KINETIC DESCRIPTION of PAIR CREATION from VACUUM under the ACTION of STRONG FIELDS

S.A. Smolyansky[1]

V.V. Dmitriev[1], A.M. Fedotov[2], A.D. Panferov[1]

1) Saratov State University, Russia

2) Moscow Engineering Physics Institute, Russia

Ginzburg Centennial Conference on Physics,

May 29 – June 3, 2017, Moscow

1. Introduction
2. Basic KE
3. Order parameter
4. Residual currents
5. Particles and quasiparticles
6. Non-monotone entropy growth
7. Strong nonequilibrium
8. Field induced phase transition
9. Widening the scope of the basic kinetic theory
10. Conclusion: the nearest and further perspectives

1. Introduction

Topic: Kinetic description of vacuum EPP creation in the D=3+1 QED

Historical reference

- QED:

[1] I. Bialynicky-Birula, P. Gornicki, and J. Rafelski, Phys. Rev. **D 44**, 1825 (1991).

[2] S. Schmidt, D. Blaschke, G. Roepke, S.A. Smolyansky, A.V. Prozorkevich, IJMP **E7**, 709 (1998).

[3] N.N. Pervushin, V.V. Skokov, Acta Phys. Polon. **37**, 2587 (2006).

[4] A.M. Fedotov, E.G. Gelfer, K. Yu. Korolev, S.A. Smolyansky, Phys. Rev. **D 83**, 025011 (2011).

- Cosmology:

[1] Ja.B. Zeldovich, A.A. Starobinsky, JETP, **61**, 2161 (1971).

[2] S.G. Mamaev, V.M. Mostepanenko, V.M. Frolov, Phys. Atomic. Nucl. **26**, 215 (1977).

[3] A.V. Veriaskin, V.G. Lapchinski, V.A. Rubakov, Preprint INR П-0198 M. (1981).

- Condensed Matter - ?

2. Basic KE

Field model: linearly polarized electric field in the Hamiltonian gauge

The basic KE for D=3+1 QED

$$A^{\mu}(t) = \left(0,0,0,\vec{A}(t)\right)$$

$$\dot{f}(\vec{p},t) = \frac{1}{2}\lambda^{\pm}(\vec{p},t)\int_{t_0}^{t} dt'\lambda^{\pm}(\vec{p},t')\left[1 \pm 2f(\vec{p},t')\right]\cos\theta(t,t'),$$

where

$$\theta(t,t') = 2\int_{t'}^{t} d\tau\varepsilon(\vec{p},t),$$

$$\varepsilon(\vec{p},t) = \sqrt{\varepsilon_{\perp}^{2}(\vec{p}) + P^{2}},$$

$$\lambda^{+}(\vec{p},t) = eE(t)P/\varepsilon^{2}(\vec{p},t),$$

$$\varepsilon_{\perp} = \sqrt{m^{2} + p_{\perp}^{2}},$$

$$\lambda^{-}(\vec{p},t) = eE(t)\varepsilon_{\perp}/\varepsilon^{2}(\vec{p},t),$$

$$P = p_{||} - eA(t).$$

Equivalent ODE's system

$$\dot{f} = \frac{1}{2}\lambda^{\pm}u, \quad \dot{u} = \lambda^{\pm}\left(1 \pm 2f\right) - 2\varepsilon v, \quad \dot{v} = 2\varepsilon u$$

3. Order parameter
Quantum field interpretation:

Foundation: transition in the quasiparticle representation, where the Hamiltonian is diagonal and the states with positive and negative energies are separated

The creation and annihilation operations of the electrons and positrons are time dependent and are defined under the t – dependent vacuum. So it is a theory with unstable vacuum.

Then the general distribution function is

$$f(\vec{p},t) = \langle \mathrm{in} | a^{+}(\vec{p},t) a(\vec{p},t) | \mathrm{in} \rangle = \langle \mathrm{in} | b^{+}(-\vec{p},t) b(-\vec{p},t) | \mathrm{in} \rangle$$

under the electroneutrality condition and functions of vacuum polarization

$$u = 2\Re\mathrm{e}\, f^{(+)} = 2\Re\mathrm{e}\, f^{(-)}, \quad v = 2\Im\mathrm{m}\, f^{(+)} = -2\Im\mathrm{m}\, f^{(-)}$$

where

$$f^{(+)}(\vec{p},t) = \langle \mathrm{in} | a^{+}(\vec{p},t) b^{+}(-\vec{p},t) | \mathrm{in} \rangle,$$

$$f^{(-)}(\vec{p},t) = \langle \mathrm{in} | b^{-}(-\vec{p},t) a^{-}(-\vec{p},t) | \mathrm{in} \rangle$$

are the anomalous averages.

3. Order parameter

$$\Phi \equiv u + i v = 2 f^{(+)} = 2 f^{(-)*}$$

Equation of motion $\dot{\Phi} + 2i\varepsilon\Phi = \lambda(1 - 2f)$

has the solution $\Phi(t) = \int_{t_0}^{t} dt' \lambda(t')[1 - 2f(t')] e^{-2i\int_{t'}^{t} d\tau \varepsilon(\tau)}$

in according with the initial condition $u(t_0) = v(t_0) = 0,$

that corresponds to $\Phi(t_0) = 0.$

Let us consider a finite field $E(t > t_{out}) = 0,$ so $\lambda(t > t_{out}) = 0.$

Then

$$\Phi(t > t_{out}) \sim \exp[-2i\varepsilon_{out}(t - t_{out})],$$

$$\varepsilon_{out} = \varepsilon(t > t_{out}) = \sqrt{\varepsilon_{\perp}^{2} + (p_{||} - eA_{out})^{2}},$$

that is the order parameter survives after shutdown of an external field: it is breathing oscillations of the out-vacuum.

3. Order parameter

Exact information can be obtained from the solution for $\Phi(t)$ at $t > t_{out}: \quad \Phi(t > t_{out}) = \Phi_{out}(\vec{p}) \exp\left[-2i\varepsilon_{out}(t - t_{out})\right]$

with the amplitude

$$\Phi_{out}(\vec{p}) = \int_{t_0}^{t_{out}} dt' \lambda(t') \left[1 - 2f(t')\right] e^{-2i\int_{t'}^{t} d\tau \varepsilon(\tau)}$$

contains all information about evolution of the system in an external field.

Thus, $\left|\Phi(t > t_{out})\right|^2 = \left|\Phi(\vec{p})\right|^2 = \text{const.}$

A glance forward : now it is understandable that the oscillations $\Phi(t > t_{out})$ will bring to oscillations of the polarization current $j_{pol}(t)$ on background of the constant conductivity current. Let us consider it rather more detailed.

4. Residual currents

Quasiparticle representation allows to select the conduction and polarization currents: $j(t) = j_{cond}(t) + j_{pol}(t)$

$$j_{cond}(t) = 2e\int_B \frac{d^3p}{(2\pi)^3} f(\vec{p},t)\frac{P}{\varepsilon}, \qquad j_{pol}(t) = e\int_B \frac{d^3p}{(2\pi)^3} u_R(\vec{p},t)\frac{\varepsilon_\perp}{\varepsilon}.$$

$u_R(\vec{p},t)$ is the regularized vacuum polarization function.

Now we can investigate evolution of the currents after shutdown of external field at $t > t_{out}$. According to the basic KE for $t > t_{out}$

$$\dot{f}(t) = 0, \qquad f(t > t_{out}) = \mathrm{const}.$$

It brings to constant conduction current $j_{con}(t > t_{out}) = \mathrm{const}.$

Because the order parameter survives, we obtain for the polarization current in this asymptotic domain

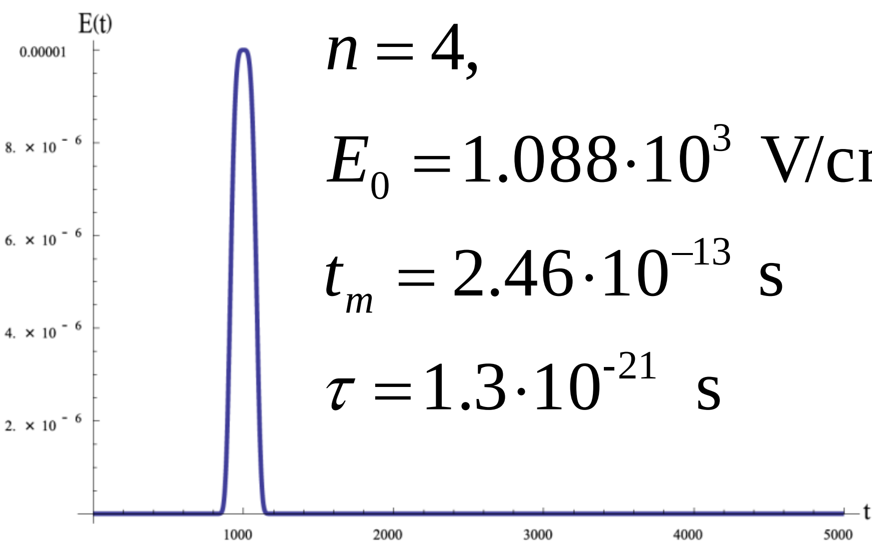
$$j_{pol}(t > t_{out}) = e\int_B \frac{d^3p}{(2\pi)^3}\frac{\varepsilon_\perp}{\varepsilon}\Phi_{out}(\vec{p})\cos\left[2\varepsilon_{out}(\vec{p})(t - t_{out})\right].$$

This expression is nonperturbative.

4. Residual currents

Illustration: graphene

Supergaussian field pulse $E(t)=E_0\exp\left[-(t-t_m)^n/2\tau^n\right]$

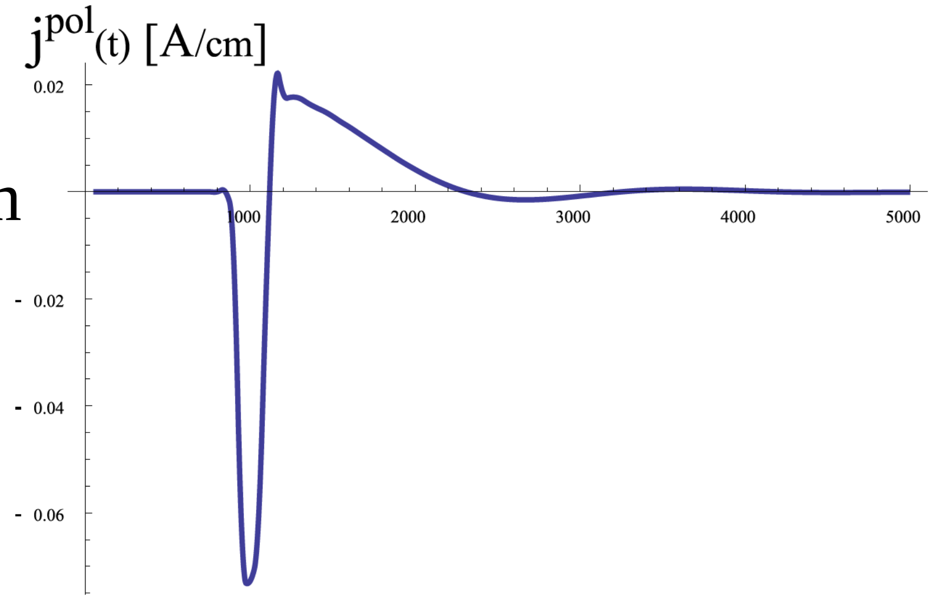


$n=4,$

$E_0=1.088\cdot10^3$ V/cm

$t_m=2.46\cdot10^{-13}$ s

$\tau=1.3\cdot10^{-21}$ s



Conclusions:

1) long tail of the damping oscillations of $j_{pol}(t>t_{out})$ on the background of constant condition currents;

2) domination of the polarization current $|j_{pol}|>>|j_{cond}|$;

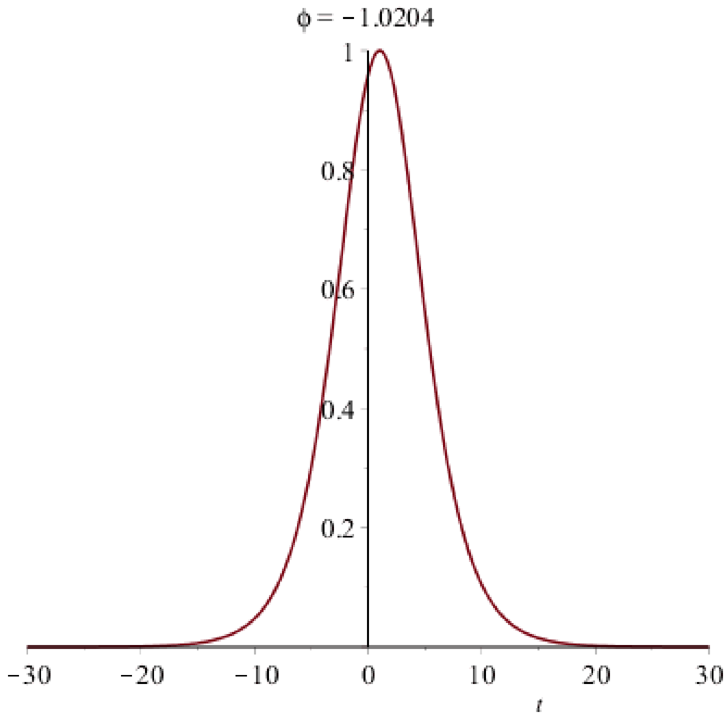
3) apparently, this effects is available to experimental proof (S.A. Smolyansky, D.V. Churochkin, V.V. Dmitriev, A.D. Panferov, B. Kaempfer, ISHEPP-2016)

5. Particles and quasiparticles

Two finite external field models will be used bellow:

1) the Eckart-Sauter field (one sheeted model), it allows an exact solution (Nikishov, Narozhnyi)

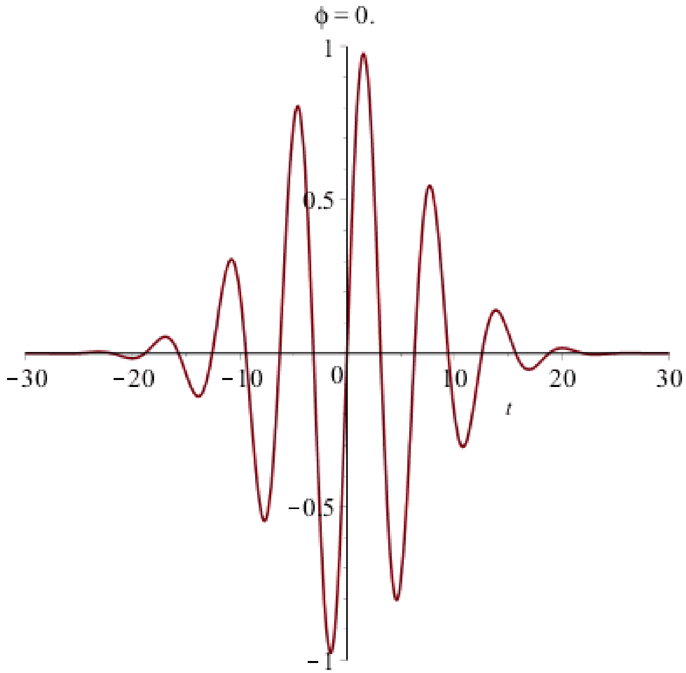
$$E(t) = E_0 \cosh^{-2}\left[t/\tau\right]$$



2) the Gaussian model of laser pulse (many-sheeted model)

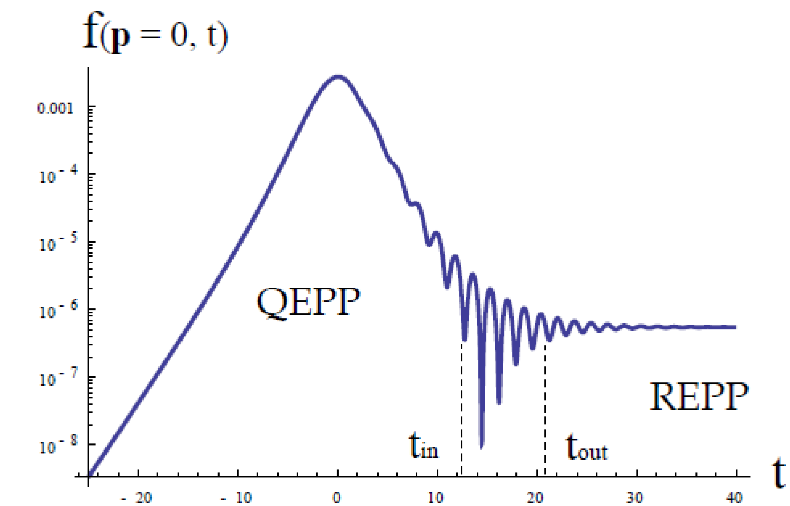
$$\tau >> 1/\omega >> 1/m$$

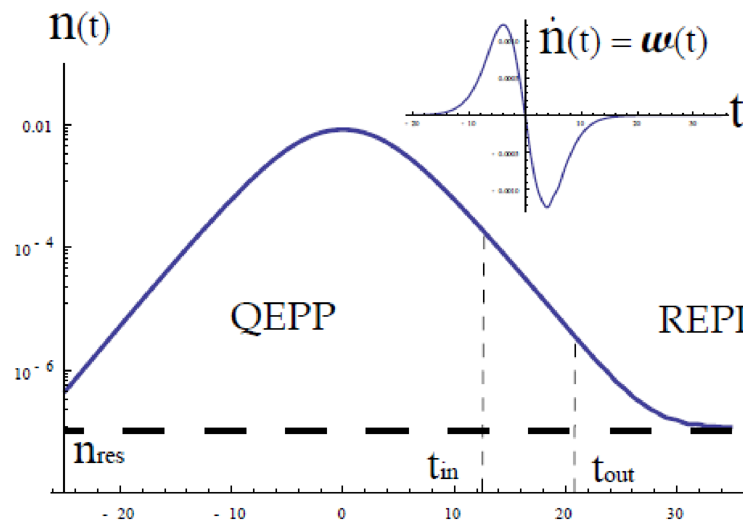
$$E(t) = E_0 \cos(\omega t)\exp\left[-t^2/2\tau^2\right]$$



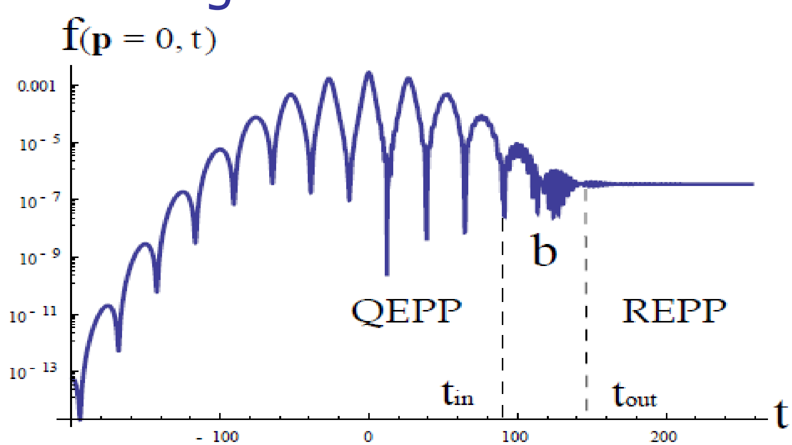
5. Particles and quasiparticles

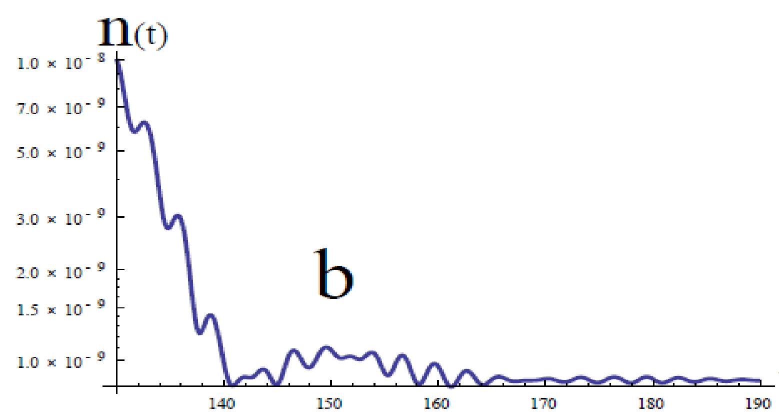
Three basic stages of evolution for the Eckart-Sauter field model: QEPP, transient, REPP





Stages of evolution for the Gaussian laser field model

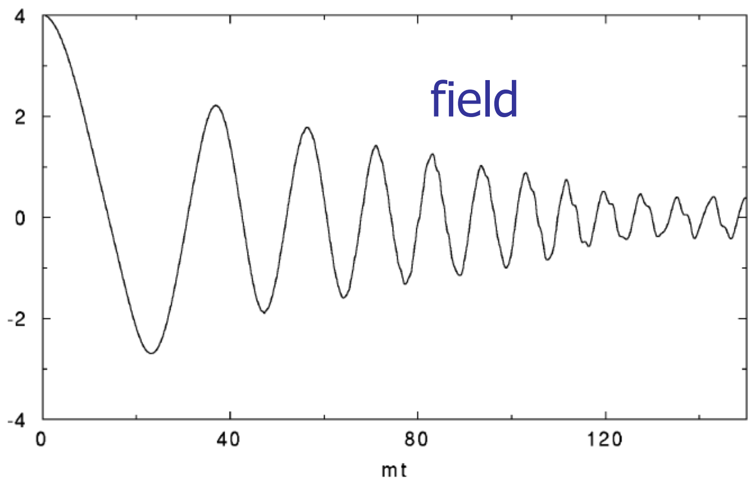


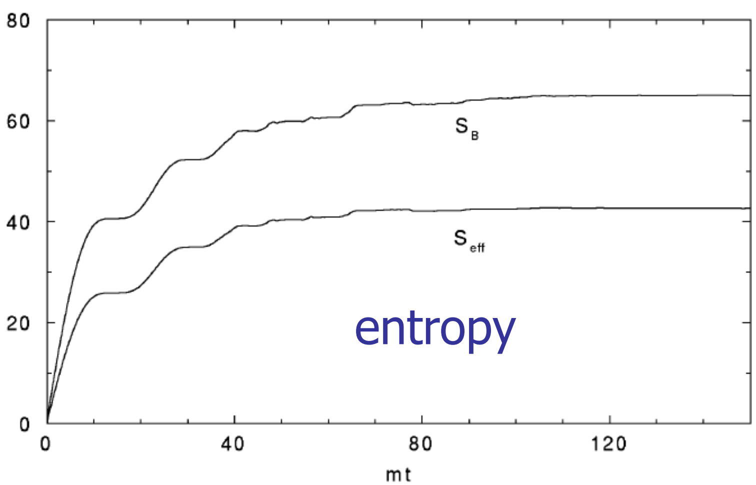


The all three stages give their specific contributions in radiation from EPP (Smolyansky, PRD-2013).

6. Non-monotone entropy growth

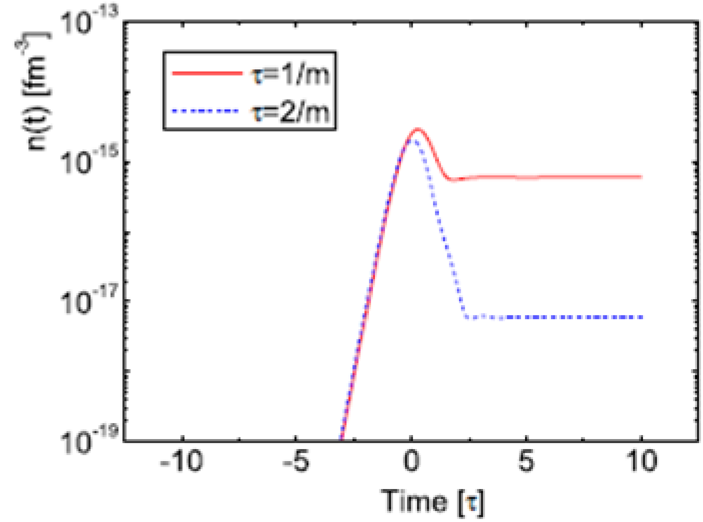
$$S(t_0) = 0$$

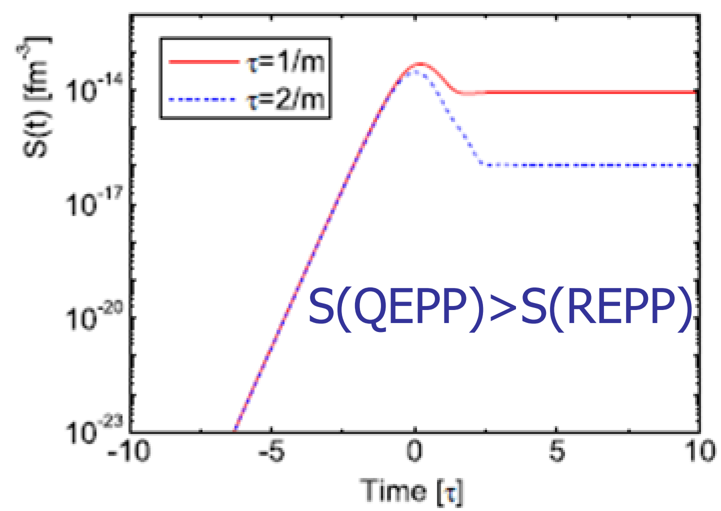




S. Habib, Y. Kluger, E. Mottola and J.P. Paz // Phys. Rev. L **76** 4660 (1996).

The Eckart-Sauter field model





S.A. Smolyansky, A.D. Panferov, A.V. Prozorkevich and M. Bonitz // P-Adic Numbers, Ultrametric Analysis and Applications **4**, 319 (2012).

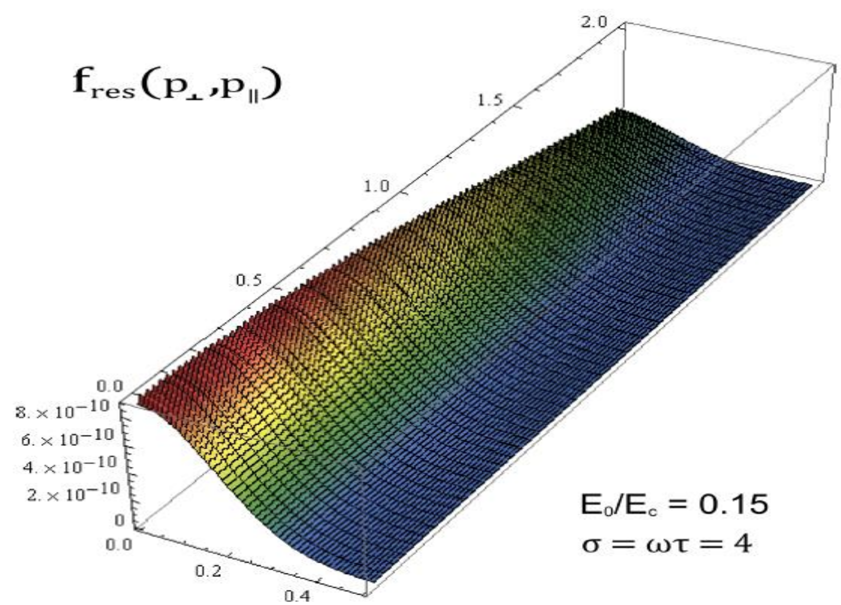
7. Strong nonequilibrium

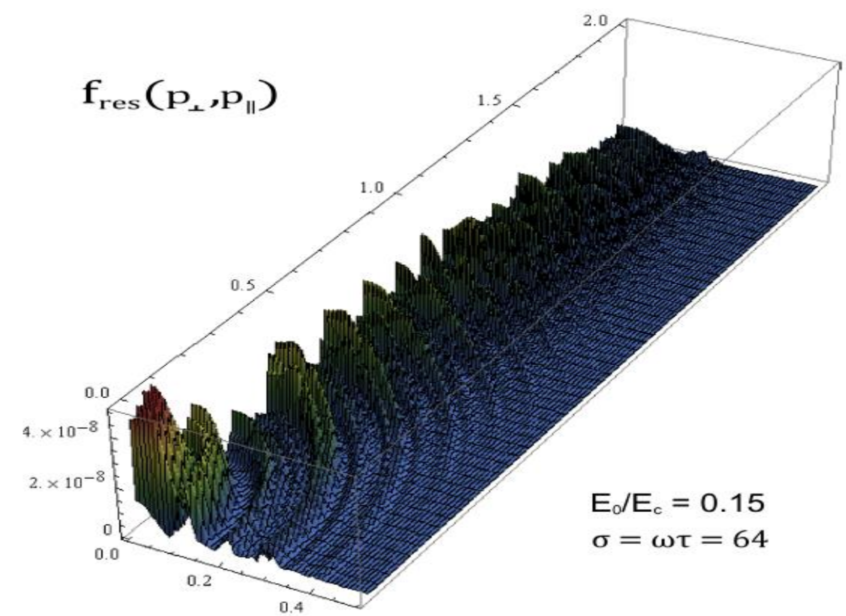
Linearly polarized electric fields brings to strong anisotropic (and hence strong nonequilibrium) distributions both in the QP-state EPP and in the residual state. Examples:

1) E(t)=const, exact solution

$$f_{out}(\vec{p}) = f_{out}(p_\perp) \sim \exp\left\{-\frac{E_c}{E_0}\left(\frac{\varepsilon_\perp}{m}\right)^2\right\}, \qquad p_{||} = p_3 \text{ - degeneration}$$

2) the laser pulse field model





8. Field induced phase transition

The general feature (linear polarization):

1) Existence of the three basic stages of EPP evolution in an one sheeted field pulse: QEPP, transient and REPP;

2) Existence of stable oscillations of the vacuum polarization in the out-state (residual oscillations of the order parameter), that brings to the damping oscillations of the polarization current on the background of the conductivity current;

3) Non-monotonic entropy growth ant strong non-equilibration of the EPP evolution.

All these features are reflected on character of electromagnetic radiation from EPP (Smolyansky et al, PRD (2013)).

8. Field induced phase transition

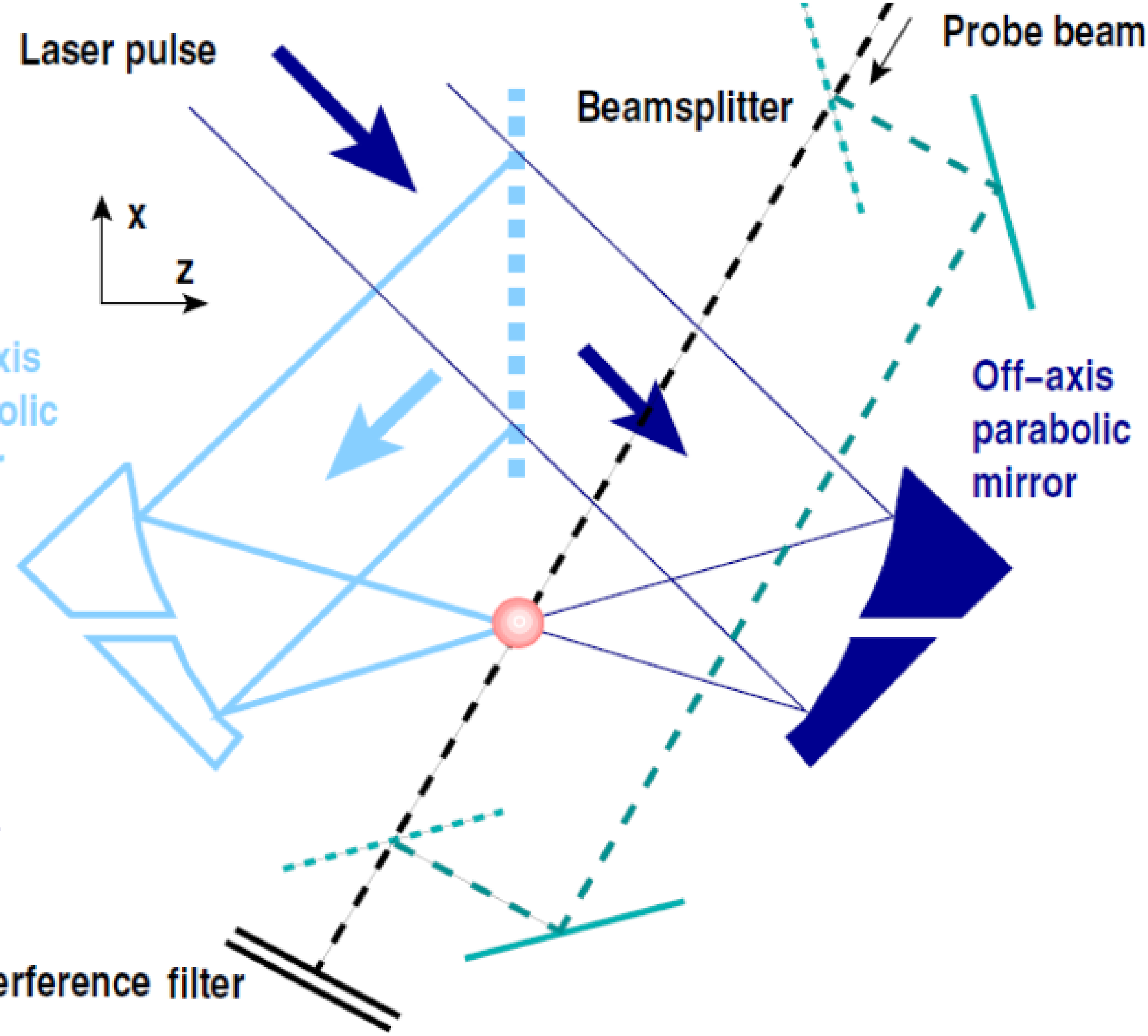
There are a set of the QF-systems, where the vacuum creation processes are described by KE's having the same form as the basic KE.

Inner degrees of freedom of such kind QF-systems either are absent (scalar QED) or are hidden (spinor QED with the linear polarization, graphene) or are separated (massive vector bosons).

The FRW cosmological models bring also to the analogous classes of KE's. Isotropic specific of these models needs in special investigation. See, however, the exact solved model (Pascoal et al, 2007) that leads to a strong nonequilibrium state. Thus, it can draw a conclusion that rather large class of QF systems leads to the form invariant basic KE.

It allows to unite the vacuum creation processes in these systems in the common class of the field induced phase transitions (the definition is adopted from the condensed matter physics, e.g., Oka (2012)).

9. Widening the scope of the kinetic theory



Jena's experiment

Conditions of the planed experiments assume, that the strong laser field in the focus spot is very inhomogeneous and complicated polarized.

9. Widening the scope of the kinetic theory

Restrictions of the present kinetic theory:

- linear polarization of the external electric field
- spatial homogeneous.

The first step: transition to the case of arbitrary polarization. The corresponding generalization: Pervushin, Skokov (2006); Smolyansky, Prozorkevich (2006).

KE system

$$\dot{f}_0 = -2\Xi\vec{r}^{\,+},$$

$$\dot{\vec{f}} = -2\left[\vec{f}\times\Theta\right] + 2\left[\vec{r}^{\,-}\times\Xi\right] - 2r_0^{+}\Xi,$$

$$\dot{r}_0^{+} = 2\omega r_0^{-} + 2\Xi\vec{f},$$

$$\dot{\vec{r}}^{\,+} = 2\omega\vec{r}^{\,-} - 2\left[\vec{r}^{\,+}\times\Theta\right] + \Xi(2f_0 - 1),$$

$$\dot{r}_0^{-} = -2\omega r_0^{+},$$

$$\dot{\vec{r}}^{\,-} = -2\omega\vec{r}^{\,+} - 2\left[\vec{r}^{\,-}\times\Theta\right].$$

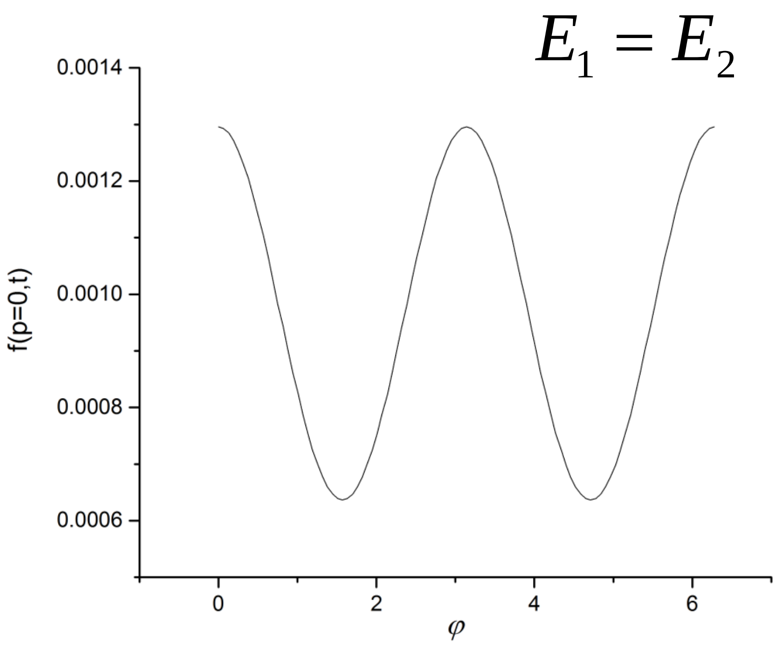
9. Widening the scope of the basic kinetic theory

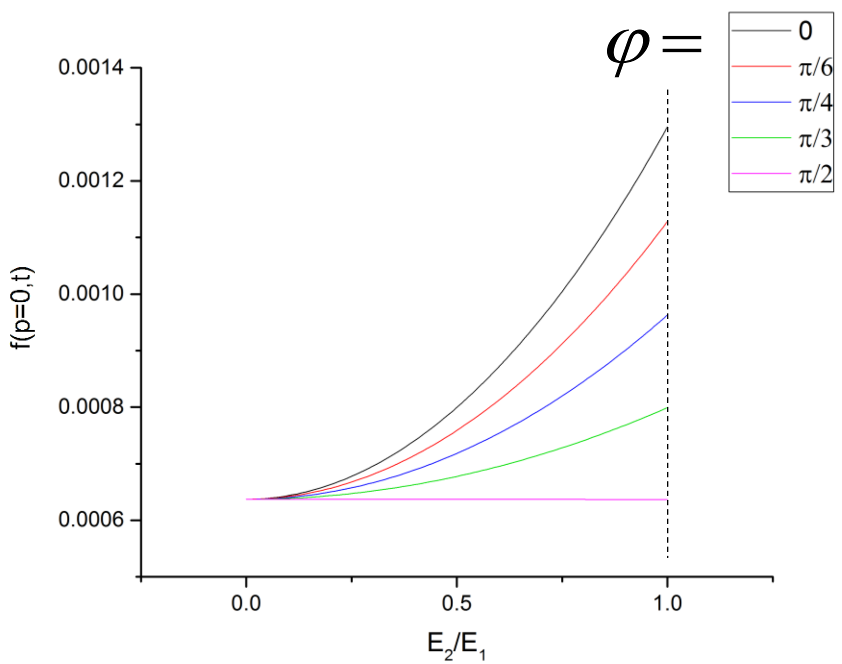
Preliminary results.

Field model $E_{1,2}(t) = E_{1,2}^{0} \cos(\omega t + \varphi_{1,2}) \exp\left[-t^2 / 2\tau^2\right]$

dependence on phase

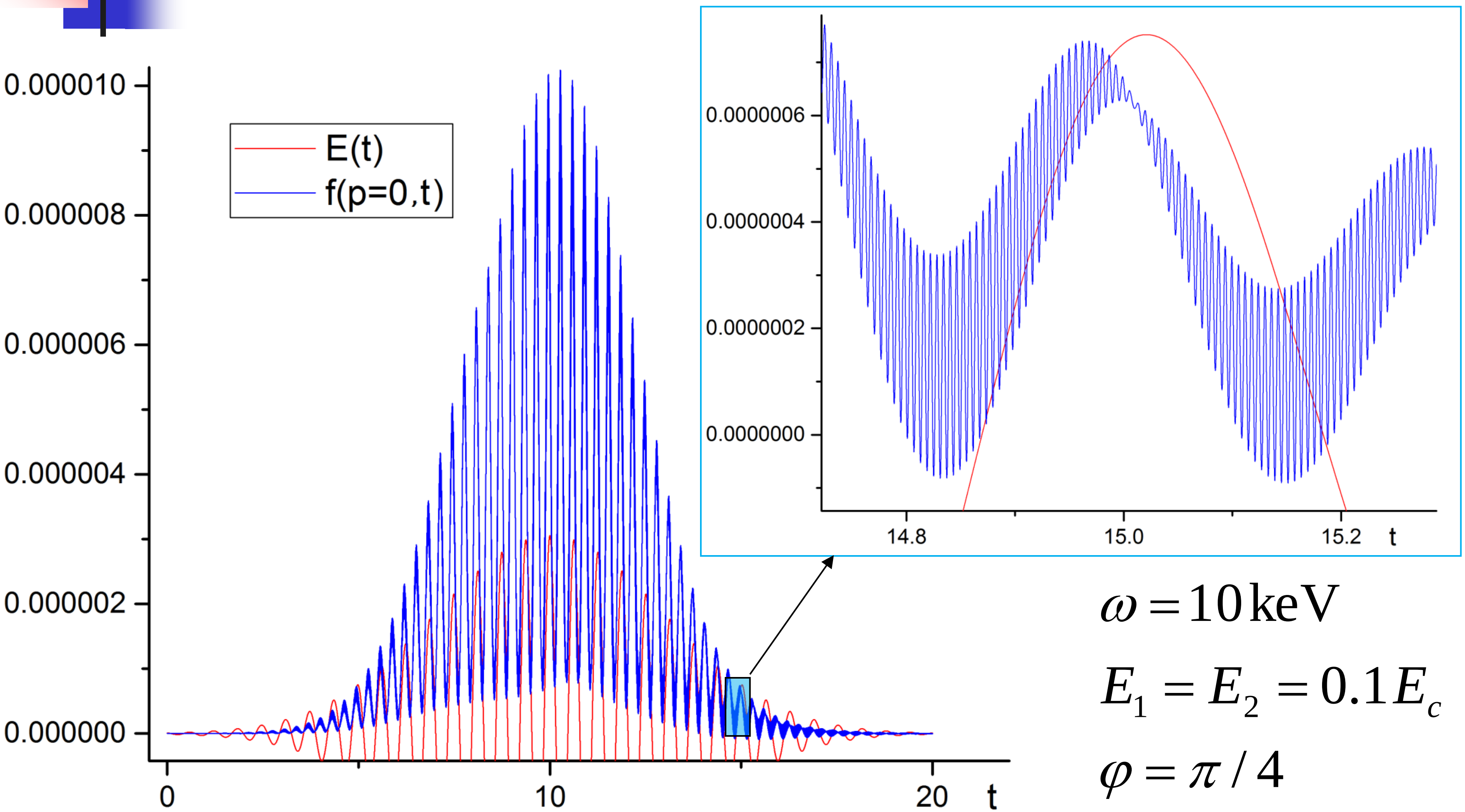
dependence on polarization





9. Widening the scope of the kinetic theory

Real x-rays laser simulation



$$\omega = 10\,\mathrm{keV}$$

$$E_1 = E_2 = 0.1E_c$$

$$\varphi = \pi / 4$$

10. Conclusion

The nearest problems:

1) development of the kinetics of EPP creation in the electric fields of arbitrary polarization;

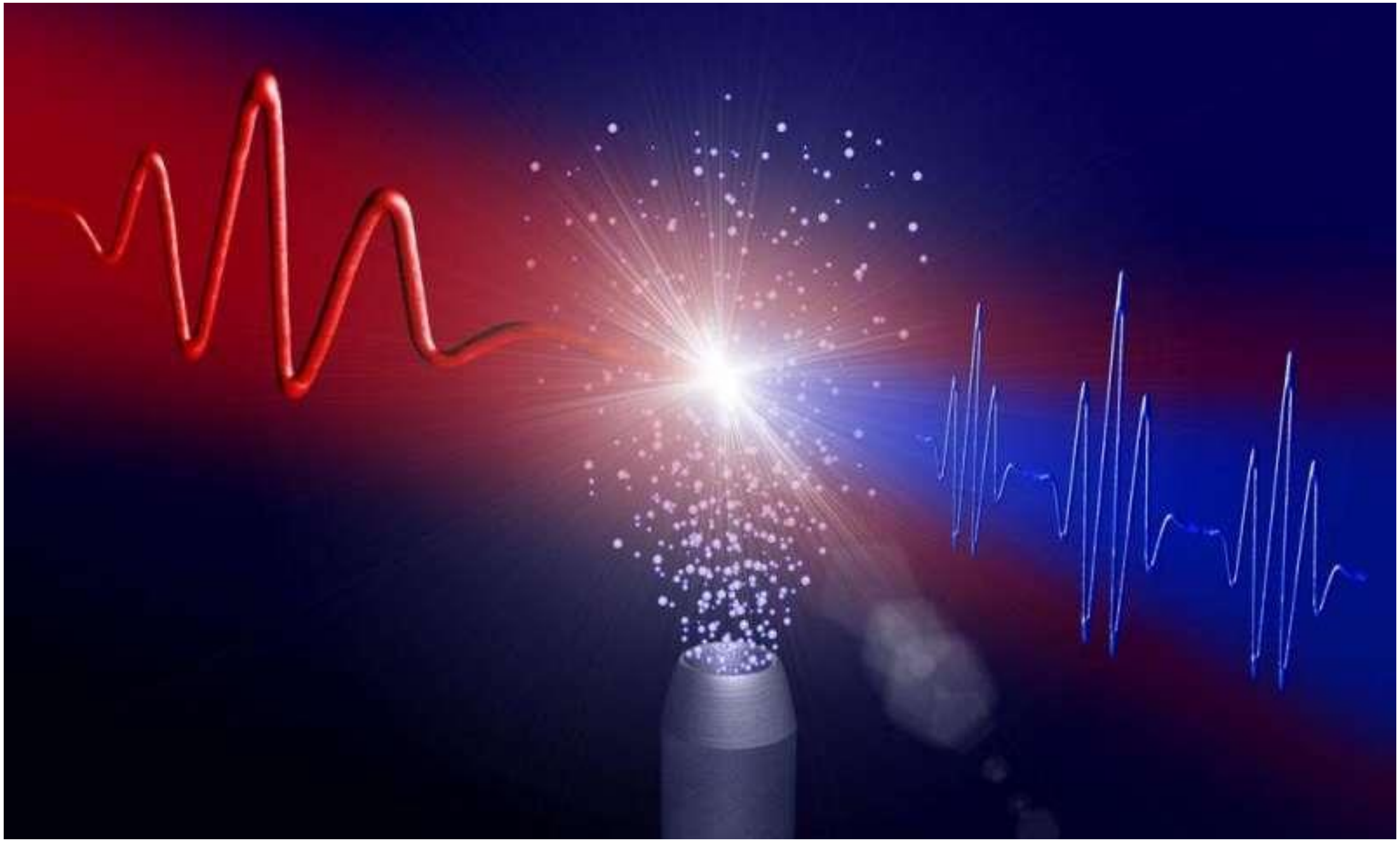
2) description of cascade processes;

3) kinetic description of vacuum creation of QGP etc.

The further perspective:

construction of the quantum hydrodynamic of the EPP vacuum creation.

The reported study was funded by RFBR according to the research project № 17-02-00375



THANK YOU FOR ATTENTION