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SMALL-SCALE DYNAMO, HELICITY FLUCTUATIONS AND MAGNETIC FIELD GENERATION IN CELESTIAL BODIES

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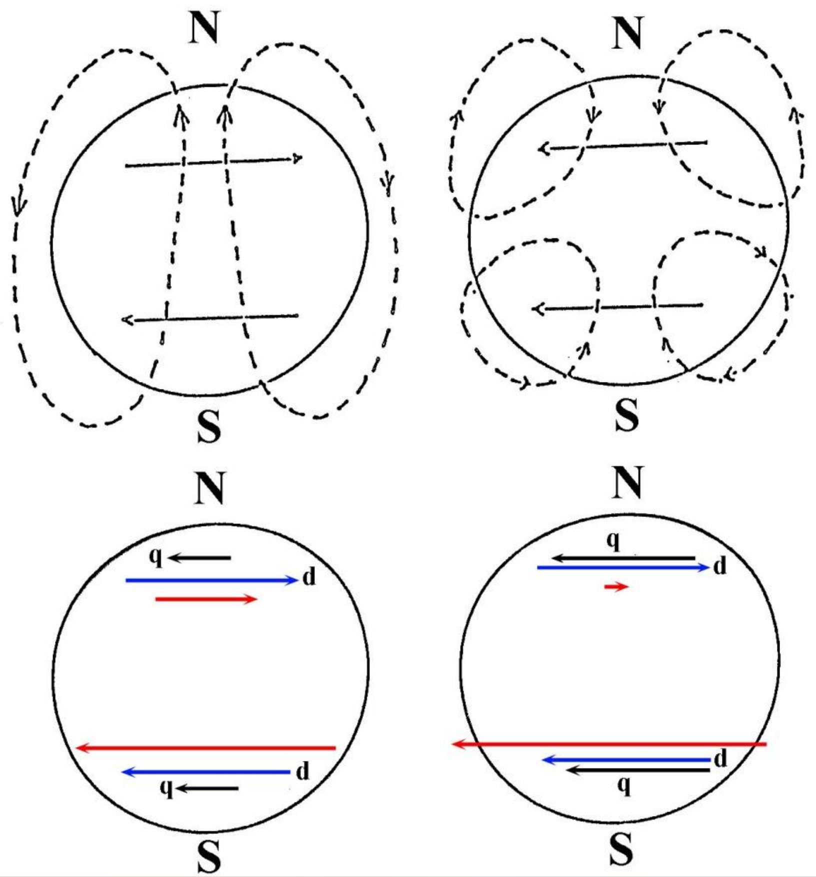
Mean-fed dynamo

$$B_P \xrightarrow{\Omega} B_T$$

Differential rotatiom

$$B_T \xrightarrow{\alpha} B_P$$

Mirror asymmatry



This dynamo produces mean as well as small-scale magnetic fields

Cycles in spherical shells

There is another type of dynamo: small-scale one.
No cycle, small-scale magnetic field only.

H=B+h+b

Dynamo instability. Exponential growth have to be somehow saturated.

Model of saturation: a balance relation.

An obvious (however not a very deep) idea – concervation of energy.

General rotation – plenty of energy. Better to compare magnet energy and energy of DIFFERENTIAL rotation or turbulence.

Not very clear how it works.

Another option – magnetic helicity concervation.

Helicity is a pseudoscalar and can affect directly mirror asymmetry (alpha-effect) which is a weak part of the dynamo self-excitation
chane.

Last 15 – 20 years this is a paradigm in dynamo studies.

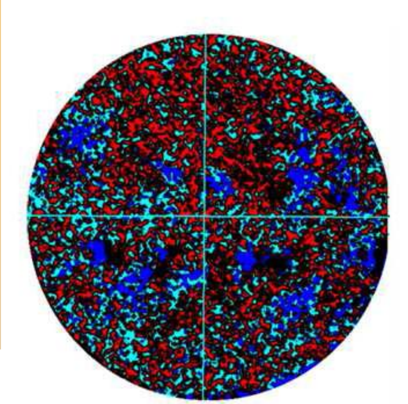
How it works?

Large-scale B growths.
B is helical.
Corrspondinng magnetic helicity growths.
It is inviscid invariant of motion.
Small-scale magnetic helicity have to compencate large-scale one. Helicity of b
kills alpha.

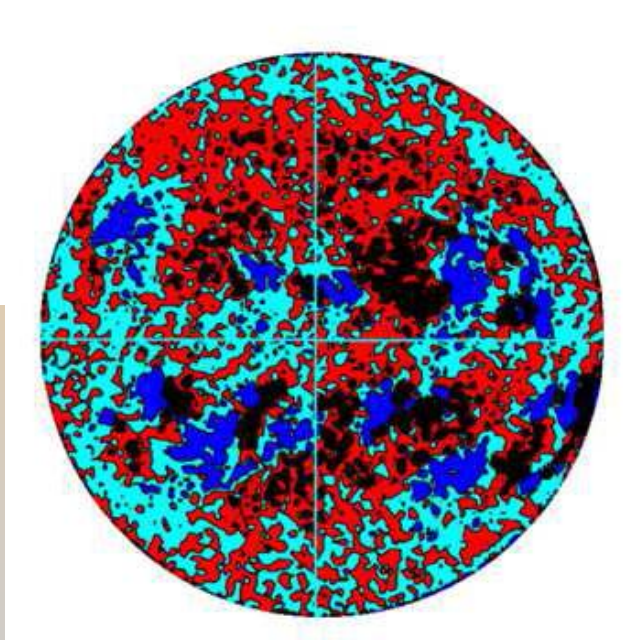
This scenario supposes that h is non-helical.

This can be however wrong.

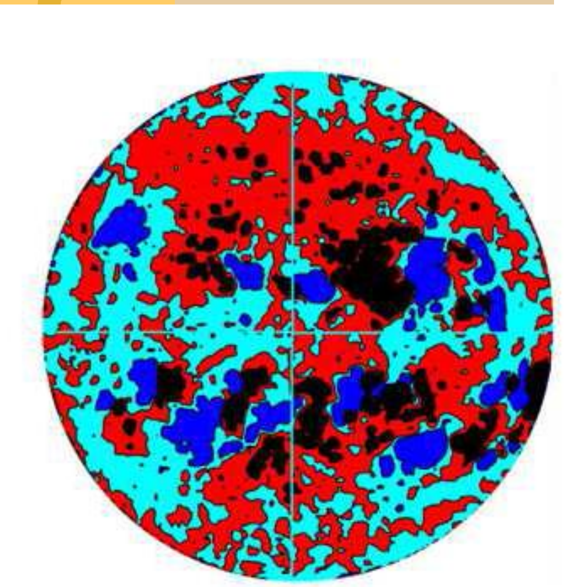
Solar magnetograms with various resolutions



a

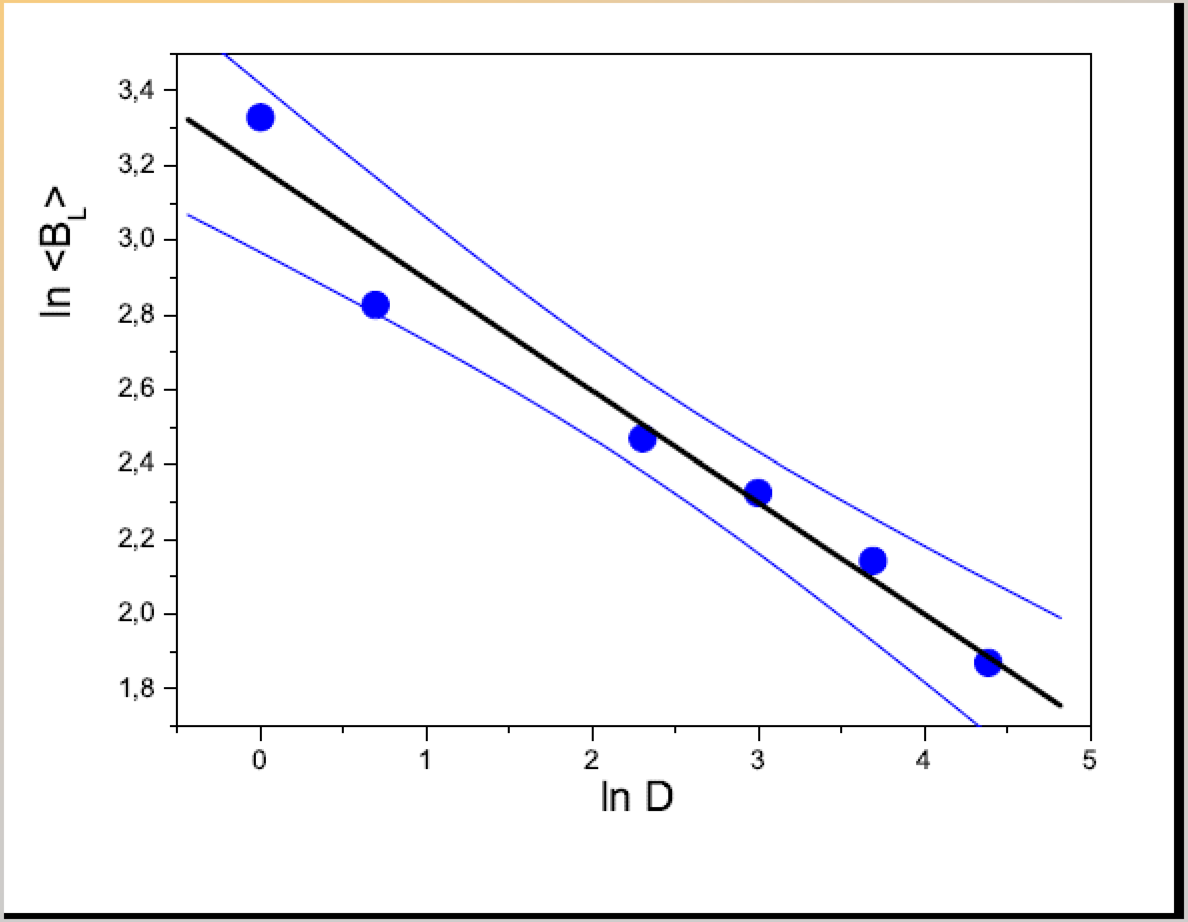


b

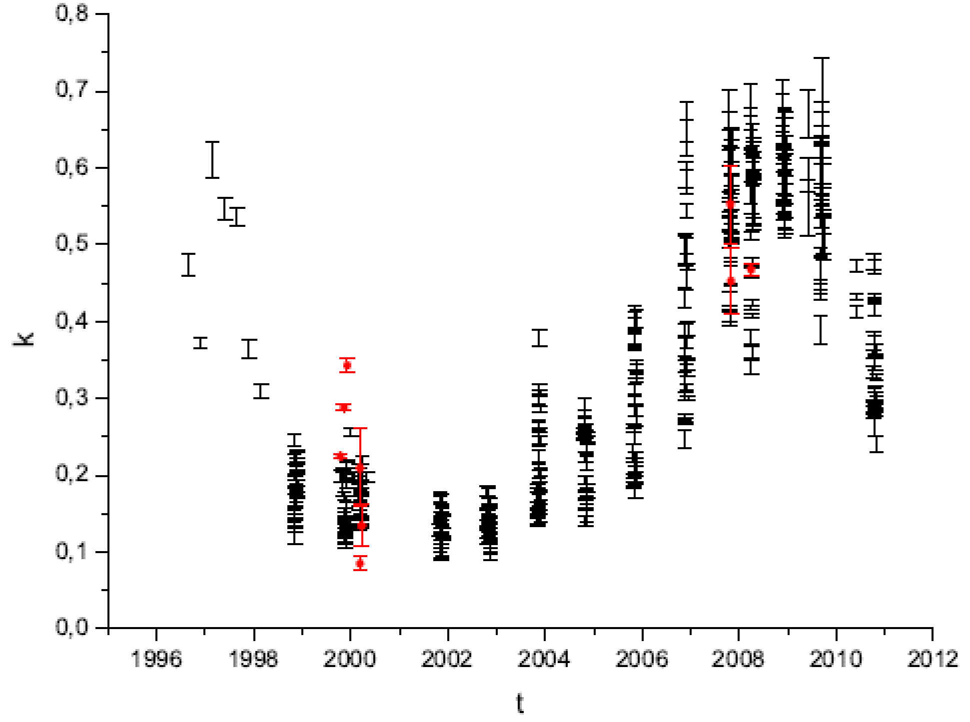


c

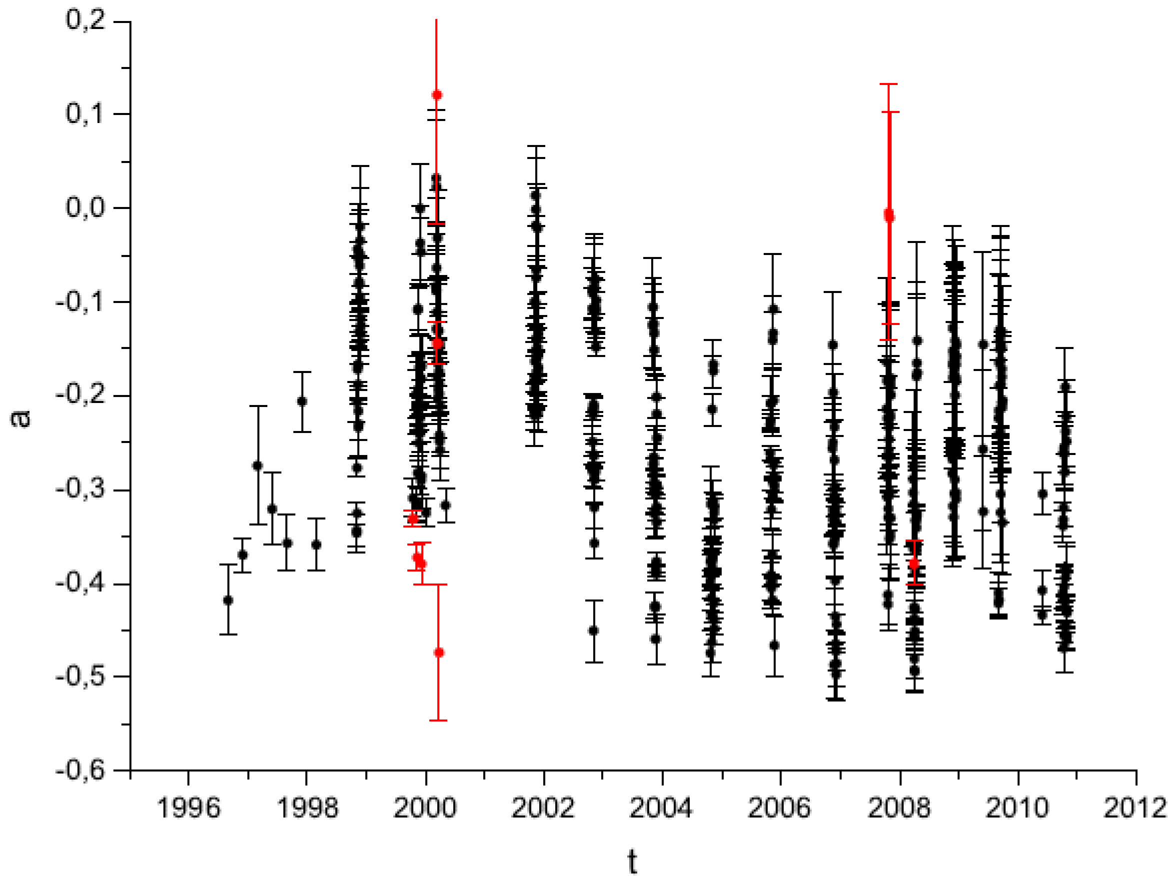
Fractal property of solar magnetic field



Hausdorf dimension of solar magnetic field in solar cycle
Shibalova, Obridko, Sokoloff, Solar Phys., 2017



Hausdorf measure (almost cycle independent)



One have to solve Kazantsev equation in a mirror-asymmetric flow.

For a mirror-symmetric flow h is non-helical (80-th).
We are interesting however in a mirror-asymmetric case..

People (e.g. Boldyrev) considered the case and clamed that h is helical..

However

A. Dynamo community ignored the claim.
B. B. He considered models inconsistent with assumptions under which Kazantsev equation is obtained. This equation is VEEERY delicate. One could hope that helicity of h is an artefact.

We revisited the point and analysed the problem as self-consistent as possible.

We obtain

1. h is helical indeed
2. Impute of helicity occurs in the dissipation scale so conservation law does not held.
3. Nonlinear agnetic field evolution in such a circumstances is almost not investigated (a rare exception is Frick et al., ApJ).

New field for investigation.

Kazantsev (1968)

$$\langle v^i(r,t)\cdot v^j(0,0)\rangle$$

Homogeneous and isotropic

Short-correlated

$$\left((F+rF_r/2)(\delta^{ij}-r^ir^j/r^2)+Fr^ir^j/r^2+G\epsilon^{ijk}r^k\right)\cdot\delta(t)$$

$$\langle B^i(r,t)\cdot B^j(0,0)\rangle$$

Correlations of magnetic helicity

Nonhelical part

$$(M+rM_r/2)(\delta^{ij}-r^ir^j/r^2)+Mr^ir^j/r^2+K\epsilon^{ijk}r^k$$

Vainshtein-Kitchatinov (1986)

$$M_t=2r^{-4}(r^4\eta M_r)_r+2Mr^{-4}(r^4\eta_r)_r-4\alpha K$$

$$K_t=r^{-4}(r^4(\alpha M+2\eta K)_r)_r,$$

$$\alpha(r)=G(0)-G(r)$$

$$\eta=\frac{1}{Rm}+\frac{F(0)-F(r)}{3}$$

Asymptotic soution

$$\gamma\phi = \eta\phi_{rr} + \left[\frac{\eta_{rr}}{2} + \frac{2\eta_r}{r} - \frac{2\eta}{r^2} + \frac{\eta_r^2}{4\eta}\right]\phi - \delta\eta(\theta_{rr} - 2r^{-2}\theta),$$

$$\gamma\theta = \eta\left[\theta_{rr} - 2r^{-2}\theta\right] + \delta\phi.$$

$$r \in [0, \sqrt{\varepsilon}]$$

$$\varepsilon(\phi_{rr} - 2r^{-2}\phi) - \varepsilon\delta(\theta_{rr} - 2r^{-2}\theta) = 0,$$

$$\varepsilon(\theta_{rr} - 2r^{-2}\theta) + \delta\phi = 0.$$

$$\eta = \varepsilon + \frac{r^2}{5} - \frac{3r^4}{50} + o(r^4)$$

$$\phi_{rr} - (2r^{-2} + \delta^2/\varepsilon)\phi = 0.$$

$$\phi \sim r^2$$

$$\theta \sim r^2$$

Задача вблизи нуля

$$\gamma\phi = \eta\phi_{rr} + \left[\frac{\eta_{rr}}{2} + \frac{2\eta_r}{r} - \frac{2\eta}{r^2} + \frac{\eta_r^2}{4\eta}\right]\phi - \delta\eta(\theta_{rr} - 2r^{-2}\theta),$$

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$$\phi_{rr} - (2r^{-2} + \delta^2/\varepsilon)\phi = 0.$$

$$\phi \sim r^2$$

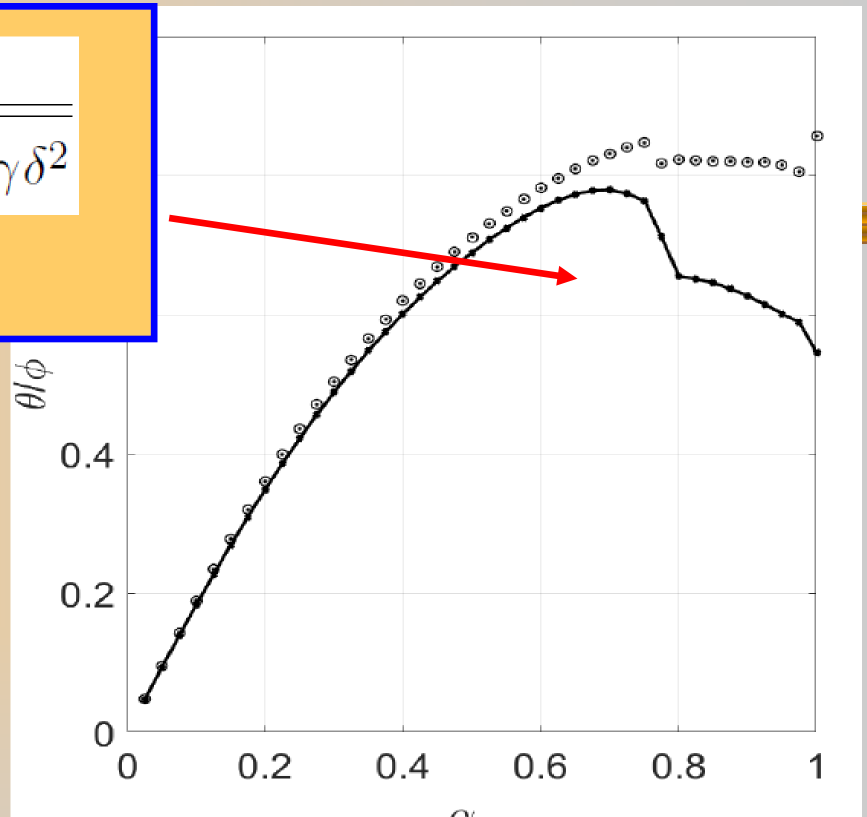
$$\theta \sim r^2$$

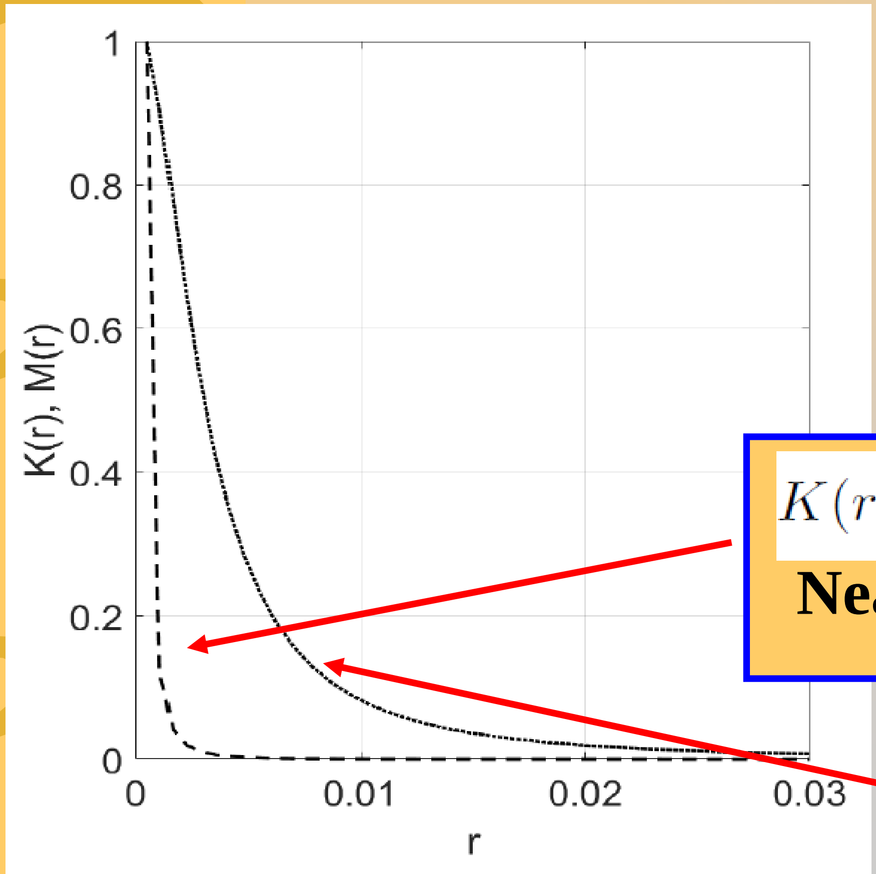
Nearby r=0

Symmetric and asymmetric parts

$$\frac{\theta(r)}{\varphi(r)} = \frac{5\delta}{5\delta^2/2+3+\sqrt{(5\delta^2/2+3)^2-25\gamma\delta^2}}$$

For weak mirror-asymmetry





$$K(r,t) = 2^{-1}\exp(2\gamma t)r^{-4}(r^4(r^{-2}\theta(r))_r)_r$$

Nearby r=0

$$M(r,t) = \exp(2\gamma t)r^{-2}\eta^{-1/2}\phi(r)$$

Conclusion

- About $10/Rm^{1/2}$ part of magnetic energy produced by small-sca;e dynamo is helical.