Generality of inflation in observation motivated models

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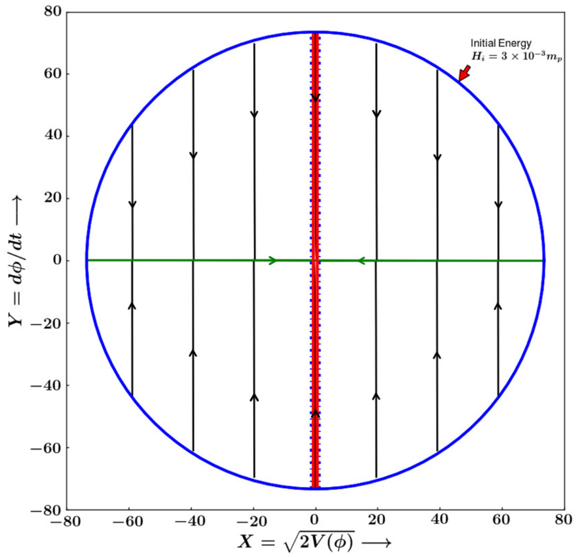

Figure 1:

Current observation shows that inflation, most probably, is one of the following types:

- Minimally coupled scalar field with flat potential
- Higgs field with non-minimal coupling
- R^2 inflatioin

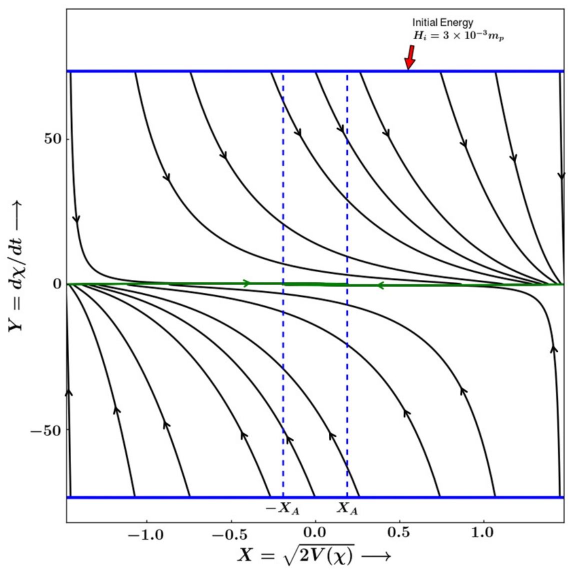

Figure 2:

This difference in generality of inflation in power-law and asymptotically flat potentials needs a qualitative explaination. The question that we would like to understand is why asymptotically flat potentials have these particular properties! The actual reason is, however, that these two cases are not qualitatively different. The peculiarity of the latter case manifests only if we, as usual, need not only inflation to happen, but also with atleast 60 e-foldings. Suppose, however, that we lift this condition of 60 e-foldings and constrain ourselves to only finding initial conditions that causes universe to inflate. Then what range of initial conditions are the worst for inflation? Contrary to wide spread opinion, it is not actually $\phi = 0$. To explain this point more clearly, let us consider the case of a contracting universe. A typical contracting universe with a massive scalar field has an equation of state of a steep matter i.e $w = 1$ as an attractor solution with the field growing fast fast while universe approaches the singularity. Now imagine that we stop the evolution at some energy level and reverse the direction of time. Universe will follow the same trajectory while expanding. Obviously, the equation of state is invariant under time reversal, so the universe will expand with no inflation at all and initial conditions located on a typically contracting trajectory (after changing the sign of $\dot{\phi}$) give us the worst initial conditions for inflation to occure. It is clear that ϕ can be considerably large, in particular, for a massive scalar field on a Planckean boundary, ϕ can reach the value about $2m_p$. As it is known, $w = 1$ at expansion (in contrast to contraction) requires a set of very special conditions, so for $\dot{\phi}$ different enough from the one constructed above, inflation becomes possible. Still this leads to about 25 e-foldings which is less than the required 60. That is why this influence of e-folding number on the initial value of $\dot{\phi}$, though existing, is not so interesting for realistic models. On the other hand, we can start from $\phi = 0$ and with a Plankean kinetic energy. Numerical simulations show that for $m \simeq 6 \times 10^{-6}\ m_p$, the field climbs up and gives about 11 e-foldings after that. This is still not enough, however, it shows the effect of initial kinetic term. The same situation happens for an asymptotically flat potential and in this case, this effect is important for realistic models also.

Let us consider the same way the dynamics in $R + R^2$ theory. What is the past attractor for the isotropic Universe? This is "false radiation" regime $a \sim \sqrt{t}$. At this regime $Q_1 = \dot{H}/H^2 = 2$. This means that starting from $Q_1 = -2$ we do not get inflation for sure. Numerical integration shows that inflation does not occure for $Q_1 < -2$.

Now we suppose a solution of this particular line element $ds^2 = -d\tau^2 + \tau^{2p_1}dx^2 + \tau^{2p_2}dy^2 + \tau^{2p_3}dz^2$. By direct substitution of this metric into the field equations for vacuum source, a purely algebraic equation is obtained

$$p_2^2 + p_2(-1 + p_1 + p_3) - (p_1 + p_3 - p_1^2 - p_3^2 - p_1 p_3) = 0, \tag{1}$$

whose solution is

$$p_2 = \frac{1}{2}\left(1 - p_1 - p_3 \pm \sqrt{1 + 2p_1 + 2p_3 - 3p_1^2 - 2p_1p_3 - 3p_3^2}\right). \tag{2}$$

Remembering the definition of Q_1 and introducing $\Sigma_\pm = \frac{\sigma_\pm}{H^2}$ we may rewrite the solution as

$$\begin{aligned} Q_1 &= -\frac{3}{p_1 + p_2 + p_3} \\ \Sigma_+ &= \frac{-3p_1 + (p_1 + p_2 + p_3)}{2(p_1 + p_2 + p_3)} \\ \Sigma_- &= \frac{\sqrt{3}(p_2 - p_3)}{2(p_1 + p_2 + p_3)}. \end{aligned} \tag{3}$$

In these variables, Kasner's solution is given by $Q_1 = -3$, $\Sigma_+^2 + \Sigma_-^2 = 1$, and the isotropic vacuum solution $Q_2 = -2$ and $\Sigma_+ = \Sigma_- = 0$.

In this solution

$$Q_1 = -2 - \Sigma_+^2 - \Sigma_-^2$$

CONCLUSIONS

In Einstein frame the initial conditions good for inflation for asymptorically flat potential have the form different from known for a massive scalar field.

The initial conditions good for Starobinsky inflation have simple form in Jordan frame:

$$Q_1 > -2 - \Sigma^2$$