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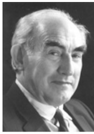

Violation of the Goldreich-Julian relation in a neutron star

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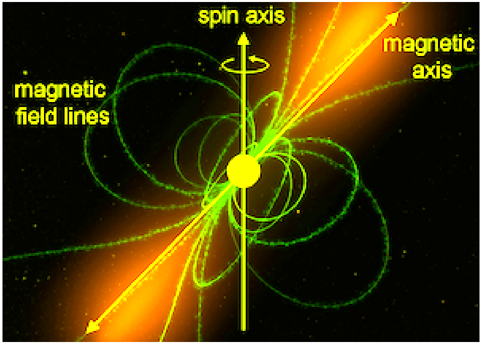
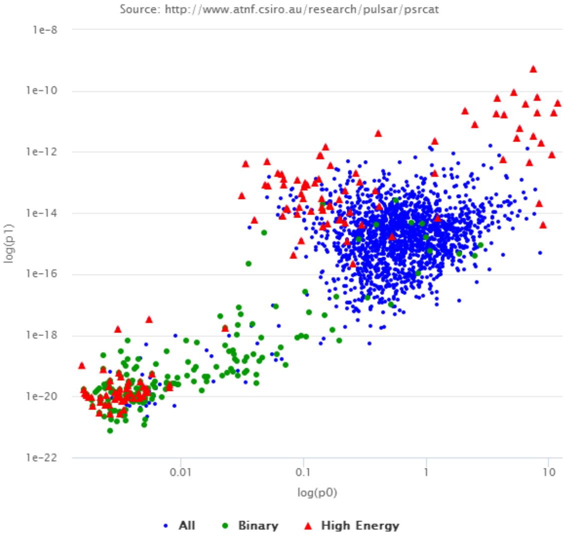
Neutron stars

- A compact massive rotating magnetized star
- Classical observational manifestation—the radio pulsar (Beskin 1999)





Multifarious zoo

- magnetars (Mereghetti 2008)
- gamma-ray pulsars (Caraveo 2014)
- rotating radio transients (RRATs) (McLaughlin et al. 2006)
- extreme nullers (Wang et al. 2007)
- hybrids of the above objects (Burke-Spolaor & Bailes 2010)

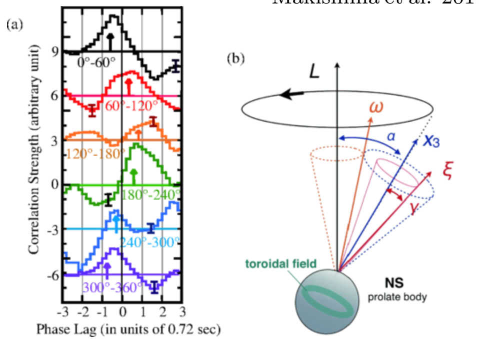
Exterior—a magnetosphere

- vacuum (Deutsch 1955)
- plasma-filled (Goldreich & Julian 1969)
- nonstationary—switching between the above cases (Istomin & Sob'yanin 2011)

Manifestations of the interior?

- ▻ precession (Link 2007)
- ▻ glitches (Espinoza et al. 2011)
- ▻ bursts (Deibel et al. 2014)

Makishima et al. 2014



Possible mechanism

- ▻ relation to deformation (Duncan 1998; Makishima et al. 2014; Haskell & Melatos 2015)
- ▻ role of the internal magnetic field (Cutler 2002; Lander et al. 2015; García & Ranea-Sandoval 2015)
- ▻ mechanics as a mediator between internal processes and observations

Manifestations beyond mechanics?

- example—generation of currents by a changing field
- reflection via crust heating and concomitant X-ray and radio emission
- requires the possibility of a change in the charge density

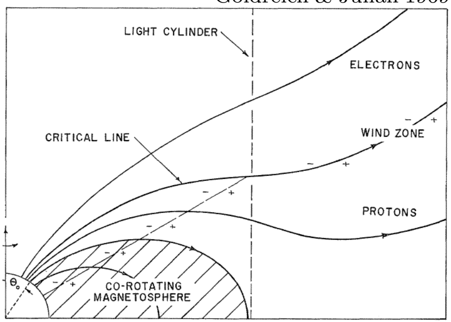
Common assumptions

The internal charge density is

- bounded
- unchanged
- equal to the Goldreich-Julian density

$$\rho_{\rm GJ} = \frac{-\mathbf{\Omega}\cdot\mathbf{B}/2\pi}{1-v^2}$$

Goldreich & Julian 1969



Neutron star

- arbitrary form
- arbitrary rigid rotation around a fixed point

$$\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r}$$

- Maxwell's equations

$$\operatorname{div} \mathbf{E} = 4\pi\rho, \quad \operatorname{div} \mathbf{B} = 0$$
$$\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{curl} \mathbf{B} = 4\pi\mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}$$

- infinite conductivity ("Ohm's law")

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

- latter follows from the relativistic Ohm law

$$\mathbf{j} = \sigma\gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B} - \mathbf{v}\mathbf{v} \cdot \mathbf{E}) + \rho\mathbf{v}$$

when $\sigma \to \infty$

Magnetic field

- freezing-in condition

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}(\mathbf{v} \times \mathbf{B})$$

- governing equation

$$\frac{d\mathbf{B}}{dt} = \mathbf{\Omega} \times \mathbf{B}$$

  with the full time derivative $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$

- similarity to the equation for the radius vector

$$\frac{d\mathbf{r}}{dt} = \mathbf{\Omega} \times \mathbf{r}$$

- magnetic field vector rotates analogously to the radius vector

Quaternions

- describe rotation of a rigid body
- in some sense resemble complex numbers

$$\Lambda = \cos\frac{\alpha}{2} + \boldsymbol{\zeta}\sin\frac{\alpha}{2}$$

- corresponds to rotation around an axis $\boldsymbol{\zeta}$ through an angle α
- generally time dependent, $\boldsymbol{\zeta} = \boldsymbol{\zeta}(t)$ and $\alpha = \alpha(t)$
- final radius vector is related to the initial radius vector via

$$\mathbf{r} = \Lambda \circ \mathbf{r}_0 \circ \bar{\Lambda}$$

- product of quaternions $\mathrm{M} = \mu_0 + \boldsymbol{\mu}$ and $\mathrm{N} = \nu_0 + \boldsymbol{\nu}$ is

$$\mathrm{M} \circ \mathrm{N} = \mu_0\nu_0 - \boldsymbol{\mu}\cdot\boldsymbol{\nu} + \mu_0\boldsymbol{\nu} + \nu_0\boldsymbol{\mu} + \boldsymbol{\mu}\times\boldsymbol{\nu}$$

- the product is associative but not commutative

$$\mathrm{M} \circ \mathrm{N} \neq \mathrm{N} \circ \mathrm{M}$$

Magnetic field

- rotation for an arbitrarily changing angular velocity

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}(t) = 2\dot{\Lambda} \circ \bar{\Lambda}$$

gives

$$\mathbf{B}(\mathbf{r}, t) = \Lambda \circ \mathbf{B}(\bar{\Lambda} \circ \mathbf{r} \circ \Lambda, 0) \circ \bar{\Lambda}$$

- corotation always

Electric field

- governing equation

$$\frac{d\mathbf{E}}{dt} = \boldsymbol{\Omega} \times \mathbf{E} - \mathbf{w} \times \mathbf{B}$$

with the rotational acceleration $\mathbf{w} = \dot{\boldsymbol{\Omega}} \times \mathbf{r}$

- corotation for a constant angular velocity

Charges and currents

$$\rho = \frac{\operatorname{div}\mathbf{E}}{4\pi}$$

$$\mathbf{j} = \frac{1}{4\pi}\left(\operatorname{curl}\mathbf{B} - \frac{\partial\mathbf{E}}{\partial t}\right)$$

- charge-current relation

$$\rho = \rho_{\mathrm{GJ}\,0} + \mathbf{j}_{\mathrm{m}} \cdot \mathbf{v}$$

- Goldreich-Julian density for zero velocity

$$\rho_{\mathrm{GJ}\,0} = -\frac{\mathbf{\Omega} \cdot \mathbf{B}}{2\pi}$$

- charge density depends on the azimuthal component of the magnetization current (not the total current)

$$\mathbf{j}_{\mathrm{m}} = \frac{\operatorname{curl}\mathbf{B}}{4\pi}$$

Charge density

- magnetization current corotates analogously to the magnetic field

$$\frac{d\mathbf{j}_{\mathrm{m}}}{dt} = \mathbf{\Omega} \times \mathbf{j}_{\mathrm{m}}$$

- Were $\mathbf{j}_{\mathrm{m}\,\phi} = \rho\mathbf{v}$, we would have the standard Goldreich-Julian density

$$\rho_{\mathrm{GJ}} = \frac{\rho_{\mathrm{GJ}\,0}}{1 - v^2}$$

- magnetization current is independent of $\rho\mathbf{v}$
- Goldreich-Julian relation does not hold

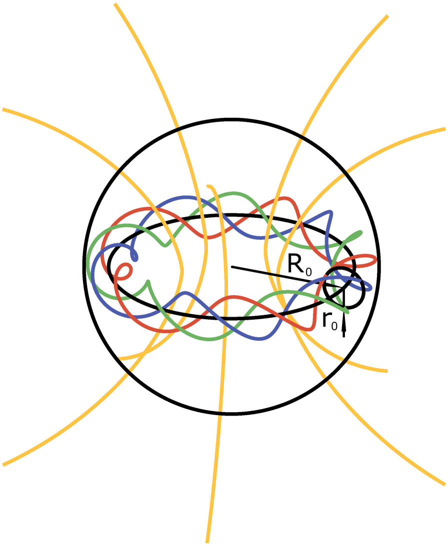
$$\rho \neq \rho_{\mathrm{GJ}}$$

- importance of the magnetic field topology
- twisting magnetic field lines results in locally accumulating charge

Super-Goldreich-Julian density

- charge accumulated due to twisting can significantly exceed the standard Goldreich-Julian value
- example—a twisted torus
- the charge density is $R_0/r_0 \gg 1$ times $\rho_{\mathrm{GJ}\,0}$
- for $r_0 \approx 1$ km and $R_0 \approx 10$ km the charge density is an order of magnitude higher
- possible structure of the internal magnetic field for the magnetar (Braithwaite and Nordlund 2006)

Sob'yanin 2016



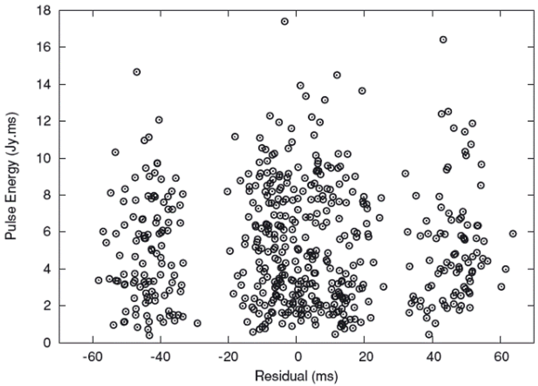
Crust heating

- twisting or untwisting magnetic field lines results in the appearance of currents
- charge accumulation in the twisted torus gives an extra electric field energy $\varepsilon \sim q^2/2R$ with the charge $q \sim (R_0/r_0)\rho_{\mathrm{GJ}\,0} V$
- energy release $\varepsilon \sim 10^{39}$ erg even when the magnetic field is unchanged
- heating due to formation of large stationary magnetization currents
- example—the magnetar crust
- thermal emission is provided by the energy release $H \sim 10^{20}\ \mathrm{erg\,cm^{-3}\,s^{-1}}$ (Kaminker et al. 2012)
- for the conductivity $\sigma \sim 10^{22}\ \mathrm{s^{-1}}$ the current density $j \sim \sqrt{\sigma H} \sim 10^{21}$ cgs units
- the current density can be obtained due to the electromagnetic field rearrangement accompanied by the appearance of a large charge density $\rho = \lambda \rho_{\mathrm{GJ}}$ with $\lambda \sim 100$
- change in the crustal magnetic field at small spatial scales $R/\lambda \sim 100$ m

Observational consequences: RRATs

- rotating radio transients as separate, sparse, short, relatively bright radio bursts
- typical burst rate 1 min^{-1}–1 h^{-1}
- intensity of single radio bursts from 100 mJy to 10 Jy at 1.4 GHz (Keane et al. 2010)
- phase is approximately retained
- underlying periodicity 0.1–6.7 s
- RRAT as a pulsar lightning (Istomin & Sob'yanin 2011)
- further application of the idea to FRBs (Katz 2017)

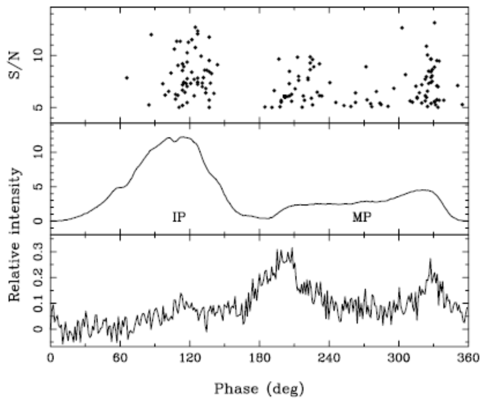
Lyne et al. 2009



Observational consequences: RRATs

- rearrangement of the internal magnetic field can manifest itself via crust heating by external magnetospheric effects related to radio emission
- heating can initiate a transition from a RRAT to pulsar state of the neutron star (Istomin & Sob'yanin 2011)
- observed in two hybrid radio sources PSR J0941-39 and PSR B0826-34 (Burke-Spolaor & Bailes 2010; Esamdin et al. 2012)

Esamdin et al. 2012



Summary

- neutron star rotating in an arbitrary way is considered
- charge density is not equal to and can exceed significantly the common Goldreich-Julian density
- charge distribution is connected with the magnetic field topology
- rearrangement of the internal magnetic field is potentially observable
- twisting and untwisting magnetic field lines causes internal currents that can heat the crust and change observational properties of neutron stars