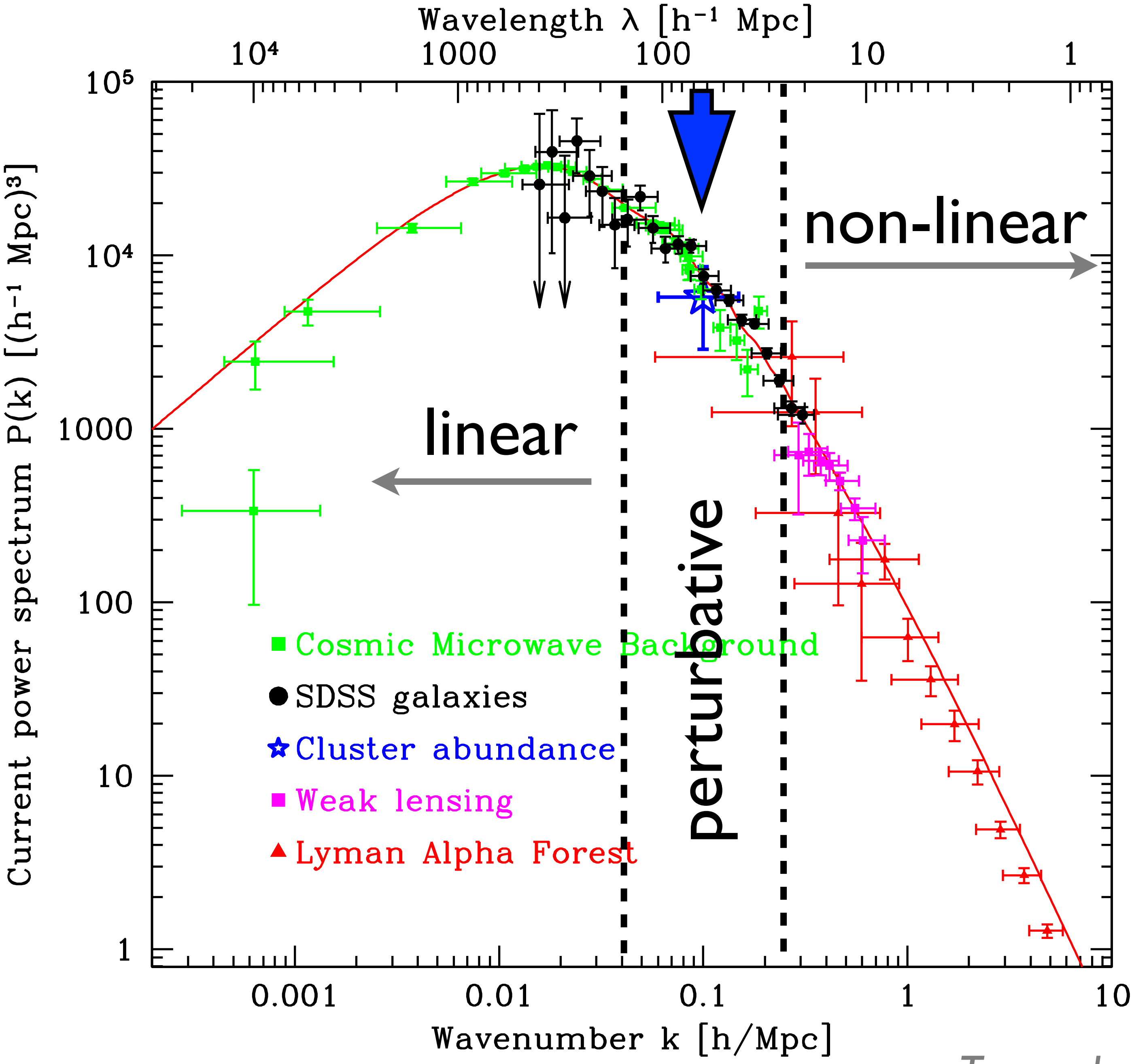


statistical error

$$\propto (k_{max})^{-3/2}$$

Tegmark et al. (2004)

About scales



statistical error

$$\propto (k_{max})^{-3/2}$$

NB. perturbative region increases at $z > 0$

Tegmark et al. (2004)

Challenges to theorists

The fundamental description is known (?): collisionless particles interacting through gravity

Vlasov -- Poisson system for the distribution function $f(\mathbf{x}, \mathbf{v}, t)$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \ , \qquad \nabla^2 \phi = 4\pi G \int f \ d^3\mathbf{v}$$

Challenges to theorists

The fundamental description is known (?): collisionless particles interacting through gravity

Vlasov -- Poisson system for the distribution function $f(\mathbf{x}, \mathbf{v}, t)$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \ , \qquad \nabla^2 \phi = 4\pi G \int f \, d^3\mathbf{v}$$

- numerical solution: N-body simulations
  - + valid up to arbitrary k
  - – costly, scanning over theory parameters is time-consuming, non-standard models are hard to implement

Challenges to theorists

The fundamental description is known (?): collisionless particles interacting through gravity

Vlasov -- Poisson system for the distribution function $f(\mathbf{x}, \mathbf{v}, t)$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \nabla\phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \;, \qquad \nabla^2\phi = 4\pi G \int f \, d^3\mathbf{v}$$

- numerical solution: N-body simulations

  \+ valid up to arbitrary k

  – costly, scanning over theory parameters is time-consuming, non-standard models are hard to implement

- analytical perturbative methods at $k \lesssim 0.3 \, \mathrm{h}^{-1}\mathrm{Mpc}$

  – are approximate

  \+ theoretical control of physical processes, flexibility

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4 \text{ Mpc}$

DM particles move by $uH^{-1} \sim 10 \text{ Mpc}$

10^{-3}

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4 \text{ Mpc}$

DM particles move by $uH^{-1} \sim 10 \text{ Mpc}$

10^{-3}

➡ nonrelativistic fluid at $10 \text{ Mpc} \ll l \ll 10^4 \text{ Mpc}$

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4\ \mathrm{Mpc}$

DM particles move by $\underset{\overset{\nearrow}{10^{-3}}}{u}H^{-1} \sim 10\ \mathrm{Mpc}$

➡ nonrelativistic fluid at $10\ \mathrm{Mpc} \ll l \ll 10^4\ \mathrm{Mpc}$

$$\frac{\partial \delta_\rho}{\partial \tau} + \nabla[\mathbf{u} + \delta_\rho \mathbf{u}] = 0$$

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta_\rho$$

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4\ \mathrm{Mpc}$

DM particles move by $uH^{-1} \sim 10\ \mathrm{Mpc}$

10^{-3}

➡ nonrelativistic fluid at $10\ \mathrm{Mpc} \ll l \ll 10^4\ \mathrm{Mpc}$

$$\frac{\partial \delta_\rho}{\partial \tau} + \nabla[\mathbf{u} + \delta_\rho \mathbf{u}] = 0$$

treat as perturbations

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta_\rho$$

Simplifying the problem

Newtonian approximation at $l \ll H^{-1} \sim 10^4 \ \mathrm{Mpc}$

DM particles move by $uH^{-1} \sim 10 \ \mathrm{Mpc}$

10^{-3}

➡ nonrelativistic fluid at $10 \ \mathrm{Mpc} \ll l \ll 10^4 \ \mathrm{Mpc}$

$$\frac{\partial \delta_\rho}{\partial \tau} + \nabla[\mathbf{u} + \delta_\rho \mathbf{u}] = 0$$

treat as perturbations

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta_\rho$$

vorticity decays at linear level ➡ work with $\theta \propto \nabla \cdot \mathbf{u}$

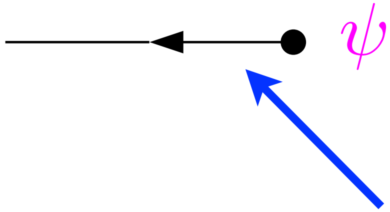
Standard perturbation theory (SPT)

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

Standard perturbation theory (SPT)

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

$\psi^{(1)} =$

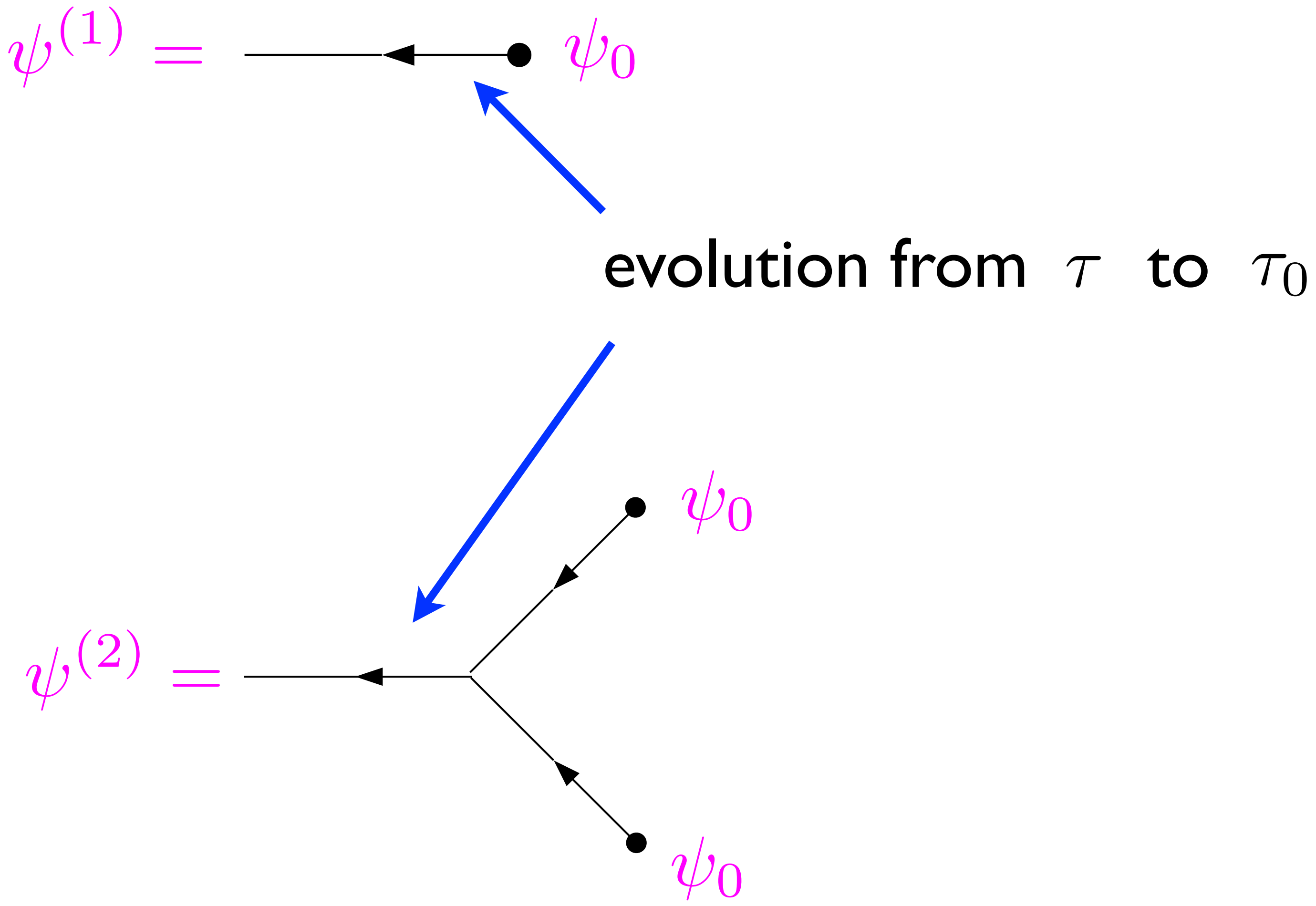


ψ_0

evolution from τ to τ_0

Standard perturbation theory (SPT)

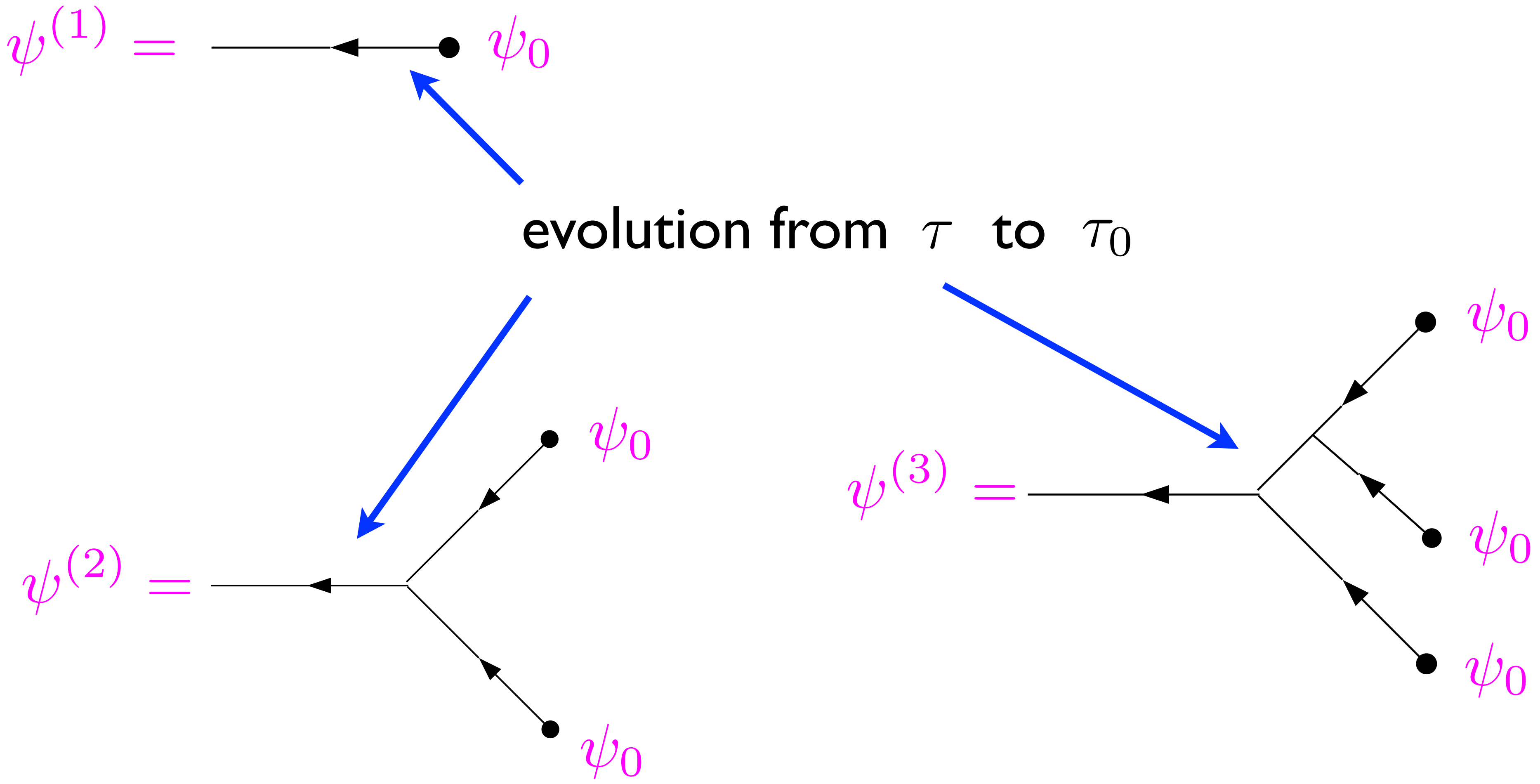
Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$



evolution from τ to τ_0

Standard perturbation theory (SPT)

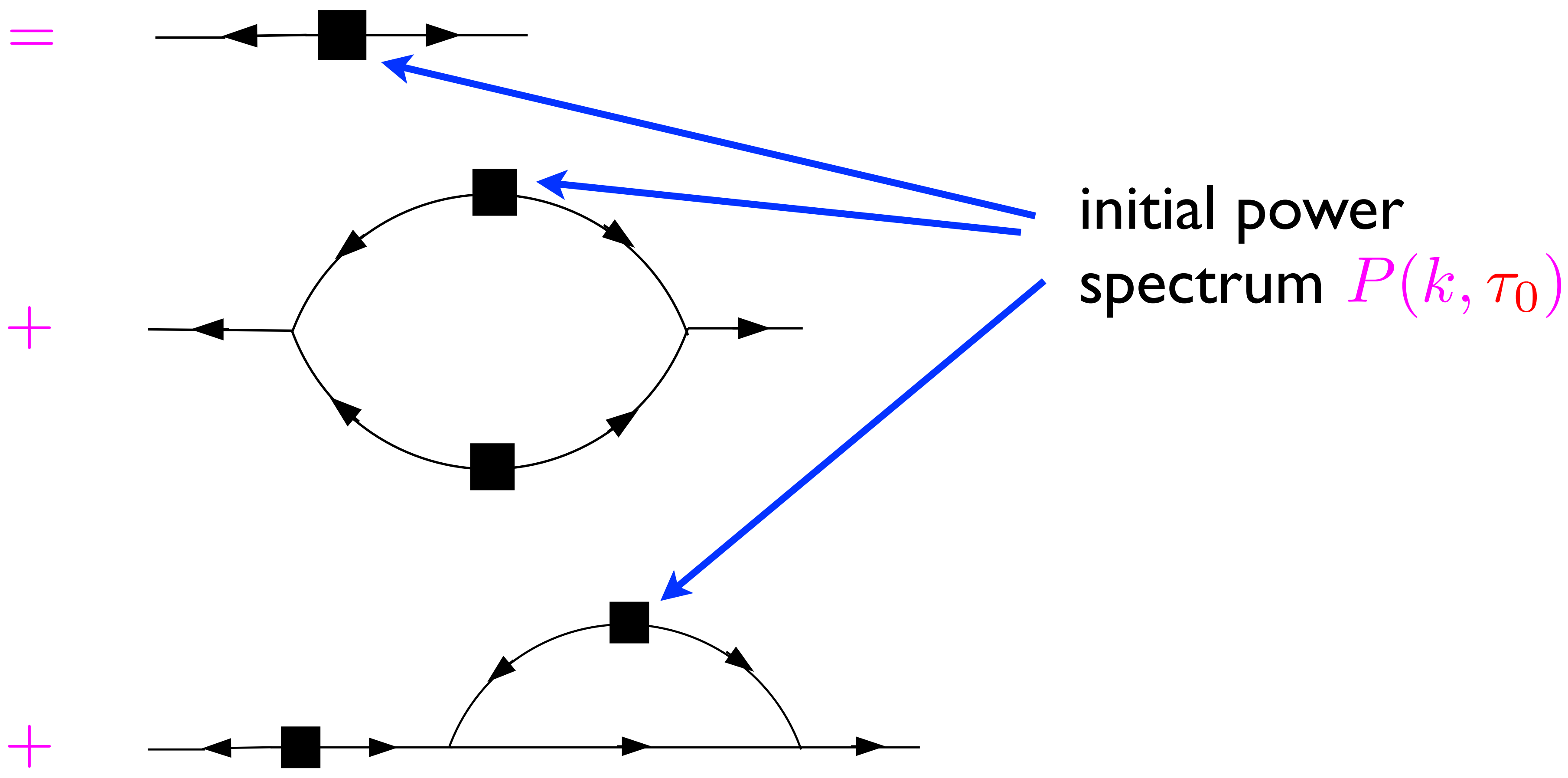
Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$



evolution from τ to τ_0

Average over the ensemble of initial conditions:

$$\langle \psi(k_1,\tau)\psi(k_2,\tau)\rangle = \langle \psi^{(1)}\psi^{(1)}\rangle + \langle \psi^{(2)}\psi^{(2)}\rangle + 2\langle \psi^{(1)}\psi^{(3)}\rangle + \ldots =$$



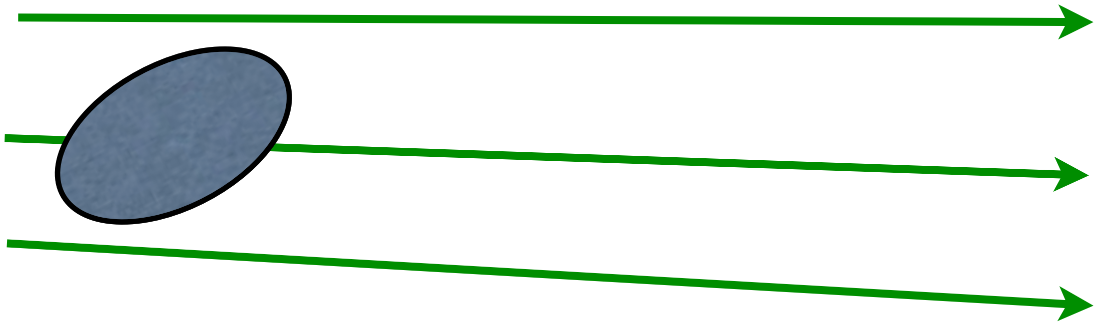
initial power spectrum $P(k,\tau_0)$

Problems of SPT

“Infrared” Individual loop diagrams diverge at small momenta. When summed, the divergences cancel in equal-time correlators

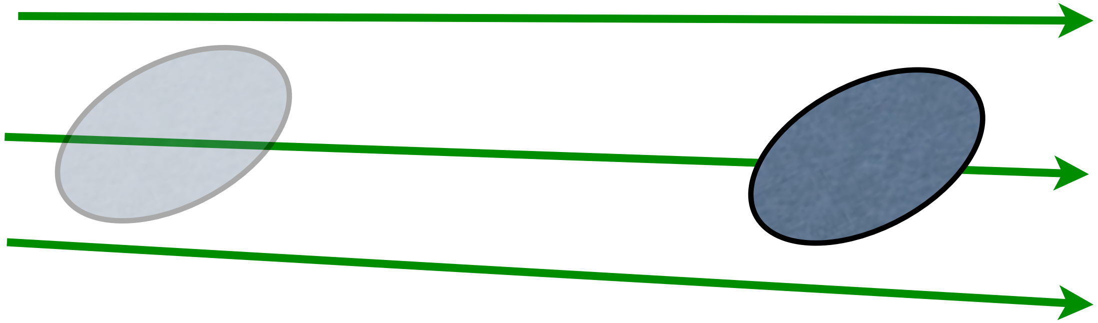
Problems of SPT

"Infrared" Individual loop diagrams diverge at small momenta. When summed, the divergences cancel in equal-time correlators



Problems of SPT

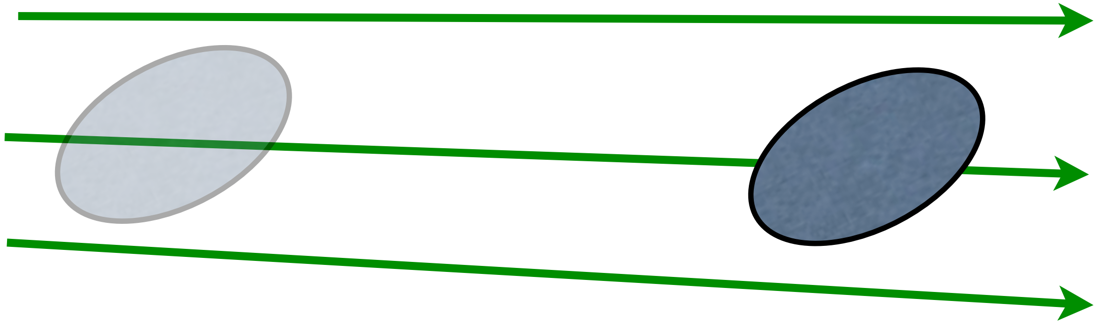
"Infrared" Individual loop diagrams diverge at small momenta. When summed, the divergences cancel in equal-time correlators



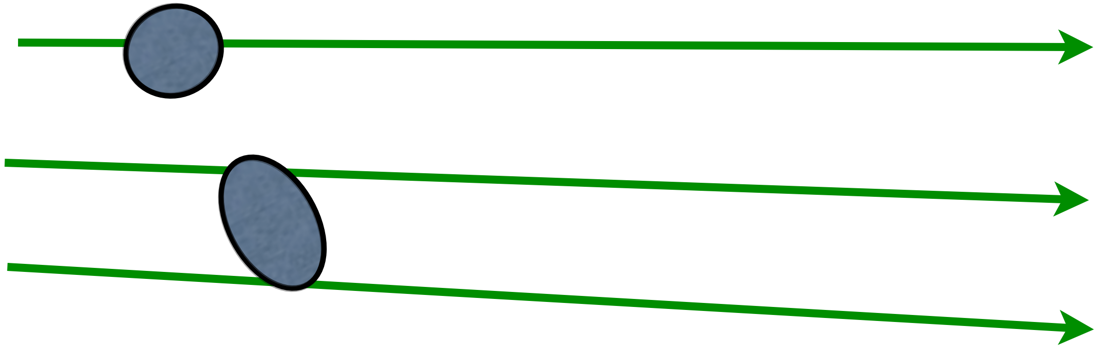
overdensity is moved by an almost homogeneous flow, accumulation of the effect with time

Problems of SPT

“Infrared” Individual loop diagrams diverge at small momenta. When summed, the divergences cancel in equal-time correlators

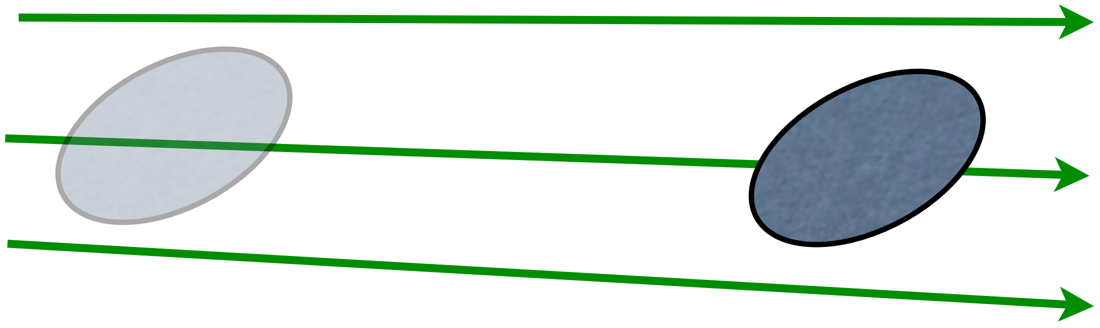


overdensity is moved by an almost homogeneous flow,
accumulation of the effect with time

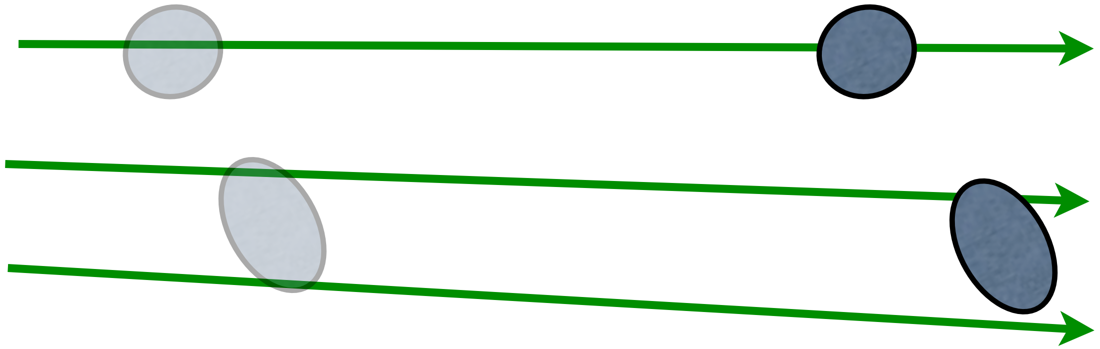


Problems of SPT

"Infrared" Individual loop diagrams diverge at small momenta. When summed, the divergences cancel in equal-time correlators



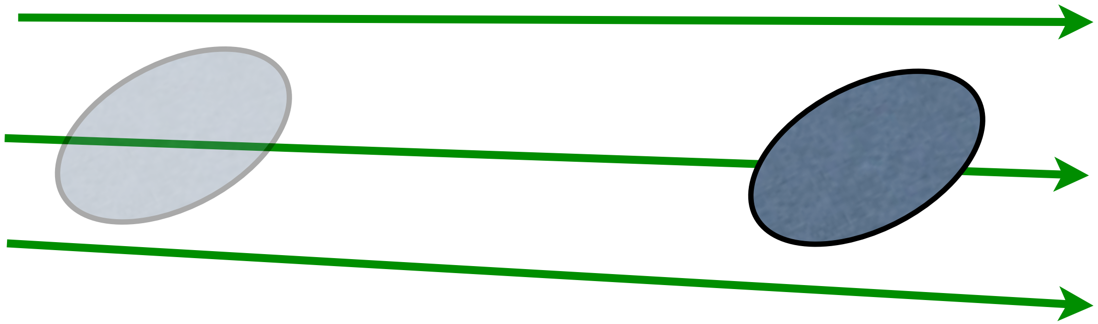
overdensity is moved by an almost homogeneous flow, accumulation of the effect with time



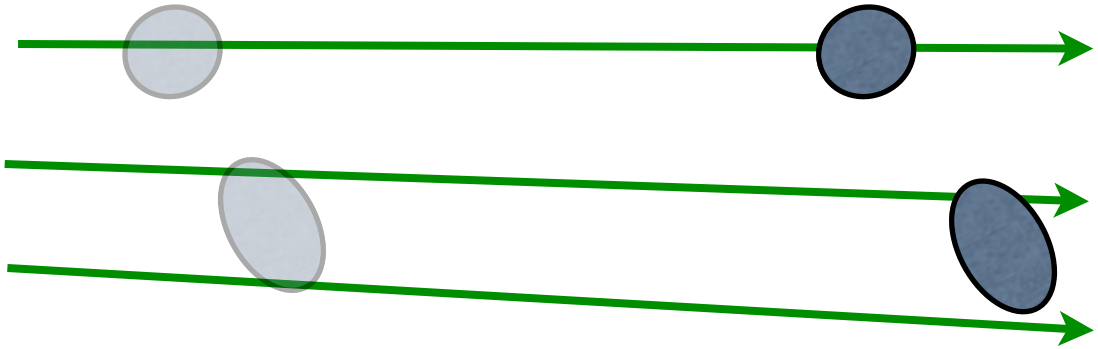
two overdensities will move (almost) identically, cancellation in equal-time correlators

Problems of SPT

"Infrared" Individual loop diagrams diverge at small momenta. When summed, the divergences cancel in equal-time correlators



overdensity is moved by an almost homogeneous flow,
accumulation of the effect with time



two overdensities will move (almost) identically,
cancellation in equal-time correlators

Possible way out: Lagrangian picture

Matsubara (2007), Padmanabhan et al. (2008)

Problems of SPT

“Ultraviolet” Loop integrals run over all momenta including short modes where the fluid description is not applicable.

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

Problems of SPT

“Ultraviolet” Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

3) add counterterms into the equations of motion to account for deviations from fluid description

Problems of SPT

“Ultraviolet” Loop integrals run over all momenta including short modes where the fluid description is not applicable.

1) introduce a UV cutoff Λ

2) renormalize the interaction vertices to ensure that the physical observables are Λ-independent

3) add counterterms into the equations of motion to account for deviations from fluid description

➡ EFT of LSS

Baumann, Nicolis, Senatore, Zaldarriaga (2010)
Carrasco, Hertzberg, Senatore (2012)
Pajer, Zaldarriaga (2013)

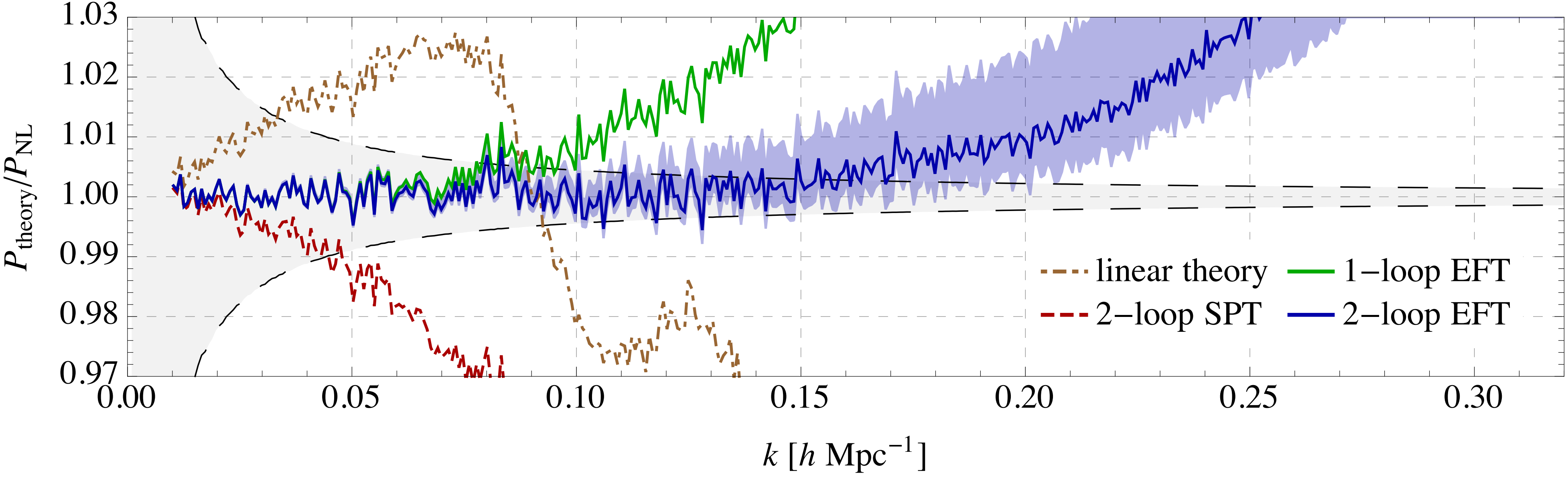
+ many more

$$\dot{u}^i + \mathcal{H}u^i + u^j\nabla_j u^i + \nabla\phi = -\frac{1}{\rho}\partial_j\tau^{ij}$$

$$\tau^{ij}_{vis} + \tau^{ij}_{stoch}$$

$$\begin{aligned}\tau^{ij}_{vis} = &- c_s^2\delta^{ij}\delta_\rho + \tilde{c}\delta^{ij}\Delta\delta_\rho \\ &+ c_1\delta^{ij}(\Delta\phi)^2 + c_2\partial^i\partial^j\phi\Delta\phi + c_3\partial^i\partial_k\phi\partial^j\partial_k\phi + \dots\end{aligned}$$

from Foreman, Perrier, Senatore (2015)

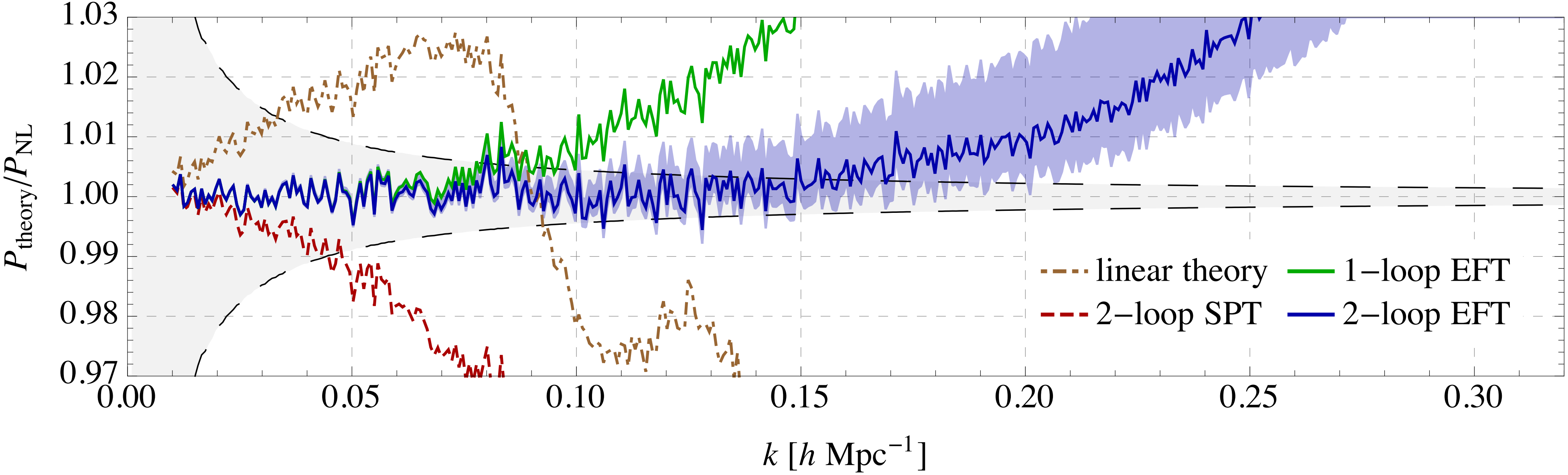


$$\dot{u}^i + \mathcal{H}u^i + u^j \nabla_j u^i + \nabla\phi = -\frac{1}{\rho}\partial_j \tau^{ij}$$

$$\tau^{ij}_{vis} + \tau^{ij}_{stoch}$$

$$\tau^{ij}_{vis} = -c_s^2 \delta^{ij} \delta_\rho + \tilde{c}\delta^{ij} \Delta\delta_\rho + c_1 \delta^{ij} (\Delta\phi)^2 + c_2 \partial^i \partial^j \phi \Delta\phi + c_3 \partial^i \partial_k \phi \partial^j \partial_k \phi + \dots$$

from Foreman, Perrier, Senatore (2015)



Complications:

- coefficients of the counterterms have non-local time-dependence

Abolhasani, Mirbabayi, Pajer (2015)

- treatment of stochastic terms is unclear

In approaches operating with the equations of motion IR and UV issues are mixed

In approaches operating with the equations of motion IR and UV issues are mixed

To clear up

➡ use the methods of QFT / statistical mechanics

Example: resummation of IR divergences in QED is clearly separated from UV renormalization

TSPT: time-sliced perturbation theory

Valageas (2004)

Blas, Garny, Ivanov, S.S. (2015,2016)

Main ideas: Focus on equal-time correlators

Instead of evolving fields, evolve the probability distribution function

TSPT: time-sliced perturbation theory

Valageas (2004)
Blas, Garny, Ivanov, S.S. (2015,2016)

Main ideas: Focus on equal-time correlators

Instead of evolving fields, evolve the probability distribution function

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Longrightarrow \quad \psi(\tau;\psi_0)$$

TSPT: time-sliced perturbation theory

Valageas (2004)
Blas, Garny, Ivanov, S.S. (2015,2016)

Main ideas: Focus on equal-time correlators

Instead of evolving fields, evolve the probability distribution function

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Longrightarrow \quad \psi(\tau;\psi_0)$$

SPT: $\int d\psi_0 \, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2$

$$\Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$$

TSPT: time-sliced perturbation theory

Valageas (2004)

Blas, Garny, Ivanov, S.S. (2015,2016)

Main ideas: Focus on equal-time correlators

Instead of evolving fields, evolve the probability distribution function

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Longrightarrow \quad \psi(\tau;\psi_0)$$

SPT: $\int d\psi_0 \, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2$ $\qquad \Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$

TSPT: $\int d\psi \, e^{-\Gamma[\psi;\tau]}\psi^2$ $\qquad \Gamma[\psi;\tau] = \sum_n \frac{\Gamma_n(\tau)}{n!} \, \psi^n$

Two integrals must coincide

➡ equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

➡ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1}}_{\text{contains only } \Gamma_{n'} \text{ with } n' < n} + A_{n+1}$$

Two integrals must coincide

➡ equation for the “vertices”

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

➡ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1}}_{\text{contains only } \Gamma_{n'} \text{ with } n' < n} + A_{n+1}$$

The same logic for fields in space with the substitution:
integral $\Longrightarrow$ path integral

Generating functional for cosmological correlators

$$Z[J,\tau] = \int [\mathcal{D}\delta_\rho] \exp\left\{ -\Gamma[\delta_\rho;\tau] + \int J\delta_\rho \right\}$$

$$\Gamma = \frac{1}{2}\int \frac{\delta_\rho^2}{P(k)} + \sum_{n=3}^{\infty} \frac{1}{n!}\int \Gamma_n(\tau)\delta_\rho^n$$

NB. Γ is an action of a (nonlocal) 3d Euclidean QFT;

τ --- an external parameter

Advantages

- For gaussian initial conditions the time dependence factorize

$$\Gamma = \frac{1}{g^2(\tau)}\bar{\Gamma}$$

effective coupling constant

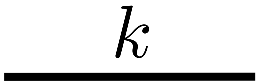
NB. For primordial NG

$$\Gamma = \frac{1}{g^2}\bar{\Gamma} + \frac{1}{g^3}\hat{\Gamma}$$

$\sim f_{NL} g_0$

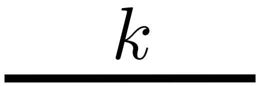
- Simplified diagrammatic technique

- Simplified diagrammatic technique

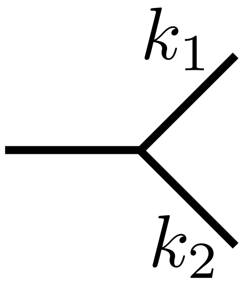


$$= g^2 \bar{P}(k)$$

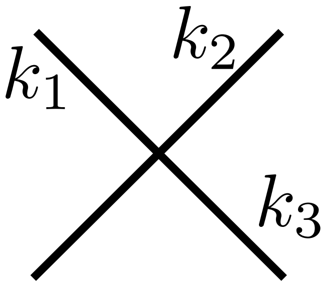
- Simplified diagrammatic technique



$$= g^2 \bar{P}(k)$$



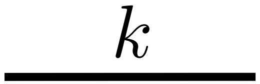
$$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$$



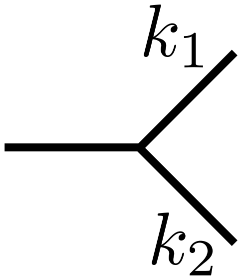
$$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$$

……………………………………

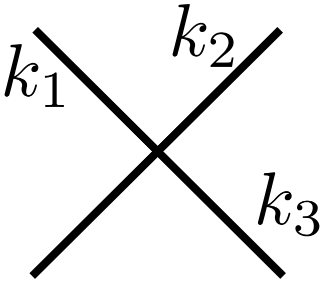
- Simplified diagrammatic technique



$$= g^2 \bar{P}(k)$$



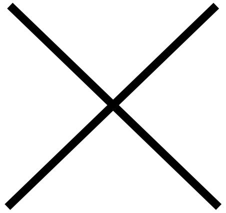
$$= \frac{1}{g^2} \bar{\Gamma}_3(k_1, k_2)$$



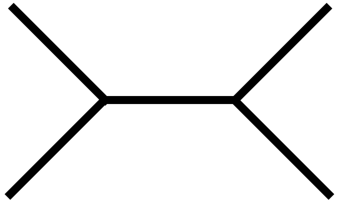
$$= \frac{1}{g^2} \bar{\Gamma}_4(k_1, k_2, k_3)$$

……

$$\langle \delta_\rho \delta_\rho \delta_\rho \delta_\rho \rangle =$$



+



IR safety

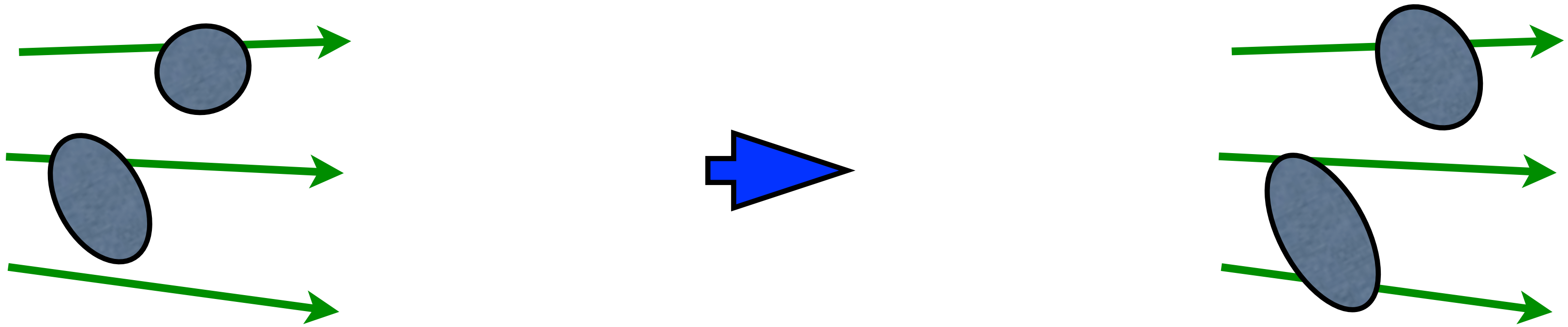
All Γ_n, K_n are finite for soft momenta

$$\lim_{\epsilon \longrightarrow 0} \Gamma_n(k_1, \ldots, k_l, \epsilon q_1, \ldots, \epsilon q_{n-l}) < \infty$$

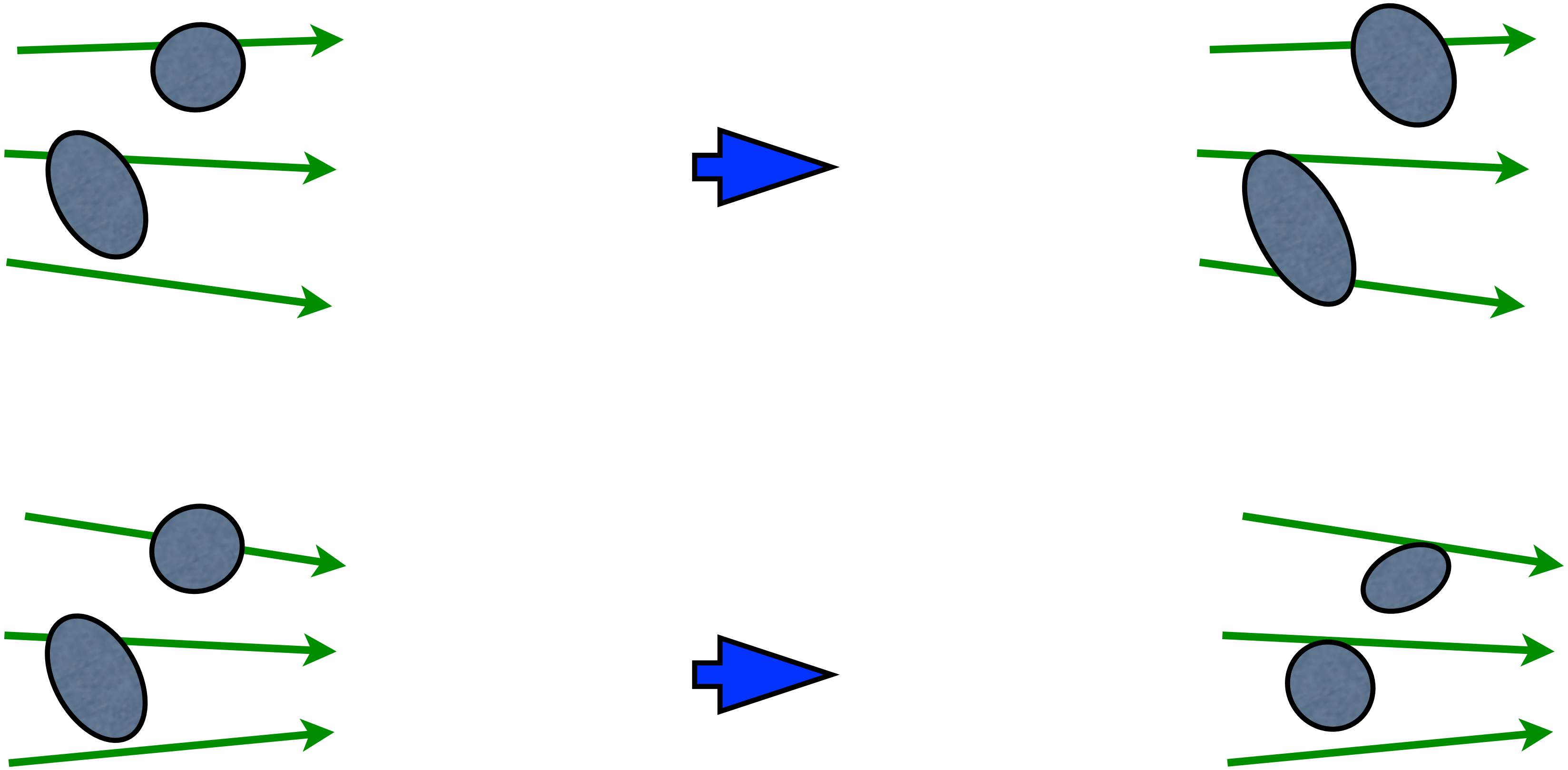
➡ no IR divergences in the individual loop diagrams

NB. Can be related to the equivalence principle / Galilean invariance of Γ through Ward identities

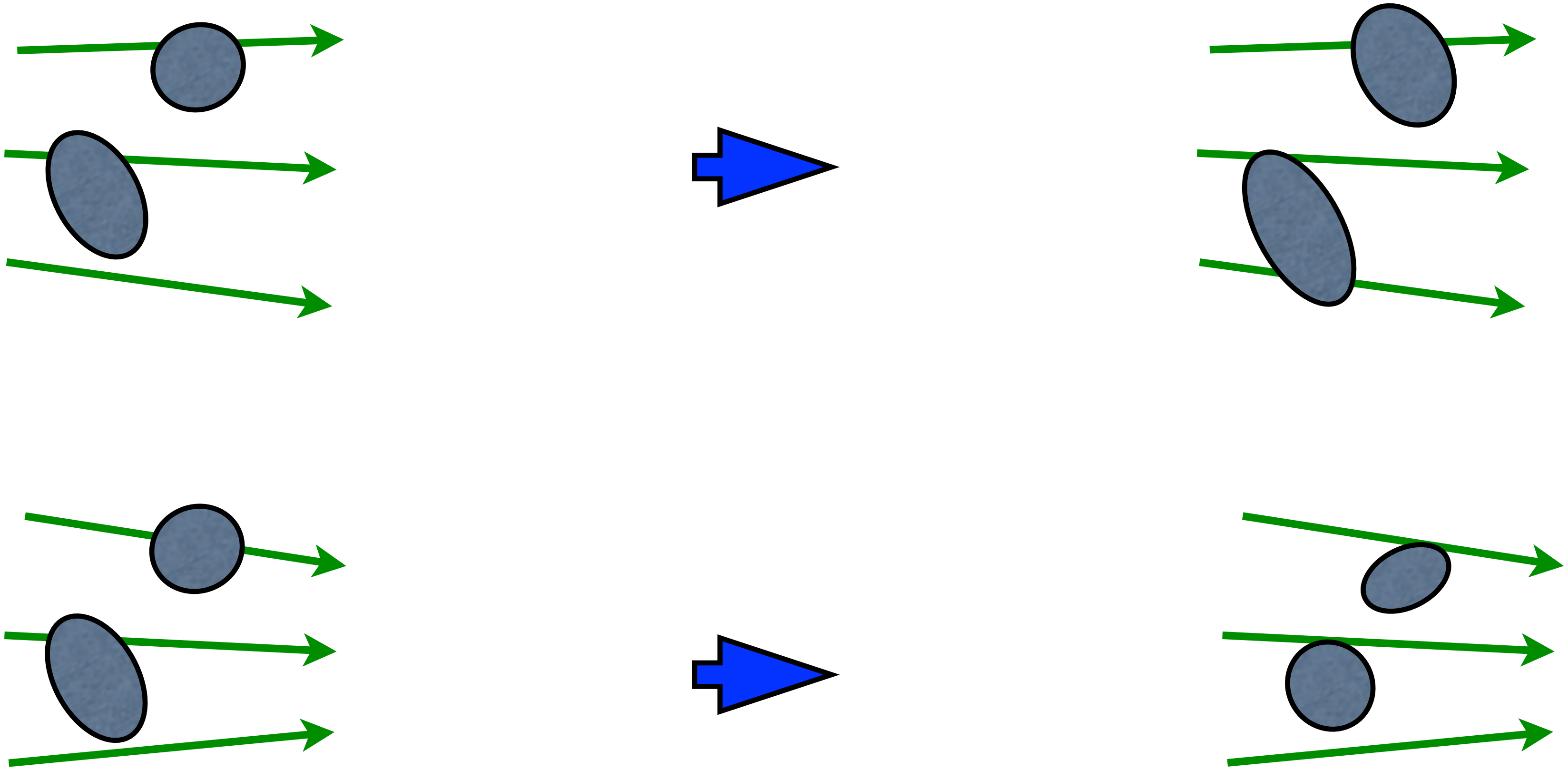
IR enhanced effects due to flow gradients



IR enhanced effects due to flow gradients



IR enhanced effects due to flow gradients



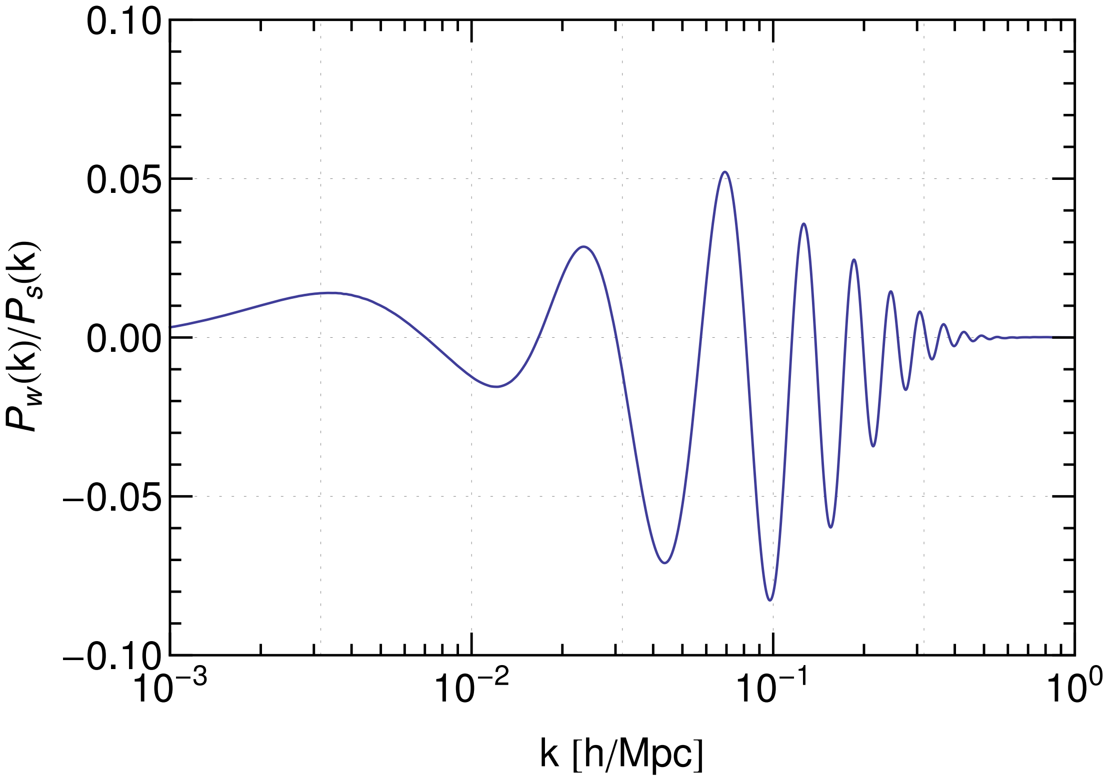
smearing of the BAO feature in the correlation functions

IR resummation

In TSPT large IR contributions can be systematically resummed

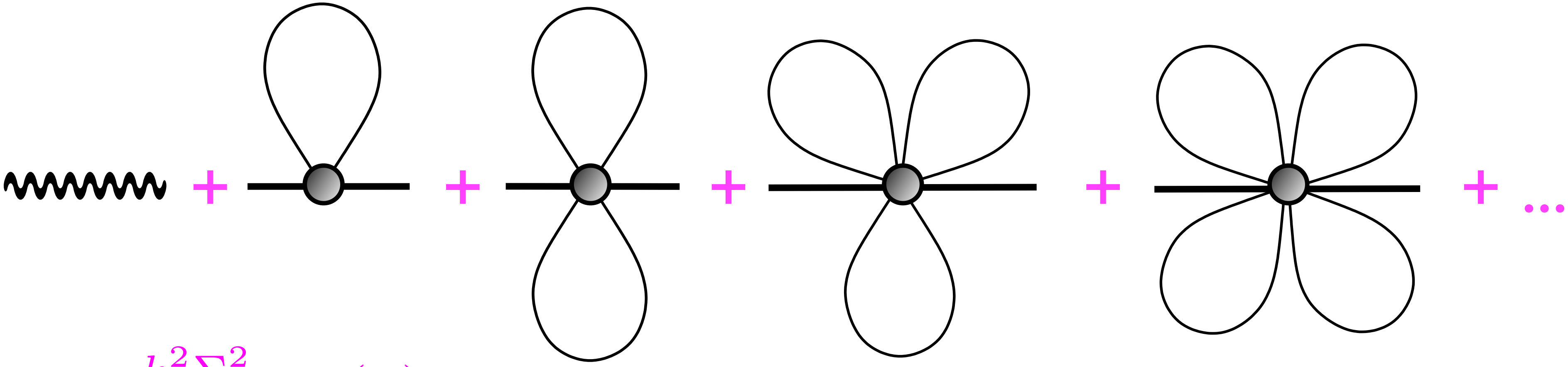
Step 1: smooth + wiggly decomposition

$$P(k) = P_s(k) + P_w(k) \quad \Longrightarrow \quad \Gamma(k) = \Gamma_s(k) + \Gamma_w(k)$$



Step II: identification of leading diagrams correcting the wiggly part ➡ daisies

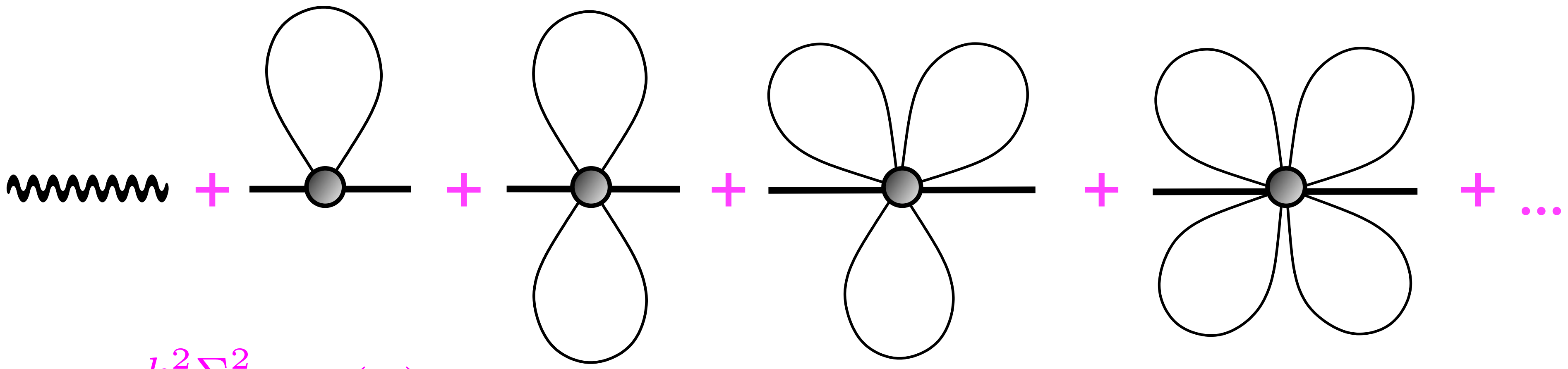
$$P_w^{\text{dressed}} =$$



$$= \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

Step II: identification of leading diagrams correcting the wiggly part → daisies

$$P_w^{\text{dressed}} =$$



$$= \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

BAO wavelength

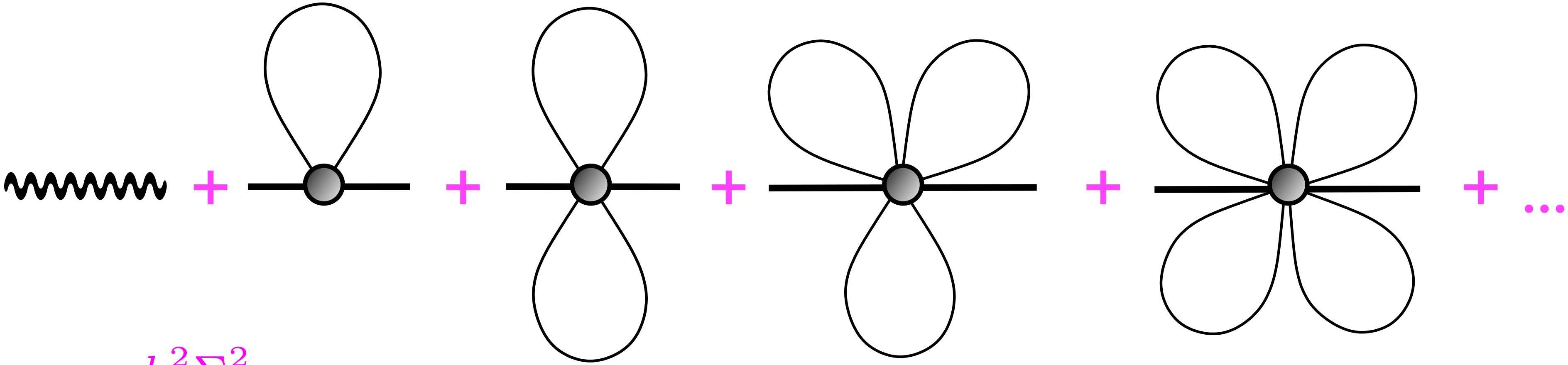
$$\Sigma_L^2 = \frac{4\pi}{3}\int_0^{k_L} dq\; P_s(q)\;\left(1 - j_0(qr_s) + 2j_2(qr_s)\right)$$

Baldauf et al. (2015)

Blas, Garny, Ivanov, S.S. (2016)

Step II: identification of leading diagrams correcting the wiggly part ➡ daisies

$$P_w^{\text{dressed}} =$$



$$= \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

separation between hard and IR momenta

BAO wavelength

$$\Sigma_L^2 = \frac{4\pi}{3}\int_0^{\boxed{k_L}} dq\; P_s(q)\; \left(1 - j_0(qr_s) + 2j_2(qr_s)\right)$$

Baldauf et al. (2015)

Blas, Garny, Ivanov, S.S. (2016)

Step III: add the smooth part

$$P(k) = P_s(k) + \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

Step III: add the smooth part

$$P(k) = P_s(k) + \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

Step IV: compute to the desired order in hard loops using the dressed power spectrum

NB. Valid for any correlation function

Step III: add the smooth part

$$P(k) = P_s(k) + \mathrm{e}^{-k^2\Sigma_L^2} P_w(k)$$

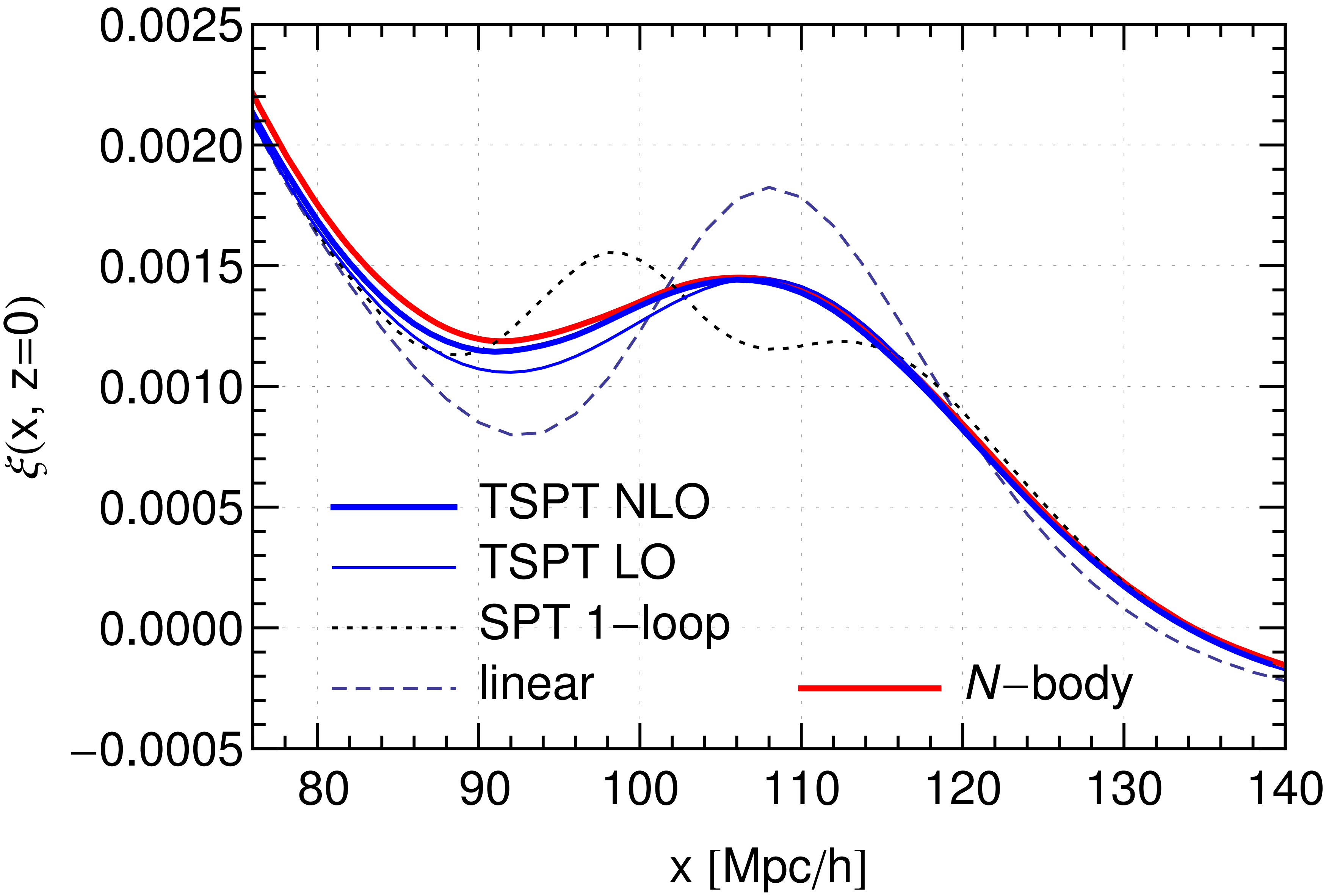
Step IV: compute to the desired order in hard loops using the dressed power spectrum

NB. Valid for any correlation function

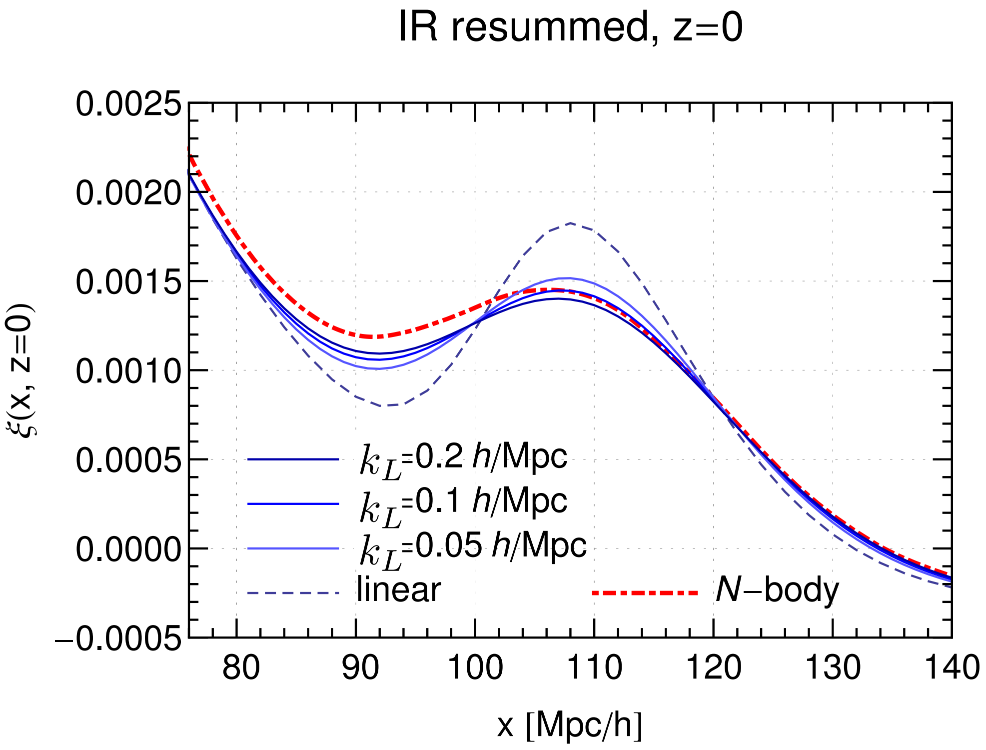
Further developments:

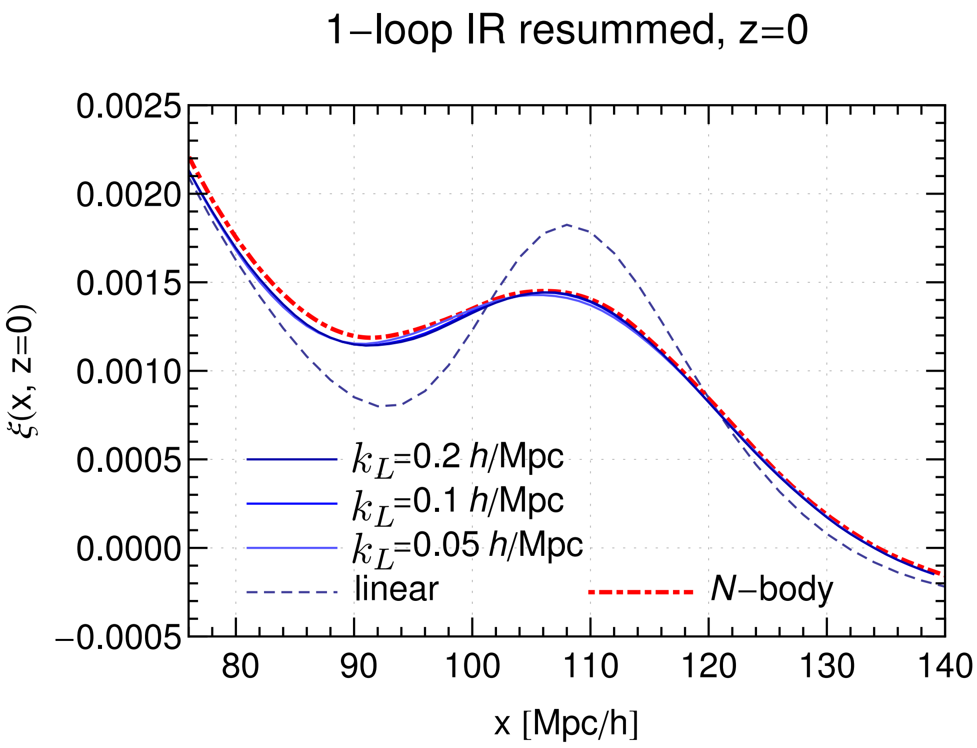
- NLO IR corrections. Important for the shift of BAO peak

Comparison with N-body



Sensitivity to the IR separation scale: LO vs NLO



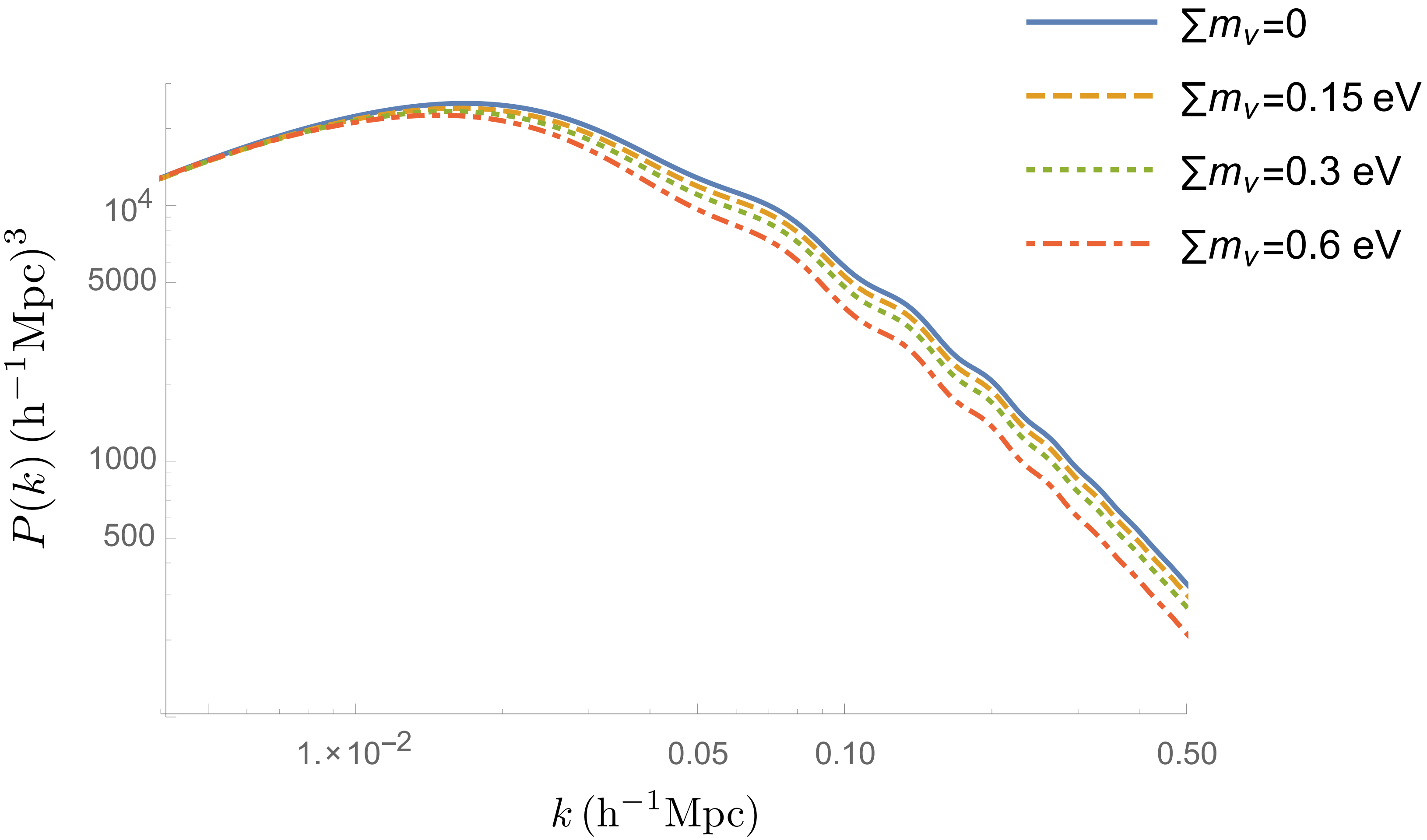


dependence on k_L decreases with the loop order

Residual dependence gives an estimate of the error ~ 2% in the BAO range

BAO and the neutrino mass

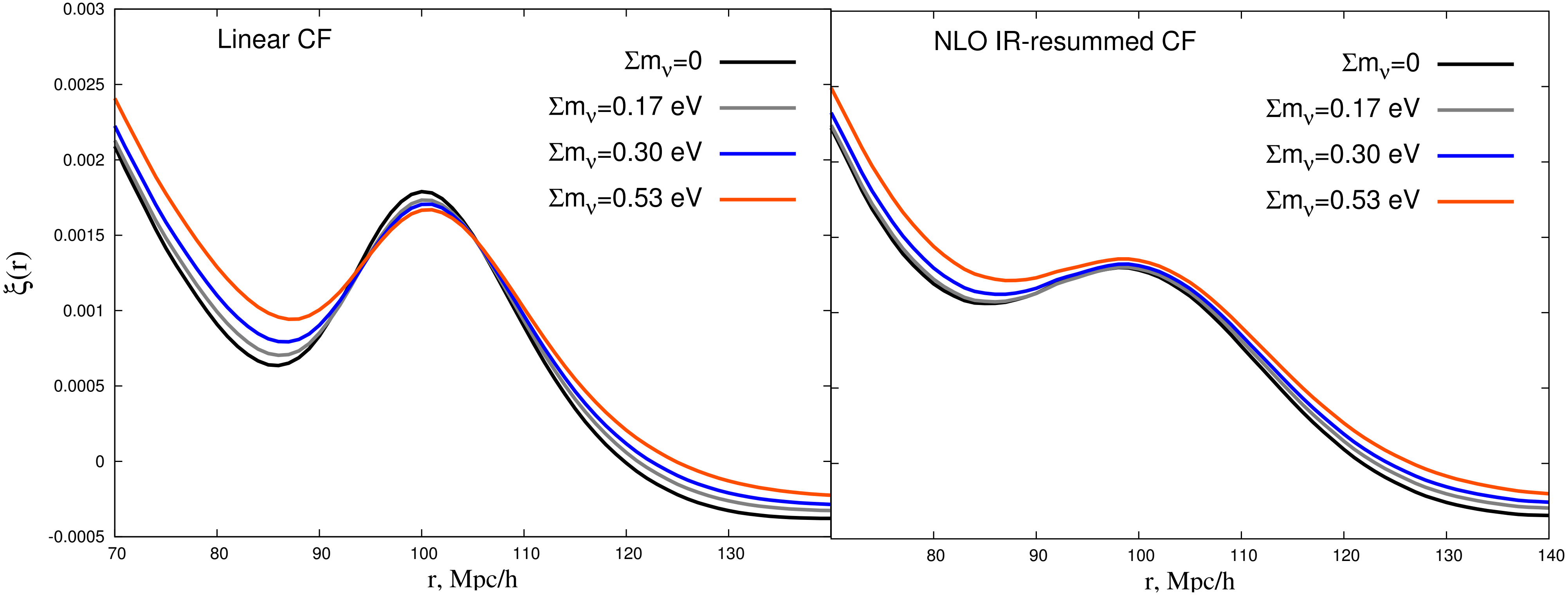
Effect on linear PS:



At $k > 0.05\ \mathrm{h}^{-1}\mathrm{Mpc}$ degenerate with the overall normalization

BAO and the neutrino mass

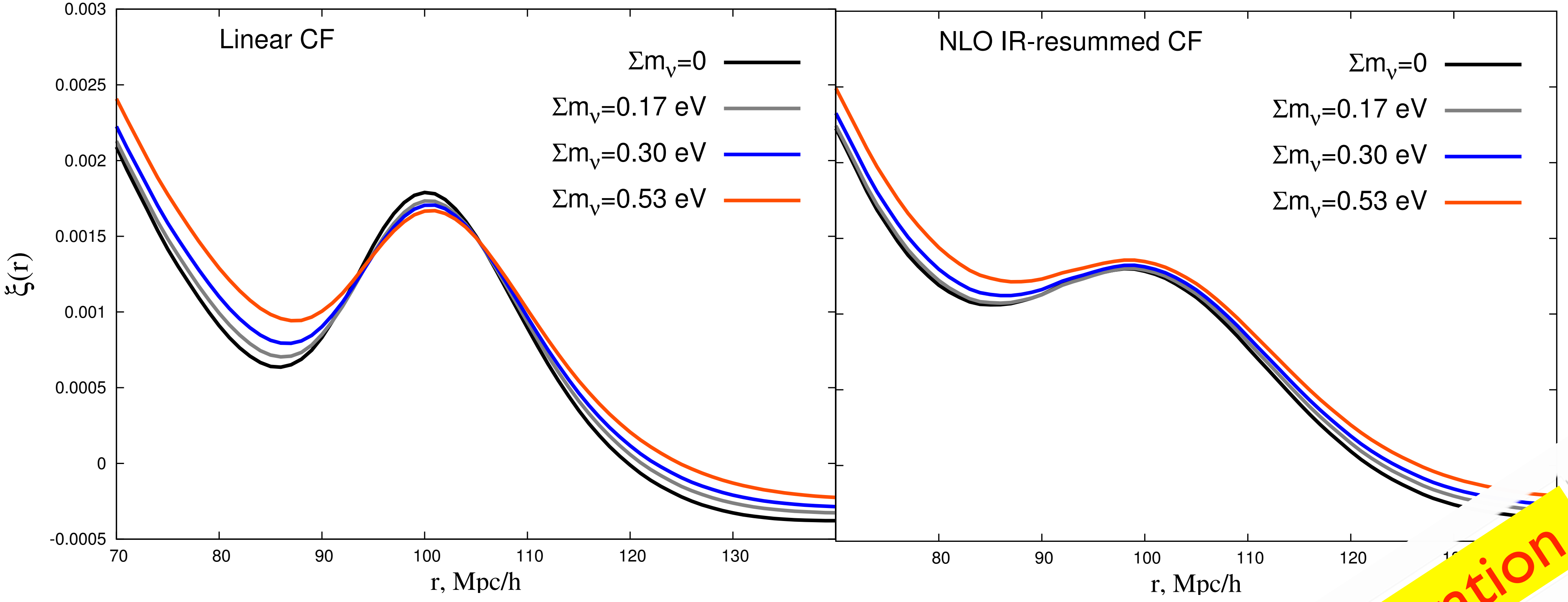
Non-linear effects remove the degeneracy



A probe of m_ν alternative to CMB and Lyα ?

BAO and the neutrino mass

Non-linear effects remove the degeneracy



A probe of m_ν alternative to CMB and Lyα

under investigation

UV renormalization in TSPT

Introduce a cutoff:

$$P(k) \mapsto P^{\Lambda}(k) = \begin{cases} P(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

$$\Gamma_n \mapsto \Gamma_n^{\Lambda}$$

UV renormalization in TSPT

Introduce a cutoff:

$$P(k) \mapsto P^{\Lambda}(k) = \begin{cases} P(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

$$\Gamma_n \mapsto \Gamma_n^{\Lambda}$$

Wilsonian renormalization group:

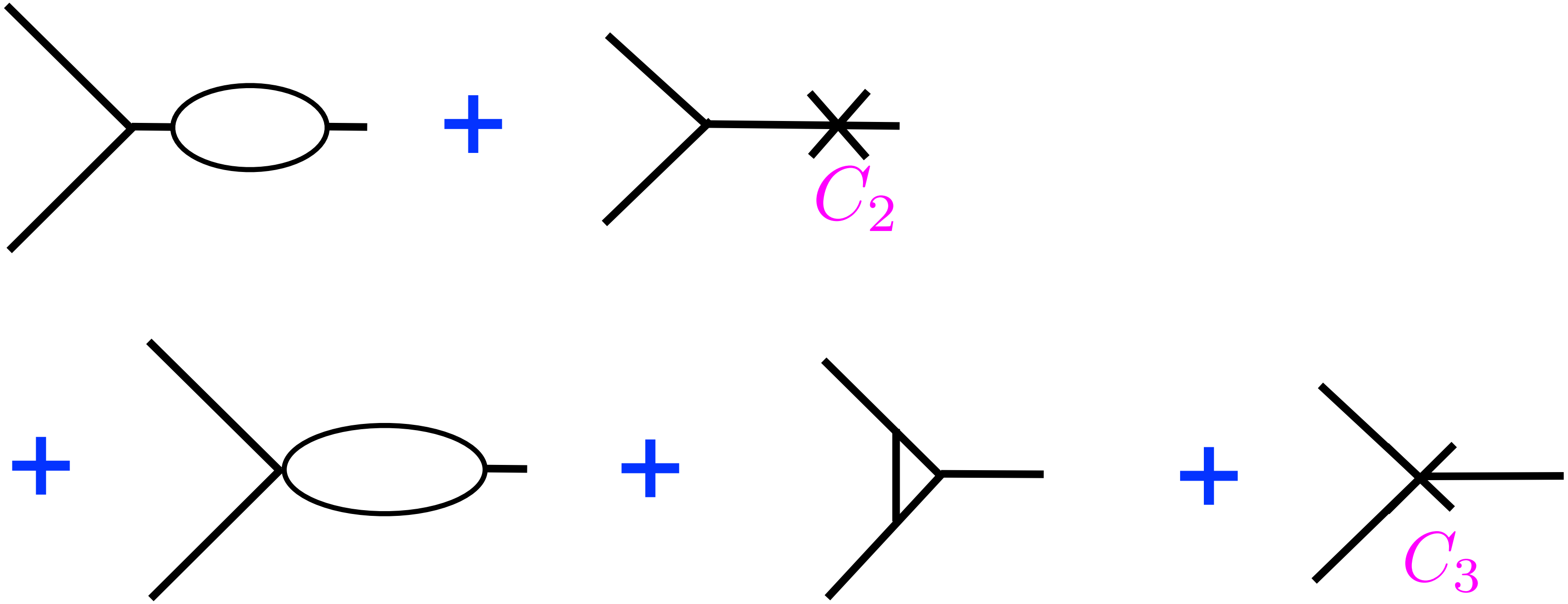
$$\frac{d\Gamma_n^{\Lambda}}{d\Lambda} = \mathcal{F}_n[P^{\Lambda}, \Gamma^{\Lambda}]$$

Boundary conditions = counterterms C_n encapsulating the effects of short modes

UV renormalization in TSPT

- $C_n(\{k\},\tau)$ local in time by construction

- clear separation between PR and PI counterterms



- stochastic contributions are at the same footing as viscous ones

– spatial locality is not manifest

⟺ $C_n(\{k\},\tau)$ are non-polynomial in momenta

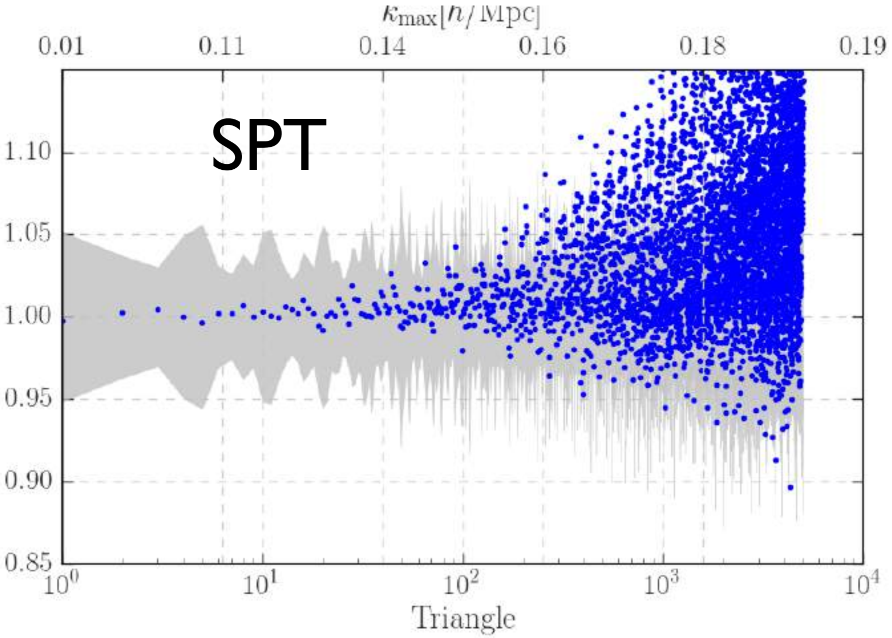
What fixes the structure of the counterterms and how many of them are needed ?

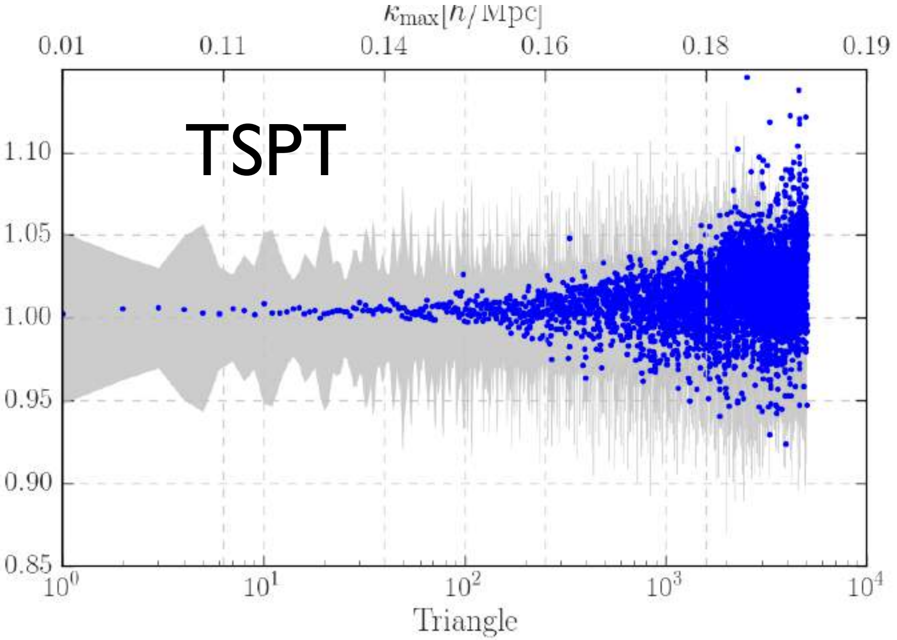
– spatial locality is not manifest

⟺ $C_n(\{k\},\tau)$ are non-polynomial in momenta

What fixes the structure of the counterterms and how many of them are needed ?

Empirically, a single $C_2 \propto k^2/P(k)$ is enough to improve agreement with the N-body data of 2- and 3-point correlators



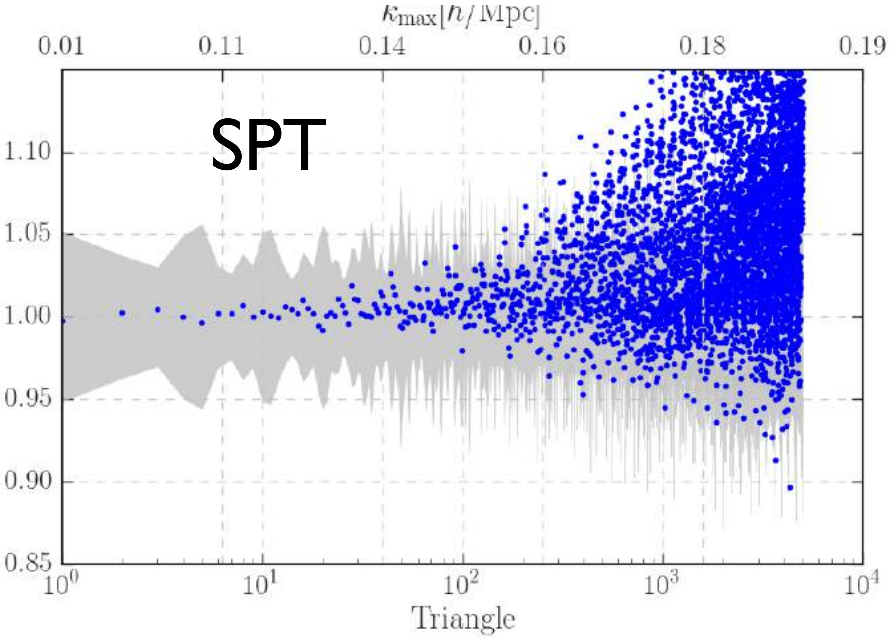


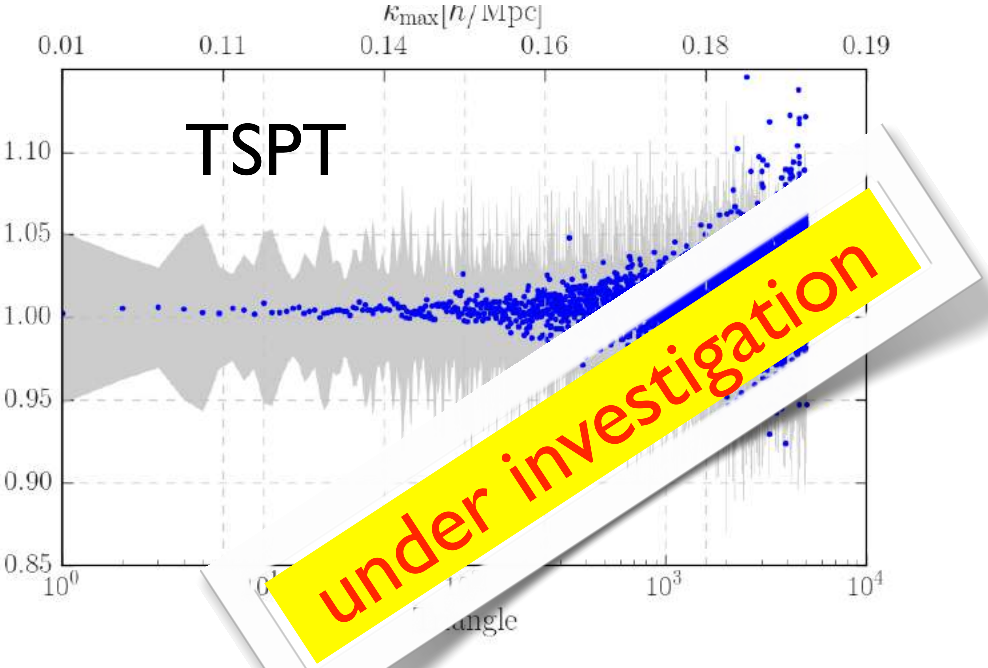
- spatial locality is not manifest

⟺ $C_n(\{k\},\tau)$ are non-polynomial in momenta

What fixes the structure of the counterterms and how many of them are needed ?

Empirically, a single $C_2 \propto k^2/P(k)$ is enough to improve agreement with the N-body data of 2- and 3-point correlators





CiC statistics

CiC statistics

$$\delta_W = \frac{1}{\rho_0} \int d\mathbf{x}\, \underset{\uparrow \atop \frac{3}{4\pi R^3}\theta(R - |\mathbf{x}|)}{W(\mathbf{x})} (\rho(\mathbf{x}) - \rho_0)$$

CiC statistics

$$\delta_W = \frac{1}{\rho_0} \int d\mathbf{x}\, W(\mathbf{x}) (\rho(\mathbf{x}) - \rho_0)$$

$$\frac{3}{4\pi R^3} \theta(R - |\mathbf{x}|)$$

CiC statistics

$$\delta_W = \frac{1}{\rho_0}\int d\mathbf{x}\, \underset{\uparrow}{W(\mathbf{x})}(\rho(\mathbf{x})-\rho_0)$$

$$\frac{3}{4\pi R^3}\theta(R-|\mathbf{x}|)$$

CiC statistics

$$\delta_W = \frac{1}{\rho_0} \int d\mathbf{x}\, W(\mathbf{x})(\rho(\mathbf{x}) - \rho_0)$$

$$\frac{3}{4\pi R^3}\theta(R - |\mathbf{x}|)$$

$$\Downarrow$$

$$P(\delta_W)$$

CiC statistics

$$\delta_W = \frac{1}{\rho_0}\int d\mathbf{x}\, W(\mathbf{x})(\rho(\mathbf{x}) - \rho_0)$$

$$\frac{3}{4\pi R^3}\theta(R - |\mathbf{x}|)$$

$$\boxed{P(\delta_W)}$$

Theoretical framework

... P. Valageas, F. Berdardeau, C. Pichon, ...

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2}\ \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) - \delta_W\right]$$

$$= \int \frac{d\lambda}{2\pi g^2} e^{-\lambda\delta_W/g^2} \int [D\delta(\mathbf{x})] \exp\left[-\frac{1}{g^2}\Gamma[\delta(\mathbf{x})] + \frac{\lambda}{g^2}\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x})\right]$$

Theoretical framework

... P. Valageas, F. Berdardeau, C. Pichon, ...

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2} \; \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x}) \delta(\mathbf{x}) - \delta_W \right]$$

$$= \int \frac{d\lambda}{2\pi g^2} e^{-\lambda \delta_W / g^2} \int [D\delta(\mathbf{x})] \exp\left[-\frac{1}{g^2} \Gamma[\delta(\mathbf{x})] + \frac{\lambda}{g^2} \int d\mathbf{x} W(\mathbf{x}) \delta(\mathbf{x}) \right]$$

formally $g^2 \ll 1$ ➡ use semiclassical expansion (saddle-point approximation, steepest descent)

Theoretical framework

... P. Valageas, F. Berdardeau, C. Pichon, ...

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2} \; \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) - \delta_W \right]$$

$$= \int \frac{d\lambda}{2\pi g^2} e^{-\lambda\delta_W/g^2} \int [D\delta(\mathbf{x})] \exp\left[-\frac{1}{g^2}\Gamma[\delta(\mathbf{x})] + \frac{\lambda}{g^2}\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) \right]$$

formally $g^2 \ll 1$ → use semiclassical expansion (saddle-point approximation, steepest descent)

$$P(\delta_W) = \mathcal{A}\, e^{-\Gamma[\delta_*(\mathbf{x})]/g^2}$$

saddle-point configuration, spherical if so is W

Theoretical framework

... P. Valageas, F. Berdardeau, C. Pichon, ...

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2}\ \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) - \delta_W\right]$$

$$= \int \frac{d\lambda}{2\pi g^2} e^{-\lambda\delta_W/g^2} \int [D\delta(\mathbf{x})] \exp\left[-\frac{1}{g^2}\Gamma[\delta(\mathbf{x})] + \frac{\lambda}{g^2}\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x})\right]$$

formally $g^2 \ll 1$ → use semiclassical expansion (saddle-point approximation, steepest descent)

$$P(\delta_W) = \mathcal{A}\, e^{-\Gamma[\delta_*(\mathbf{x})]/g^2}$$

$$\propto \left(\det \frac{\delta^2\Gamma}{\delta\delta(\mathbf{x})\delta\delta(\mathbf{x}')}\bigg|_{\delta_*} \right)^{-1/2}$$

saddle-point configuration, spherical if so is W

Theoretical framework

... P. Valageas, F. Berdardeau, C. Pichon, ...

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2} \; \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) - \delta_W\right]$$

$$= \int \frac{d\lambda}{2\pi g^2} e^{-\lambda\delta_W/g^2} \int [D\delta(\mathbf{x})] \exp\left[-\frac{1}{g^2}\Gamma[\delta(\mathbf{x})] + \frac{\lambda}{g^2}\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x})\right]$$

formally $g^2 \ll 1$ → use semiclassical expansion (saddle-point approximation, steepest descent)

$$P(\delta_W) = \mathcal{A}\, e^{-\Gamma[\delta_*(\mathbf{x})]/g^2}$$

$$\propto \left(\det \left.\frac{\delta^2\Gamma}{\delta\delta(\mathbf{x})\delta\delta(\mathbf{x}')}\right|_{\delta_*}\right)^{-1/2}$$

saddle-point configuration, spherical if so is W

NB. δ_W can be large → sensitive to nonlinear dynamics of DM

Theoretical framework

... P.Valageas, F. Berdardeau, C. Pichon, ...

$$P(\delta_W) = \int [D\delta(\mathbf{x})] e^{-\Gamma[\delta(\mathbf{x})]/g^2} \; \delta^{(1)}\left[\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) - \delta_W \right]$$

$$= \int \frac{d\lambda}{2\pi g^2} e^{-\lambda\delta_W/g^2} \int [D\delta(\mathbf{x})] \exp\left[-\frac{1}{g^2}\Gamma[\delta(\mathbf{x})] + \frac{\lambda}{g^2}\int d\mathbf{x} W(\mathbf{x})\delta(\mathbf{x}) \right]$$

formally $g^2 \ll 1$ ➡ use semiclassical expansion (saddle-point approximation, steepest descent)

$$P(\delta_W) = \mathcal{A}\, e^{-\Gamma[\delta_*(\mathbf{x})]/g^2}$$

$$\propto \left(\det \frac{\delta^2\Gamma}{\delta\delta(\mathbf{x})\delta\delta(\mathbf{x}')}\bigg|_{\delta_*} \right)^{-1/2}$$

saddle-point configuration, spherical if so i [illegible]

in progress

NB. δ_W can be large ➡ sensitive to nonlinear d [illegible] of DM

Summary and Outlook

- perturbative methods are essential to fully exploit the potential of LSS surveys (m_ν, f_{NL} , properties of DM and DE)
- time-sliced perturbation theory (TSPT) casts the theory of cosmic structure in the language of (3d Euclidean) QFT
- clean derivation of known results and new insights (diagrammatic resummation of IR-enhanced contributions into BAO, UV renormalization à la Wilsonian RG, large deviation statistics as semiclassical approximation)

Summary and Outlook

- perturbative methods are essential to fully exploit the potential of LSS surveys (m_ν, f_{NL} , properties of DM and DE)
- time-sliced perturbation theory (TSPT) casts the theory of cosmic structure in the language of (3d Euclidean) QFT
- clean derivation of known results and new insights (diagrammatic resummation of IR-enhanced contributions into BAO, UV renormalization à la Wilsonian RG, large deviation statistics as semiclassical approximation)
- classification of UV counterterms
- inclusion of "astrophysical" effects (biases, redshift space distortion, baryons)
- comparison with the data, searches for new physics