Holographic Reconstruction

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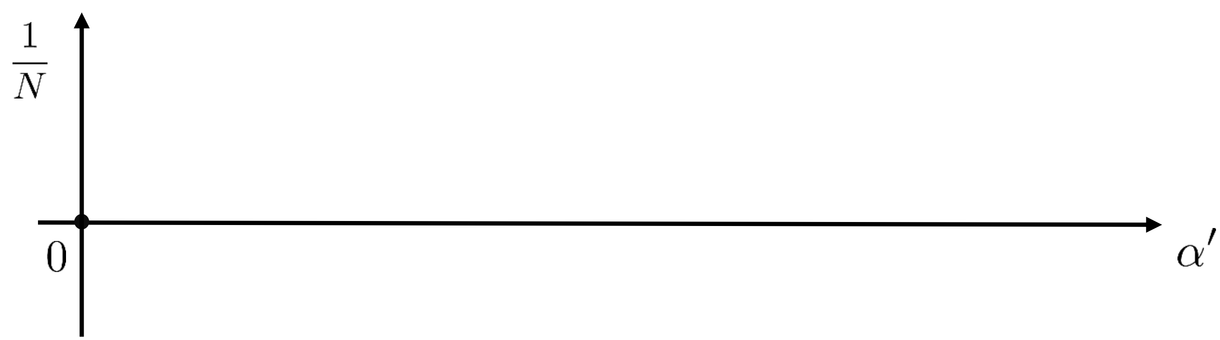
Based on: arXiv:1603.00022, 1609.00991, 1702.08619, 1704.07859
(with Charlotte Sleight)

Tensionless Limit & AdS

- The standard AdS/CFT dictionary provides the basic mapping of parameters:

$$N \sim \left(\frac{L}{l_p}\right)^d \qquad\qquad \lambda = N g_{YM}^2 \sim \left(\frac{L^2}{\alpha'}\right)^{d/2}$$

- strong/weak duality ***vs.*** weak/weak duality:



[Haggi-Mani & Sundborg, Witten; Klebanov et al.; ...]

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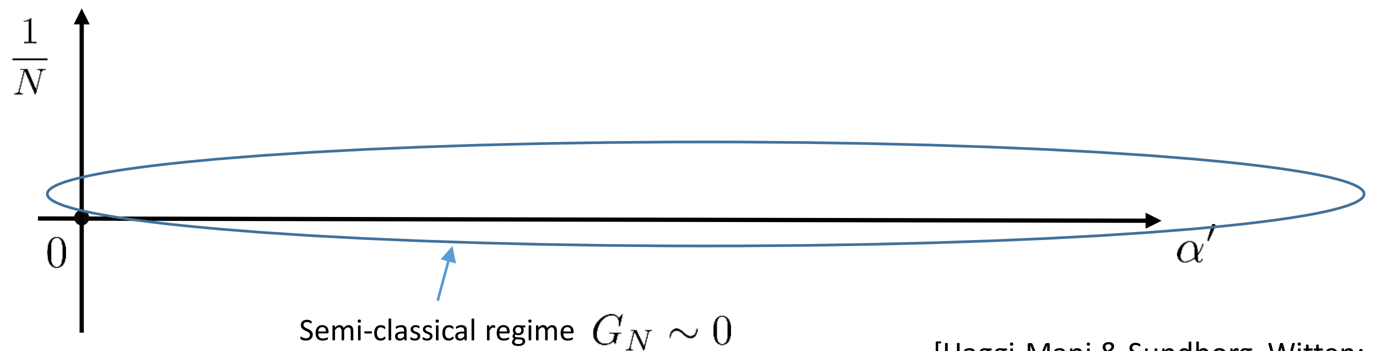

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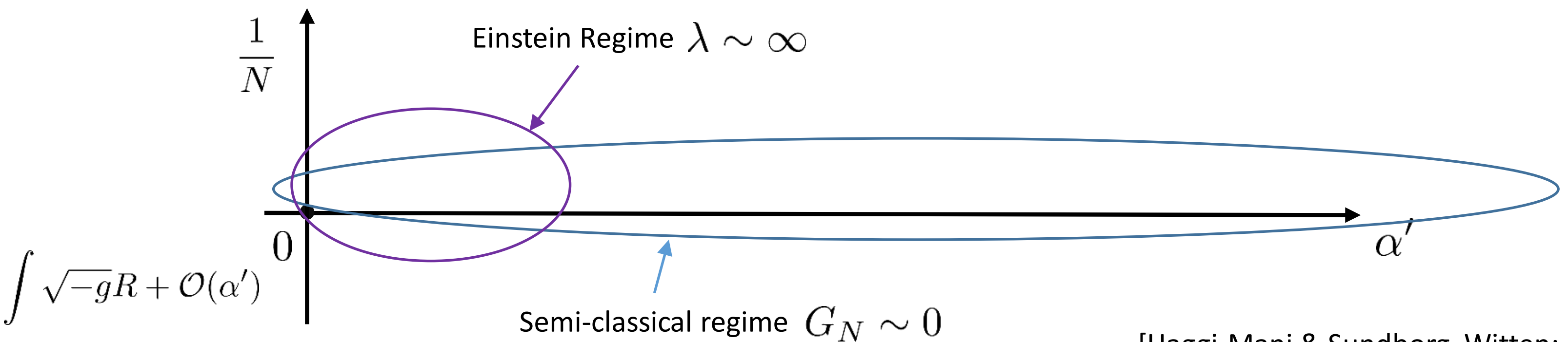

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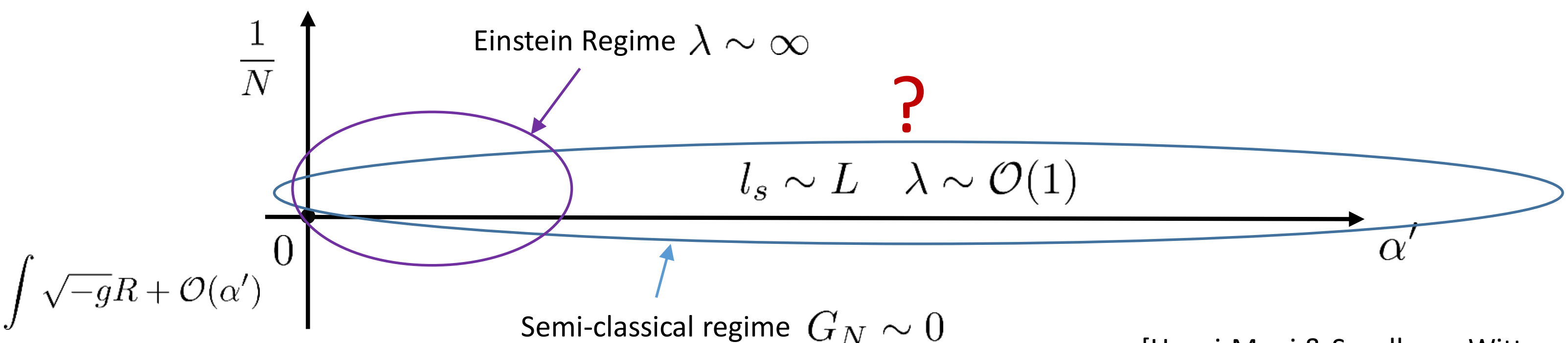

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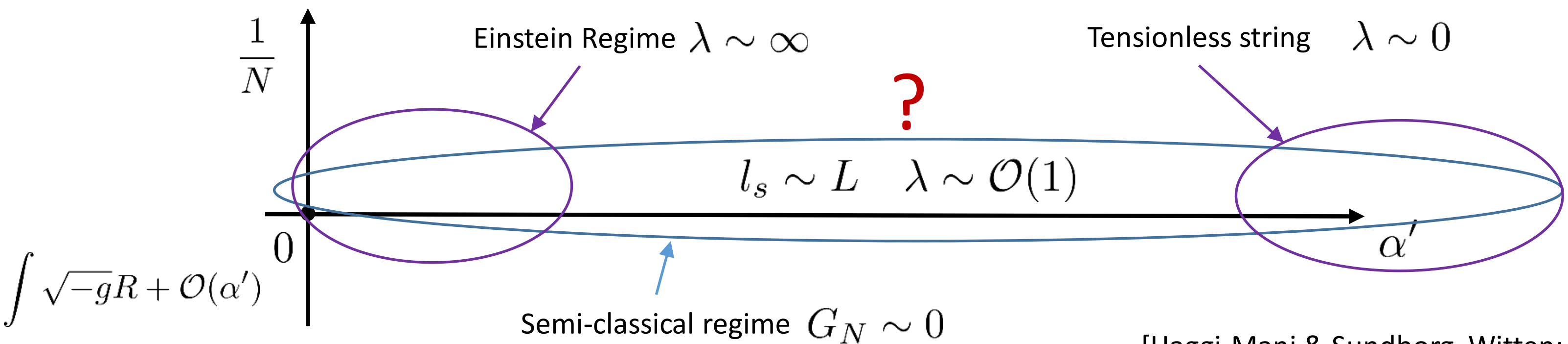

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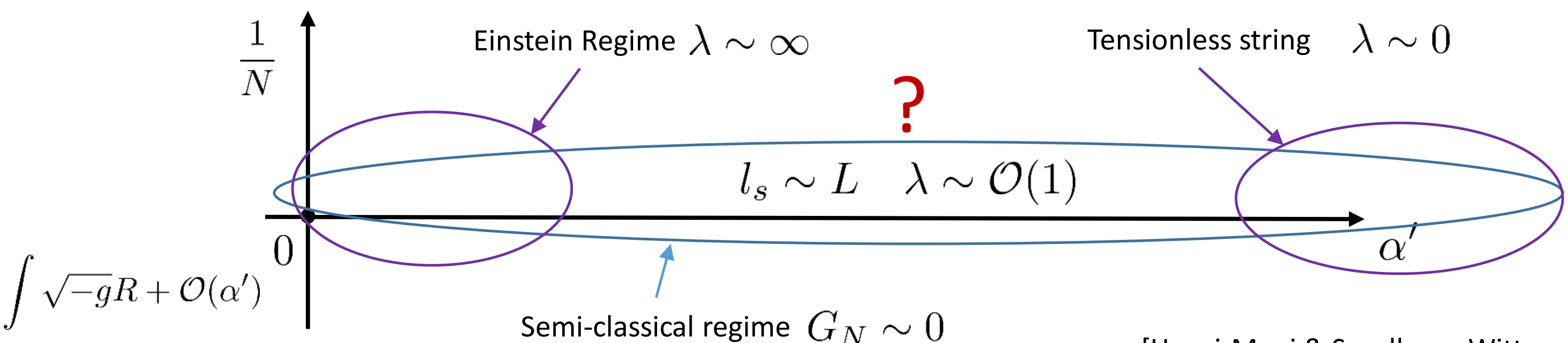
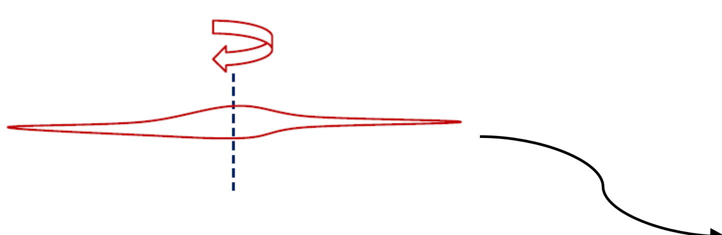


[Haggi-Mani & Sundborg, Witten; Klebanov et al.; ...]

- In **AdS$_{\mathbf{d+1}}$** cosmological constant gives and handle on tensionless limit:



(rotating string in a ``Box'')

- Short string: $m^2 \sim (s-1)/\alpha'$
- Long string: $mL \sim s$

First Regge trajectory becomes massless!
(HS gauge fields)

The Fronsdal Program

- In the tensionless limit the boundary side acquires an infinite dimensional global symmetry: HS symmetry
- On the bulk side such symmetry is gauged and the corresponding bulk theory should be a solution to the ``Fronsdal Program'':

[Fronsdal '78, Fradkin-Vasiliev '80s, ...]

$$S_s^{(2)} = \int \frac{1}{2}\, \varphi^{\mu_1 \dots \mu_s} \Box \varphi_{\mu_1 \dots \mu_s} + \dots$$

$$\delta^{(0)} \varphi_{\mu(s)} = \nabla_\mu \xi_{\mu(s-1)}$$

Fronsdal Program:
Find a fully-non-linear action based on gauge principle which in a weak field expansion reduces to sums of the latter quadratic action:

$$S[\phi, g]\Big|_{g\sim 0} = \sum_{s\in I} S_s^{(2)} + \mathcal{O}(g)$$

Conventional Approach: Noether

Take as starting point the Fronsdal Lagrangian [Fronsdal '78]

$$S^{(2)} = \sum_s \int \frac{1}{2}\, \varphi^{\mu_1 \dots \mu_s} \Box \varphi_{\mu_1 \dots \mu_s} + \dots$$

$$\delta^{(0)} \varphi_{\mu(s)} = \nabla_\mu \xi_{\mu(s-1)}$$

Consider a **weak field expansion** of a would be non-linear action and enforce gauge invariance order by order (Berends, Burgers & van Dam '80; Barnich & Henneaux '90):

$$\begin{aligned} S &= S^{(2)} + S^{(3)} + S^{(4)} + \dots \\ \delta\varphi &= \delta^{(0)}\varphi + \delta^{(1)}\varphi + \dots \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \delta^{(0)} S^{(2)} &= 0 \\ \delta^{(1)} S^{(2)} + \delta^{(0)} S^{(3)} &= 0 \\ \delta^{(2)} S^{(2)} + \delta^{(1)} S^{(3)} + \delta^{(0)} S^{(4)} &= 0 \\ &\dots \end{aligned}$$

With HS, becomes more and more **involved** beyond the cubic order

[Boulanger, Leclercq, Sundell 2008, M.T. 2011; Boulanger, Kessel, Skvortsov & M.T. 2015; Bekaert, Erdmenger, Ponomarev & Sleight 2015; M.T. 2016, 2017; ...]

Noether, HS & Bulk locality

There is an important subtlety when HS are included:

$$\delta^{(0)} S^{(3)} \approx 0 \qquad \Longrightarrow \qquad \#\partial \geq s_{\text{max}} + s_{\text{mid}} - s_{\text{min}} \geq s_{\text{min}}$$

Metsaev's bound

[Metsaev '05, Joung & M.T. '12]

The above & HS algebra $[T_{s_1}, T_{s_2}] = \sum_{i=|s_1-s_2|}^{s_1+s_2} T_i$ implies:

$$\mathcal{V} \sim \sum_{s_1,s_2,s_3=0}^{\infty} g_{s_1,s_2,s_3} \nabla^{s_1}\phi \nabla^{s_2}\phi \nabla^{s_3}\phi$$

pseudo-local!

For cubic observables this problem can be swept under the rug avoiding to sum over spins...

Why (pseudo-)locality is Important?

Without any restriction on the functional space of pseudo-local functionals Noether procedure is empty (Barnich & Henneaux ’93; M.T. ’11; Kessel, Gomez, Skvortsov & M.T. ‘15)

$$\delta^{(0)} S^{(3)} \approx 0 \qquad\qquad S^{(3)} \sim S^{(3)} + f[\nabla^n \Phi](\Box + a)\Phi$$

If the functional space allows $f \sim \dfrac{1}{\Box + a}\Phi$, bulk interactions are trivial at the classical level up to boundary terms...

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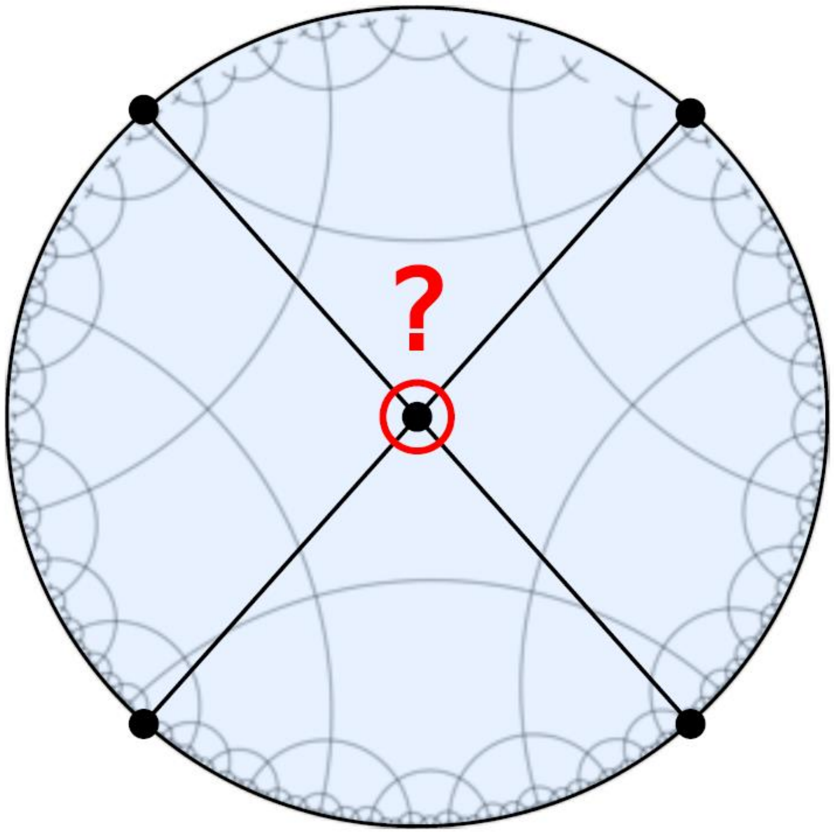
(Naive) pseudo-locality assumption:

In the context of pseudo-local field theories we consider functions of derivatives which have a finite radius of convergence as non-admissible

Open Problem: is there a proper pseudo-local functional class which allows to define HS interactions without invoking string theory?

Can holography help us understand higher-spin Interactions?



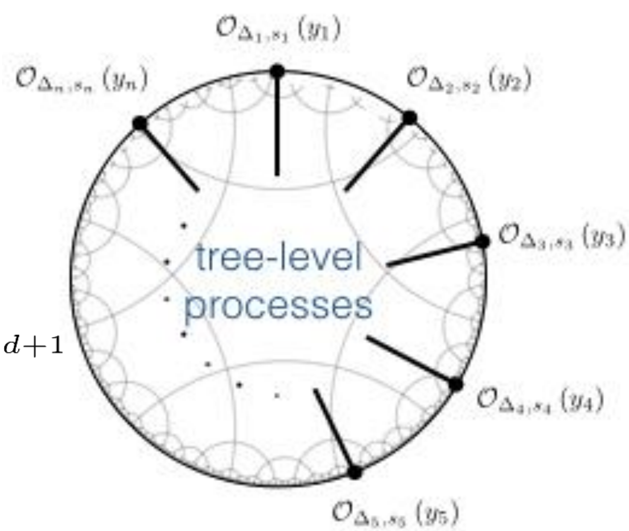
Erdmenger, Bekaert, Ponomarev, Sleight & M.T. et al. ’15-16-17, D. Gross & Rosenhaus ’17, Gubser et al. ‘17

Holographic Reconstruction

Higher-spin theory on AdS_{d+1} ⟷ Free O(N) vector model

[Sezgin-Sundell, Klebanov-Polyakov, '02]

$$\int_{\mathrm{AdS}_{d+1}}$$

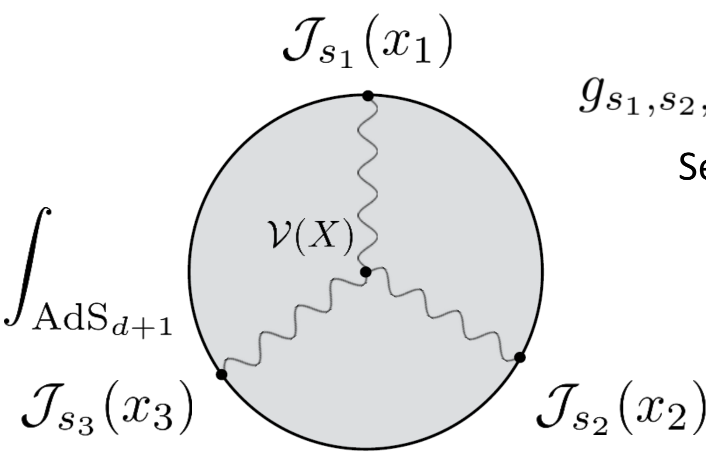


$$= \langle \mathcal{O}_{\Delta_1,s_1}(y_1)\ldots\mathcal{O}_{\Delta_n,s_n}(y_n)\rangle$$

$$\approx -\frac{1}{G}\left(\prod_{i=1}^{n}\frac{\delta}{\delta\bar{\varphi}_{s_i}(y_i)}\right) S_{AdS}[\varphi_i, \varphi_i|_{\partial AdS=\bar{\varphi}_i}]$$

Solve the above equation for the bulk vertices $\mathcal{V}(X)$ and check that the CFT gives a solution to the Noether procedure

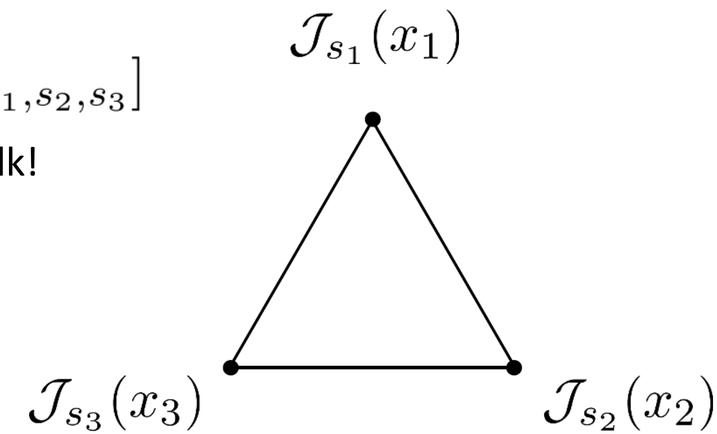
Complete Higher-Spin Cubic Action(s)



$$g_{s_1,s_2,s_3} = \mathcal{O}^{-1}[c_{s_1,s_2,s_3}]$$

See Charlotte's talk!

$$=$$



$$\mathcal{V} = \sum_{s_1,s_2,s_3} g_{s_1,s_2,s_3} I_{s_1,s_2,s_3} \qquad g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N}} \frac{\pi^{\frac{d-3}{4}} 2^{\frac{3d-1+s_1+s_2+s_3}{2}}}{\Gamma(d+s_1+s_2+s_3-3)} \prod_{i=1}^{3} \sqrt{\frac{\Gamma(s_i+\frac{d-1}{2})}{\Gamma(s_i+1)}}$$

$$I_{s_1,s_2,s_3}(\varphi_i) = (\nabla^{N_3(s_3)}\varphi_{N_1(s_1)})(\nabla^{N_1(s_1)}\varphi_{N_2(s_2)})(\nabla^{N_2(s_2)}\varphi_{N_3(s_3)}) + \mathcal{O}(\Lambda)$$

Assuming locality, we obtain the **complete higher-spin cubic action** (for any CFT!)

[C. Sleight & M.T. 1603.00022, 1702.08619]

Full Cubic Test of HS duality

A test in this context goes backwards: we have the cubic action and we can test that it solves the above necessary conditions (**``locality'' is assumed**)

[C.Sleight & M.T. 1609.00991]

$$\int \left[(\delta^{(1)}\Phi)\Box\Phi + \delta^{(0)}\mathcal{V}\right] = 0$$

$$\delta^{(0)}_{[\epsilon_1}\delta^{(1)}_{\epsilon_2]} \approx \delta^{(0)}_{[\![\epsilon_1,\epsilon_2]\!]^{(0)}}$$

Jacobi:

[Fradking & Vasiliev; Boulanger, Ponomarev, Skvortsov & MT]

$$[\![\epsilon_1, [\![\epsilon_2, \epsilon_3]\!]^{(0)}\,]\!]^{(0)} + \text{cyclic} = 0$$

Admissibility:

[Konstein & Vasiliev; Boulanger, Kessel, Skvortsov & MT]

$$\delta^{(1)}_{[\epsilon_1}\delta^{(1)}_{\epsilon_2]} \approx \delta^{(1)}_{[\![\epsilon_1,\epsilon_2]\!]^{(0)}}$$

Cubic covariance:

[Sleight & MT]

$$\delta^{(1)}_{\epsilon}S^{(3)} \approx 0$$

These completely fix $S^{(3)}$

We can also test **critical model** holography using the result of Charlotte's talk and so far this is the most general cubic test available

[C.Sleight & M.T. 1702.08619]

And beyond cubic?

At cubic order the HS duality is kinematical (HS Ward identities):

$$\int_{AdS} \delta^{(1)}_{\bar{\xi}} \mathcal{V}^{(3)} \approx 0 \quad \leftrightarrow \quad 0 = \sum_i \langle \mathcal{O}_1 \ldots \left[Q_{\bar{\xi}}, \mathcal{O}_i \right] \ldots \mathcal{O}_3 \rangle$$

At higher orders also the HS duality appears kinematical:

$$\int_{AdS} \delta^{(1)}_{\bar{\xi}} \mathcal{A}^{(n)} \approx 0 \quad \leftrightarrow \quad 0 = \sum_i \langle \mathcal{O}_1 \ldots \left[Q_{\bar{\xi}}, \mathcal{O}_i \right] \ldots \mathcal{O}_n \rangle$$

$$\mathcal{V}^{(4)} \approx \mathcal{A}^{(4)} - \left(\mathcal{V}^{(3)} \frac{1}{\Box} \mathcal{V}^{(3)} + \text{t,u} \right)$$

O(N) model correlator
[1603.00022 C. Sleight & M.T.]

$\mathcal{A}^{(n)}$ is uniquely fixed up to on-shell vanishing terms by Ward identities

[1704.07859, C. Sleight & M.T.]

$$\mathcal{A}^{(n)} = \text{Tr}\Big[\underbrace{C \star \tilde{C} \star \cdots \star C}_{n} \Big]$$

Noether, HS & Bulk locality

With HS symmetry matching CFT correlator is kinematics, **the crux** is if we can make sense of the corresponding bulk interactions in a field theory sense!

The quartic solution to Noether can be related on-shell to S-matrix-like observables
[1107.5843, 1701.05772, M.T. 1704.07859 C. Sleight & M.T.]

$$\mathcal{V}^{(4)} \approx \mathcal{A}^{(4)} - \left(\mathcal{V}^{(3)}\frac{1}{\Box}\mathcal{V}^{(3)} + \text{t,u-channels}\right) = \sum_{s,l=0}^{\infty} g_{s,l}(\phi\nabla^s\phi)\Box^l(\phi\nabla^s\phi)$$

[C. Sleight & M.T. 1702.08619]

pseudo-locality: all 1/□ contained in the exchange are cancelled!

Are HS theories pseudo-local **field theories**? In which sense bulk interactions are non-trivial? Does it exist a functional space of non-localities allowing to make sense of HS theories as **field theories**?

A bit of CFT jargon

Bulk non-localities in AdS can be identified from the conformal partial wave expansion:

Exchange in AdS:

OPE coefficients

$$\mathcal{E}^{\mathsf{s}}_{0,0|s|0,0} = \mathsf{c}_{\mathcal{O}\mathcal{O}\mathcal{J}_s}\mathsf{c}^{\mathcal{J}_s}{}_{\mathcal{O}\mathcal{O}}\underbrace{\mathcal{W}_{(0,0|s|0,0)}\left(y_1,y_2;y_3,y_4\right)} + \sum_n \sum_{l=0}^{s} d_{l,n}\underbrace{W_{[\mathcal{O}\mathcal{O}]_{l,n}}\left(y_1,y_2;y_3,y_4\right)}$$

Single trace contribution signal the exchange of a physical particle

Double trace blocks correspond to bound states and signal contact terms

Exchange in flat:

$$\mathcal{E}^{\mathsf{s}}_{0,0|s|0,0} = \mathsf{g}_{\Phi\Phi\varphi_s}\mathsf{g}^{\varphi_s}{}_{\Phi\Phi}\frac{(\mathsf{u}-\mathsf{t})^s}{\mathsf{s}-m^2} + \sum_n \sum_{l=0}^{s} d_{l,n}(\mathsf{t}-\mathsf{u})^l\mathsf{s}^n$$

Locality at quartic order

The simplest example is the quartic scalar self-interaction: [1704.07859, C. Sleight & M.T.]

CFT:

$$\int_{AdS} \mathcal{A}^{(4)} = \frac{1}{N}\frac{1}{(y_{12}^2 y_{34}^2)^{d-2}}\left[u^{\frac{d}{2}-1} + \left(\frac{u}{v}\right)^{\frac{d}{2}-1} + u^{\frac{d}{2}-1}\left(\frac{u}{v}\right)^{\frac{d}{2}-1}\right] \sim \frac{1}{\Box}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Exchange sum:

$$\sum_{r=0}^{\infty} \mathcal{E}_r^{(\mathsf{s})} = \frac{1}{N}\frac{1}{(y_{12}^2 y_{34}^2)^{d-2}}\left[u^{\frac{d}{2}-1} + \left(\frac{u}{v}\right)^{\frac{d}{2}-1}\right] + \text{contact} \sim \frac{1}{\Box} + \text{contact}$$

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⬇

Contact term:

$$S^{(4)} = \int \left[\mathcal{A}^{(4)} - \sum_r (\mathcal{E}_r^{(\mathsf{s})} + \mathcal{E}_r^{(\mathsf{t})} + \mathcal{E}_r^{(\mathsf{u})})\right] = -\int \mathcal{A}^{(4)} + \text{contact}$$

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$\Downarrow$

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It contains $\frac{1}{\Box}$

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Both of them should be contact terms

NO-GO: either $S^{(4)}$ or the improvement terms in the exchange or both contain sums over spin and derivatives with **finite radius of convergence**

The role of crossing

$$f\left(u,v\right)=v^{\Delta}G\left(u,v\right)=u^{\Delta}G\left(v,u\right) \qquad\qquad u,v\to 0$$

Crossing admits very simple solutions in the double-light cone limit [Alday & Zhiboedov]

$$f\left(u,v\right)=\sum_{i,j}c_{ij}\,u^{\frac{\tau_i}{2}}v^{\frac{\tau_j}{2}}, \qquad c_{ij}=c_{ji}$$

$$\text{twist }\tau_1 \quad \overset{\text{crossing}}{\leftrightarrow} \quad \text{twist }\tau_2 \qquad\qquad \tau=\Delta-s$$

In **field theory** (including strings in point-like limit $\alpha'\to 0$) the **standard** behaviour is:

$$\text{single-trace} \overset{\text{crossing}}{\leftrightarrow} \sum\text{double-trace} \qquad \sum\text{double-trace} \overset{\text{crossing}}{\leftrightarrow} \sum\text{double-trace}$$

Such behaviour is equivalent to the possibility of **cancelling** all 1/□ (emergence of pseudo-locality in the bulk)

[1704.07859, C. Sleight & M.T.]

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In the holographic dual to the O(N) model we identify two contributions

$$f_{d-2}(u,v) = \frac{1}{N} u^{\frac{d}{2}-1} v^{\frac{d}{2}-1} \qquad f_{2(d-2)}(u,v) = \frac{1}{N}\left(u^{\frac{d}{2}-1} v^{d-2} + v^{\frac{d}{2}-1} u^{d-2}\right)$$

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$$\text{Regge}_{\text{single-trace}} \quad \overset{\text{crossing}}{\leftrightarrow} \quad \text{Regge}_{\text{double-trace}}$$

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$$\text{twist } \tau_1 \quad \overset{\text{crossing}}{\leftrightarrow} \quad \text{twist } \tau_2 \qquad\qquad \tau = \Delta - s$$

In the holographic dual to the O(N) model we identify two contributions

$$f_{d-2}(u,v) = \frac{1}{N} u^{\frac{d}{2}-1} v^{\frac{d}{2}-1} \qquad f_{2(d-2)}(u,v) = \frac{1}{N}\left(u^{\frac{d}{2}-1} v^{d-2} + v^{\frac{d}{2}-1} u^{d-2}\right)$$

$$\text{Regge}_{\text{single-trace}} \quad \overset{\text{crossing}}{\leftrightarrow} \quad \text{Regge}_{\text{single-trace}}$$

This contribution is the very reason of the non-local obstruction we find

Summary & Outlook

- We have argued that there is a **non-local** obstruction to the Fronsdal program in a field theory context at the classical level (like in flat space)
- Bulk locality is shown to be linked to a very precise property of conformal block expansions under crossing
- The problem arise at quartic order where sums over spins and derivatives are shown to have a **finite** radius of convergence
- This results explains the **strongly coupled** series in derivatives already appearing at cubic order from Vasiliev's equations:

$$\Box\Phi_{\mu_1\mu_2} + \ldots \sim \sum_l (\Lambda\Box)^{-1}\left[(\nabla_{\mu_1\mu_2}\Phi)\Phi + \ldots\right]$$

[Skvortsov & M.T. '15, M.T. '16]

- Is string theory the only HS theory? How can we make sense of HS interactions without invoking string theory? Can string theory provide guidance?