Spinning Witten Diagrams

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based on work with M. Taronna

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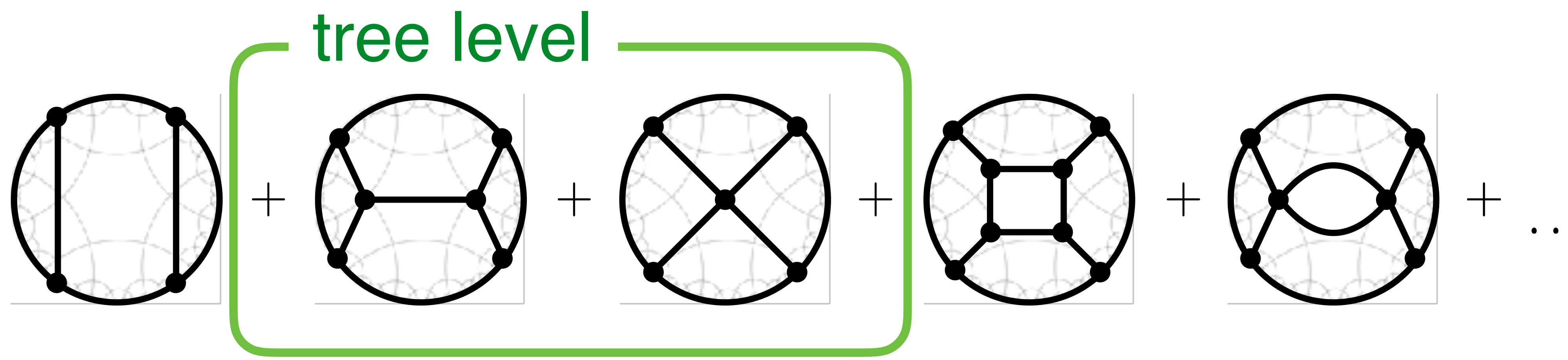
Introduction

[Gubser et al., Witten]

CFT$_d$ correlation functions ⟷ AdS$_{d+1}$ scattering processes

$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle =$



- For external scalars: Well understood how to evaluate

  [`98 Mück et al., Freedman et al., Liu et al., Arutyunov et al., Petkou et al., …]

- More recently, the underlying structure: CPWE of Witten diagrams

  [Costa et al.; Bekaert, Erdmenger, Ponomarev, C.S.; Hijano et al.]

Introduction

[Gubser et al., Witten]

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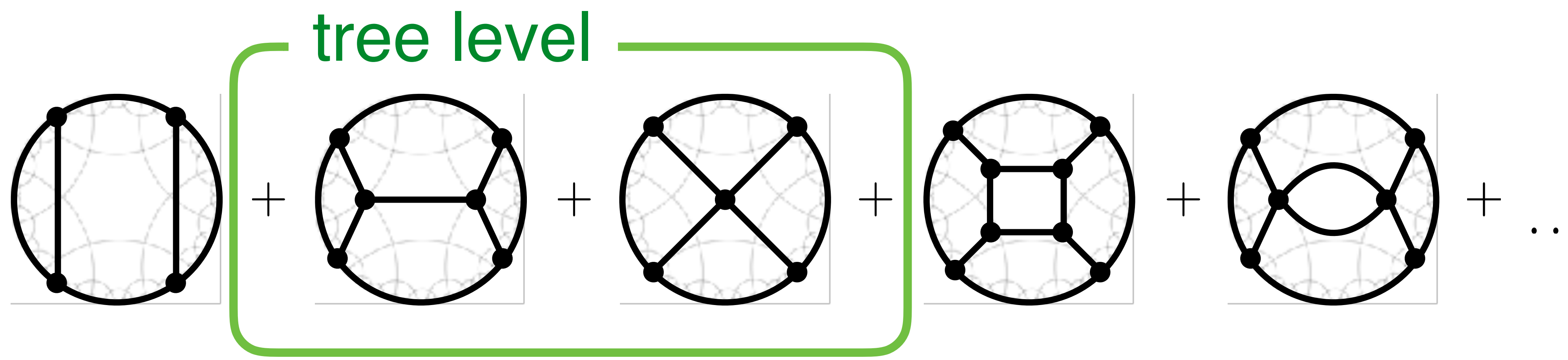

Today: With spinning external (& internal) legs!

- For external scalars: Well understood how to evaluate
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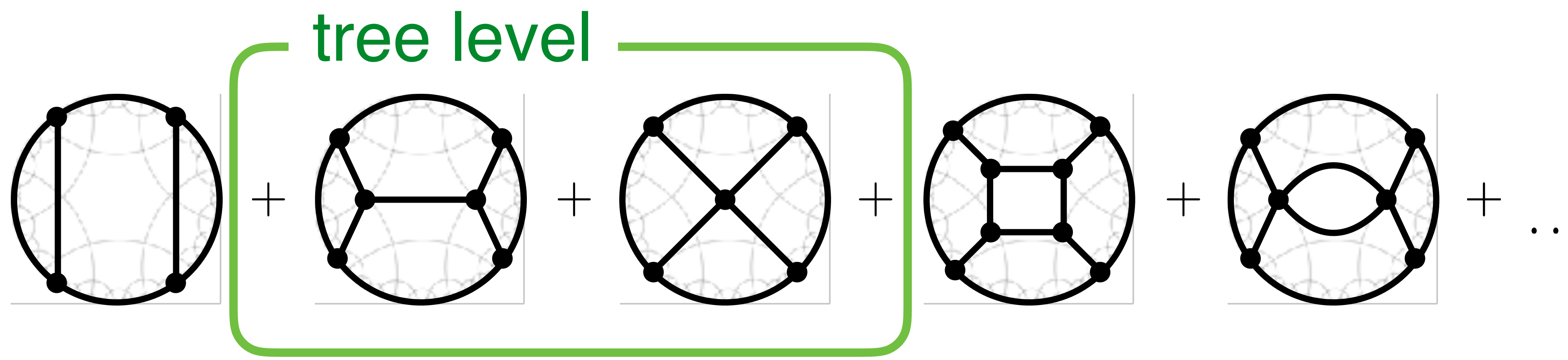


Today: Spinning external (& internal) legs!

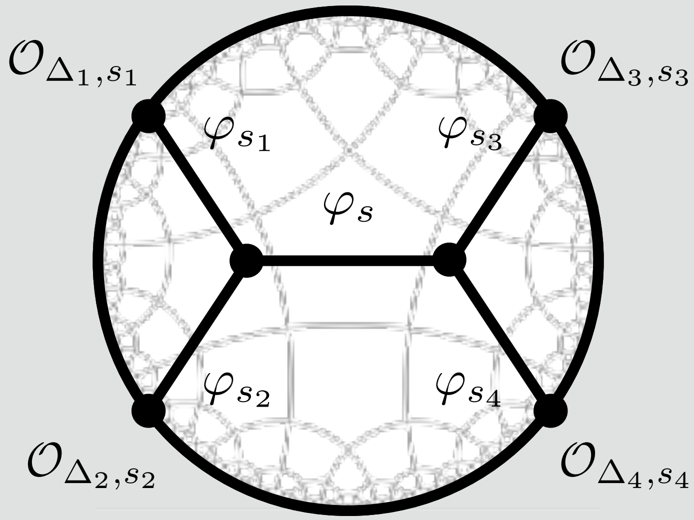
Motivations:

- Holographic reconstruction
- Conformal Bootstrap

Outline

- Conformal Partial Wave Expansion (CPWE) of Spinning Witten Diagrams
- Spinning 3pt
- Spinning 4pt exchange diagrams

Goal

Decompose  into Conformal Partial Waves

Totally symmetric external fields φ_{s_i} and internal field φ_s

Field - Operator map:

single-trace operators $\longleftrightarrow$ **bulk fields (single-particles)**

$\mathcal{O}_{\Delta,s}$ (Scaling dim. Δ, spin s) $\longleftrightarrow$ φ_s (spin s)

mass: $m_s^2 R^2 = \Delta\left(\Delta - d\right) - s$

General Approach

CFT side.

General Approach

Review: CPWE of 4pt CFT correlators

Δ and s = scaling dim. and spin of $\mathcal{O}$

$$\langle \mathcal{O}_1(y_1)\,\mathcal{O}_2(y_2)\,\mathcal{O}_3(y_3)\,\mathcal{O}_4(y_4)\rangle = \sum_{\{\Delta,s\}} \mathsf{c}_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2}\mathsf{c}_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4} W_{\Delta,s}(y_1,y_2;y_3,y_4)$$

- Operator Product Expansion (OPE) coefficients

$$\mathcal{O}_1 \times \mathcal{O}_2 \sim \mathsf{c}_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2}\mathcal{O} + \ldots \quad \text{and} \quad \mathcal{O}_3 \times \mathcal{O}_4 \sim \mathsf{c}_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4}\mathcal{O} + \ldots$$

- Conformal Partial Waves (CPWs) $W_{\Delta,s}$:

  — Re-sums contributions from $\mathcal{O}$ and all descendants $\partial ... \partial\mathcal{O}$

  — For external scalars: Explicit expressions in terms of elementary functions only possible in even d

  — For spinning external operators: Comparably little known

General Approach

Shadow Formalism [Ferrara et al.]

$$\tilde{\mathcal{O}}(y) = \kappa \int d^d y' \frac{I(y-y')}{\left[(y-y')^2\right]^{d-\Delta}} \mathcal{O}(y')$$

Primary operator		Shadow operator
$\mathcal{O}$	$\longrightarrow$	$\tilde{\mathcal{O}}$
$\Delta,\, s$		$d-\Delta,\, s$

Projector: $\mathcal{P}_{\Delta,s} = \kappa \int d^d y\, \mathcal{O}(y)\, |0\rangle\langle 0| \tilde{\mathcal{O}}(y)$

- dimensionless
- conformally invariant

Projects onto multiplet of $\mathcal{O}$ and $\tilde{\mathcal{O}}$:

→ $\langle \mathcal{O}_1(y_1)\, \mathcal{O}_2(y_2)\, \mathcal{P}_{\Delta,s} \mathcal{O}_3(y_3)\, \mathcal{O}_4(y_4)\rangle = \mathsf{c}_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2} \mathsf{c}_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4} W_{\Delta,s}(y_1, y_2; y_3, y_4) + \text{shadow}$

↓

$\kappa \int d^d y \,\langle \mathcal{O}_1(y_1)\, \mathcal{O}_2(y_2)\, \mathcal{O}(y)\rangle \langle \tilde{\mathcal{O}}(y)\, \mathcal{O}_3(y_3)\, \mathcal{O}_4(y_4)\rangle$ factorisation into 3pt correlators!

General Approach

Shadow Formalism [Ferrara et al.]

$$\tilde{\mathcal{O}}\left(y\right)=\kappa\int d^{d}y'\,\frac{I\left(y-y'\right)}{\left[\left(y-y'\right)^{2}\right]^{d-\Delta}}\mathcal{O}\left(y'\right)$$

Primary operator		Shadow operator
$\mathcal{O}$	$\longrightarrow$	$\tilde{\mathcal{O}}$
$\Delta,\, s$		$d-\Delta,\, s$

3pt conformal structure

[notation: $\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}\rangle = c_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2}\langle\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}\rangle\rangle$]

Leads to integral expression for CPWs:

$$W_{\Delta,s}\,+\,W_{d-\Delta,s}\;=\;\int d^{d}y\,\langle\langle\mathcal{O}_{1}\mathcal{O}_{2}\mathcal{O}\left(y\right)\rangle\rangle\langle\langle\tilde{\mathcal{O}}\left(y\right)\mathcal{O}_{3}\mathcal{O}_{4}\rangle\rangle$$

Useful because:

- Universal: Holds for any d, any $\mathcal{O}$ and any external primary operators $\mathcal{O}_i$
- In terms of 3pt conformal structures
- Naturally arise from harmonic function decomposition of Witten diagrams

General Approach

Shadow Formalism for Spinning CPWs

$$\tilde{\mathcal{O}}(y) = \kappa \int d^d y' \frac{I(y-y')}{\left[(y-y')^2\right]^{d-\Delta}} \mathcal{O}(y')$$

$$W^{\mathbf{m},\mathbf{n}}_{\Delta,s} + W^{\mathbf{m},\mathbf{n}}_{d-\Delta,s} = \kappa \int d^d y \, \langle\langle \mathcal{O}_{\Delta_1,s_1} \mathcal{O}_{\Delta_2,s_2} \mathcal{O}(y) \rangle\rangle^{(\mathbf{m})} \langle\langle \tilde{\mathcal{O}}(y) \mathcal{O}_{\Delta_3,s_3} \mathcal{O}_{\Delta_4,s_4} \rangle\rangle^{(\mathbf{n})}$$

[notation: $\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O} \rangle = \mathsf{c}_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2} \langle\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O} \rangle\rangle$]

Basis of 3pt spinning conformal structures:

$\tau_i = \Delta_i - s_i$

$$\langle\langle \mathcal{O}_{\Delta_1,s_1}(y_1) \mathcal{O}_{\Delta_2,s_2}(y_2) \mathcal{O}_{\Delta_3,s_3}(y_3) \rangle\rangle^{(\mathbf{n})} = \frac{\mathsf{Y}_1^{s_1-n_2-n_3} \mathsf{Y}_2^{s_2-n_1-n_3} \mathsf{Y}_3^{s_3-n_1-n_2} \mathsf{H}_1^{n_1} \mathsf{H}_2^{n_2} \mathsf{H}_3^{n_3}}{(y_{12}^2)^{\frac{\tau_1+\tau_2-\tau_3}{2}} (y_{13}^2)^{\frac{\tau_1+\tau_3-\tau_2}{2}} (y_{23}^2)^{\frac{\tau_2+\tau_3-\tau_1}{2}}}$$

$\mathbf{n} = (n_1, n_2, n_3)$ $\quad 0 \leq n_1 \leq \min\{s_2 - n_3, s_3 - n_2\}$ $\quad 0 \leq n_2 \leq \min\{s_1 - n_3, s_3\}$ $\quad 0 \leq n_3 \leq \min\{s_1, s_2\}$

Six conformally covariant building blocks:

$$(\mathsf{Y}_i)_\mu = \frac{(y_i - y_{i+1})_\mu}{(y_i - y_{i+1})^2} - \frac{(y_i - y_{i+2})_\mu}{(y_i - y_{i+2})^2}, \qquad (\mathsf{H}_i)_{\mu\nu} = \frac{1}{(y_{i+1} - y_{i+2})^2} \left(\delta_{\mu\nu} + 2 \frac{(y_{i+1} - y_{i+2})_\mu (y_{i+1} - y_{i+2})_\nu}{(y_{i+1} - y_{i+2})^2} \right)$$

$i = 1, 2, 3 \quad i \cong i+3$

General Approach

Projecting out the shadow

$$\tilde{\mathcal{O}}(y) = \kappa \int d^d y' \frac{I(y-y')}{\left[(y-y')^2\right]^{d-\Delta}} \mathcal{O}(y')$$

Spectral representation

$$W_{\Delta,s} = \int_{-\infty}^{\infty} \frac{d\nu}{\nu^2 + \left(\Delta - \frac{d}{2}\right)^2} W_{\frac{d}{2}+i\nu,s} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{d\nu}{\nu^2 + \left(\Delta - \frac{d}{2}\right)^2}\left[W_{\frac{d}{2}+i\nu,s} + W_{\frac{d}{2}-i\nu,s}\right]$$

pole in spectral function at: $\frac{d}{2} + i\nu = \Delta$

$$\int d^d y \langle\langle \mathcal{O}_{\Delta_1,s_1}\mathcal{O}_{\Delta_2,s_2}\mathcal{O}_{\frac{d}{2}+i\nu,s}(y)\rangle\rangle\langle\langle \tilde{\mathcal{O}}_{\frac{d}{2}+i\nu,s}(y)\mathcal{O}_{\Delta_3,s_3}\mathcal{O}_{\Delta_4,s_4}\rangle\rangle$$

Gives rise to useful representation of CPWE:

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle = \sum_s \int_{-\infty}^{\infty} d\nu\, c_s(\nu)\, W_{\frac{d}{2}+i\nu,s}$$

Contribution from some primary operator $\mathcal{O}$:

$$c_s(\nu) \sim \frac{c_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2} c_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4}}{\nu^2 + \left(\Delta - \frac{d}{2}\right)^2} + \ldots$$

Bulk side.

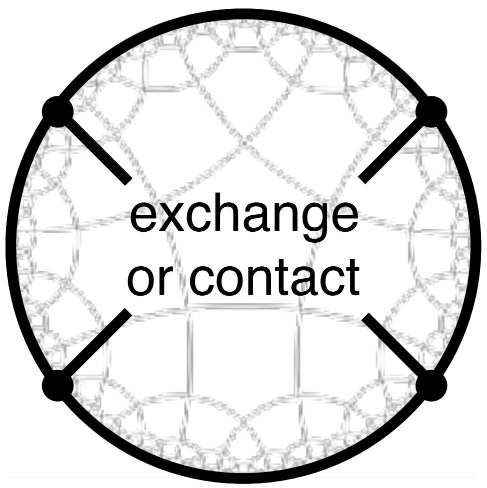
General Approach

Harmonic function decomposition of Witten diagrams

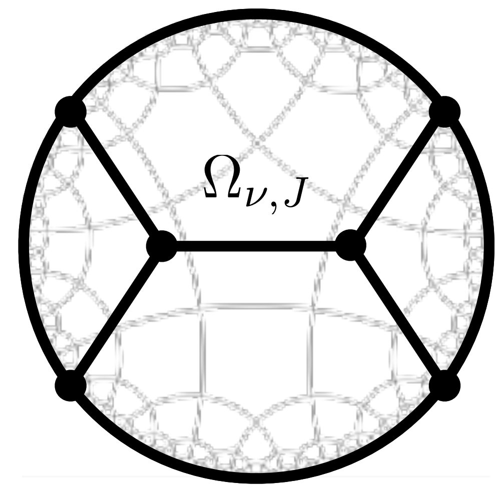
$$\Box \Omega_{\Delta,s}(x_1, x_2) \propto \Omega_{\Delta,s}(x_1, x_2), \qquad \nabla \cdot \Omega_{\Delta,s} = 0, \qquad (g \cdot \Omega_{\Delta,s}) = 0$$

divergenceless   traceless

Furnish complete basis of bi-tensors in AdS

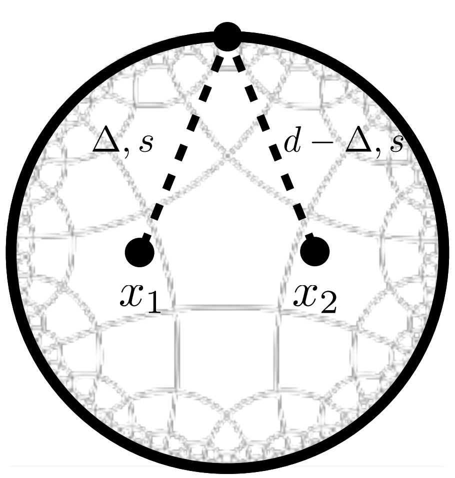


$$= \sum_J \int_{-\infty}^{\infty} d\nu$$



Factorisation of harmonic functions:

$$\Omega_{\Delta,s}(x_1, x_2) =$$



related to projector in shadow formalism

$$\mathcal{P}_{\Delta,s} = \int d^d y \, \mathcal{O}_{\Delta,s}(y) \, |0\rangle\langle 0| \tilde{\mathcal{O}}_{\Delta,s}(y)$$

General Approach

Harmonic function decomposition of Witten diagrams

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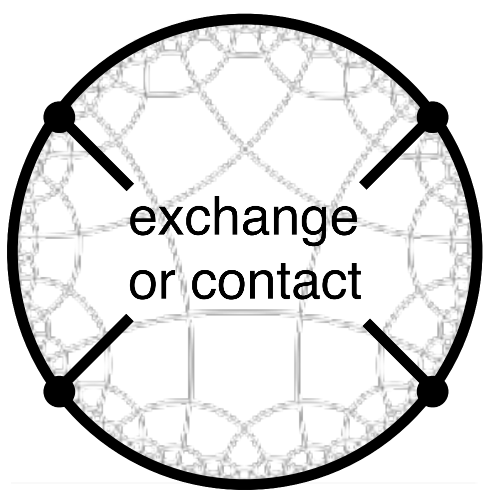

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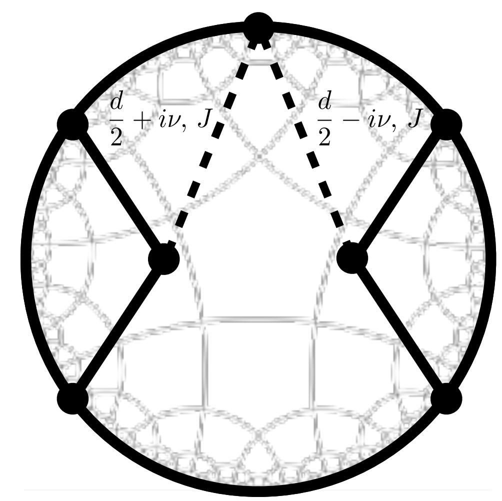

Factorise into 3pt Witten diagrams!

Factorisation of harmonic functions:

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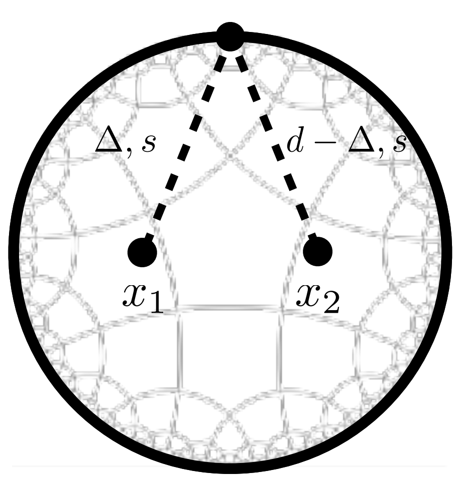

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General Approach

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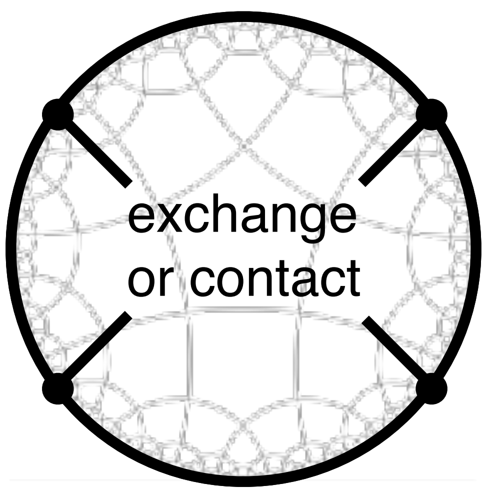

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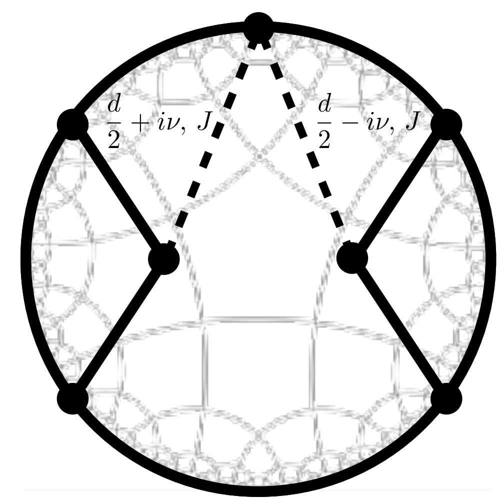
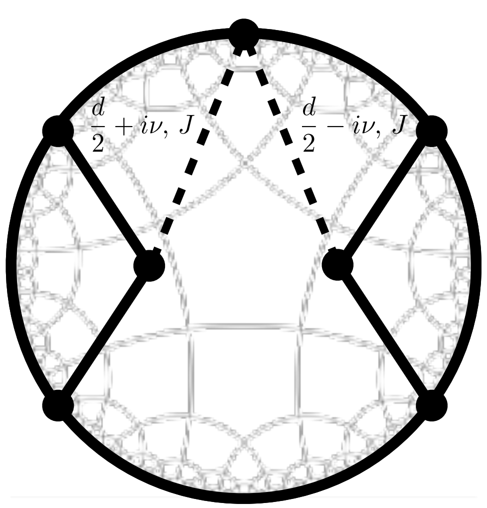


Evaluate 3pt Witten diagrams:



$$= c_J(\nu)\int d^d y\, \langle\langle \mathcal{O}_{\Delta_1,s_1}\mathcal{O}_{\Delta_2,s_2}\mathcal{O}_{\frac{d}{2}+i\nu,J}(y)\rangle\rangle\langle\langle\tilde{\mathcal{O}}_{\frac{d}{2}+i\nu,J}(y)\,\mathcal{O}_{\Delta_3,s_3}\mathcal{O}_{\Delta_4,s_4}\rangle\rangle$$

$\downarrow$

$$W_{\frac{d}{2}+i\nu,J}+W_{\frac{d}{2}-i\nu,J}$$

General Approach

Harmonic function decomposition of Witten diagrams

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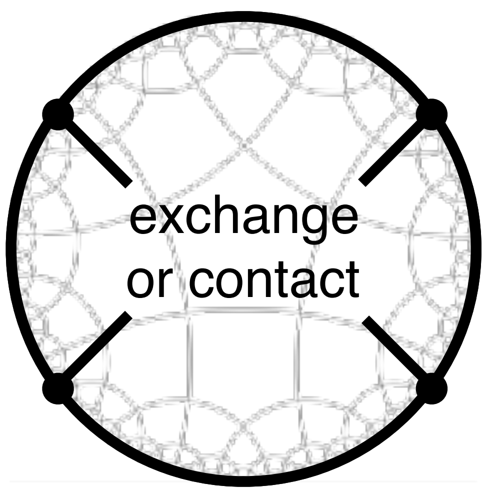
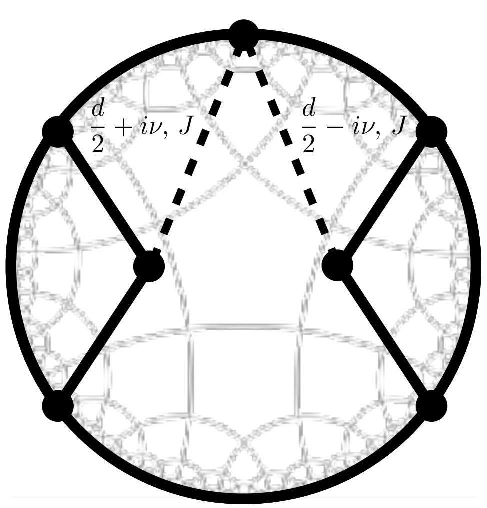
divergenceless traceless

Furnish complete basis of bi-tensors in AdS



$$= \sum_J \int_{-\infty}^{\infty} d\nu\, c_J\left(\nu\right) W_{\frac{d}{2}+i\nu,J}$$

Evaluate 3pt Witten diagrams:



$$= c_J\left(\nu\right)\int d^d y\,\langle\langle\mathcal{O}_{\Delta_1,s_1}\mathcal{O}_{\Delta_2,s_2}\mathcal{O}_{\frac{d}{2}+i\nu,J}\left(y\right)\rangle\rangle\langle\langle\tilde{\mathcal{O}}_{\frac{d}{2}+i\nu,J}\left(y\right)\mathcal{O}_{\Delta_3,s_3}\mathcal{O}_{\Delta_4,s_4}\rangle\rangle$$

↓

$$W_{\frac{d}{2}+i\nu,J}+W_{\frac{d}{2}-i\nu,J}$$

Outline

- General approach
- Spinning 3pt
- Spinning 4pt exchange diagrams

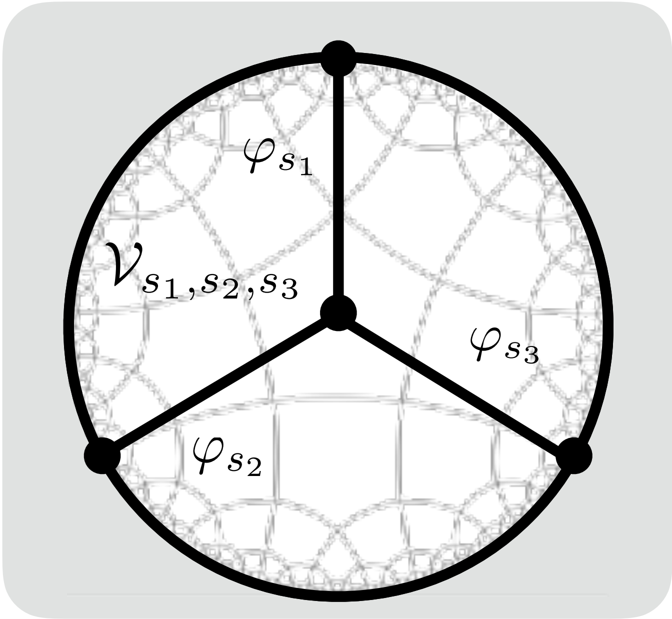
Spinning 3pt Diagrams

Basis of bulk cubic couplings

$$\mathcal{V}_{s_1,s_2,s_3} = \sum_{n_i} g^{n_1,n_2,n_3}_{s_1,s_2,s_3}\, \mathcal{V}^{n_1,n_2,n_3}_{s_1,s_2,s_3}$$

$0 \leq n_1 \leq \min\{s_2 - n_3, s_3 - n_2\}$ $\quad 0 \leq n_2 \leq \min\{s_1 - n_3, s_3\}$ $\quad 0 \leq n_3 \leq \min\{s_1, s_2\}$

Want to evaluate:



φ_{s_i} of mass $m_i^2 R^2 = \Delta_i (\Delta_i - d) - s_i$

with basis elements:

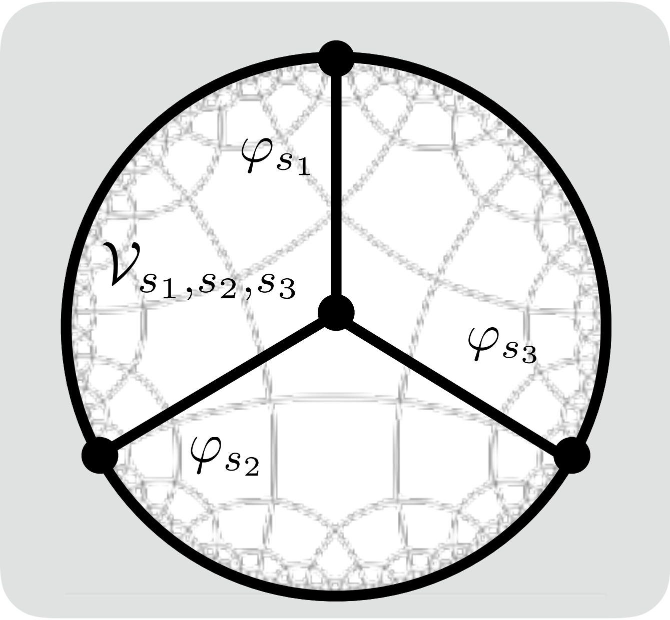
$$\mathcal{V}^{n_1,n_2,n_3}_{s_1,s_2,s_3} = g^{\mu(n_3)\nu(n_3)} g^{\nu(n_1)\sigma(n_1)} g^{\sigma(n_2)\mu(n_2)} \nabla^{k(\bar{s}_3)} \varphi_{\mu(n_2+n_3)p(\bar{s}_1)} \times \nabla^{p(\bar{s}_1)} \varphi_{\nu(n_3+n_1)q(\bar{s}_2)} \nabla^{q(\bar{s}_2)} \varphi_{\sigma(n_1+n_2)k(\bar{s}_3)}$$

Notation

$\bar{s}_i = s_i - (n_{i-1} + n_{i+1})$ $\quad \varphi_{\mu(s)} = \varphi_{\mu_1 \ldots \mu_s}$ $\quad g^{\mu(n)\nu(n)} = g^{\mu_1\nu_1} \ldots g^{\mu_n\nu_n}$ $\quad \nabla^{\mu(n)} = \nabla^{\mu_1} \ldots \nabla^{\mu_n}$

Spinning 3pt Diagrams

Want to evaluate:



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$0 \leq n_1 \leq \min\{s_2 - n_3, s_3 - n_2\}$ $\quad 0 \leq n_2 \leq \min\{s_1 - n_3, s_3\}$ $\quad 0 \leq n_3 \leq \min\{s_1, s_2\}$

with basis elements: (6 basic contractions)

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Spinning 3pt Diagrams

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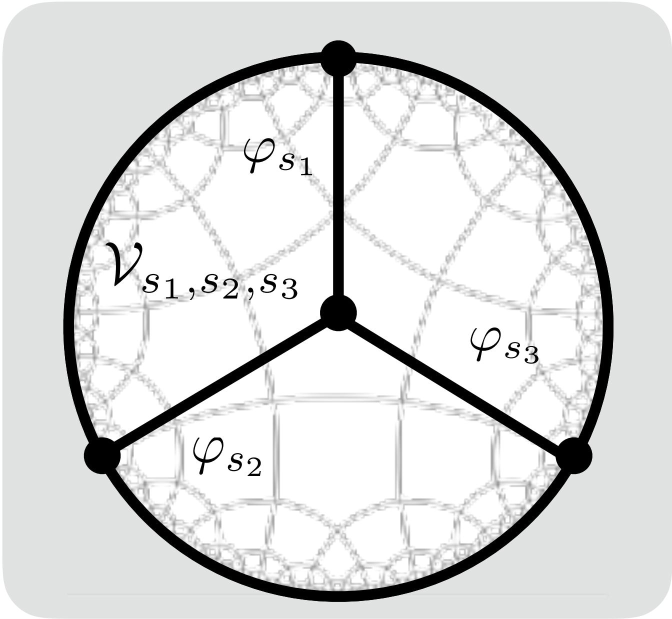

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Notation

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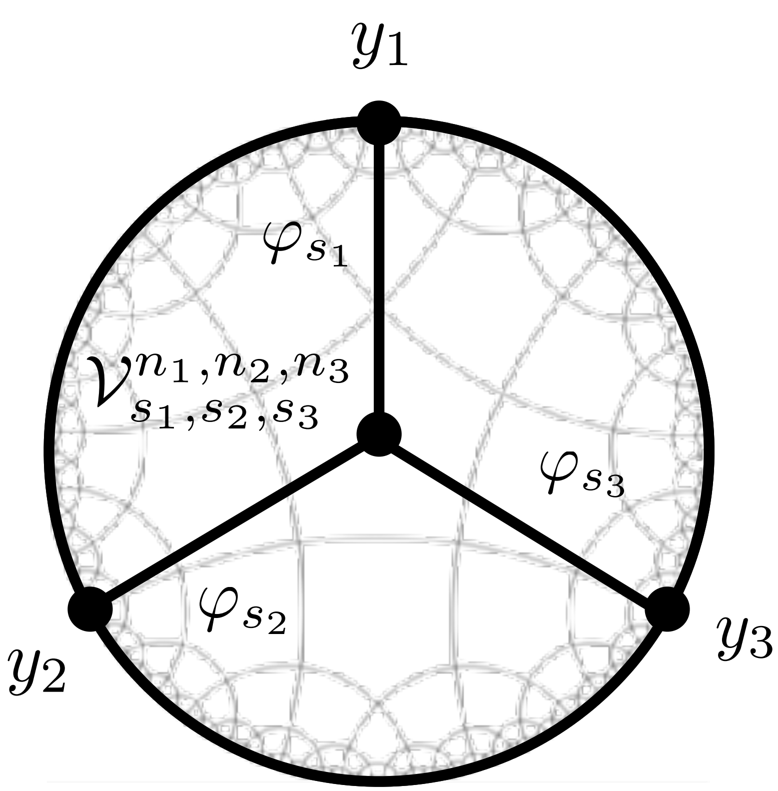
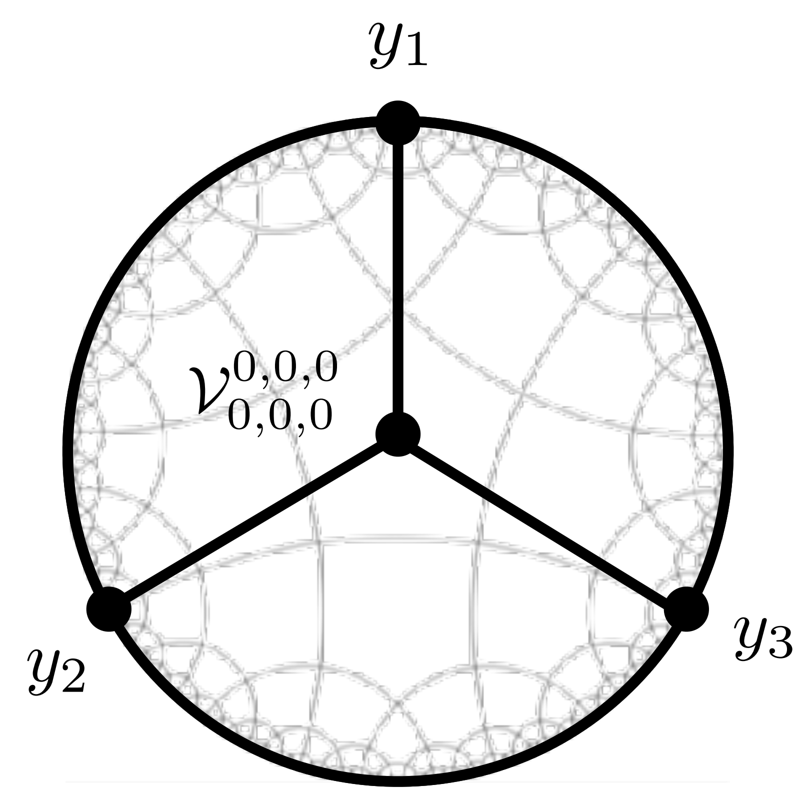
Spinning 3pt Diagrams

Integral over AdS

Trick: Reduce integral over AdS to its scalar seed



$$= \quad \mathcal{F}\left(y_i, \partial_{y_i}\right) \Bigg[\quad \Bigg]$$



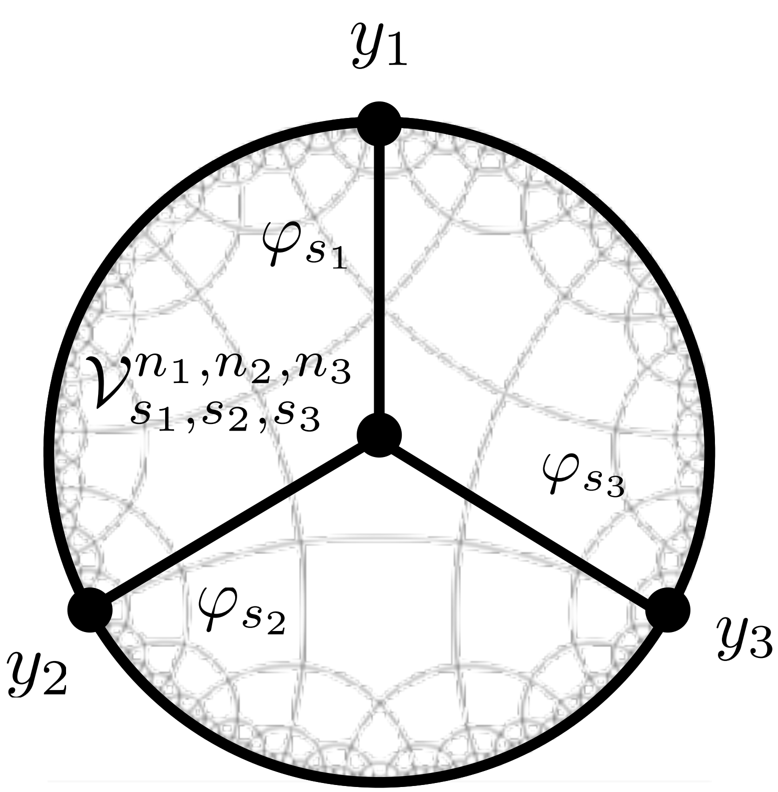
Well known how to compute

[Mück et al.; Freedman et al. `98]

Spinning 3pt Diagrams

Integral over AdS

Result:



$$= \sum_{m_i} C^{n_1,n_2,n_3}_{m_1,m_2,m_3} \langle\langle \mathcal{O}_{\Delta_1,s_1}(y_1)\, \mathcal{O}_{\Delta_2,s_2}(y_2)\, \mathcal{O}_{\Delta_3,s_3}(y_3) \rangle\rangle^{(\mathbf{m})}$$

Recall: Basis of 3pt spinning conformal structures $\quad \tau_i = \Delta_i - s_i$

$$\langle\langle \mathcal{O}_{\Delta_1,s_1}(y_1)\, \mathcal{O}_{\Delta_2,s_2}(y_2)\, \mathcal{O}_{\Delta_3,s_3}(y_3) \rangle\rangle^{(\mathbf{n})} = \frac{\mathsf{Y}_1^{s_1-n_2-n_3}\mathsf{Y}_2^{s_2-n_1-n_3}\mathsf{Y}_3^{s_3-n_1-n_2}\mathsf{H}_1^{n_1}\mathsf{H}_2^{n_2}\mathsf{H}_3^{n_3}}{(y_{12}^2)^{\frac{\tau_1+\tau_2-\tau_3}{2}}(y_{13}^2)^{\frac{\tau_1+\tau_3-\tau_2}{2}}(y_{23}^2)^{\frac{\tau_2+\tau_3-\tau_1}{2}}}$$

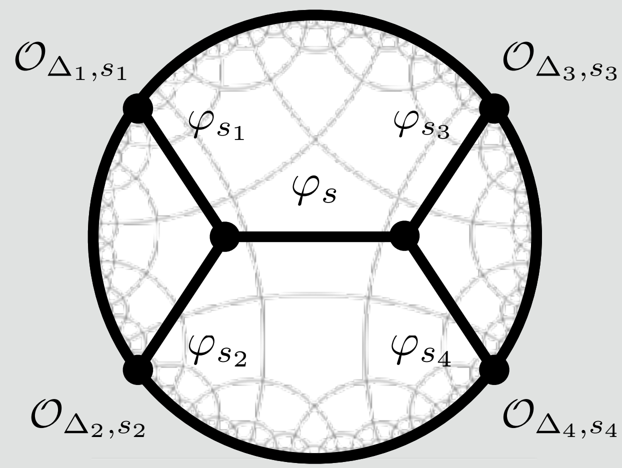
Outline

- General approach
- Spinning 3pt
- Spinning 4pt exchange diagrams

Spinning Exchange Diagrams

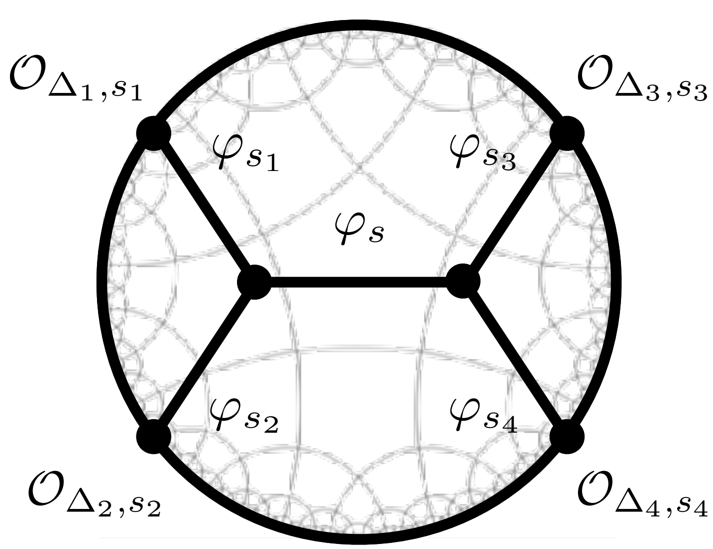
Goal:

Decompose

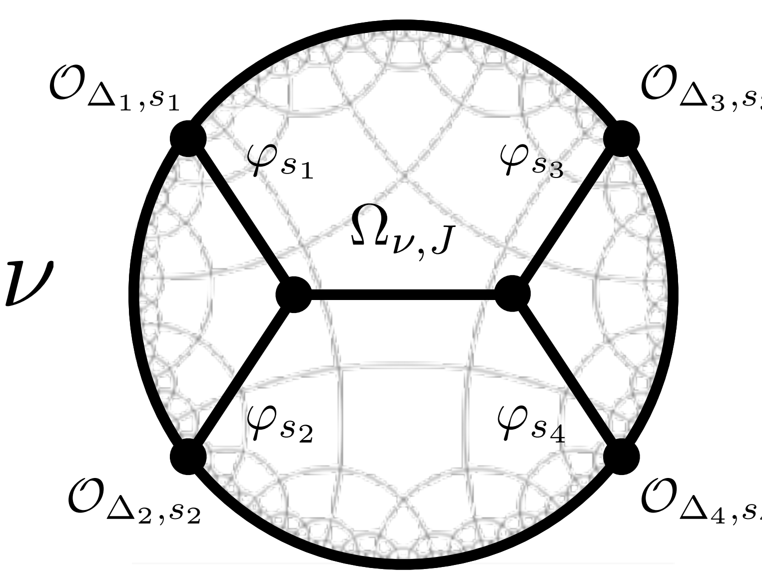


into Conformal Partial Waves

First step: Decomposition into harmonic functions $\Omega_{\nu,J}$

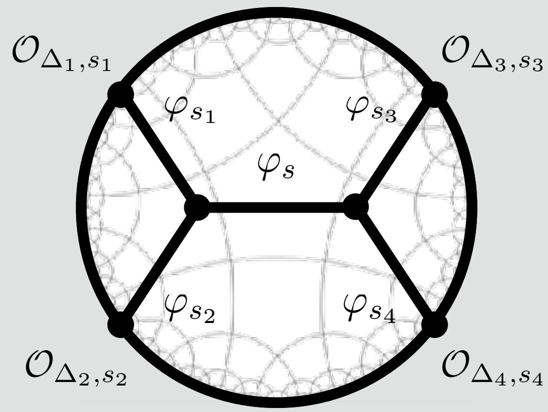


$$= \sum_J \int_{-\infty}^{\infty} d\nu$$

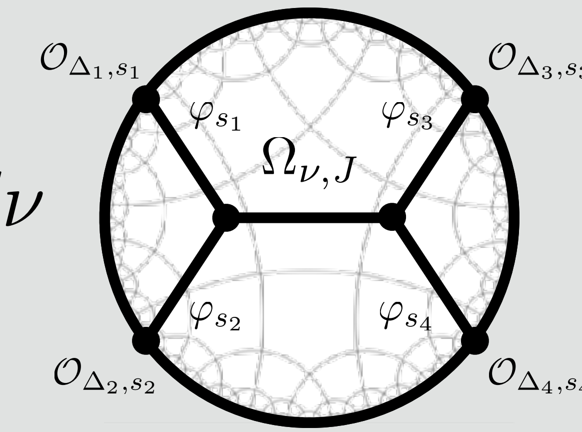


Spinning Exchange Diagrams

First step: Harmonic function decomposition



$$= \sum_J \int_{-\infty}^{\infty} d\nu$$



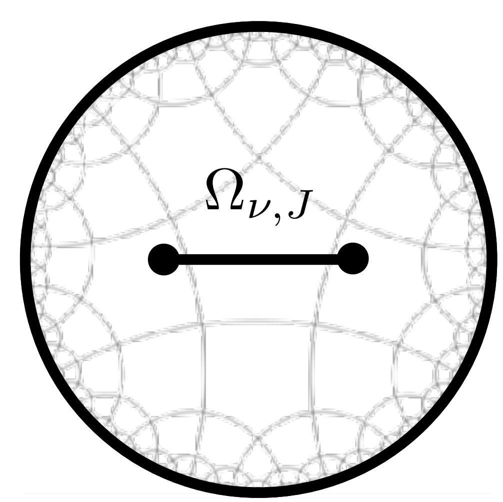
Bulk-to-bulk propagator Π_s

$$m_s^2 R^2 = \Delta\left(\Delta - d\right) - s$$

$$\varphi_s\left(x\right) = \int_{\mathrm{AdS}} d^{d+1}x'\,\Pi_s\left(x,x'\right)\cdot J_s\left(x'\right), \qquad \left(\Box_1 - m_s^2 + \ldots\right)\Pi_s\left(x_1,x_2\right) = -\delta^{d+1}\left(x_1,x_2\right)$$

Express in basis of harmonic functions $\Omega_{\nu,J}$:

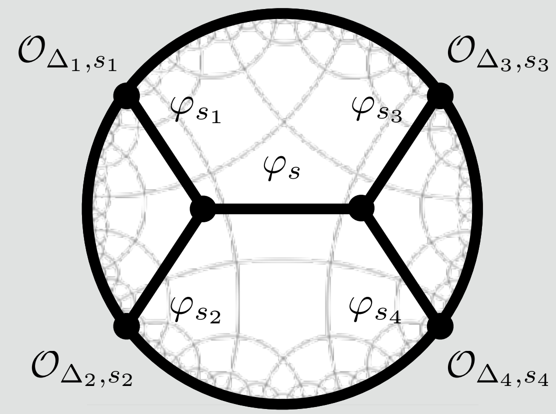
$$\Pi_s\left(x_1,x_2\right) \;=\; \sum_{J=0}^{s}\int_{-\infty}^{\infty} d\nu\, f_J\left(\nu\right)$$



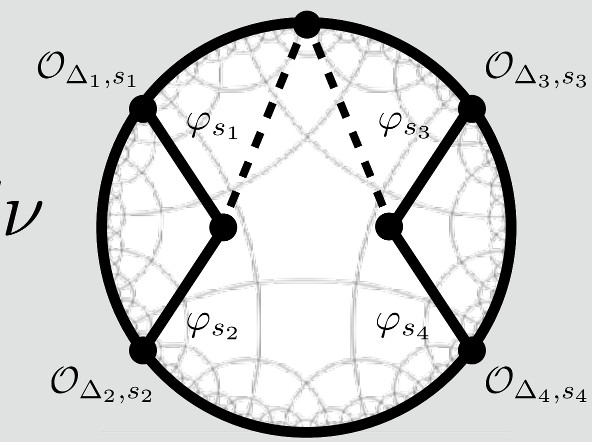
[Bekaert, Erdmenger, Ponomarev, C.S. `14]

Spinning Exchange Diagrams

First step: Harmonic function decomposition



$$= \sum_{J=0}^{s} \int_{-\infty}^{\infty} d\nu$$



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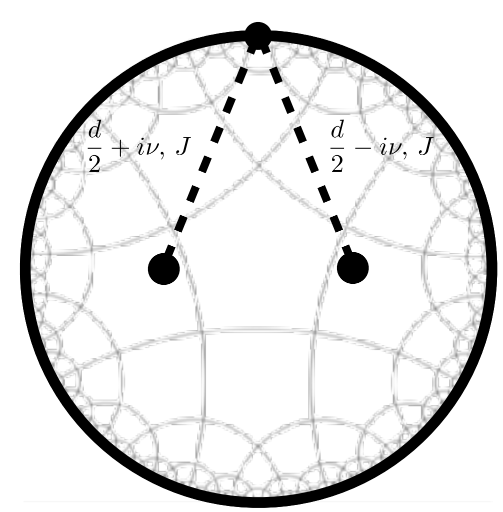
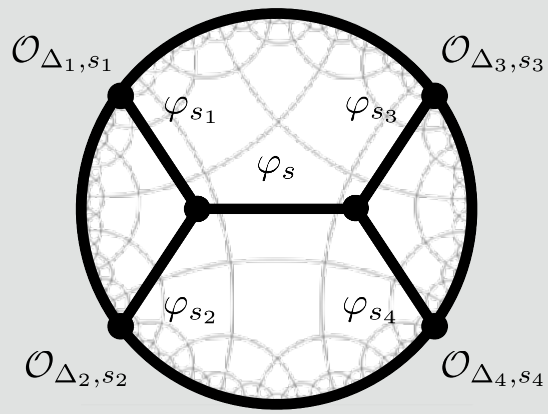


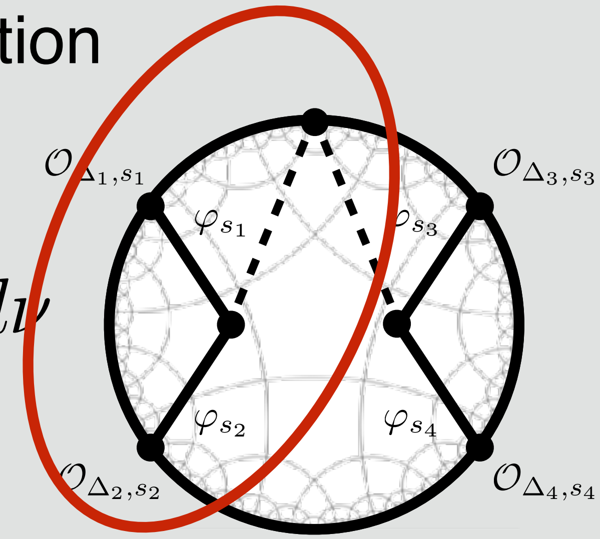
[Bekaert, Erdmenger, Ponomarev, C.S. `14]

Spinning Exchange Diagrams

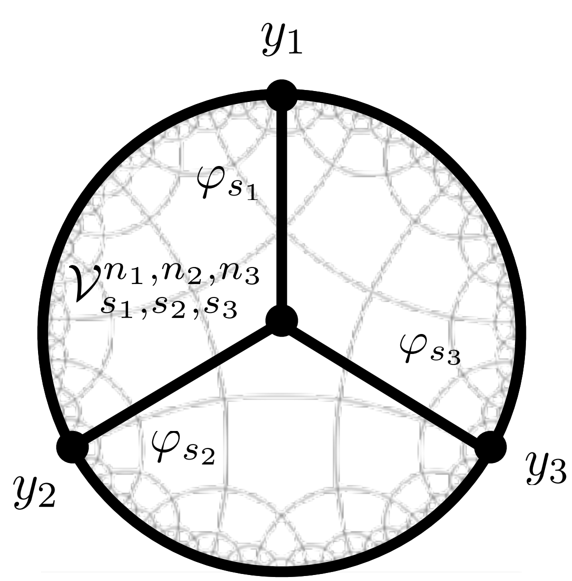
First step: Harmonic function decomposition



$$= \sum_{J=0}^{s} \int_{-\infty}^{\infty} d\nu$$



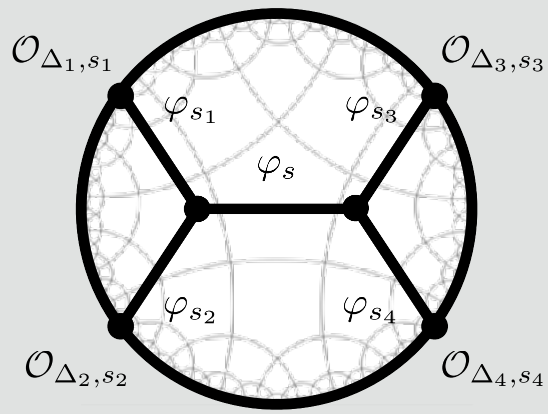
Second step: Evaluate the 3pt Witten diagrams



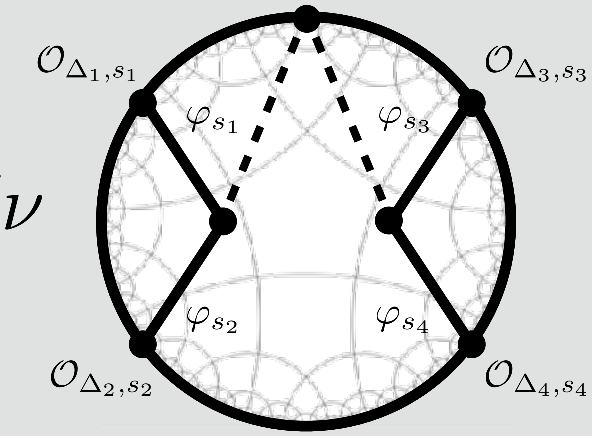
$$= \sum_{m_i} C^{n_1,n_2,n_3}_{m_1,m_2,m_3} \langle\langle \mathcal{O}_{\Delta_1,s_1}(y_1)\, \mathcal{O}_{\Delta_2,s_2}(y_2)\, \mathcal{O}_{\Delta_3,s_3}(y_3) \rangle\rangle^{(\mathbf{m})}$$

Spinning Exchange Diagrams

First step: Harmonic function decomposition

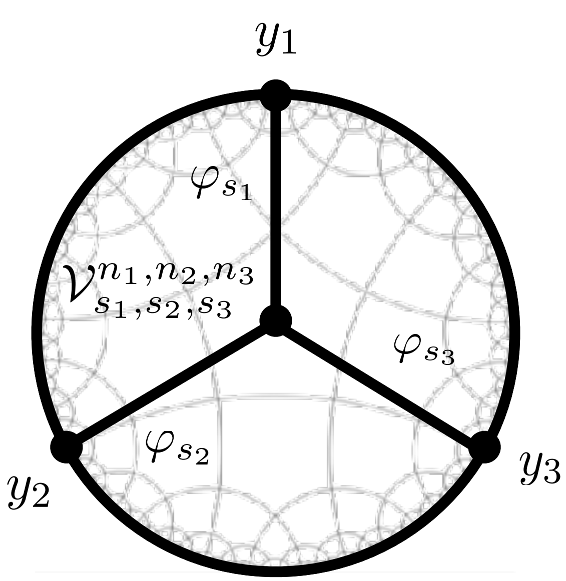


$$= \sum_{J=0}^{s} \int_{-\infty}^{\infty} d\nu$$



$$\int d^d y \langle\langle \mathcal{O}_{\Delta_1,s_1} \mathcal{O}_{\Delta_2,s_2} \mathcal{O}_{\frac{d}{2}+i\nu,J}\left(y\right)\rangle\rangle^{(\mathbf{m})} \langle\langle \tilde{\mathcal{O}}_{\frac{d}{2}+i\nu,J}\left(y\right) \mathcal{O}_{\Delta_3,s_3} \mathcal{O}_{\Delta_4,s_4}\rangle\rangle^{(\mathbf{n})}$$

Second step: Evaluate the 3pt Witten diagrams



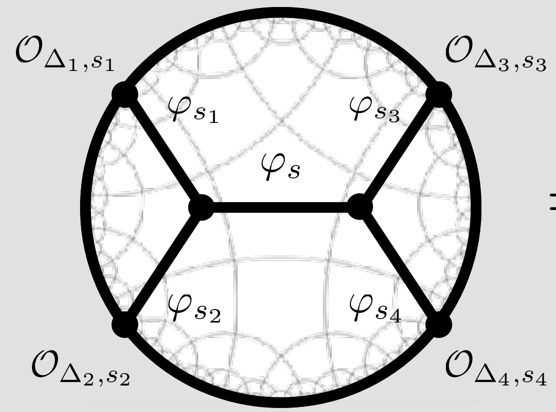
$$= \sum_{m_i} C^{n_1,n_2,n_3}_{m_1,m_2,m_3} \langle\langle \mathcal{O}_{\Delta_1,s_1}\left(y_1\right) \mathcal{O}_{\Delta_2,s_2}\left(y_2\right) \mathcal{O}_{\Delta_3,s_3}\left(y_3\right)\rangle\rangle^{(\mathbf{m})}$$

To determine the CPWE, recall:

$$W^{\mathbf{m},\mathbf{n}}_{\Delta,s} + W^{\mathbf{m},\mathbf{n}}_{d-\Delta,s} = \kappa \int d^d y \, \langle\langle \mathcal{O}_{\Delta_1,s_1} \mathcal{O}_{\Delta_2,s_2} \mathcal{O}\left(y\right)\rangle\rangle^{(\mathbf{m})} \langle\langle \tilde{\mathcal{O}}\left(y\right) \mathcal{O}_{\Delta_3,s_3} \mathcal{O}_{\Delta_4,s_4}\rangle\rangle^{(\mathbf{n})}$$

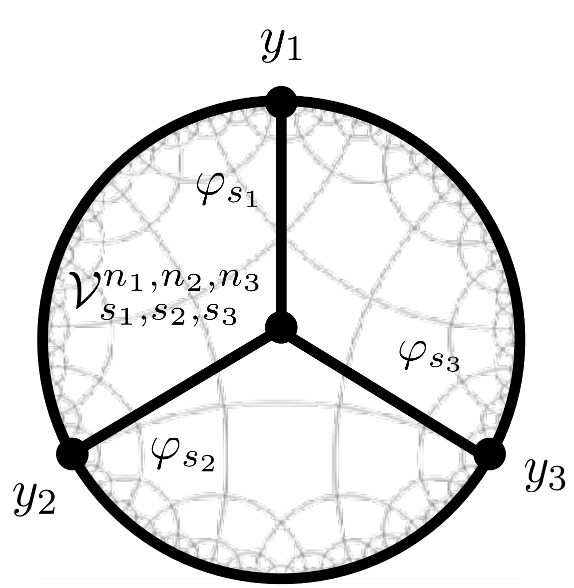
Spinning Exchange Diagrams

Final step: Conformal Partial Wave Expansion



$$= \sum_{J=0}^{s} \int_{-\infty}^{\infty} d\nu \, c_J^{\mathbf{m},\mathbf{n}}(\nu) \, W^{\mathbf{m},\mathbf{n}}_{\frac{d}{2}+i\nu,J}$$

Second step: Evaluate the 3pt Witten diagrams

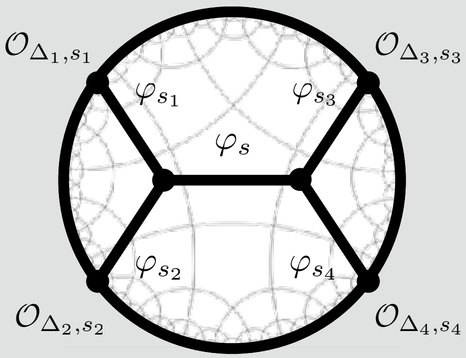


$$= \sum_{m_i} C^{n_1,n_2,n_3}_{m_1,m_2,m_3} \langle\langle \mathcal{O}_{\Delta_1,s_1}(y_1)\, \mathcal{O}_{\Delta_2,s_2}(y_2)\, \mathcal{O}_{\Delta_3,s_3}(y_3) \rangle\rangle^{(\mathbf{m})}$$

To determine the CPWE, recall:

$$W^{\mathbf{m},\mathbf{n}}_{\Delta,s} + W^{\mathbf{m},\mathbf{n}}_{d-\Delta,s} = \kappa \int d^d y \, \langle\langle \mathcal{O}_{\Delta_1,s_1} \mathcal{O}_{\Delta_2,s_2} \mathcal{O}(y) \rangle\rangle^{(\mathbf{m})} \langle\langle \tilde{\mathcal{O}}(y)\, \mathcal{O}_{\Delta_3,s_3} \mathcal{O}_{\Delta_4,s_4} \rangle\rangle^{(\mathbf{n})}$$

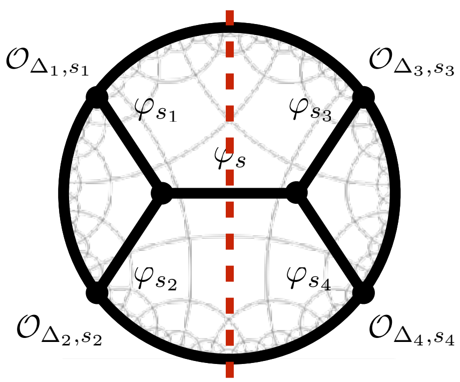
Spinning Exchange Diagrams



$$= \sum_{J=0}^{s} \int_{-\infty}^{\infty} d\nu \, c_J^{\mathbf{m},\mathbf{n}}(\nu) \, W^{\mathbf{m},\mathbf{n}}_{\frac{d}{2}+i\nu,J}$$

Two types of contributions:

1. Single trace operator $\mathcal{O}_{\Delta,s}$ dual to exchanged single-particle state φ_s



non-coincident bulk points

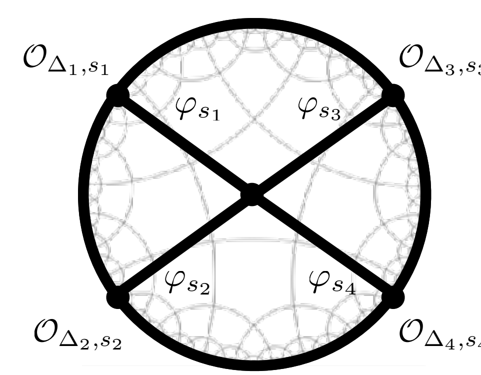
$$c_s^{\mathbf{m},\mathbf{n}}(\nu) \sim \frac{\mathsf{c}^{\mathbf{m}}_{\mathcal{O}\mathcal{O}_1\mathcal{O}_2}\mathsf{c}^{\mathbf{n}}_{\mathcal{O}\mathcal{O}_3\mathcal{O}_4}}{\nu^2 + \left(\Delta - \frac{d}{2}\right)^2} + \ldots$$

2. Double-trace operators $[\mathcal{O}_{\Delta_1,s_1}\mathcal{O}_{\Delta_2,s_2}]_J$ and $[\mathcal{O}_{\Delta_3,s_3}\mathcal{O}_{\Delta_4,s_4}]_J$ of spin $J = 0, \ldots, s$

Scaling dimensions:

$$\Delta_1 + \Delta_2 - (s_1 + s_2) + J + 2n\,, \qquad \Delta_3 + \Delta_4 - (s_3 + s_4) + J + 2n$$

$$n = 0, 1, 2, 3 \ldots$$



arise from contact terms in the exchange

Summary

- New tools to evaluate 3pt and 4pt Spinning Witten diagrams

- Spells out the map between OPE coeffs. and bulk couplings of totally symmetric fields

- Given any CFT_d, this provides holographic reconstruction of cubic and quartic interactions of totally symmetric fields on AdS_{d+1}

Some concrete applications so far:

- Computation of all 3pt diagrams and 4pt exchange diagrams in type A minimal higher-spin gauge theory on AdS_{d+1}

- Holographic reconstruction of cubic and quartic interactions in type A higher-spin gauge theory

1603.0022, 1702.08619 [hep-th]