

Vorticity and Polarization in Heavy-Ion Collisions: From **Micro-** to **Macrophysics** and back again

GINZBURG CENTENNIAL CONFERENCE on
PHYSICS

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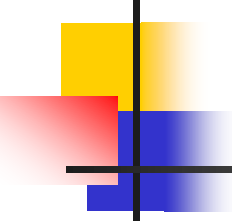
Phys.Rev. C88 (2013) 061901, C92 (2015) 014906,
C93 (2016) 031902, C95 (2017) 011902 ,

ArXiv 1701.00923, 1705.01650 and work in progress

Oleg Teryaev
(JINR)

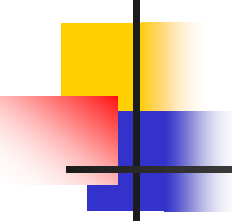
In collaboration with Mircea Baznat (also at JINR) and
Konstantin Gudima (IAP, Chisinau);

Alexander Sorin (JINR); Valentin I. Zakharov (ITEP)



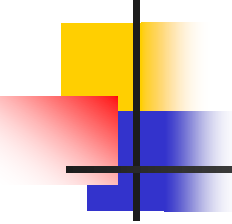
Main Topics

- Micro to macro: Rotation in heavy-ion collisions
- Angular momentum and vortical structures
- Hydrodynamical helicity and its separation
- Polarization: from nucleons to ions
- Macro back to micro: Anomalous mechanism: 4-velocity as a gauge field
- Chemical potential and Energy dependence
- Polarization enhancement for antihyperons
- QCD and rotation as GR effect
- Conclusions



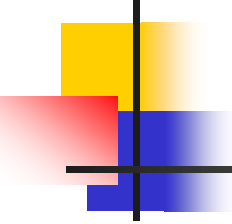
Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity $\sim c/\text{Compton wavelength}$)
- ~ 25 orders of magnitude faster than Earth's rotation
- Differential rotation – vorticity
- P-odd : May lead to various P-odd effects
- Calculation in kinetic quark - gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes):
Baznat, Gudima, Sorin, OT, PRC'13,16; HSD-OT, Usubov'15



Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L = \sum \mathbf{r} \times \mathbf{p}$
- Differential pseudovector characteristics – vorticity
- $\omega = \text{curl } \mathbf{v}$
- Pseudoscalar – helicity
- $H \sim \langle (\mathbf{v} \cdot \text{curl } \mathbf{v}) \rangle$
- Maximal helicity – Beltrami chaotic flows
 $\mathbf{v} \parallel \text{curl } \mathbf{v}$



Simulation in QGSM (Kinetics -> HD)

$50 \times 50 \times 100$ cells $dx = dy = 0.6$ fm, $dz = 0.6/\gamma$ fm

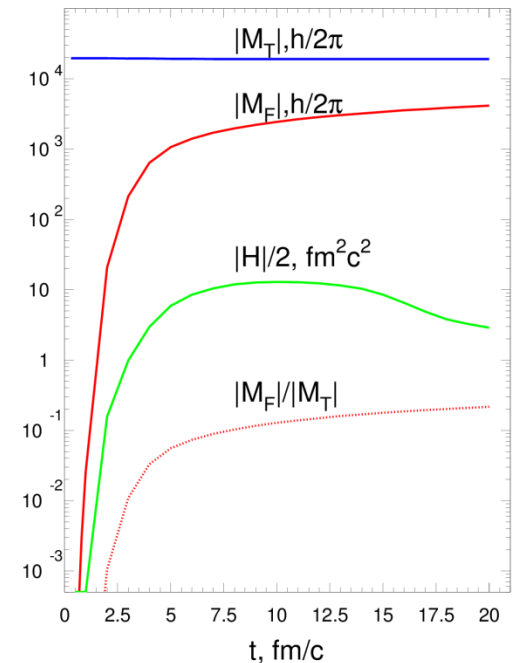
- Velocity

$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

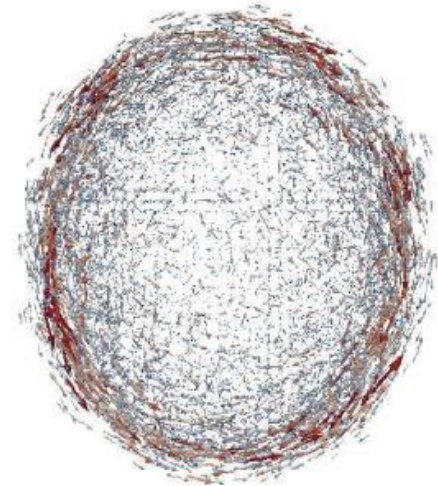
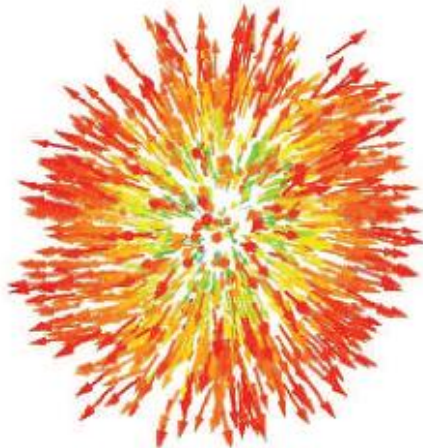
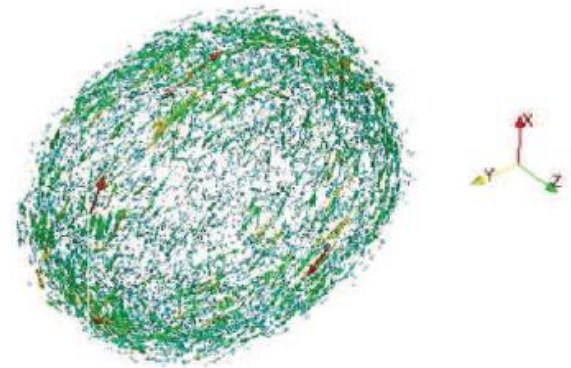
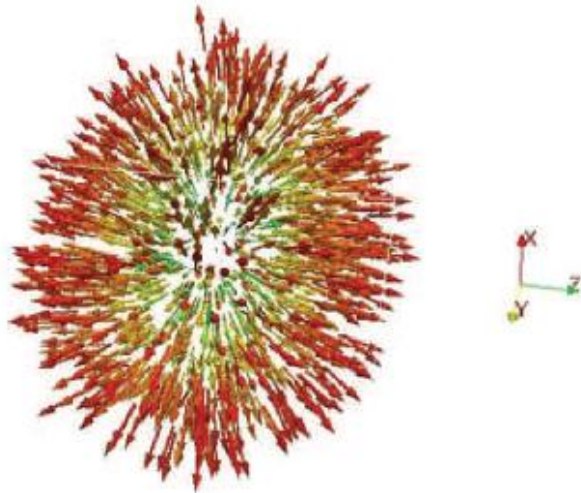
- Vorticity – from discrete partial derivatives

Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- ($\sim 10\%$ of total)
- Conservation of OAM with a good accuracy!

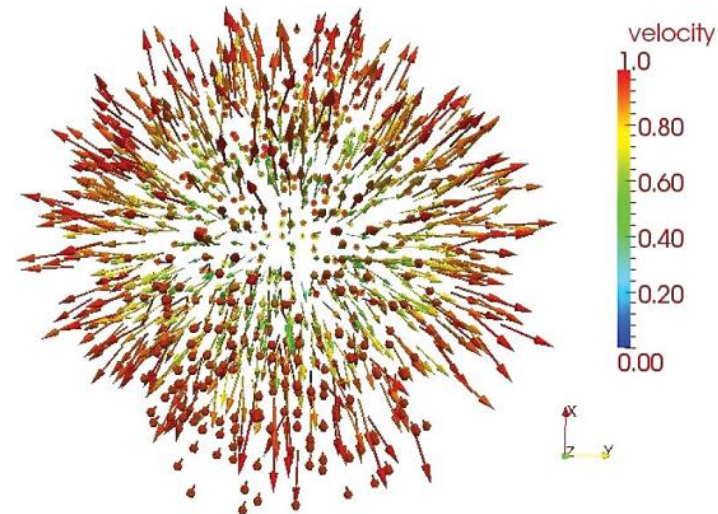
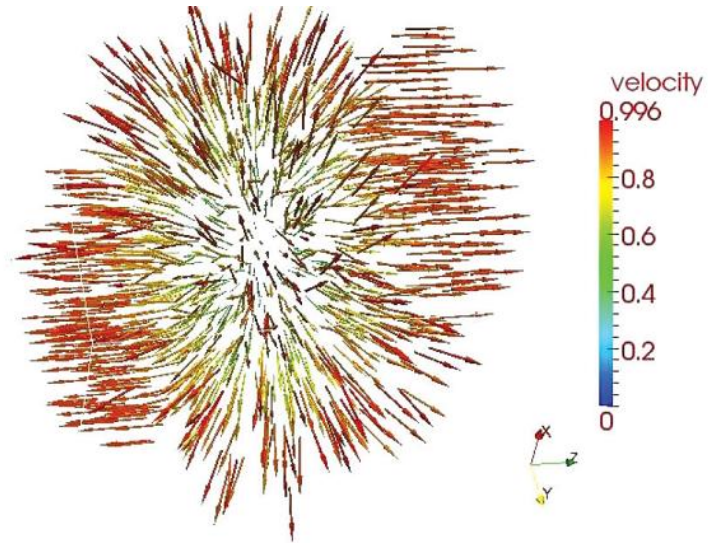


Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



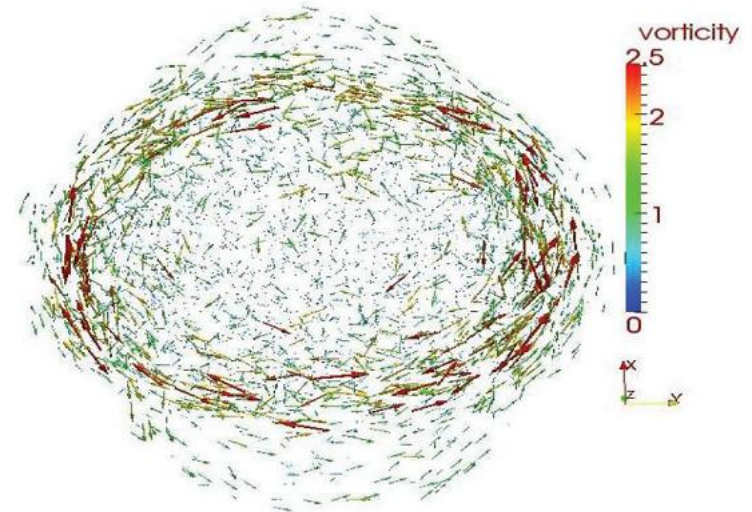
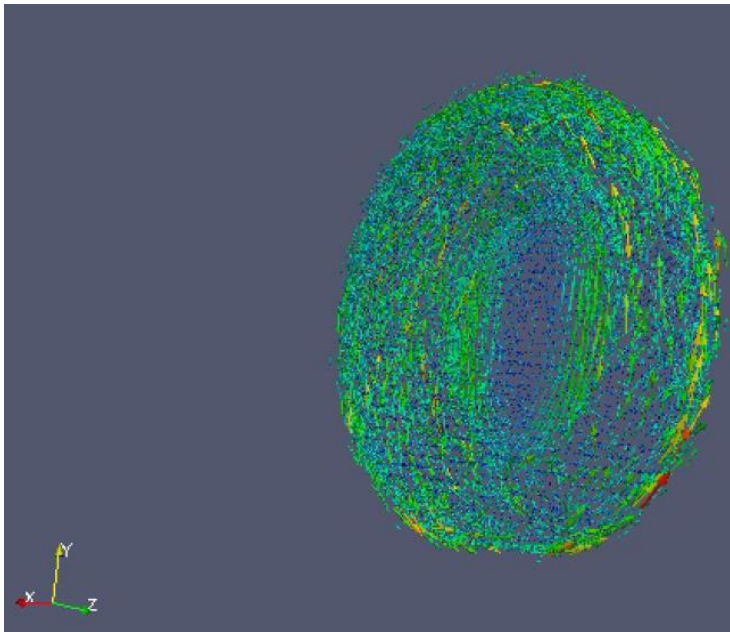
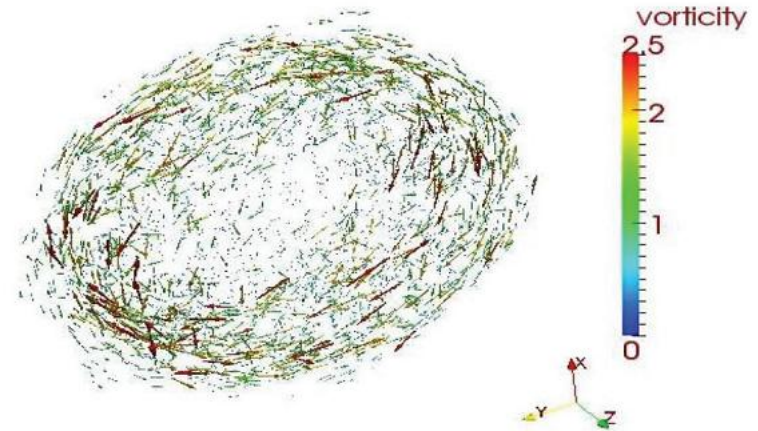
Distribution of velocity ("Small Bang")

- 3D/2D projection
- z-beams direction
- x-impact parameter



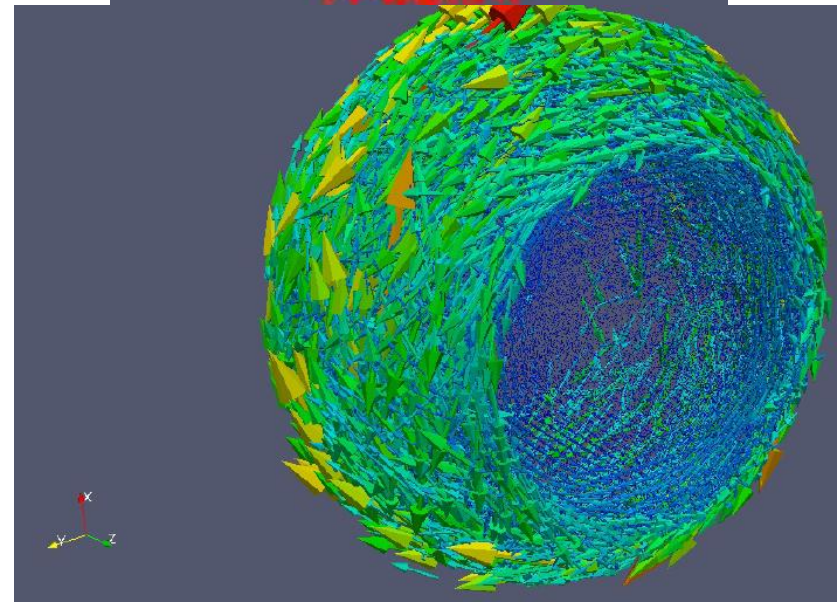
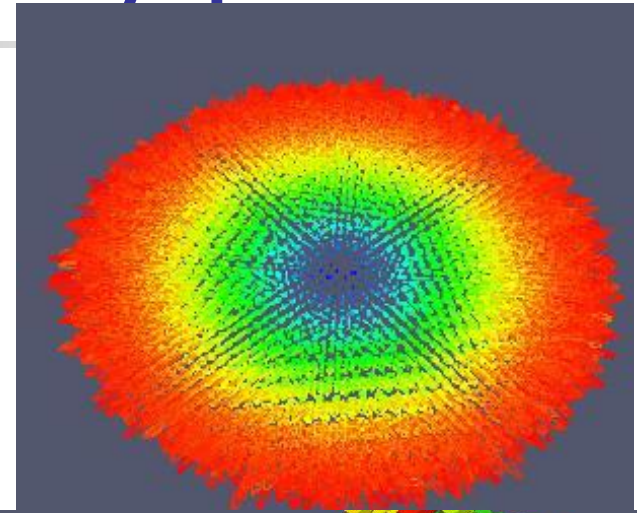
Distribution of vorticity (“small galaxies”)

- Layer (on core - corona borderline) patterns

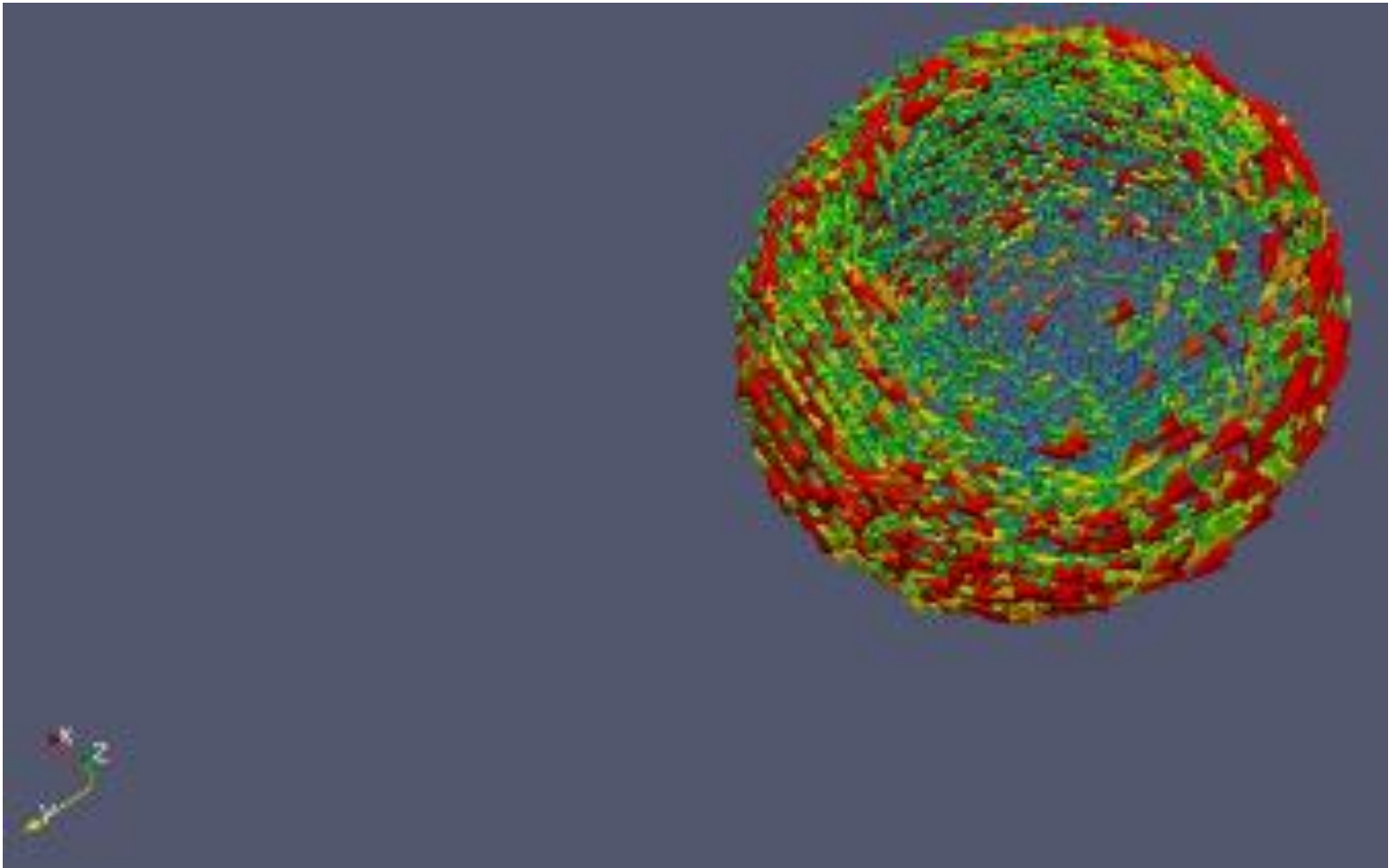


Velocity and vorticity patterns

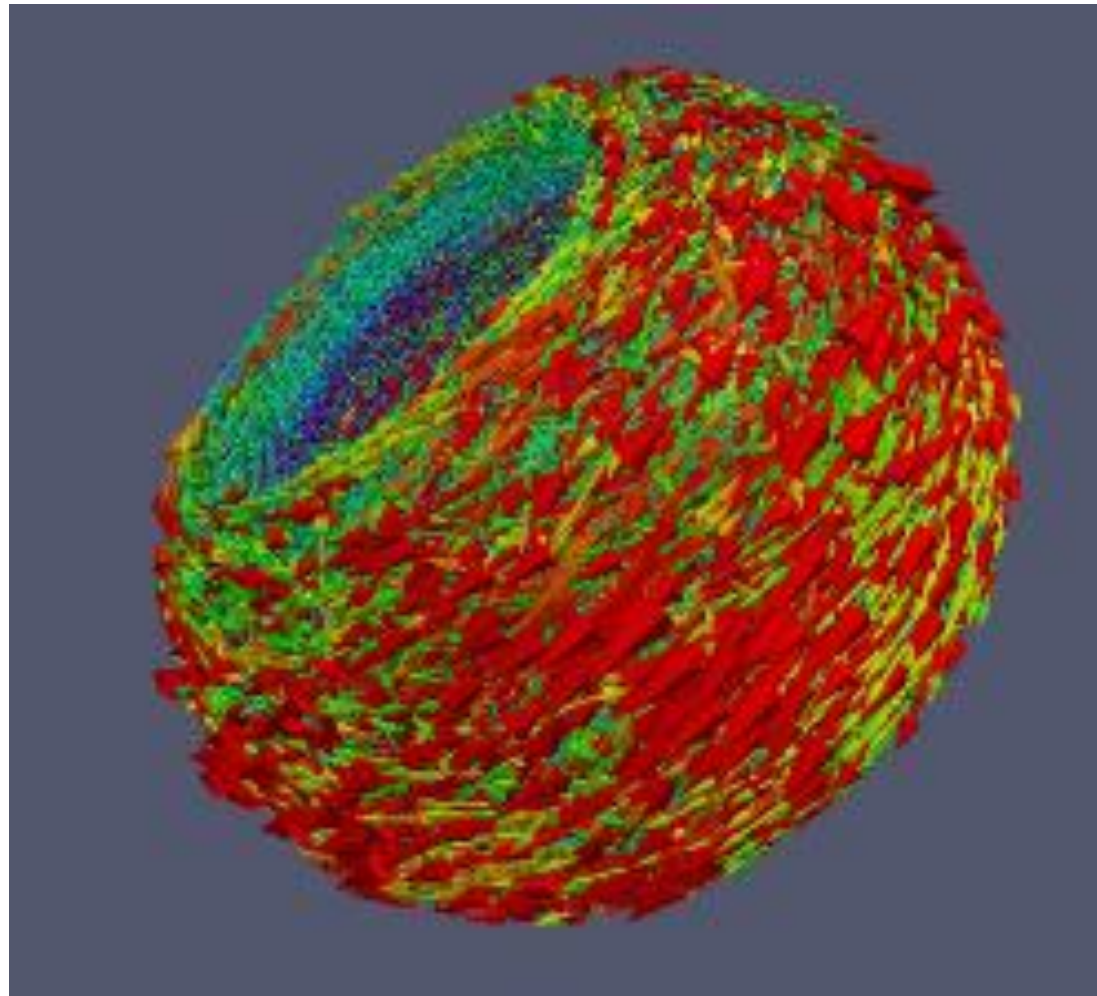
- Velocity
- Vorticity pattern –
vortex sheets –
due to L BUT
cylinder symmetry!



Vortex sheet (fixed direction of L)

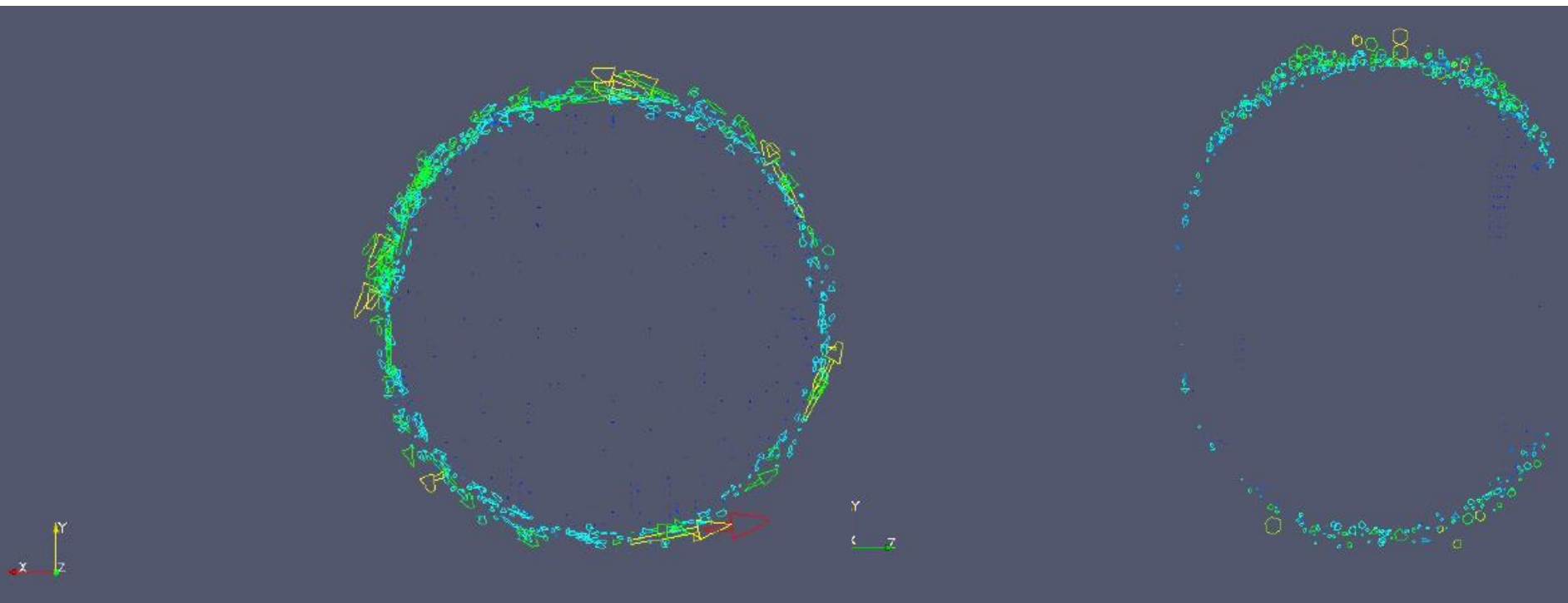


Vortex sheet (Average over L directions)



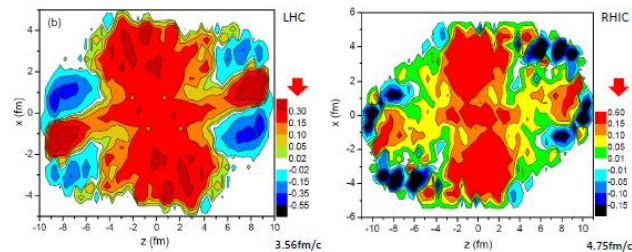
Sections of vorticity patterns

- Front and side views

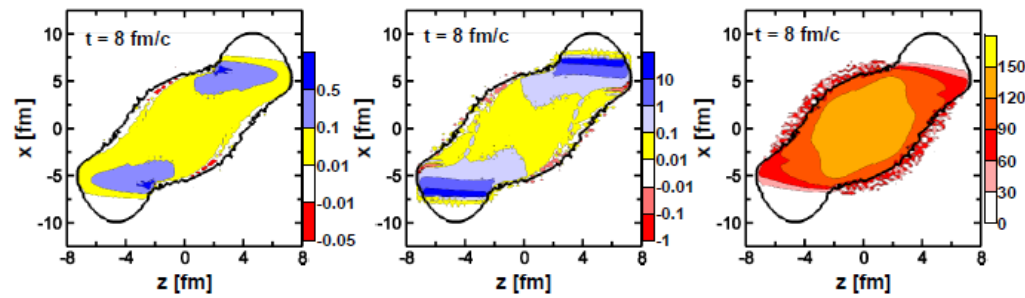


Vortex sheets

- Naturally appears in kinetic models
- Absent in viscous HD (L. Csernai et al)



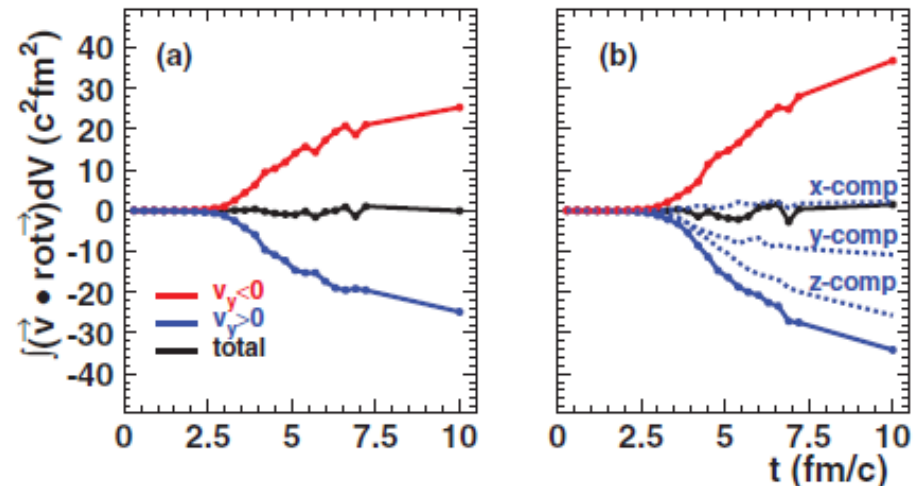
- Appears in 3 fluid dynamics model :
Yu. Ivanov, A. Soldatov, [arXiv:1701.01319](https://arxiv.org/abs/1701.01319)



Helicity separation in QGSM

PRC88 (2013) 061901

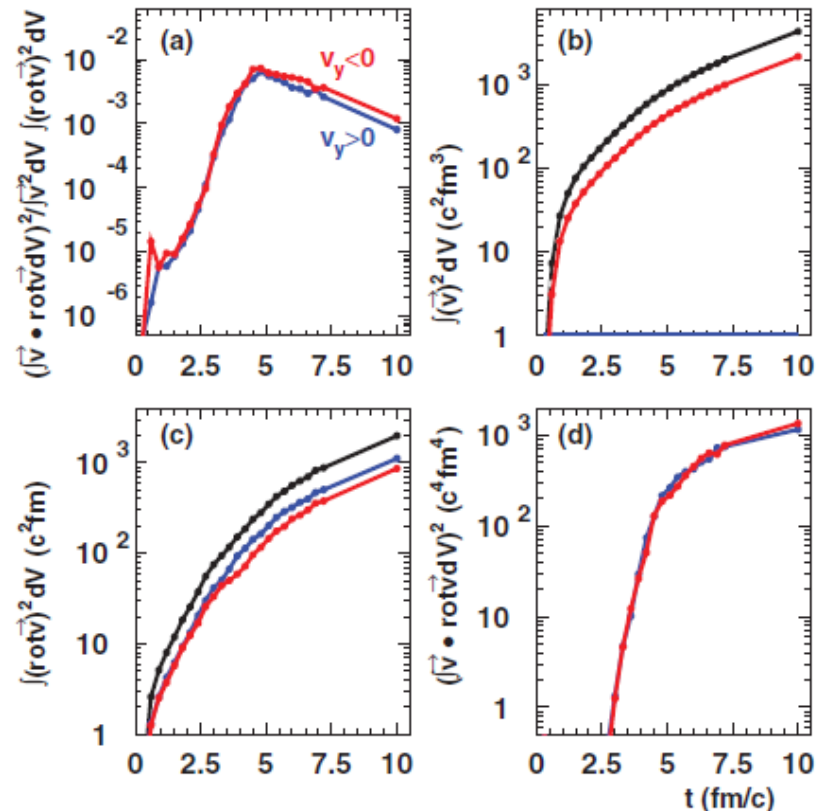
- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (OT, Usubov, PRC92 (2015) 014906



What is the relative orientation of velocity and vorticity?

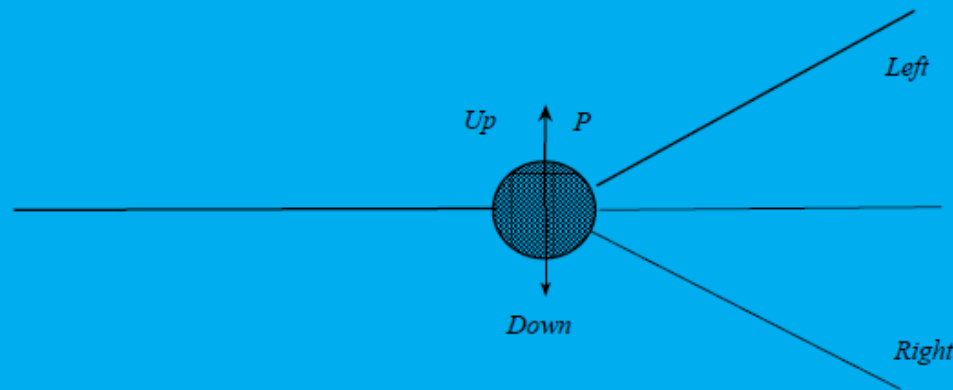
- Measure – Cauchy-Schwarz inequality
- Small but non-negligible correlation
- Maximal correlation - Beltrami flows

PHYSICAL REVIEW C 88, 061901(R) (2013)



Single Spin Asymmetries: indirect probe of vorticity

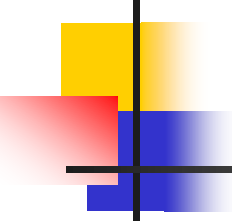
Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi \vec{N} \rightarrow \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

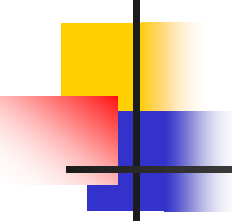
Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2\text{Im}(ab^*)}{|a|^2 + |b|^2}$



Λ -polarisation

- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: '87): Randomization – smearing – no direction normal to the scattering plane



Global polarization

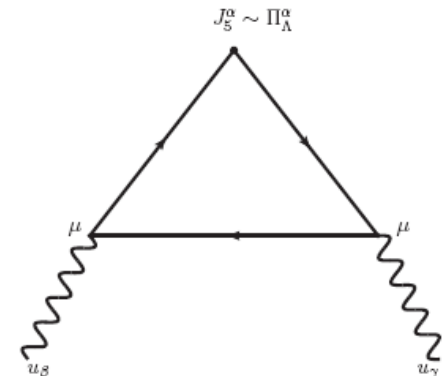
- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?
(Macro to Micro back again)

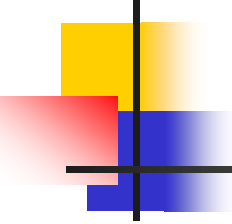
Anomalous mechanism – polarization similar to CM(V)E

- 4-Velocity is also a **GAUGE FIELD**
(V.I. Zakharov)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- Triangle anomaly leads to polarization of quarks and hyperons
(Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin
(Efremov, OT'88)
- **4-velocity instead of gluon field!**



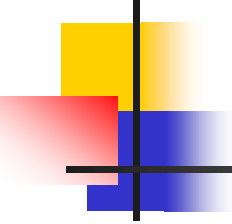


Anomaly for polarization

- Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Neglect axial chemical potential
- T-dependent term- related to gravitational anomaly
- Lattice simulation: suppressed due to collective effects



Energy dependence

- Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2} \int d^3x \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Large chemical potential: appropriate for NICA/FAIR energies

One might compare the prediction below with the right panel figures

*O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortical effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)*

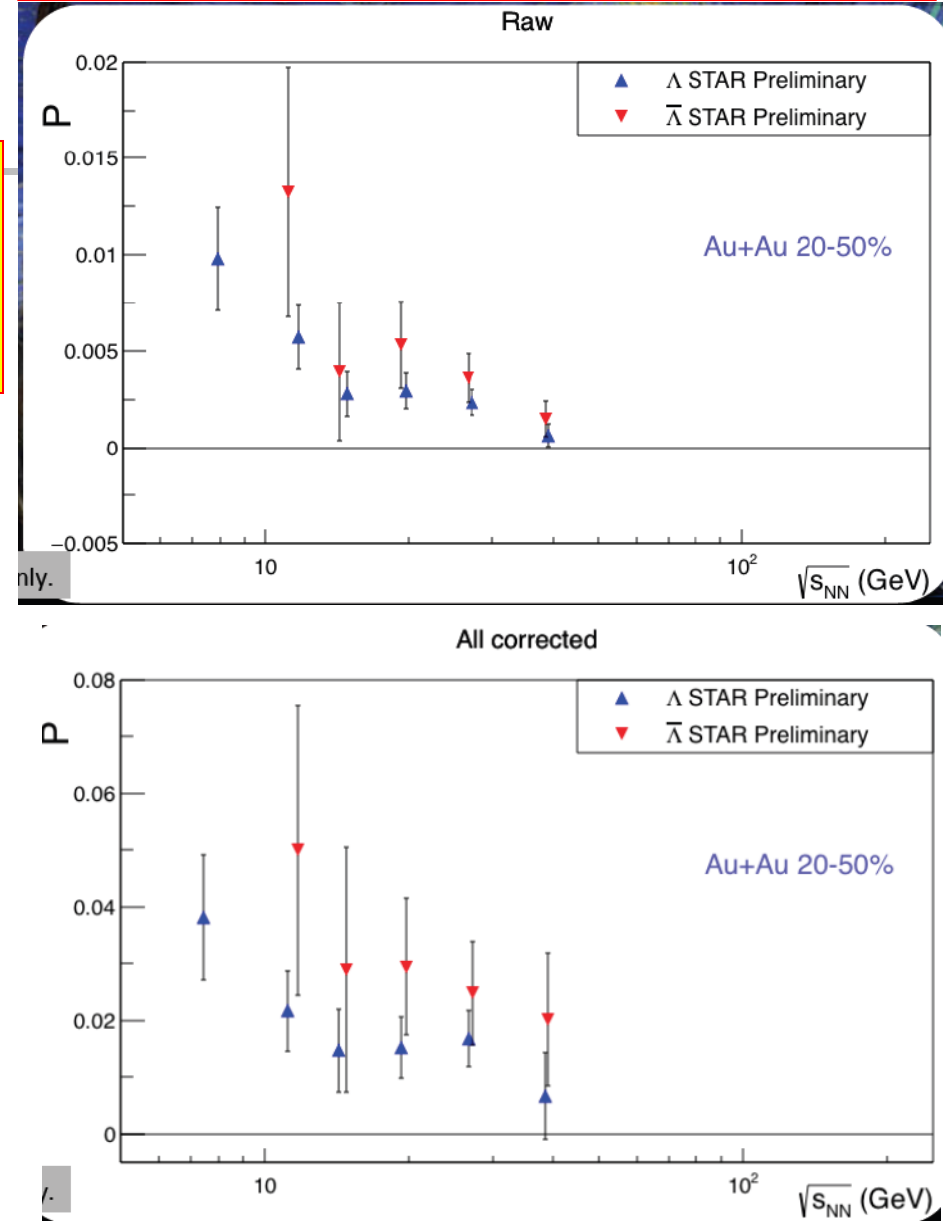
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)} \right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \quad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

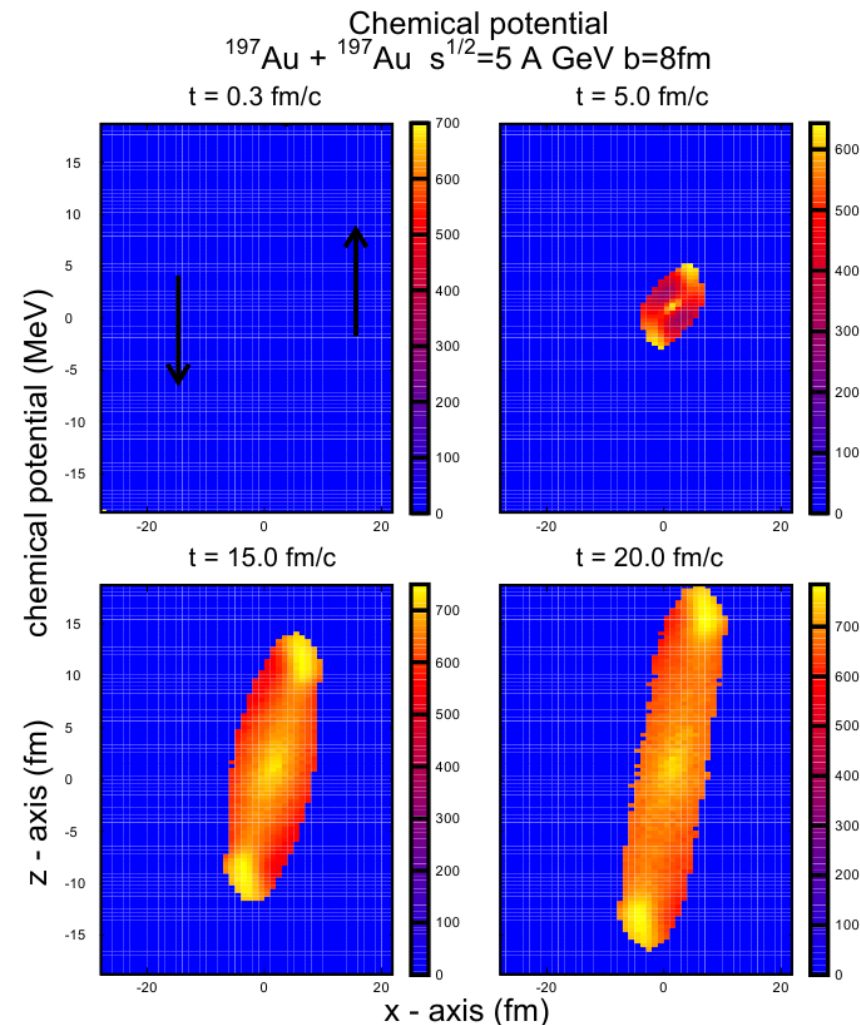
*M. Lisa, for the STAR collaboration, QCD Chirality Workshop, UCLA, February 2016;
SQM2016, Berkeley, June 2016*



Chemical potential : Kinetics

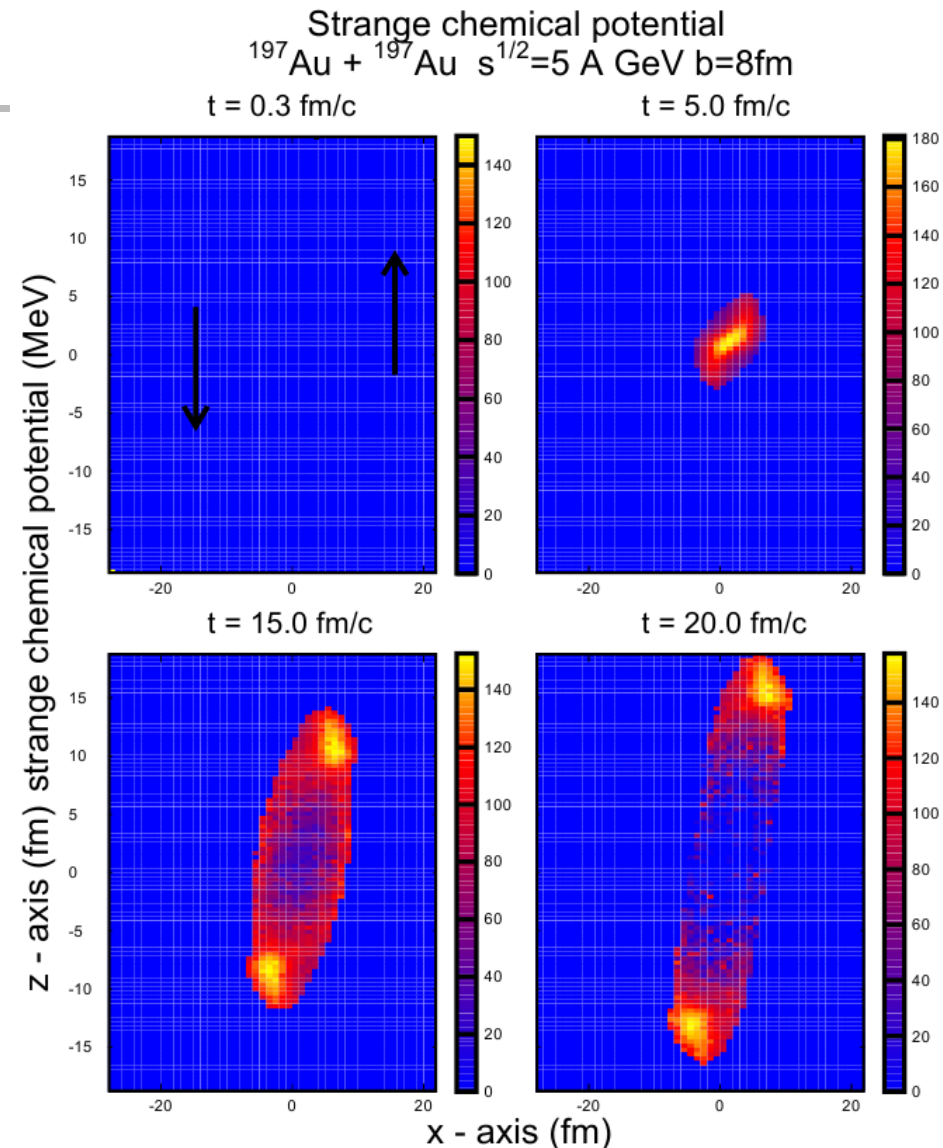
-> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: $y=0$

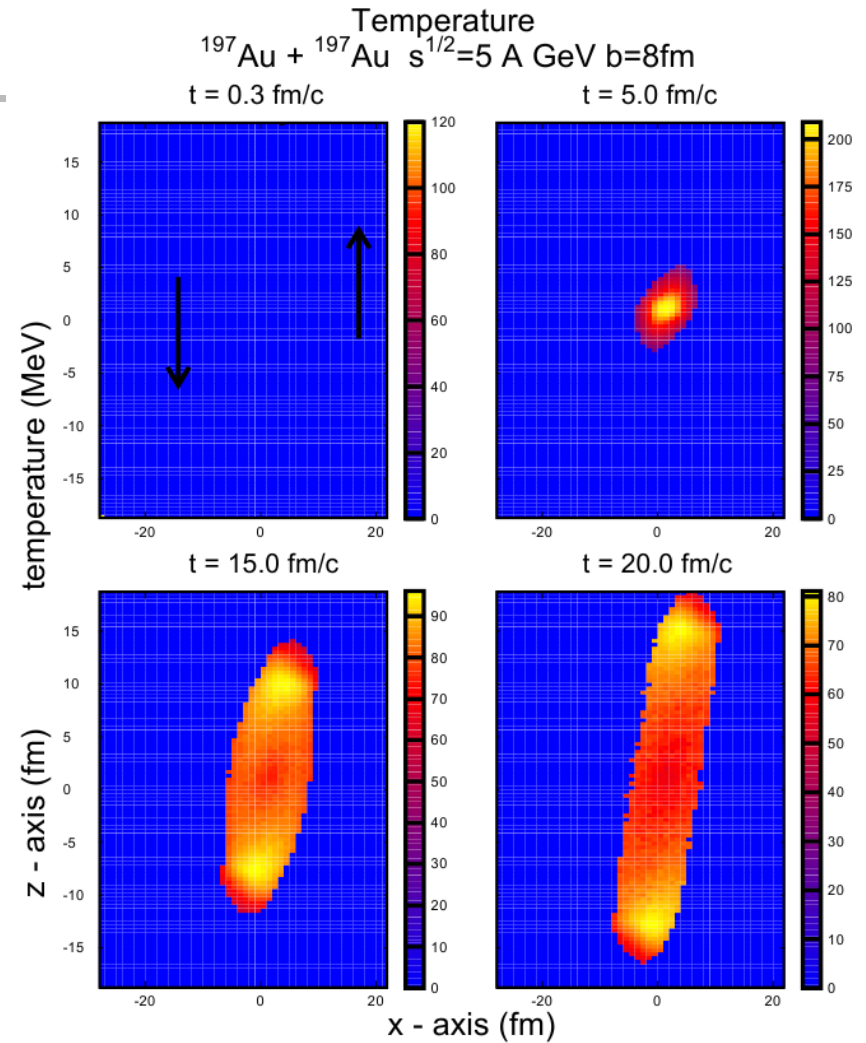


Strange chemical potential (polarization of Lambda is carried by strange quark!)

- Non-uniform in space and time



Temperature



From quark axial charge to hyperon polarization (sort of q/H duality - bypass of confinement problem)

- Analogy of matrix elements and classical averages

$$\langle p_n | j^0(0) | p_n \rangle = 2p_n^0 Q_n \quad \langle Q \rangle \equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x j_{class}^0(x)}{N}$$

- Lorentz boost: compensate the sign of helicity

$$\Pi^{\Lambda,lab} = (\Pi_0^{\Lambda,lab}, \Pi_x^{\Lambda,lab}, \Pi_y^{\Lambda,lab}, \Pi_z^{\Lambda,lab}) = \frac{\Pi_0^{\Lambda}}{m_{\Lambda}} (p_y, 0, p_0, 0)$$

$$\langle \Pi_0^{\Lambda} \rangle = \frac{m_{\Lambda} \Pi_0^{\Lambda,lab}}{p_y} = \langle \frac{m_{\Lambda}}{N_{\Lambda} p_y} \rangle Q_5^s \equiv \langle \frac{m_{\Lambda}}{N_{\Lambda} p_y} \rangle \frac{N_c}{2\pi^2} \int d^3x \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

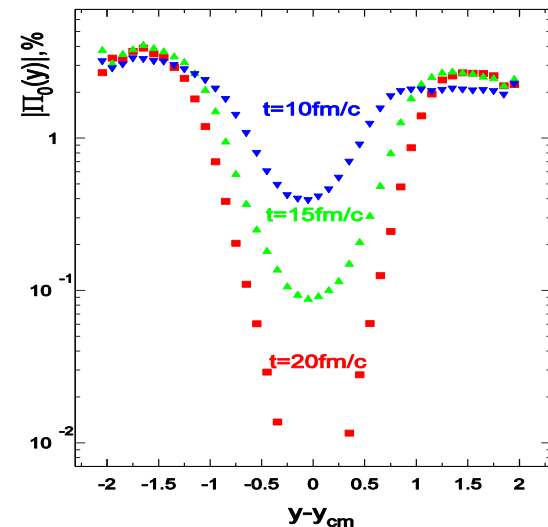
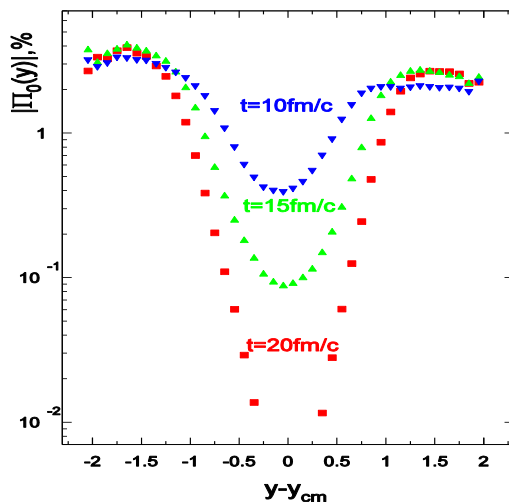
- Antihyperons (smaller N) : same sign and larger value (confirmed by STAR)

Helicity -> rest frame polarization

- Helicity $\sim$ 0th component of polarization in lab. frame – effect of boost to Lambda rest frame – various options

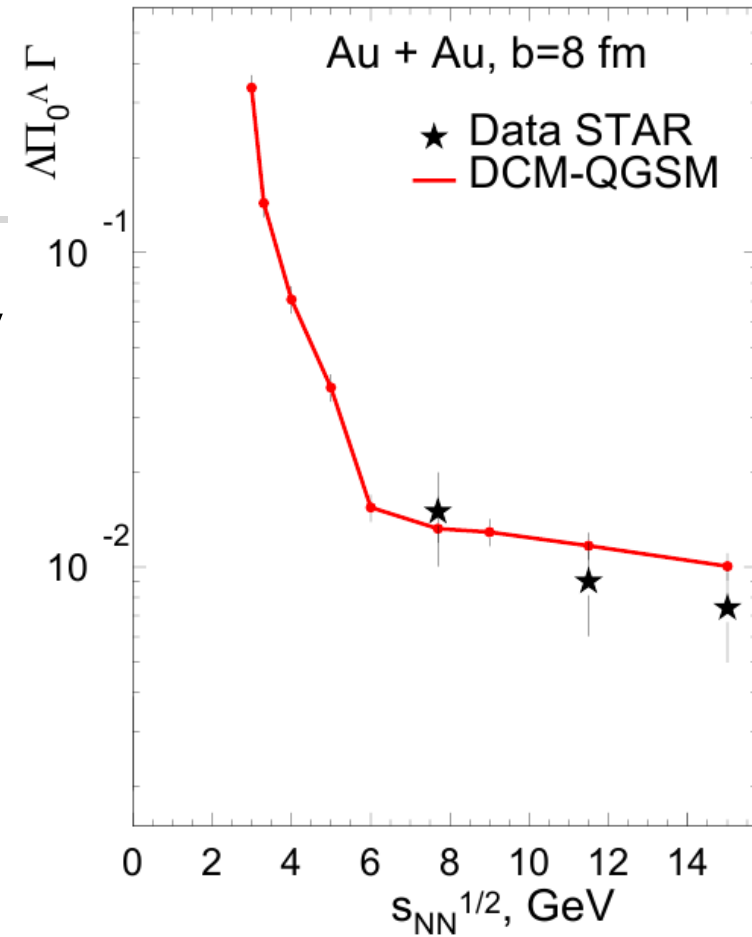
$$\Pi_0(y) = \frac{1}{(4\pi^2)} \int y^2(x) \mu_s^2(x) |\mathbf{v} \cdot \text{rot}(\mathbf{v})| n_\Lambda(y, x) w_1 d^3x / \int n_\Lambda(y, x) w_2 d^3x$$

$w_1=1, w_2=1$ $w_1=1, w_2=p_y/m$



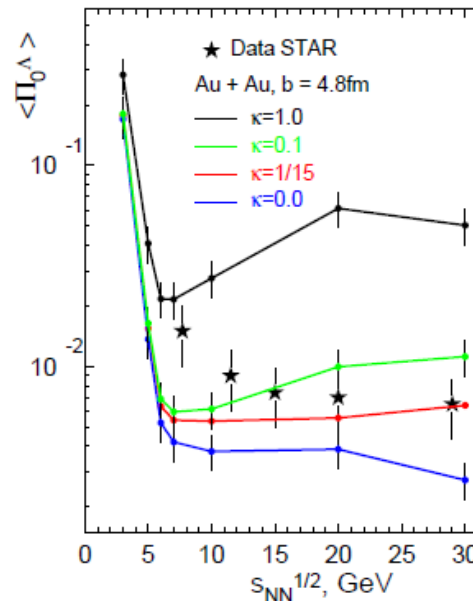
Energy dependence

- Growth at low energy
- Surprisingly close to STAR data!



The role of (gravitational anomaly related) T^2 term

- Different values of coefficient probed



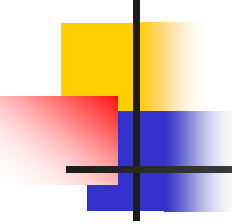
- LQCD suppression by collective effects supported

Other approach to confinement: vortices in pionic superfluid (V.I. Zakharov, OT:1705.01650)

- Pions may carry the axial current due to quantized vortices in pionic superfluid (Kirilin, Sadofyev, Zakharov'12)

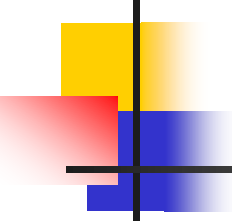
$$j_5^\mu = \frac{1}{4\pi^2 f_\pi^2} \epsilon^{\mu\nu\rho\sigma} (\partial_\nu \pi^0) (\partial_\rho \partial_\sigma \pi^0) \quad \frac{\pi_0}{f_\pi} = \mu \cdot t + \varphi(x_i) \quad \oint \partial_i \varphi dx_i = 2\pi n$$
$$\partial_i \varphi = \mu v_i$$

- Suggestion: core of the vortex- baryonic degrees of freedom- polarization
- (Quantum) Macro to micro at short distances



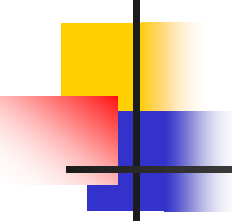
Conclusions/Outlook-fast rotation

- Vorticity – femto-vortex sheet formation(QGSM+; HSD?) , helicity separation (QGSM+,HSD)
- Polarization – new probe of anomaly in quark-gluon matter (to be studied at NICA!?)
- Energy dependence predicted and confirmed
- Same energy dependence in Regge theory: survival of Pomeron only at high energy – no polarization
- Possible “duality” between statistical and dynamical (Macro and Micro) description (cf “Ridge” in pp)



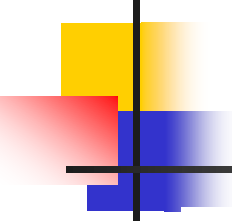
Slow rotation

- CAN STRONG INTERACTIONS AND QCD BE MANIFESTED IN INERTIAL/GRAVITY EFFECTS?



Spin-gravity/rotation (~ 25 orders of magnitude slower!) interactions

- How to describe hadron spin/gravity(inertia) couplings?
- Matrix elements of Energy-Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case



Gravitational Formfactors

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M \right] u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Distributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin, OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x, b) = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q} \cdot \vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1 + \tau}$$

$$\rho_0^{\text{Gr}}(b) = \frac{1}{2\pi} \int_0^\infty dq q J_0(qb) A(q^2)$$

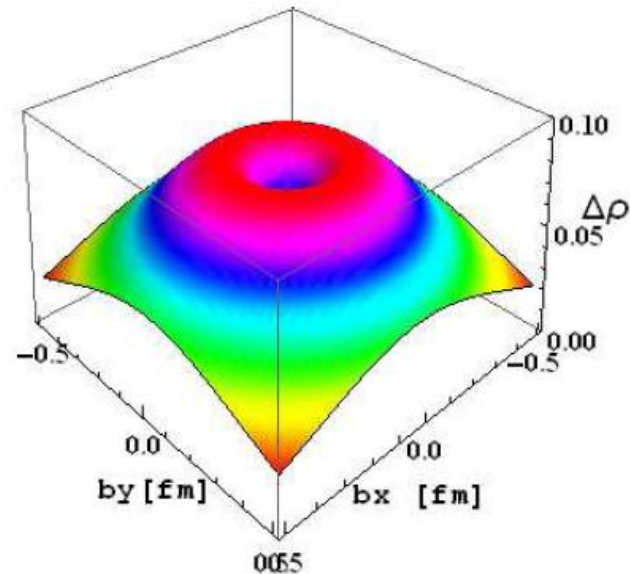
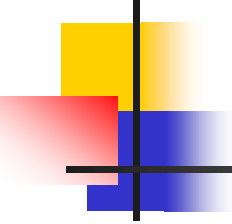


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)



Electromagnetism vs Gravity

- Interaction – field vs metric deviation

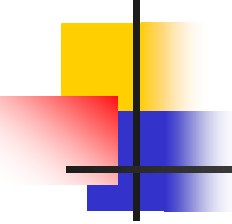
$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu \qquad \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$
$$h_{00} = 2\phi(x)$$

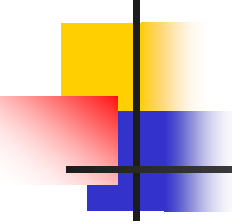
$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle
(Einstein '10-11, Praha)



Equivalence principle

- Newtonian – “Falling elevator” – well known and checked with high accuracy (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’ ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservation laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- For GEDM –checked with sometimes controversial results
- For AGM not checked on purpose but in fact checked in the same atomic spins experiments at % level (Silenko, OT’07)



Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

$$\vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i}$$

spin dragging twice
smaller than EM

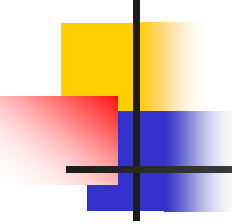
- Lorentz force – similar to EM case: factor $\frac{1}{2}$ cancelled with 2 from frequency same as EM

$$h_{00} = 2\phi(x)$$

Larmor

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \quad \vec{H}_L = \text{rot} \vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle



Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

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LETTERS

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Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

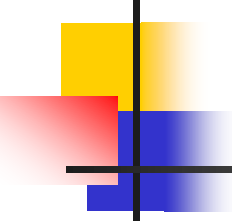
B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

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- If (CP-odd!) $G_{EDM}=0 \rightarrow$ constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \mathbf{B} \cdot \mathbf{S} - \zeta \hbar \boldsymbol{\omega} \cdot \mathbf{S}, \quad \zeta = 1 + \chi$$

$$|\chi(^{201}\text{Hg}) + 0.369\chi(^{199}\text{Hg})| < 0.042 \quad (95\% \text{C.L.})$$



Equivalence principle for moving particles

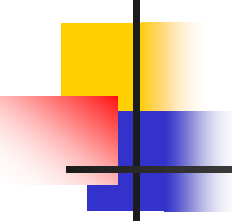
- Compare gravity and acceleration: gravity provides EXTRA space components of metrics

$$h_{zz} = h_{xx} = h_{yy} = h_{00}$$

- Matrix elements DIFFER

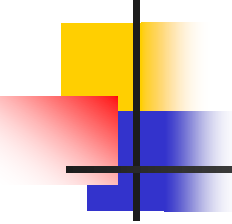
$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \quad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov (TALK), Silenko, OT '09, '11, '13



Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim \mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99)

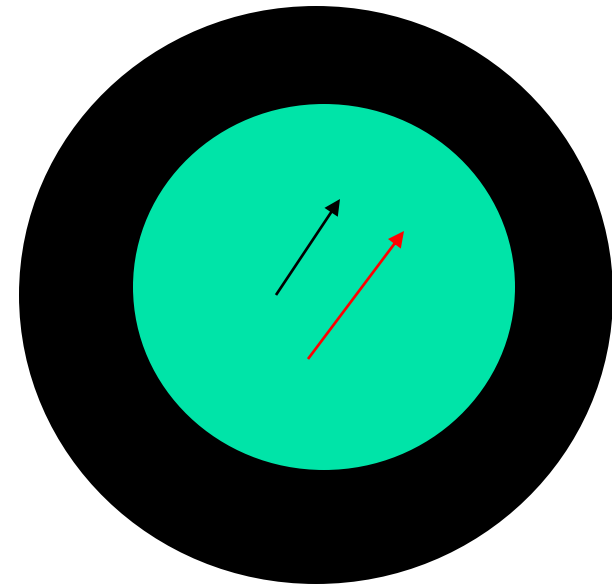


Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $\langle P+q | T^{mn} | P-q \rangle = P^{\{m} \langle P+q | J^{n\} \rangle | P-q \rangle / e$ up to the terms linear in q
- Special role of $g=2$ for any spin (asymptotic freedom for vector bosons)
- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also $g=2$ for Black Holes. Indication of “quantum” nature?!

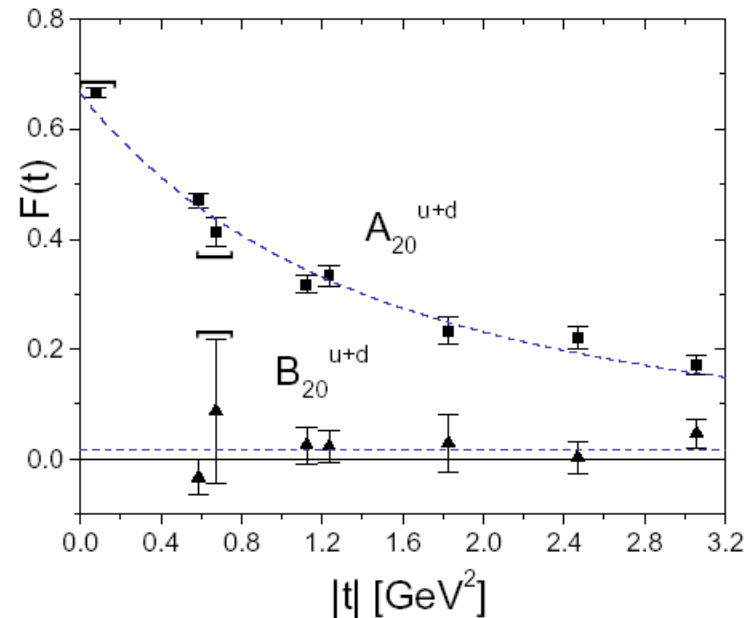
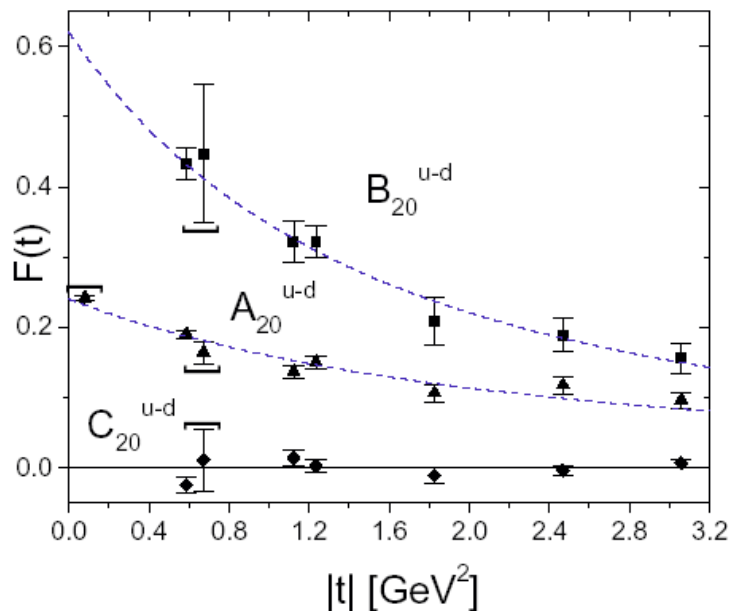
Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For **flat** "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and **quantum** rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



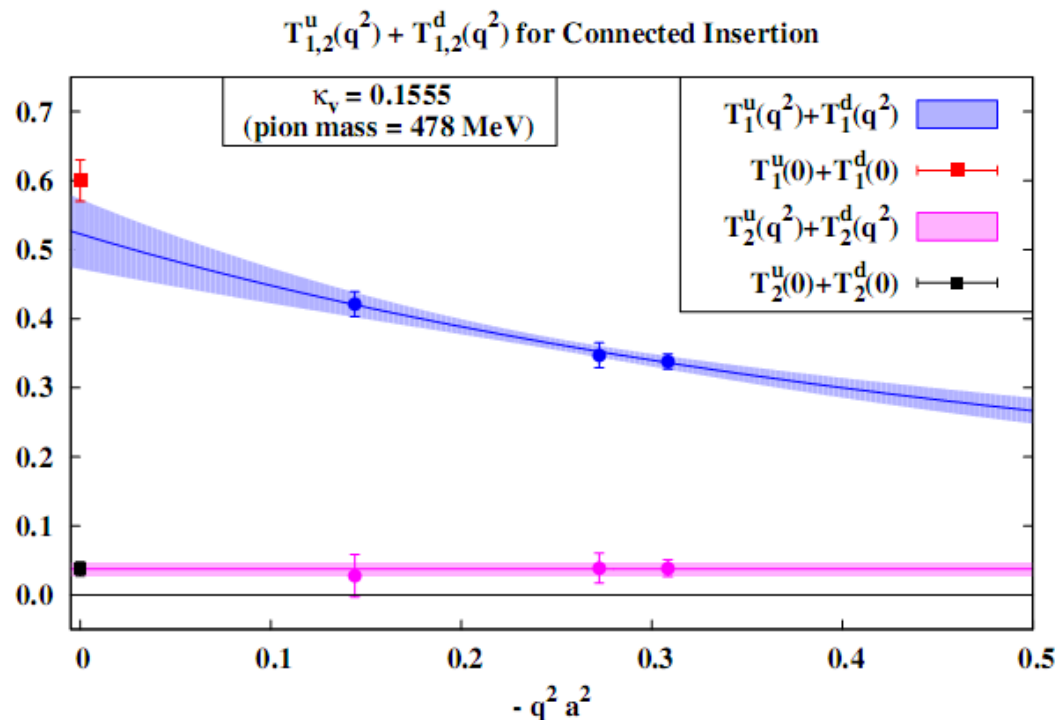
Generalization of Equivalence principle

- Various arguments: $AGM \approx 0$ separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Recent lattice study (M. Deka et al. [arXiv:1312.4816](https://arxiv.org/abs/1312.4816))

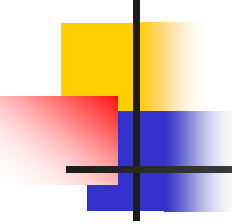
- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence

Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive enetering Black Hole?)?!



Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P, S | \bar{\psi}(0) \gamma^\nu D^{\nu_1} \dots D^{\nu_n} \psi(0) | P, S \rangle_{\mu^2} = i^{-n} M^2 S^{\nu\nu_1} P^{\nu_2} \dots P^{\nu_n} \int_0^1 C_q^T(x) x^n dx$$

$$\sum_q \langle P, S | T_i^{\mu\nu} | P, S \rangle_{\mu^2} = 2P^\mu P^\nu (1 - \delta(\mu^2)) + 2M^2 S^{\mu\nu} \delta_1(\mu^2)$$

$$\langle P, S | T_g^{\mu\nu} | P, S \rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu} \delta_1(\mu^2)$$

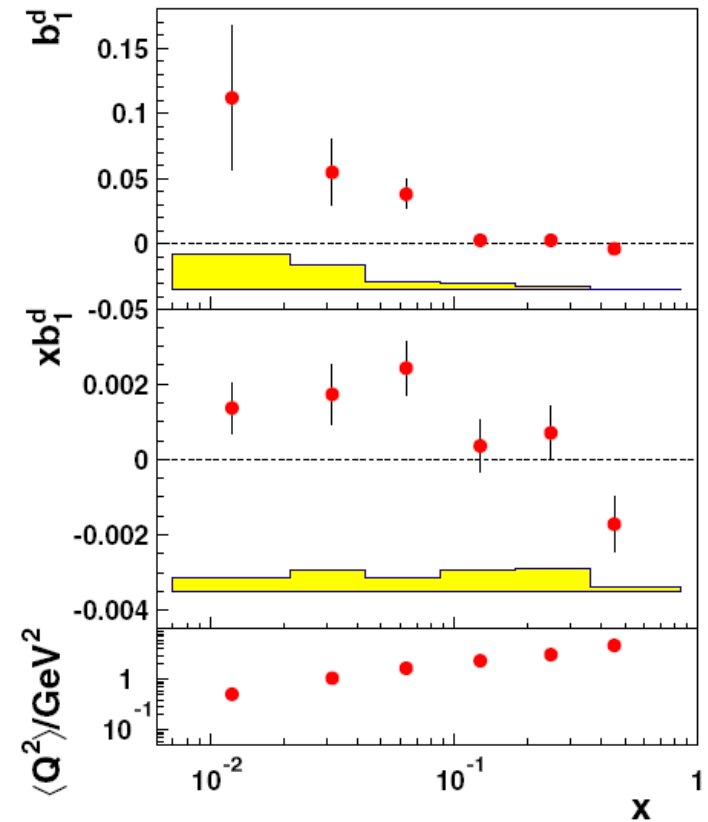
$$\sum_q \int_0^1 C_i^T(x) x dx = \delta_1(\mu^2) = 0 \text{ for ExEP}$$

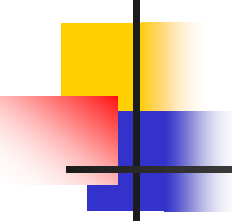
HERMES – data on tensor spin structure function

PRL 95, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence:

$$\int_0^1 C_i^T(x) dx = 0.$$

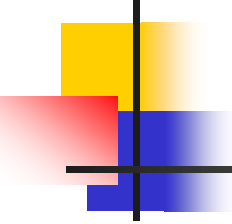




Conclusions (slow rotation)

- Probe of equivalence principle for spin
- May be tested in EDM search experiments
- Extension of EP –validity separately for quarks and gluons

- **BACKUP SLIDES**



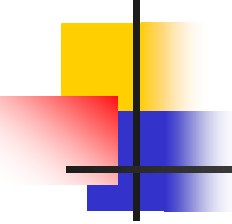
Sum rules for EMT (and OAM)

- First (seminal) example: X. Ji's sum rule ('96). Gravity counterpart – OT'99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT'14)

Pole prescription and Burkardt SR

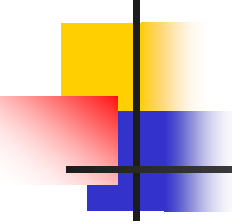
- Pole prescription (dynamics!) provides ("T-odd") symmetric part!
- SR: $\sum \int dx T(x, x) = 0$ (but relation of gluon Sivers to twist 3 still not found – prediction!)

$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!



Are more accurate data possible?

- HERMES – unlikely
- JLab may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle



EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides $g=2$ identically!
- Experimental test at time –like region possible