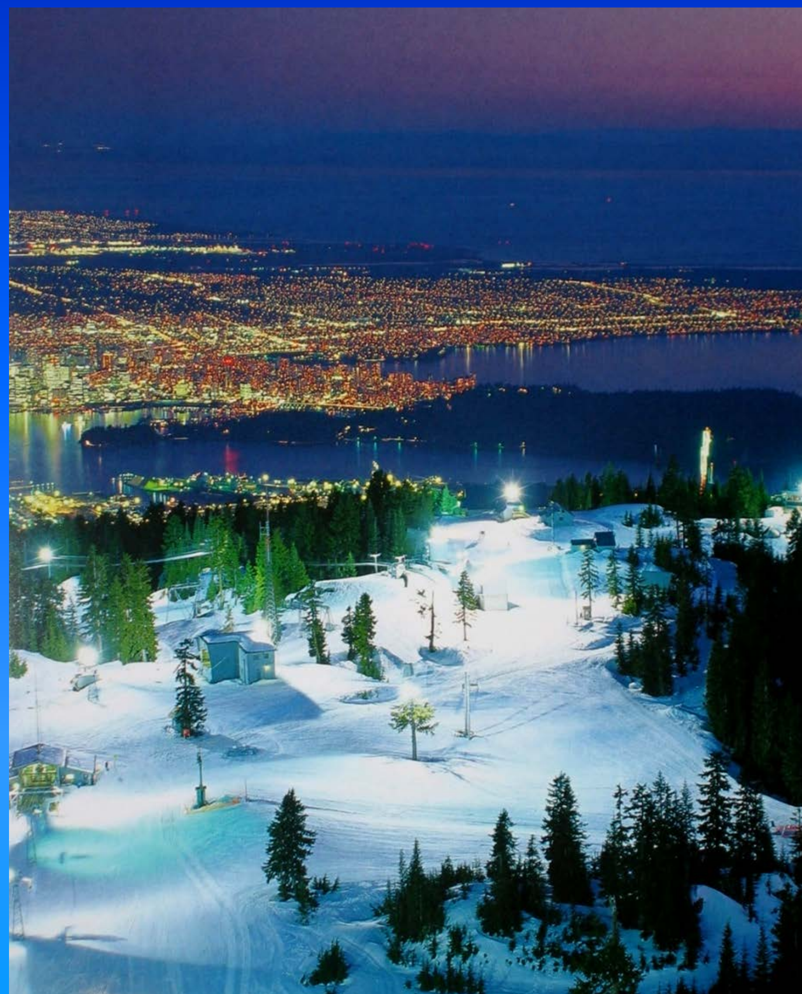
CORRELATED WORLDLINE THEORY of QUANTUM GRAVITY

P.C.E. STAMP

Ginzburg Centennial, Moscow
30 May 2017

Physics & Astronomy
UBC
Vancouver



Pacific Institute
for
Theoretical Physics

The Problem of Quantum Gravity

Researchers working on Quantum Gravity are after a really big question:
How to unify the 2 greatest theories of the 20th century.

Neither theory has ever failed;
but there are severe incompatibilities between the two.

Some idea of the divergence of opinions can
be found from the following recent reviews:

(1) "Quantum Gravity: A Brief History of Ideas and Some Prospects "
S Carlip, D-WChiou, W-T Ni, R Woodard /arXiv 1507.08194

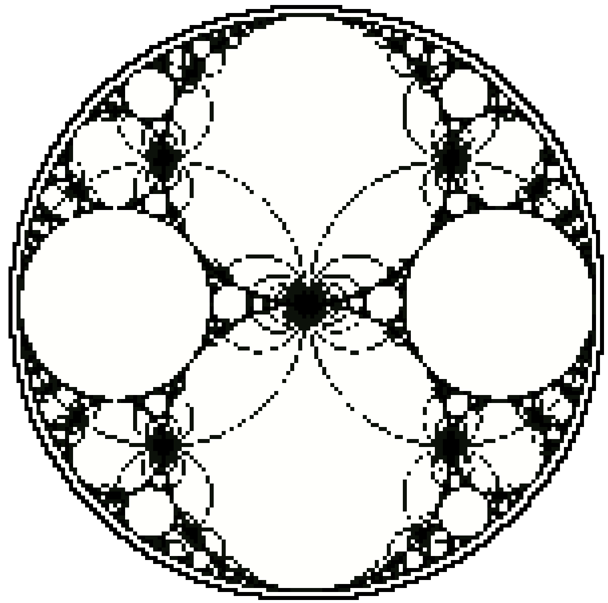
(2) "Information Loss" WG Unruh RM Wald /arXiv 1703.02140

(3) "The Black Hole information problem: past, present, future" D Marolf /arXiv 1703.02143

Currently, the two main lines of thought are
STRING THEORY, & LOOP QUANTUM GRAVITY

Are these really the only two approaches that one can adopt?

The Correlated Worldline Theory of Quantum Gravity



This talk briefly describes a theory which tries a different approach – I also discuss experimental tests of the theory

For details:

PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
“ , New J. Phys. 17, 06517 (2015)
A Barvinsky, D Carney, PCE Stamp, to be published

NSERC CRSNG

John Templeton Foundation

ACKNOWLEDGEMENTS:

A BARVINSKY D CARNEY R PENROSE H BROWN
G Semenoff M Aspelmeyer RM Wald

Question: HOW MACROSCOPIC is QUANTUM MECHANICS?

What evidence for N-particle /superpositions/entanglement for $N >> 1$?

(1) PHASE SUPERPOSITION/ENTANGLEMENT: Consider cases where the 2 states are not physically separate, but out of phase. A famous example is the SQUID macroscopic superposition experiment (Leggett). Define:

$$\Delta N_{tot} = \sum_{\mathbf{k},\sigma} \langle \circlearrowleft | \hat{c}^{\dagger}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k},\sigma} | \circlearrowleft \rangle - \langle \circlearrowright | \hat{c}^{\dagger}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k},\sigma} | \circlearrowright \rangle$$

Results from experiments (similar for experiments on superfluids):

		L	ΔI_p	$\Delta\mu$	ΔN_{tot}
SUNY	Nb	560 μm	2–3 μA	$5.5 - 8.3 \times 10^9 \mu_B$	3800–5750
Delft	Al	20 μm	900 nA	$2.4 \times 10^6 \mu_B$	42
Berkeley	Al	183 μm	292 nA	$4.23 \times 10^7 \mu_B$	124

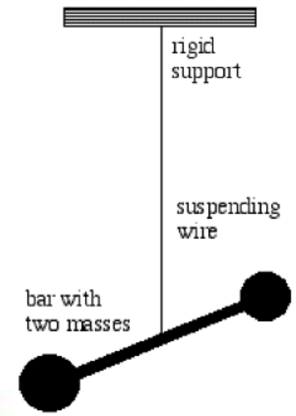
Korsbakken et al., Phys Rev A75, 042106 (2007)
Korsbakken et al., Europhys Lett 89, 30003 (2010)
Volkoff & Whaley, Phys Rev A89, 012122 (2014)

(2) MASS INTERFERENCE EXPERIMENTS: These are done by sending a mass through a 2-slit device. The largest masses for which this has so far been done are $m \sim 10^5$ AMU (ie., $m \sim 10^{-14} M_P$, where M_P is the Planck mass)

M Arndt, K Hornberger, Nat Phys 10, 271 (2014)
T Juffmann et al., Rep Prog Phys. 76, 086402 (2013)

THUS – QM has not yet been tested at the macroscopic level.

Problems with Quantum Gravity at low E



Feynman 1957, Karolhazy 1966, Eppley-Hannah 1977, KIBBLE 1978-82, Page 1981, Unruh 1984, PENROSE 1996, argued there is a basic conflict between QM & GR at ordinary 'table-top' energies.

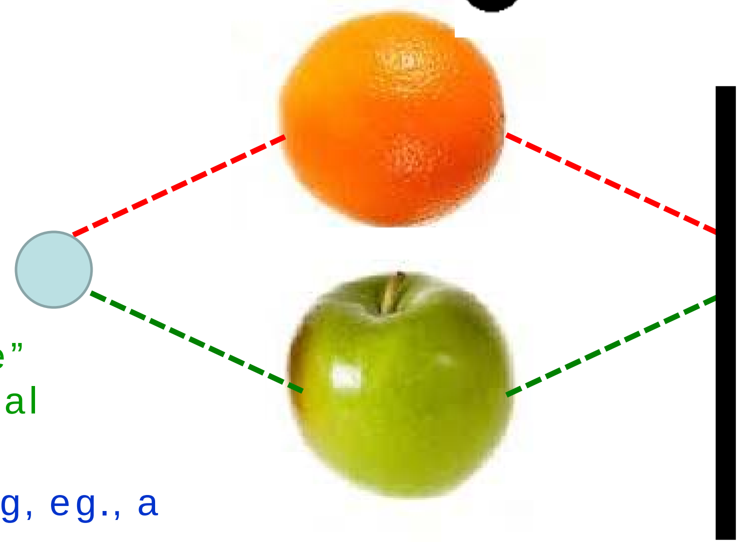
Consider a 2-slit experiment with a mass M, and a wave-fn

$$|\Psi\rangle = a_1|\Phi_1; \tilde{g}^{\mu\nu}_{(1)}(x)\rangle + a_1|\Phi_2; \tilde{g}^{\mu\nu}_{(2)}(x)\rangle$$

In a non-relativistic treatment we write

$$\Phi(\mathbf{r},t) \equiv \langle \mathbf{r}|\Phi(t)\rangle = a_1\Phi_1(\mathbf{r},t) + a_2\Phi(\mathbf{r},t)$$

and then: $\langle \Phi_1(t)|\Phi_2(t)\rangle = \int d^3r \, \langle \Phi_1^*(\mathbf{r},t)|\Phi_2(\mathbf{r},t)\rangle$



(i) FORMAL PROBLEM: There are 2 different coordinate systems, $(\mathbf{r}_j, t_j)$, defined by 2 different metrics $\tilde{g}^{\mu\nu}_{(j)}(x)$; in general their causal structure is different.

(ii) PHYSICAL PROBLEM: "Wave-function collapse" causes non-local changes, including unphysical changes in the metric.

The same problems arise in a path integral; writing, eg., a propagator

$$\mathcal{K}(x,x') = \int \mathcal{D}g^{\mu\nu}\Delta(g)\, e^{\frac{i}{\hbar}S_G[g^{\mu\nu}]} \int_{x'}^{x} \mathcal{D}q\, e^{\frac{i}{\hbar}\, S_M[q,g^{\mu\nu}]}$$

we are again superposing metrics with different lightcone structure.

SOME SUGGESTED REMEDIES:

- No wave function collapse → Many Worlds (can we take this seriously?)
- Treat Q Gravity as a low-E effective theory only (what abut low-E superpositions?)

What is needed is a concrete theory addressing these problems

CORRELATED WORLDLINE THEORY: BASIC IDEAS

1. RULES of the GAME

We want to keep the following features in the theory:

(i) connection between phase (+ connection), and action on worldlines (paths)

(ii) indistinguishability for multiple particles and/or fields

(iii) fully relativistic – obeying the weak principle of equivalence, no violation of causal structure, well-defined metric.

(iv) gravity/spacetime is treated as a quantum field as well as matter

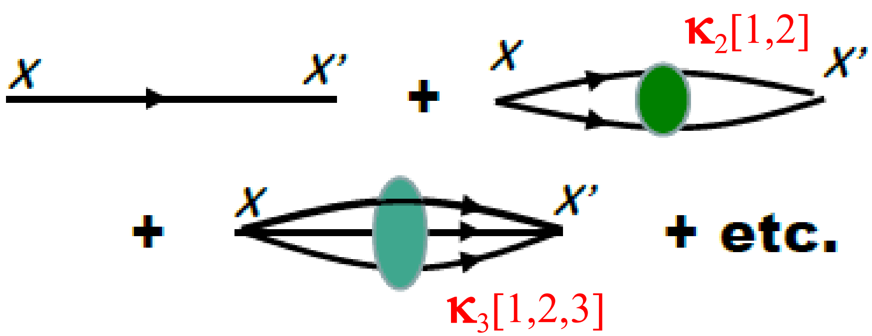
Q: How can we modify QM/QFT, in a way consistent with those features we wish to keep?

The answer goes roughly as follows; we change the mathematics to:

$$G_o(2,1) = \int_1^2 \mathcal{D}q(\tau) e^{\frac{i}{\hbar}S(2,1)} \longrightarrow \sum_{n=1}^{\infty} \prod_{k=1}^{n} \int_1^2 \mathcal{D}q_k(\tau)\, \kappa_n[\{q_k\}]\, e^{\frac{i}{\hbar}S[q_k;2,1]}$$

In other words, we allow arbitrary correlations between any number of different paths. Since the paths are no longer independent, the superposition principle is no longer valid in general !

A diagrammatic view of this is: G(x,x') =



NB: These are obviously not conventional Feynman diagrams: The rules are quite different

2: NATURE of the CORRELATORS $\kappa_n[q_1,....q_n]$ between PATHS

We treat the quantum phase along each path as physical – the relationship between them is then 'seen' by gravity. Because of indistinguishability & the equivalence principle, gravitational interactions between 2 paths for 2 different particles/fields, and 2 paths for the same particle/field, are then not distinguished.

We thus arrive at the following prescription:

Use the correlator:

$$\kappa_n = \int \mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x)\; e^{\frac{i}{\hbar}S_G}\,\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$$

ie., integrate over different spacetimes with a weighting factor

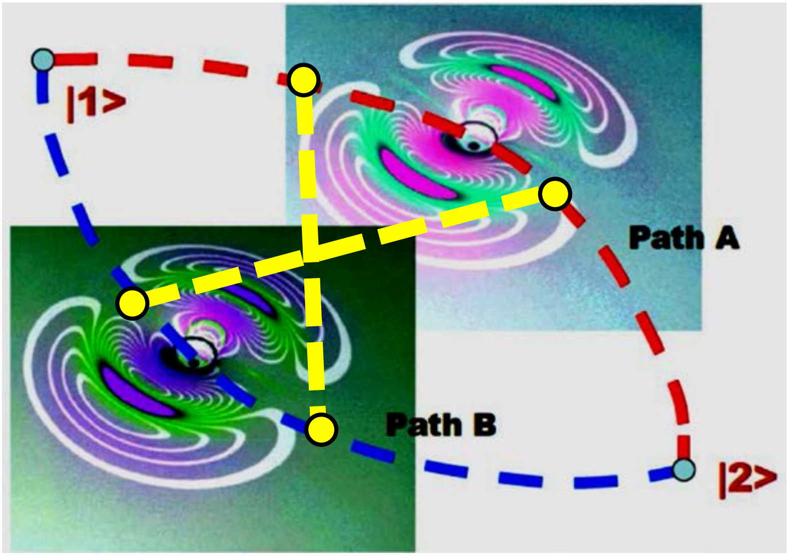
metric density → $\mathcal{D}\tilde{\mathfrak{g}}^{\mu\nu}(x)$; gravitational action → S_G; Faddeev-Popov determinant → $\Delta[\tilde{\mathfrak{g}}^{\mu\nu}(x)]$

This COMMUNICATES BETWEEN PATHS information about each path's spacetime status (<u>and</u> about the object's distortion of spacetime).

We thus get a breakdown of the QM superposition principle, at scales where gravity is important (~ M_P)

This leads to what we will call "Correlated Worldline Theories"

It turns out there is a whole class of these CWL theories



"Summation" version of CWL Theory

In this first version of the CWL theory, we sum over all the possible gravitational correlations between pairs of paths, triplets of paths, etc. Then we have the following form for the generating functional

$$\mathbb{Q}[J] = \oint \mathcal{D}g^{\mu\nu} \Delta(g)\, e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]}\, \tilde{\mathbb{Q}}[J,g]$$

$$\equiv \oint \mathcal{D}g^{\mu\nu} \Delta(g)\, e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]} \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{k=1}^{n} \oint \mathcal{D}\phi_k\, e^{\frac{i}{v_n \hbar} \sum_k \left(S_M[\phi_k, g^{\mu\nu}] + \int dt J(x)\phi_k(x)\right)}$$

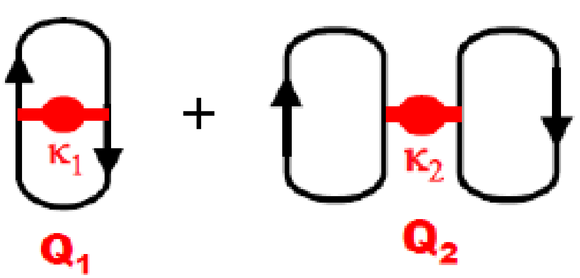
and the corresponding propagator between field configurations is

$$\mathcal{K}(\Phi, \Phi'|J) = \oint \mathcal{D}g^{\mu\nu} \Delta(g)\, e^{\frac{i}{\hbar} S_G[g^{\mu\nu}]}$$

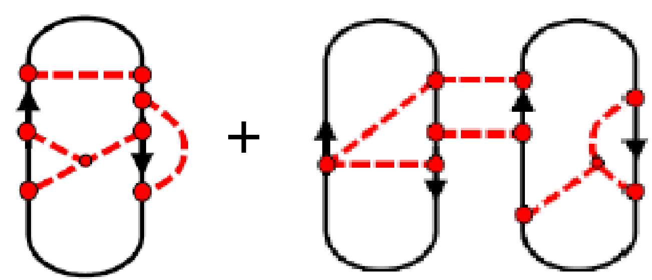
$$\times \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{k=1}^{n} \int_{\Phi'}^{\Phi} \mathcal{D}\phi_k\, e^{\frac{i}{v_n \hbar} \sum_k \left(S_M[\phi_k, g^{\mu\nu}] + \int dt J(x)\phi_k(x)\right)}$$

The function v_n is a regulator, to be discussed below (originally $v_n = 1/n$)

Diagrammatically, for the generating functional we sum:



+ etc. ➔>>>



+

"Product" version of CWL Theory

The summation form for the generating functional satisfies the usual consistency tests (Ward identities, etc), but there is a problem coming from the regulator v_n . If v_n increases with n, which helps to make the theory renormalizable, then we get divergent quantum fluctuations at large n. The classical limit is also hard to define.

The problems can be cured by writing a product form for the CWL theory, but has to take a rather special form. We now introduce not only a set of dummy or "replica" field configurations for the matter fields, but also for the gravitational field; and we write:

$$\mathbb{Q}[J] = \prod_{n=1}^{\infty} \int Dg_n e^{-nS_G[g_n]} \prod_{i=1}^{n} \int D\phi_i^{(n)} \, e^{-S[g_n,\phi_i^{(n)}]+\frac{J}{v_n}\phi_i^{(n)}}$$

$$= \prod_{n=1}^{\infty} \int Dg_n e^{-nS_G[g_n]} \left(Z\left[g_n, \frac{J}{v_n}\right]\right)^n = \prod_{n=1}^{\infty} \mathcal{Z}_n[J]$$

Notice several features of this:

- The factor n in front of the gravitational action means a gravitational coupling ~ 1/n, suppressing higher terms
- We have CWL correlations inside each n-th level, but not between different n

REMARKS on PRODUCT CWL THEORY

Let's look at the eqtns of motion. These are

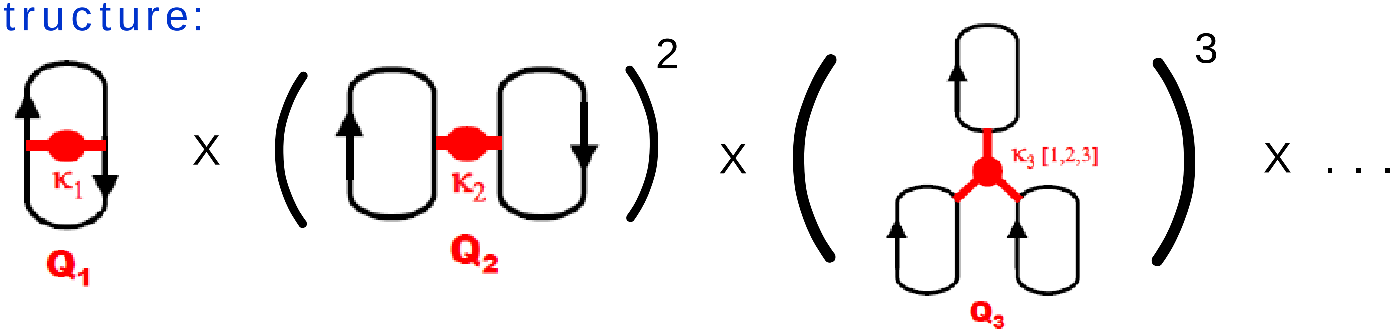
$$\frac{\delta S[g_n, \phi_i^{(n)}]}{\delta \phi_i^{(n)}} - \frac{1}{v_n} J = 0 \qquad \text{and} \qquad n\,\frac{\delta S_G[g_n]}{\delta g_n} + \sum_{i=1}^{n} \frac{\delta S[g_n, \phi_i^{(n)}]}{\delta g_n} = 0,$$

These eqtns of motion, for some specific value of n, are the same for any i.

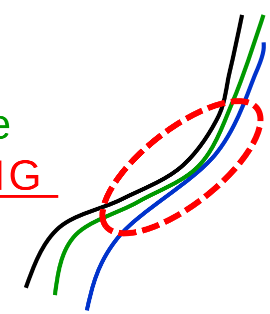
Then the stress tensors, viz., $T^{\mu\nu}[g, \phi_i^{(n)}] = 2\delta S[g, \phi_i^{(n)}]/\delta g_{\mu\nu}$ are the same for all n, and so we get

$$\frac{\delta S_G[g_n]}{\delta g_n} + \frac{\delta S[g_n, \phi^{(n)}]}{\delta g_n} = 0,$$

ie., the Einstein eqtns for all different n. We can perform semiclassical expansions around this. Diagrammatic expansions have a non-standard structure:



<u>A key physical point</u>: Gravitational "interactions" between the different paths for even a single system cause <u>PATH BUNCHING</u> - the attractive CWL correlations, for large masses, then suppress interference → classical behaviour.



CWL THEORY - SLOW DYNAMICS

The key scales in this limit are:

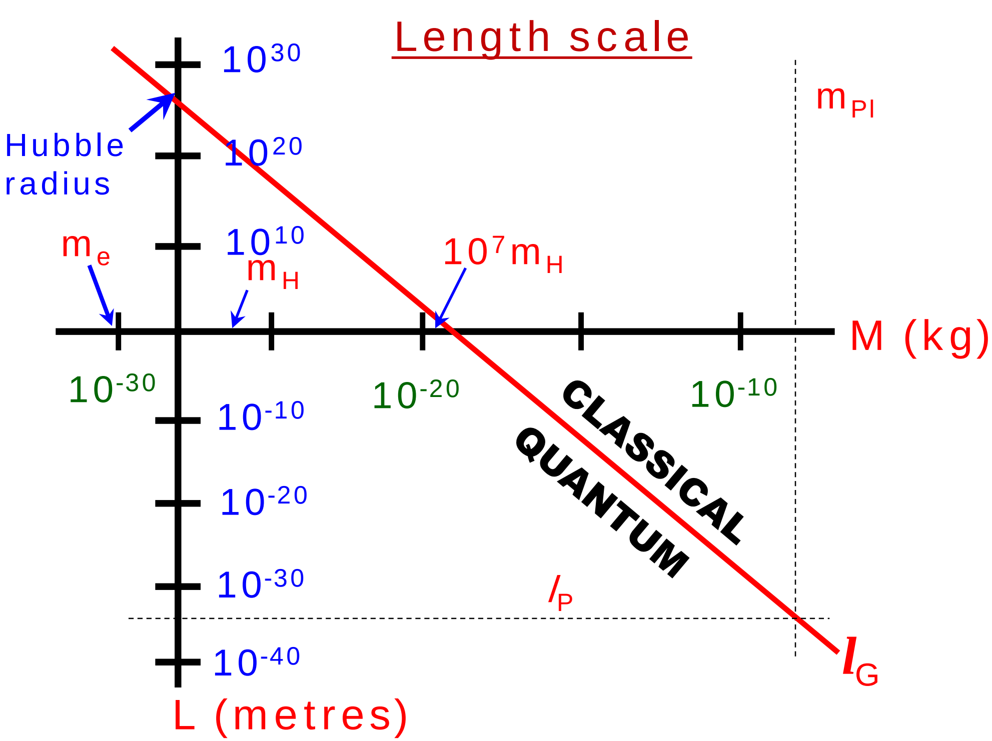
$$l_G(m) = \left(\frac{M_p}{m}\right)^3 L_p$$ Newton radius (gravitational analogue of the Bohr radius)

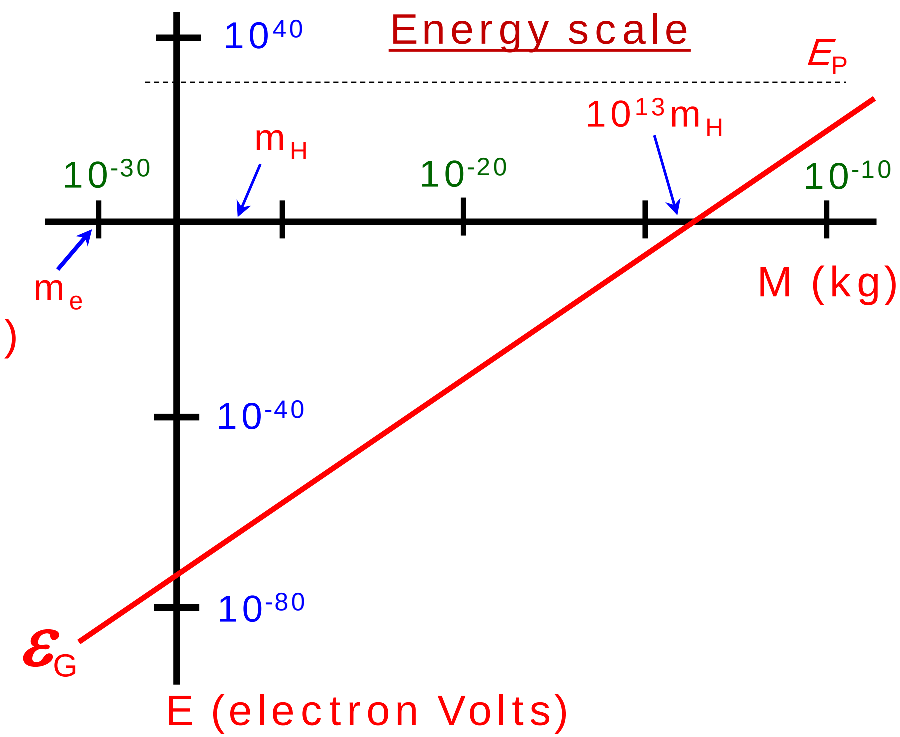
$$\epsilon_G(m) = G^2m^2/l_G(m) \equiv E_p(m/M_p)^5$$ Mutual binding energy for paths

} (QM)

$$R_s = 2Gm/c^2$$ Schwarzchild radius for the particle (Classical)

The potential well giving this 'Coulomb-Newton' attraction causes a 'path bunching'. 2 paths will bind if $\varepsilon_G > E_Q$ where E_Q is the energy scale associated with any other perturbations from impurities, phonons, photons, imperfections in any controlling potentials in the systems, and, worst of all, dynamical localized modes like defects, dislocations, paramagnetic or nuclear spins, etc.





N-PARTICLE SYSTEM (SLOW-MOVING)

write positions around the centre of mass $\mathbf{R_o}(t) = \frac{1}{N}\sum_{j=1}^{N} \mathbf{q}_j(t)$ so that $\mathbf{q}_j = \mathbf{R_o} + \mathbf{r}_j$

The effective action is then

$$S_o[\mathbf{R}_o, \{\mathbf{r}_j\}] = \int d\tau \left[\frac{M_o}{2}\dot{\mathbf{R}}_o^2 + \sum_{j=1}^{N} \frac{m_j}{2}\dot{\mathbf{r}}_j^2 - \sum_{i<j}^{N} V(\mathbf{r}_i - \mathbf{r}_j) \right]$$

Define sum & difference coordinates:

$$\begin{aligned} \mathbf{r_j} + \mathbf{r'_j} &= \mathbf{x}_j & \qquad \mathbf{R_o} + \mathbf{R'_o} &= \mathbf{X_o} \\ \mathbf{r_j} - \mathbf{r'_j} &= \xi_j & \qquad \mathbf{R_o} - \mathbf{R'_o} &= \mathbf{\Xi_o} \end{aligned}$$

Lowest-order correction to propagator:

$$\Delta\mathcal{G}(2,1) = \int \mathcal{D}\mathbf{X}_o \int \mathcal{D}\mathbf{\Xi_o} \prod_j \int \mathcal{D}\mathbf{x}_j \int \mathcal{D}\xi_j$$

$$\times \quad \kappa_2^N[\Xi_o; \{\xi_j\}] \int d\mathbf{P} d\mathbf{K} \; e^{\frac{i}{\hbar N}(\mathbf{P}\cdot\mathbf{x}_j + \mathbf{K}\cdot\xi_j)} \; e^{i\Psi_2[\Xi_o, \{\xi_j\}; \mathbf{X_o}, \{\mathbf{x}_j\}]}$$

where the C.o.m. correlates gravitationally with the individual particles according to

$$\kappa_2^N[\Xi_o, \{\xi_j\}] = \left(\exp\left[\frac{i\lambda^2}{4\pi\hbar} \int d\tau \sum_{j=1}^{N} \frac{m_j^2}{|\Xi_o + \xi_j|} \right] - \delta_{\Xi_o}\delta_{\xi_\mathbf{j}} \right)$$

PHONON EFFECTS We can understand the main effect by looking at the displacement correlator

$$\langle u_i^\alpha(t_1) u_j^\beta(t_1) \rangle = \frac{1}{N} \sum_{\mathbf{Q}\mu} \frac{\hat{e}^\alpha_{\mathbf{Q}\mu} \hat{e}^\beta_{\mathbf{Q}\mu}}{2m\omega_{\mathbf{Q}\mu}} e^{i[\mathbf{Q}\cdot\mathbf{r}_{\mathbf{ij}}^{(\mathbf{o})} - \omega_{\mathbf{Q}\mu}(t_1 - t_2)]}$$

Typical displacements: 10^{-12} -- 6×10^{-12} m

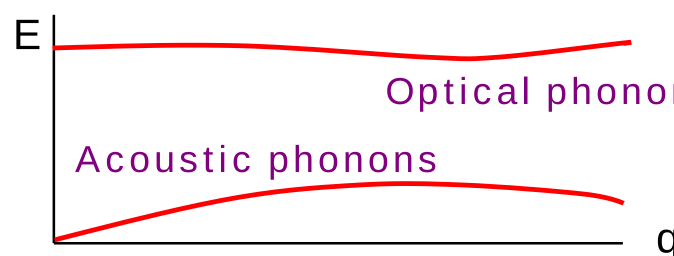


TABLE-TOP EXPERIMENTS

Only wimps specialize in the general case. Real scientists pursue examples. MV Berry (1995)

I. MECHANICAL OSCILLATOR

Now we add a term to the action: $S_U[\mathbf{R_o};\mathbf{F_o}] = -\int d\tau \left[\frac{1}{2}U_o\mathbf{R_o^2}(\tau) + \mathbf{F_o}(\tau)\cdot\mathbf{R_o}(\tau)\right]$

In the absence of any coupling between the phonons and the centre of mass, we get

$$\begin{aligned}\mathcal{G}(2,1) &= G_{osc}(2,1)\, G_c(2,1) \\ &\equiv G_{osc}(\mathbf{X_o}^{(2)},\mathbf{X_o}^{(1)}|\mathbf{F_o}(t))\;\; G_c(\mathbf{\Xi_o}^{(2)},\mathbf{\Xi_o}^{(1)};\{\xi_\mathbf{j}^{(2)},\xi_\mathbf{j}^{(1)}\})\end{aligned}$$

where the latter term incorporates the reduction of the path-bunching coming from individual ion dynamics.

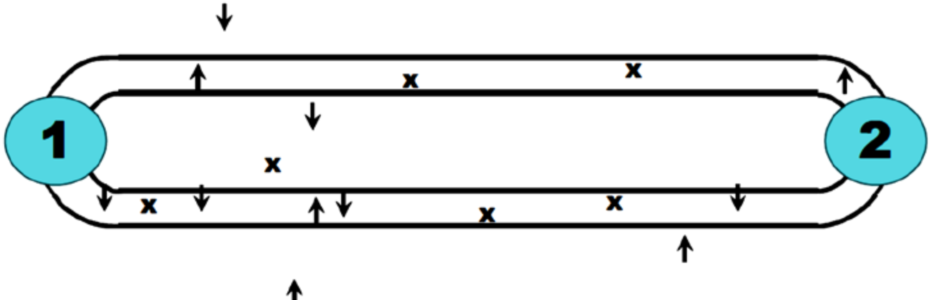
The final result depends strongly on both the phonon dynamics and on the coupling of phonons to defects and spin impurities. The onset of path bunching is now at mass scales $M \sim 10^{19}$-$10^{20}\ m_H$ with an effective path-bunching length $\sim 10^{-19}$ -10^{-16} m.

Such an experiment has many attractive features.

II. 2-SLIT EXPERIMENT

This is at first glance a very attractive experiment to analyse – but to realize it will be very difficult. For an extended mass the numbers come out similarly to those for the oscillator – but the influence of defects and impurities is much greater.

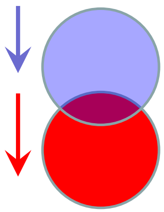
Such an experiment is likely impossible – even if one could do interference for such large objects.



CRUCIAL RESULT: The CWL CORRELATIONS & PATH BUNCHING MECHANISM DO NOT INVOLVE DECOHERENCE !!

OPTOMECHANICAL RESONATOR EXPERIMENTS

(1) The key idea is to look at interference between 2 separate states of a moving object – the 2 paths here, corresponding to the 2 different positions of the mass, will interact gravitationally in the CWL theory according to what we have seen. One can imagine lots of different ways to do this – eg., with a freely falling mass, or with a resonator put into a superposition of 2 different oscillating states.

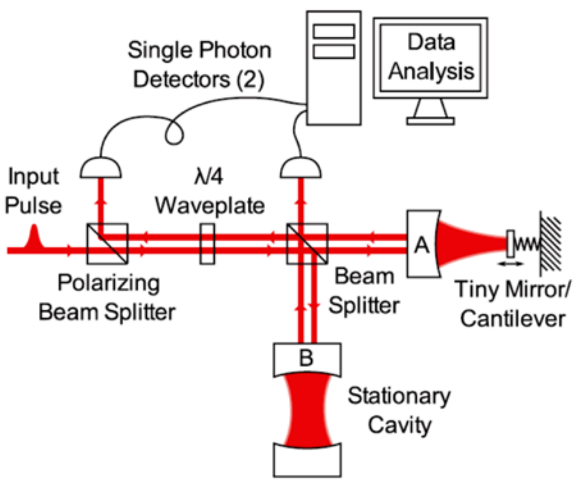


(2) Let's look 1st at interference between the 2 paths of an oscillating heavy mass. One way to do this is to entangle a photon with a heavy mirror, and then look for gravitational effects. Starting from a state

$$|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$$

we get

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B\;|\kappa(1 - e^{-i\omega_m t})\rangle_m],$$



and one looks at interference between the 2 branches. Another alternative is to look at interference between a 0-phonon and a 1-phonon state

D Kleckner et al., N J Phys 10, 095020 (2008)

I Pikowski et al., Nat Phys 8, 393 (2012)

(3) The difficulty here is to reduce environmental decoherence effects – coming from the interaction with photons, or between, eg., charged defects in the system (or spin defects/nuclear spins) and EM fields.

A KEY RESULT: Gravitational effects depend in a completely different way on system parameters than do decoherence effects.

COMPARISON with OTHER PREDICTIONS

COMPARISON with PENROSE RESULT: Penrose argues that the 2 proper times elapsed in a 2-branch superposition cannot be directly compared; there is a time uncertainty, related to an energy uncertainty given in weak field by

$$\Delta E = 2E_{1,2} - E_{1,1} - E_{2,2}.$$

$$E_{i,j} = -G \int\int d\vec{r}_1 d\vec{r}_2 \frac{\rho_i(\vec{r}_1)\rho_j(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

R Penrose Gen Rel Grav 28, 581 (1996)

W Marshall et al., PRL 91, 130401 (2003)
D Kleckner et al., NJ Phys 10, 095020 (2008)

There are 2 problems here:

(i) The density is fed in by hand – it should be calculated from the theory itself, and will depend on the UV cutoff

(ii) It is only the first term in an exponential.

To understand this, note that each individual term in our correlator is meaningless. It is not permissible to expand the exponential – if we do, each term gives a divergent contribution:

$$\kappa_2[\mathbf{r},\mathbf{r}'] = \sum_{n=1}^{\infty}\prod_{j=1}^{n}\int^{t_j} d\tau_j\, \theta(\tau_j - \tau_{j-1})\delta(t-\tau_n)\, \frac{(4\pi i G m^2)^n}{|\mathbf{r}(\tau_j) - \mathbf{r}'(\tau_j)|}$$

$$= \int^t d\tau \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|} + \int^t d\tau \int^{\tau} d\tau' \frac{4\pi i G m^2}{|\mathbf{r}(\tau) - \mathbf{r}'(\tau)|}\frac{4\pi i G m^2}{|\mathbf{r}(\tau') - \mathbf{r}'(\tau')|} + \ldots.$$

If we feed in the density by hand, the role of a UV cutoff is obvious from the results:

$$\Delta E = \frac{Gmm_1}{x_0}\left(\frac{24}{5} - \frac{1}{\sqrt{2\kappa}}\right)$$

"Zero point" estimate

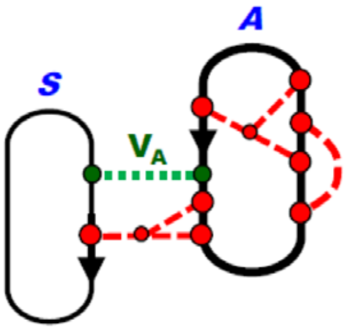
$$\Delta E = 2Gmm_1\left(\frac{6}{5a} - \frac{1}{\Delta x}\right)$$

"nuclear radius" estimate

These numbers differ by roughly 1000 !

MEASUREMENTS & PROBABILITY in CWL THEORY

The difference between standard QM and the CWL theory comes down to the path bunching effect. Let's see how this works.



The system and apparatus now couple via some measurement coupling – one useful example is

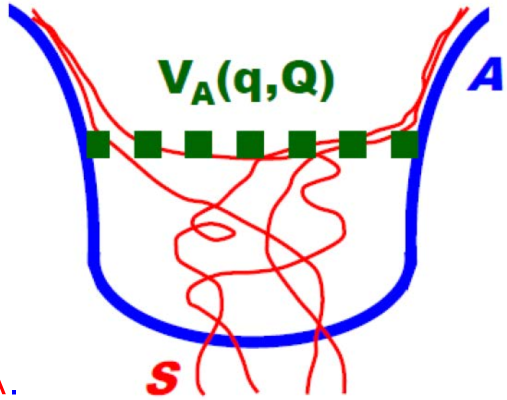
$$V_A(q, Q) = -i\hbar V(q, \partial/\partial q; t)\, \frac{\partial}{\partial Q}$$

where the coordinates Q & q refer to the apparatus *A* and the system *S* respectively. This coupling has the effect of slowly moving the apparatus coordinate Q into synchronization with the coordinate q of the system.

However although gravitational correlations between S and A are negligible, the CWL correlations in the dynamics of A are not; we assume its mass is sufficiently large so that path bunching eventually occurs in the dynamics of A (over some path bunching timescale).

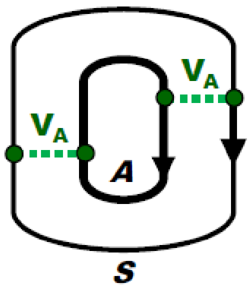
Then, as the paths of A start to diverge into 2 classes (associated here with 2 different possible paths of S, because of the coupling between S and A), we see that the path bunching stops any interference between the 2 sets of corresponding paths for A – the paths of A separate into 2 bunches.

But this also means that the paths for S will do the same; they get 'dragged' into separate bunches because of the S-A interaction . Thus interference is suppressed for both S and A.



EXPECTATION VALUES & PROBABILITIES

So far we have not discussed either wave-functions or 'projection operators' or 'measurements'. Actually there is no need to discuss these explicitly, since all we require are the correlations set up between system & apparatus – 2nd order perturbation theory is at left.



Suppose however that the apparatus is coupled to an environment; eg.,

$$L(\phi,\chi) = [\partial_\mu \phi \partial^\mu \phi - m^2\phi^2] - \frac{1}{2}\phi^2(x)\chi(x) + [\partial_\mu \chi \partial^\mu \chi - M^2\chi^2]$$

so that then $\mathcal{Q}_o[j,I] \rightarrow \oint \mathcal{D}\phi\, F[\phi,I] \exp\frac{i}{\hbar}\left[\mathcal{S}_\phi + \int d^4x\, j(x)\phi(x)\right]$

with decoherence functional $F[\phi,I] = \oint \mathcal{D}\chi \exp\frac{i}{\hbar}\left[S^o_\chi + \int d^4x\, [I(x)+\lambda\phi(x)]\chi(x)\right]$

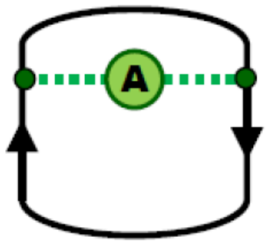
$$\rightarrow \Omega_o[\phi] \exp\frac{-i}{2\hbar}\left[\int d^4x \int d^4x'\, I(x)\tilde{\mathcal{D}}(x,x'|\phi)I(x')\right]$$

where $\Omega_o[\phi] = \exp -\frac{1}{2}[Tr \ln |\tilde{\mathcal{D}}_\chi(x,x'|\phi)/D^o_\chi(x,x')|]$,

with $[(D^o_\chi)^{-1}(x,x') + \lambda\phi^2(x)]\, \tilde{\mathcal{D}}_\chi(x,x'|\phi) = -\delta(x-x')$

The net effect of this is to reduce the coupling to a trace; in the non-relativistic limit:

$$< A(t) > = \int dx_1^+ \int dx_2^-\, A(x_1^+, x_2^-)\rho(x_2^-, x_1^+)\delta(t_1^+ - t)\delta(t_2^- - t)$$



More generally: $\langle M_j \rangle = Tr\{M_j \rho\}$

$$= \int dq \int dq'\, M_j(q,q')\rho(q',q).$$

Notice that at no point here have we written down either wave-functions, or state vectors, or operators.


THANK YOU!
PCE Stamp, Phil Trans Roy Soc A370, 4429 (2012)
“ , New J. Phys. 17, 06517 (2015)
A Barvinsky, D Carney, PCE Stamp, to be published