Three-dimensional Bosonization and Higher-Spin AdS/CFT

Ginzburg Conference on Physics, May 30, 2017

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May, 30, 2017

Main Messages

Chern-Simons vector-models is a rich class of $3d$ conformal (and not only) field theories, which capture some physics too

CS-matter theories exhibit $3d$ bosonization when CS-boson and CS-fermion stay on the opposite sides of CS-coupling Maldacena, Zhiboedov; Giombi et al; Aharony et al; Karch, Tong; Seiberg, Senthil, Wang, Witten; ...

We would like to study these theories and make some tests of the bosonisation conjecture. The results can also be phrased as predictions for quantum higher-spin theories

According to (Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou; Giombi et al) CS-matter CFT's should be dual to higher-spin theories in AdS_4

We also confirm the general higher-spin AdS/CFT duality for the CS-matter theories.

Papers

On the Higher-Spin Spectrum in Large N Chern-Simons Vector Models, Simone Giombi, V.Gurucharan, Volodya Kirilin, Shiroman Prakash, E.S., 1610.08472

Chern-Simons Matter Theories and Higher Spin Gravity, Ergin Sezgin, E.S., Yaodong Zhu, 1705.03197

Bose/Fermi Duality in Three Dimensions

Matter without Chern-Simons

Free Boson. The simplest theory ever

$$S = \int \partial\bar{\phi}^i \partial\phi_i$$

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The list of the simplest $U(N)$-singlet operators is

scalar :	$J_0 = \bar{\phi}^i \phi_i$	$\Delta = 1$
current :	$J_1 = \bar{\phi}^i \overleftrightarrow{\partial} \phi_i$	$\Delta = 2$
stress-tensor :	$J_2 = \bar{\phi}^i \overleftrightarrow{\partial} \overleftrightarrow{\partial} \phi_i + \ldots$	$\Delta = 3$
	$\ldots$	
HS current :	$J_s = \bar{\phi} \overleftrightarrow{\partial}^s \phi + \ldots$	$\Delta = s + 1$

Free theories have exact higher-spin symmetry manifested by conserved tensors. The opposite is also true (Maldacena, Zhiboedov; Boulanger et al; Alba, Diab)

HS Currents and Where to Find Them

Free Boson. The simplest theory ever

$$S = \int \partial\bar{\phi}^i \partial\phi_i$$

HS currents are given by two fields properly decorated by s derivatives (Todorov et al):

$$J = (\hat{\partial}_1 + \hat{\partial}_2)^s C_s^{\frac{d-3}{2}} \left(\frac{\hat{\partial}_1 - \hat{\partial}_2}{\hat{\partial}_1 + \hat{\partial}_2}\right) \bar{\phi}(x_1)\phi(x_2)\Big|_{x_1=x_2=x}$$

where $\hat{\partial}_i = \xi \cdot \partial_i$ and $\xi \cdot \xi = 0$ makes them traceless:

$$J(\xi|x) = \sum_s J_{a_1\ldots a_s}(x)\, \xi^{a_1}\ldots\xi^{a_s}$$

Decoration is provided by Gegenbauer polynomials

Web of Dualities and Bosonization

free boson

Matter without Chern-Simons

Critical Boson. Is the interacting IR fixed-point under $(\bar{\phi}\phi)^2$

$$S = \int \partial\bar{\phi}\partial\phi + \frac{1}{N}(\bar{\phi}\phi)\sigma$$

Can be treated by $1/N$ or $4-\epsilon$ expansions (Ising model etc). In $N=\infty$ limit the spectrum of singlets is almost the same:

scalar : $J_0 = \sigma$ $\Delta = 2 + O(\frac{1}{N})$

current : $J_1 = \bar{\phi}^i \overleftrightarrow{\partial} \phi_i$ $\Delta = 2$

stress-tensor : $J_2 = \bar{\phi}^i \overleftrightarrow{\partial} \overleftrightarrow{\partial} \phi_i + ...$ $\Delta = 3$

$...$

HS current : $J_s = \bar{\phi} \overleftrightarrow{\partial}^s \phi + ...$ $\Delta = s + 1 + O(\frac{1}{N})$

Higher-spin symmetry is broken by loops

Web of Dualities and Bosonization

free boson, $\Delta J_0 = 1$

RG flow

critical boson, $\Delta J_0 = 2$

Matter without Chern-Simons

Free Fermion. The next to the simplest theory

$$S = \int \bar{\psi}^i \slashed{\partial} \psi_i$$

The list of the simplest $U(N)$-singlets is

$$\begin{array}{rlr}
\text{scalar}: & J_0 = \bar{\psi}^i \psi_i & \Delta = 2 \\
\text{current}: & J_1 = \bar{\psi}^i \gamma \psi_i & \Delta = 2 \\
\text{stress-tensor}: & J_2 = \bar{\psi}^i \gamma \overleftrightarrow{\partial} \psi_i + ... & \Delta = 3 \\
 & ... & \\
\text{HS current}: & J_s = \bar{\psi} \gamma \overleftrightarrow{\partial}^{s-1} \psi + ... & \Delta = s + 1
\end{array}$$

Has exact higher-spin symmetry manifested by conserved tensors.

Web of Dualities and Bosonization

free boson, $\Delta J_0 = 1$

RG flow

critical boson, $\Delta J_0 = 2$

free fermion, $\Delta J_0 = 2$

Matter without Chern-Simons

Critical Fermion (Gross-Neveu). UV fixed-point under $(\bar{\psi}\psi)^2$

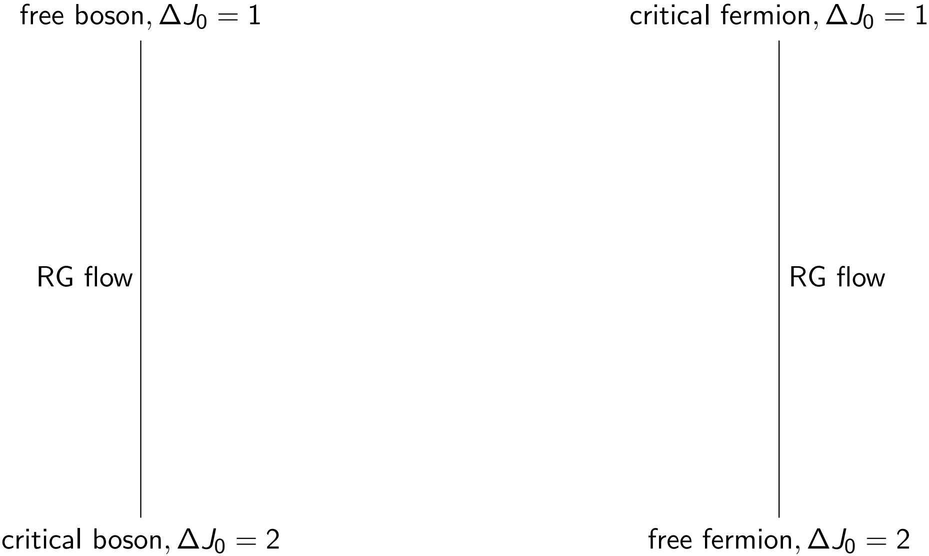
$$S = \int \bar{\psi}\not{\partial}\psi + \frac{1}{N}(\bar{\psi}\psi)\sigma$$

Can be treated by $2+\epsilon$ or large-N methods (chiral phase transition). In $N=\infty$ limit the singlets are almost the same:

$$\begin{array}{rlll}
\text{scalar}: & J_0 = \sigma & & \Delta = 1 + O(\frac{1}{N}) \\
\text{current}: & J_1 = \bar{\psi}^i \gamma \psi_i & & \Delta = 2 \\
\text{stress-tensor}: & J_2 = \bar{\psi}^i \gamma \overleftrightarrow{\partial} \psi_i + ... & & \Delta = 3 \\
& ... & & \\
\text{HS current}: & J_s = \bar{\psi} \gamma \overleftrightarrow{\partial}^{s-1} \psi + ... & & \Delta = s + 1 + O(\frac{1}{N})
\end{array}$$

Higher-spin symmetry is broken by loops

Web of Dualities and Bosonization



free boson, $\Delta J_0 = 1$

critical fermion, $\Delta J_0 = 1$

RG flow

RG flow

critical boson, $\Delta J_0 = 2$

free fermion, $\Delta J_0 = 2$

The theories group in pairs: all have HS currents, but $\Delta J_0 = 1, 2$.

Chern-Simons without Matter

Action for $U(N)$ Chern-Simons at level k is

$$S = \frac{ik}{4\pi} S_{\mathrm{CS}}$$
$$S_{\mathrm{CS}} = \int d^3x \epsilon^{\mu\nu\rho} \mathrm{Tr}(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho)\,.$$

- this is a topological field theory → the spectrum of local operators should not change much;
- k does not renormalize;
- breaks parity;
- is a building block of many other theories;
- level rank duality;

Chern-Simons with Matter

CS Boson.

$$S = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \left(D_\mu \bar{\phi} D^\mu \phi + \frac{\lambda_6}{N^2} (\bar{\phi}\phi)^3 \right)$$

Critical CS-Boson.

$$S_{\mathrm{crit}} = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \left(D_\mu \bar{\phi} D^\mu \phi + \frac{1}{N} \sigma_b \bar{\phi}\phi \right)$$

CS Fermion.

$$S = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \bar{\psi} \not{D} \psi$$

Critical CS Fermion.

$$S_{\mathrm{crit}} = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \left(\bar{\psi} \not{D} \psi + \frac{1}{N} \sigma_f \bar{\psi}\psi + g_6 \sigma_f^3 \right) ,$$

Web of Dualities and Bosonization

$$\begin{tikzcd}
\text{free boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"'] & \text{critical fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"] \\
\text{critical boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] & \text{free fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"]
\end{tikzcd}$$

Basic Properties of CS-Matter

- have large-N expansion for t'Hooft $\lambda = N/k$ fixed;
- non-SUSY CFT's with a line of fixed-points;
- solvable for any λ at large N (Giombi et al; many others);
- the spectrum of single trace operators is the 'same':

$$J_0 : \quad \Delta_0 = 1(2) + O(\frac{1}{N}) \quad J_s : \quad \Delta_s = s + 1 + O(\frac{1}{N})$$

- have an approximate higher-spin symmetry;
- parity is broken in general;
- describe some physics sometimes;
- exhibit a phenomenon of three-dimensional bosonization: CS-boson goes over into CS-fermion;
- should be dual to higher-spin theories in AdS_4 with $g \sim \frac{1}{N}$ and a parameter θ responsible for the violation of parity.

Higher-Spin Currents

We will study higher-spin currents at LO in $1/N$ but to all orders in $\lambda = N/k$. HS currents are responsible for their own non-conservation:

$$\partial \cdot J_s = \sum_{s_1,s_2} C_{s,s_1,s_2}(\lambda) \frac{1}{N} J_{s_1} J_{s_2} + F(\lambda) \frac{1}{N^2} JJJ$$

which is an exact non-perturbative quantum equation.

We will use the results of Maldacena-Zhiboedov and explicit computations of the two-point functions in order to recover the λ-dependence in $C(\lambda)$.

Then we will work out the spin-dependence using the equations of motion of CS-matter theories and compute anomalous dimensions with the help of Anselmi trick: one-loop result from zero-loop

Slightly Broken HS Symmetry

In $3d$ any 3-point function of HS currents can be decomposed

$$\langle J_{s_1} J_{s_2} J_{s_3}\rangle = \langle J_{s_1} J_{s_2} J_{s_3}\rangle_b + \langle J_{s_1} J_{s_2} J_{s_3}\rangle_f + \langle J_{s_1} J_{s_2} J_{s_3}\rangle_o$$

into structures built from free boson, fermion and an odd one.

Maldacena, Zhiboedov found out that there can be two coupling constants only $\tilde{\lambda}$, $\tilde{N}$ $(\cos^2\theta = 1/(1+\tilde{\lambda}^2))$:

$$\langle J_{s_1} J_{s_2} J_{s_3}\rangle = \tilde{N}\left(\cos^2\theta\langle J_{s_1} J_{s_2} J_{s_3}\rangle_b + \sin^2\theta\langle J_{s_1} J_{s_2} J_{s_3}\rangle_f + \cos\theta\sin\theta\langle J_{s_1} J_{s_2} J_{s_3}\rangle_o\right)$$

where $\langle TT\rangle \sim \tilde{N}$ counts effective degrees of freedom and $\tilde{\lambda}$ is a measure of the HS symmetry violation:

$$\partial\cdot J_4 = \tilde{\lambda}\left(J_2\partial J_0 - \frac{2}{5}\partial J_2 J_0\right)$$

Anomalous dimensions

CS-matter are well-defined QFT and anomalous dimensions of J_s can be computed by standard methods

We can gain one-loop if multiplet recombination occurs

$$\partial \cdot J = g\, K \qquad\qquad K \sim JJ \quad \text{in CS-matter}$$

Non-conservation can be checked in to different ways

$$\begin{tikzcd}
\langle JJ \rangle \sim \dfrac{1}{(x^2)^{\Delta_0 + g\gamma}} \arrow[r, "\text{conservation}"] \arrow[d, "\text{non-conservation}"'] & \overrightarrow{\partial} \cdot \langle JJ \rangle \cdot \overleftarrow{\partial} \sim g\gamma \arrow[d] \\
\langle \partial \cdot J\, \partial \cdot J \rangle = g^2 \langle KK \rangle \arrow[r] & \gamma \sim g \dfrac{\langle KK \rangle}{\langle JJ \rangle}
\end{tikzcd}$$

Anomalous Dimensions: Regular CS-Boson/Fermion

Combining everything together we find for $\Delta = s + 1 + \gamma_s$:

$$\gamma_s = \frac{1}{\tilde{N}} \left(a_s \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2} \right) + O(\frac{1}{N^2})$$

where

$$a_s = \begin{cases} \frac{16}{3\pi^2}\frac{s-2}{2s-1}\,, & \text{for even } s\,,\\ \frac{32}{3\pi^2}\frac{s^2-1}{4s^2-1}\,, & \text{for odd } s\,, \end{cases}$$

$$b_s = \begin{cases} \frac{2}{3\pi^2}\left(3g(s) + \frac{-38s^4+24s^3+34s^2-24s-32}{4s^4-5s^2+1}\right)\,, & \text{for even } s\,,\\ \frac{2}{3\pi^2}\left(3g(s) + \frac{20-38s^2}{4s^2-1}\right)\,, & \text{for odd } s\,, \end{cases}$$

with

$$g(s) = \sum_{n=1}^{s} \frac{1}{n-1/2} = \gamma - \psi(s) + 2\psi(2s) = H_{s-1/2} + 2\log(2)\,,$$

Anomalous Dimensions: Critical CS-Boson/Fermion

Combining everything together we find for $\Delta = s+1+\gamma_s$:

$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s\frac{1}{1+\tilde{\lambda}^2}+b_s\frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right)+O(\frac{1}{N^2})$$

where

$$a_s = \begin{cases} \frac{16}{3\pi^2}\frac{s-2}{2s-1}\,, & \text{for even } s\,, \\ \frac{32}{3\pi^2}\frac{s^2-1}{4s^2-1}\,, & \text{for odd } s\,, \end{cases}$$

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with

$$g(s)=\sum_{n=1}^{s}\frac{1}{n-1/2}=\gamma-\psi(s)+2\psi(2s)=H_{s-1/2}+2\log(2)\,,$$

Important Features

$$\gamma_s = \frac{1}{\tilde{N}} \left(a_s \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2} \right) + O(\frac{1}{N^2})$$

- independent computations for bosons and fermions give the same answer! Therefore the bosonization is confirmed;
- $b_s \sim J_{s_1} J_{s_2}$ the computations are identical in free/critical cases — σ-lines are suppressed;
- more non-trivially, a_s are the same in the dual theories;
- even strongly, the full non-conservation operators $\partial \cdot J = JJ + ...$ can be mapped into each other;
- there is $\gamma_s \sim \log s$ behaviour, which is expected for gauge theories in general (Alday, Maldacena);

Important Features

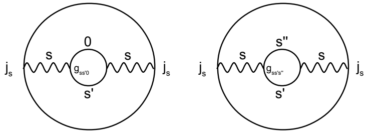
The answer for the critical cases is dual under $\lambda \leftrightarrow \lambda^{-1}$:

$$\gamma_s = \frac{1}{\tilde{N}}\left(a_s \frac{1}{1+\tilde{\lambda}^2} + b_s \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2}\right) + O(\frac{1}{N^2})$$

At weak coupling $\tilde{\lambda} \rightarrow$ we recover the Wilson-Fisher and Gross-Neveu anomalous dimensions:

$$\gamma_s^{\text{W.F.}} = \gamma_s^{\text{GN}} = \frac{1}{2N} a_s = \begin{cases} \frac{8}{3N\pi^2}\frac{s-2}{2s-1}\,, & \text{for even } s\,, \\ \frac{16}{3N\pi^2}\frac{s^2-1}{4s^2-1}\,, & \text{for odd } s\,. \end{cases}$$

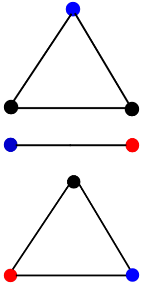
which is the same as the strong limit $\tilde{\lambda} \rightarrow \infty$ of the regular CS-matter theories.



This should correspond to one-loop corrections to the masses of higher-spin fields on the AdS side.

Three-Point Functions

in $3d$ correlators of tensor operators can always be expressed in terms of conformally-invariant P, Q, S:



Q^i_{jk} : $\langle OOJ_s\rangle \sim Q^s$

P_{ij} : $\langle J_s J_s\rangle \sim P^s$

S^i_{jk} : parity-odd

we fixed the parity-odd structures as we had to deal with them:

$$\langle j_2 j_{\tilde{0}} j_4\rangle = -\tilde{N}\tilde{\lambda}\frac{1}{|x_{12}||x_{23}||x_{31}|}\frac{iQ_3^2 S_2\left(4P_2^2+Q_1Q_3\right)}{16\pi^4}$$

AdS/CFT
Chern-Simons Matter vs. Higher-Spin Theories

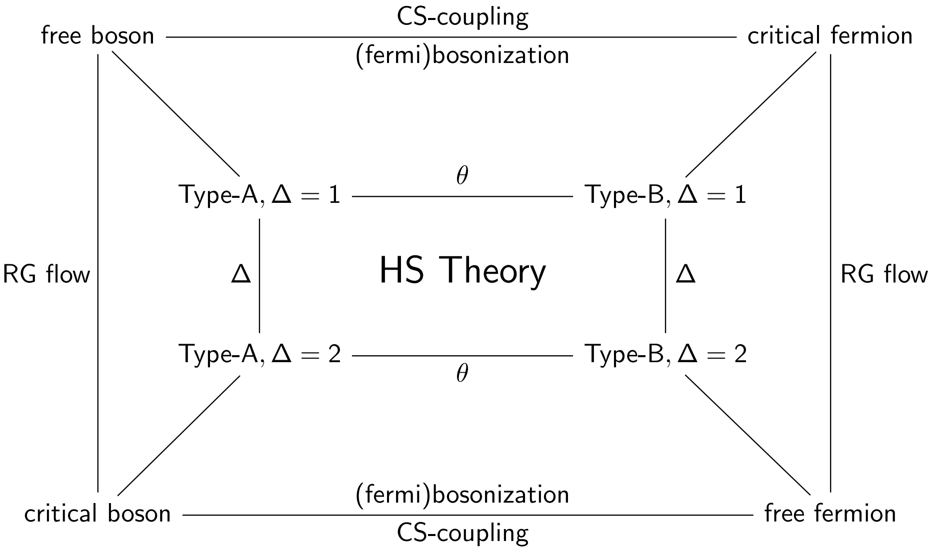
Higher-spin theories

AdS/CFT dictionary between CS-matter and HS theories

$$\begin{array}{rcl} O & \rightarrow & \phi \\ J_a & \rightarrow & A_\mu \\ T_{ab} & \rightarrow & g_{\mu\nu} \\ j_{a_1\ldots a_s} & \rightarrow & \Phi_{\mu_1\ldots\mu_s} \end{array}$$

- the minimal multiplet is still infinite $s = 0, (1), 2, (3), 4, \ldots$, but much smaller than in string theory;
- within AdS/CFT paradigm, higher-spin theories should be dual to CFT with vectorial matter: free/critical bosonic/fermionic vector-models. Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou; Giombi et al.

Web of Dualities and Bosonization



free boson

CS-coupling

(fermi)bosonization

critical fermion

Type-A, $\Delta = 1$

θ

Type-B, $\Delta = 1$

RG flow

Δ

HS Theory

Δ

RG flow

Type-A, $\Delta = 2$

θ

Type-B, $\Delta = 2$

critical boson

(fermi)bosonization

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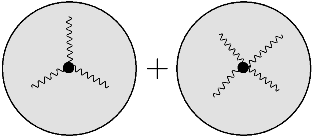
free fermion

Higher-Spin AdS/CFT

Higher-Spin Theories: Action

There are encouraging results on the action principle: (Kessel, Lucena-Gómez, E.S., Taronna; Bekaert, Ponomarev, Sleight, Erdmenger); in particular, the full cubic action is known in any *d* for the dual of Free Boson (Sleight, Taronna) and $0-0-0-0$ in $4d$ (Bekaert, Ponomarev, Sleight, Erdmenger)



This can be combined with off-shell vertices of Francia, Lo Monaco, Mkrtchyan

Higher-Spin Theories

Formally consistent equations are unfolded e.o.m. that are available directly at the 2nd order (Vasiliev, 88, 89) or can be extracted from the Vasiliev equations (Vasiliev, 90, 91)

$$\begin{aligned} d\omega &= \omega \star \omega + \mathcal{V}(\omega, \omega, C) + \mathcal{V}_2(\omega, \omega, C, C) + ... \\ dC &= \omega \star C - C \star \tilde{\omega} + \mathcal{U}(\omega, C, C) + ... \end{aligned}$$

ω — one-form in HS algebra; contains Yang-Mills potential A_μ, vielbein e^a_μ, spin-connection $\omega^{a,b}_\mu$, ...;

C— zero-form in HS algebra; contains scalar field, Maxwell $F_{\mu\nu}$, Weyl tensor, ...

HS algebra is $2d$ free QM particle or $2d$ QM harmonic oscillator $f(q,p)$: $[q^i, p_j] = \delta^i_j$, $i, j = 1, 2$

Correlators from Equations

Tree-level AdS/CFT correlators can be extracted from e.o.m.

The equations are overdetermined, it is enough to look at

$$dC^{(2)} = \omega \star C - C \star \tilde{\omega} + \mathcal{U}(h, C, C)$$

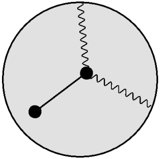
Upon solving for auxiliary fields

$$\Phi_s, ..., \nabla^{s-1}\Phi_s \in \omega \qquad\qquad \nabla^{s+\cdots}\Phi_s \in C$$

we have almost Φ^3-theory e.o.m.:

$$(\Box - \Lambda)\Phi^{(2)} = \Phi\nabla...\nabla\Phi = V(\Phi, \Phi)$$

$$\langle JJJ\rangle =$$



$$\sim \int_{AdS} \Phi V(\Phi, \Phi)$$

Old Results

Assuming that the structure is ok, compare prefactor for $0 - s_1 - s_2$ (Giombi, Yin, 09) based on ωC;

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the CC part of the equations is too non-local to be treated by field theory methods (Boulanger, Kessel, E.S., Taronna, 2015), which is consistent with above; after non-local redefinition e.o.m. (Vasiliev, 16) are free of non-localities; non-local redefinitions are dangerous (Prokushkin, Vasiliev, 98; Taronna, 16) and can give any desired or undesired result

New Results

Scalarizing the bulk integral, using the inversion trick we have

$$\begin{aligned}\langle j_{s_1} j_{s_2} j_0 \rangle_{Q.B.} &= \tilde{N}\left[\cos\theta \langle j_{s_1} j_{s_2} j_0 \rangle_{f.b.} + \sin\theta \langle j_{s_1} j_{s_2} j_0 \rangle_{odd}\right] \ , \\ \langle j_{s_1} j_{s_2} \tilde{j}_0 \rangle_{Q.F.} &= \tilde{N}\left[\cos\theta \langle j_{s_1} j_{s_2} \tilde{j}_0 \rangle_{f.f.} + \sin\theta \langle j_{s_1} j_{s_2} \tilde{j}_0 \rangle_{odd}\right] \ .\end{aligned}$$

where the free CFT's correlators are easy to give, e.g.

$$\langle j_{s_1} j_{s_2} j_0 \rangle_{f.b.} = \frac{1}{\mathrm{x}_{12}\mathrm{x}_{13}\mathrm{x}_{23}} \exp\left(\tfrac{i}{2} Q_1 + \tfrac{i}{2} Q_2\right) \cos P_{12}$$

and the 'odd' correlators — critical vector model and Gross-Neveu — are also correctly reproduced!

The CC terms for $0 - s - s$ from the new HS (Vasiliev, 16) equations agree with AdS/CFT too

Recent results: (Didenko, Vasiliev) and (Bonezzi, Boulanger, De Filippi, Sundell)

Summary

There is a rich class of bosonic/fermionic models — Chern-Simons matter theories that exhibit bose/fermi duality and we confirmed the duality based on anomalous dimensions of higher-spin currents

These theories are AdS/CFT dual to $4d$ higher-spin theories that contains massless fields with spins $s = 0, 1, 2, 3, ...$

The modified at the 2nd order HS equations seem to be ok, but still should be extended to all orders and other dimensions

The tests of the higher-spin AdS/CFT dualities are extended to the parity odd cases including the full structure of the correlators in free/critical boson/fermion models

Indicates that n-point, $n > 3$ correlators in CS-matter are of the same form: $\cos\theta/\sin\theta$ in front of several fixed structures