Effective field theory methods for classical and quantum gravity

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based on the works 1309.0804, 1410.4148 and 1410.7590, 1609.07477 1704.01624

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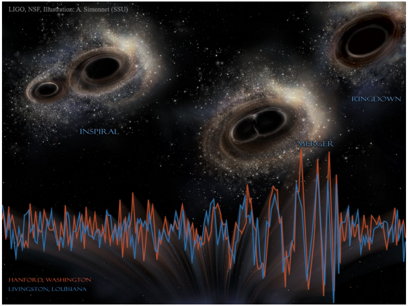
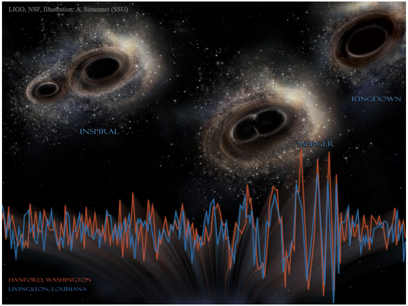
A new window on gravitation



The detection of GW150914 by LIGO has open a new window on the gravitational physics of our universe

- For the first time detection and test of GR in the strong gravity coupling regime
- For the first time dynamics of Black hole (not just static object curving space-time)

A new window on gravitation



[Yunes, Yagi, Pretorius] have listed theoretical implications of GW150914 in particular

GW150914 constrains a number of theoretical mechanisms that modify GW propagation

Effective gravity models

There are various models of modified gravity motivated by the dark energy issue

For instance models involve with higher derivative and non-compact extra dimensions

$$M_{Pl}^2 \int_{\mathcal{M}_4} d^4x \, \sqrt{g} \, \left(\mathcal{R}_{(4)} + \mathcal{O}(R^n)\right) + M_D^{D-2} \int_{\mathcal{M}_D} d^Dx \, \sqrt{G^{(D)}} \, \mathcal{R}_{(D)}$$

Treated as an effective field theory one can one constrain these models using scattering amplitude?

Embedding some model in string theory provides strong constraints `[Antoniadis, Minasian, Vanhove]` but one may want to have a purely QFT test of the various models

Quantum gravity as an effective field theory

[Donoghue] has explained that one can evaluate some long-range infra-red contributions in any quantum gravity theory and obtain reliable answers

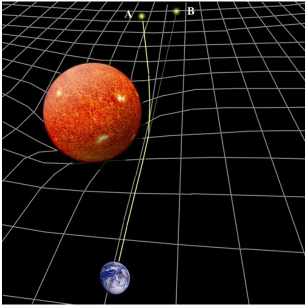
Some physical properties of quantum gravity are *universal* being independent of the UV completion

The one-loop infra-red contributions depend only the structure the low-energy fields and the classical background

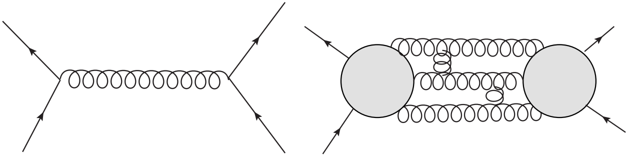
Physics of the effective field theory approach

Using the effective field theory approach to gravity one can compute

- the classical (post-Newtonian) and quantum contributions to the gravitational potential between masses
- Quantum corrections to the bending angle of massless particle by a massive classical object



Classical physics from loops



We will be considering the pure gravitational interaction between massive and massless matter of various spin

$$\mathcal{L}_{\text{EH}} \sim \int d^4x \left(-\frac{2}{\kappa^2} \mathcal{R} + \kappa h_{\mu\nu} T^{\mu\nu}_{\text{matter}} \right) ,$$

We will be considering perturbative computations $\kappa^2 = 32\pi G_N$

$$\mathfrak{M} = \frac{1}{\hbar} \mathfrak{M}^{\text{tree}} + \hbar^0 \mathfrak{M}^{1-\text{loop}} + \cdots .$$

Double expansion : classical and quantum parameters

We have two scales in the problem:

- The Schwarzschild radius

$$r_S = \frac{2G_N M}{c^2}$$

- The Compton wave-length

$$\lambdabar = \frac{\hbar}{Mc}$$

- Dual with respect to the Planck length

$$r_S \lambda = \frac{2G_N \hbar}{c^3} = 2\,\ell_P^2$$

Double expansion : classical and quantum contributions

Starting from the PPN expansion

$$V^{\text{class}}(r) = \sum_{m\geqslant 0} v_{m,0} \left(\frac{r_S}{r}\right)^m$$

If $\lambda = \hbar/(Mc)$ is the characteristic length of the quantum fluctuations we have at first order

$$\frac{1}{(r \pm \lambda\!\!\!^{-})^n} \simeq \frac{1}{r^n} \pm \frac{\lambda\!\!\!^{-}}{r^{n+1}}$$

leading to the modified potential

$$V(r \pm \lambda) \simeq \sum_m \left(v_{m,0} \frac{r_S^m}{r^m} + v_{m,1} \frac{r_S^m \lambda}{r^{m+1}} \right)$$

Double expansion : classical and quantum contributions

Since

$$r_S\lambda = \frac{2G_N\hbar}{c^3} = 2\,\ell_P^2$$

We have

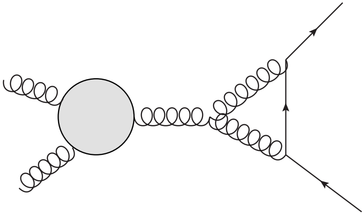
$$V(r\pm\lambda) \simeq \sum_m \left(v_{m,0}\frac{r_S^m}{r^m} + v_{m,1}\frac{r_S^{m-1}\ell_P^2}{r^{m+1}}\right)$$

This motivates the appearance of the first quantum corrections to the gravitational potential We will use scattering amplitudes to evaluate both the classical and the quantum part of the long range potential.
The Schwarzschild radius will arise from loop amplitude because

$$r_S = \frac{2G_N\ell_P^2}{c^3\lambda} = \frac{\kappa^2}{2\lambda} \qquad \kappa = \text{coupling constant}$$

Classical physics from loops

Let's consider the one-loop contribution for a say a massive scalar of mass m



Putting back the factors of $\hbar$ and c the Klein-Gordon equation reads

$$(\Box - \underbrace{\frac{m^2c^2}{\hbar^2}}_{\lambda\!\!\!^{-}{}^{-2}})\phi = 0$$

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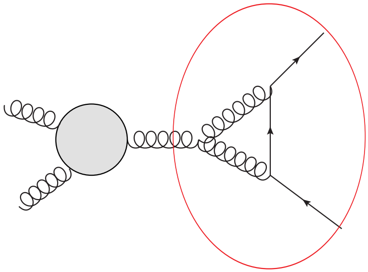

The triangle contribution with a massive leg $p_1^2 = p_2^2 = m^2$ reads

$$\int \frac{d^4\ell}{(\ell+p_1)^2(\ell^2-\frac{1}{\hbar^2})(\ell-p_2)^2}\Big|_{\text{finite part}} \sim \frac{1}{m^2}\left(\log(s)+\frac{\pi^2}{\hbar\sqrt{s}}\right)$$

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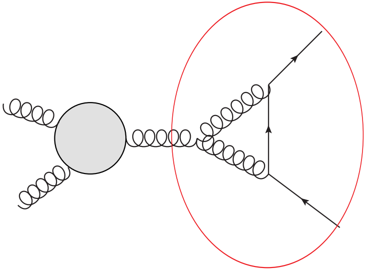

The triangle contribution with a massive leg $p_1^2 = p_2^2 = m^2$ reads

$$\frac{\kappa^2}{2\lambda\sqrt{s}} = \frac{r_S}{\sqrt{s}}$$

Fourier transformed with respect to the non-relativistic momentum transfert $|\vec{q}| = \sqrt{s}$ leads to r_S/r corrections

Classical physics from loops

The $1/\hbar$ term at one-loop contributes to the *same* order as the classical tree term [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$\mathfrak{M} = \frac{1}{\hbar}\left(\frac{G_N(m_1m_2)^2}{\vec{q}^2} + \frac{G_N^2(m_1m_2)^2(m_1+m_2)}{|\vec{q}|} + \cdots\right) + \hbar^0 G_N^2\, O(\log(\vec{q}^2)) + \cdots$$

For the scattering between a massive matter of mass m and massless matter of energy E one gets

$$\mathfrak{M} \sim \frac{1}{\hbar}\left(G_N\frac{(mE)^2}{\vec{q}^2} + G_N^2\frac{m^3E^2}{|\vec{q}|}\right) + \hbar\, G_N^2 O\left(\log(\vec{q}^2), \log^2(\vec{q}^2)\right) .$$

The mechanisms generalizes to higher loop-order amplitudes to leads to the higher order post-Newtonian corrections

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$V(r) = -\frac{G_N m_1 m_2}{r} \left(1 + C\,\frac{G_N(m_1+m_2)}{r} + Q\frac{G_N \hbar}{r^2}\right) + Q' G_N^2 m_1 m_2 \delta^3(\vec{x})$$

- C is the classical correction and Q and Q' are quantum corrections

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- C is the classical correction and Q and Q' are quantum corrections
- Q in the potential $V(r)$ is ambiguous but $V(r)$ is not observable

The coefficients of $1/\sqrt{-q^2}$ and $\log(-q^2)$ in the amplitude are unambiguously defined and depend on the long range physics

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

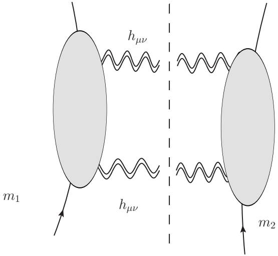
$$\mathfrak{M}^{1-\text{loop}}(q^2) = \frac{G_N (m_1 m_2)^2}{q^2} + C \frac{G_N^2 (m_1 m_2)^2 (m_1 + m_2)}{|q|} + \hbar \left(Q G_N^2 (m_1 m_2)^2 \log(-q^2) + Q' G_N^2 (m_1 m_2)^2 Q' G_N^2 (m_1 m_2)^2 \right)$$

- Q' is the short distance UV divergences of quantum gravity: need to add the R^2 term ['t Hooft-Veltman]

$$S = \int d^4x |-g|^{\frac{1}{2}} \left[\frac{2}{32\pi G_N} \mathcal{R} + c_1 \mathcal{R}^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

Loop amplitude

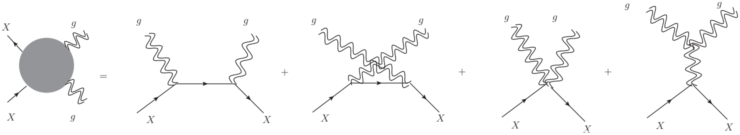
Since we are only interested in the long range graviton exchange, it is enough to just evaluate the gravitons cut



we need to know the gravitational Compton amplitudes on a particle of spin s with mass m

$$X^{s,m} + \text{graviton} \to X^{s,m} + \text{graviton}$$

Gravitational compton scattering

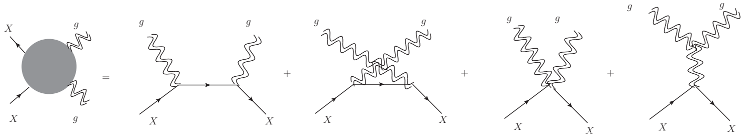


We express the gravity Compton scattering as a product of two Yang-Mills amplitudes [Kawai, Lewellen, Tye], [Bern, Carrasco, Johansson]

$$\mathfrak{M}(X^s g \to X^s g) = G_N \times (p_1 \cdot k_1)\, \mathcal{A}_s(1234) \tilde{\mathcal{A}}_0(1324)$$

$\mathcal{A}_s(1234)$ is the color ordered amplitudes scattering a gluon off a massive spin s state $X^s g \to X^s g$

Gravitational compton scattering

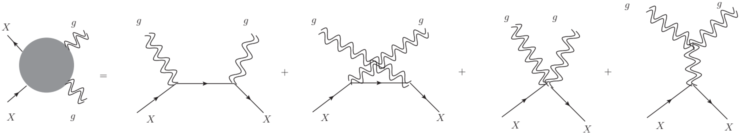


We express the gravity Compton scattering as a product of two QED Compton amplitudes using the monodromy relations [Bjerrum-Bohr, Donoghue, Vanhove]

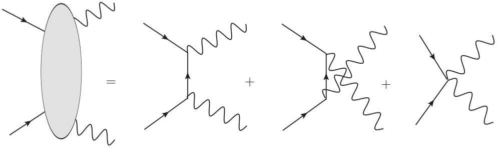
$$(k_1 \cdot k_2)\, \mathcal{A}_s(1234) = (p_1 \cdot k_2)\, \mathcal{A}_s(1324)$$

$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2}\, \mathcal{A}_s(1324)\tilde{\mathcal{A}}_0(1324)$$

Gravitational compton scattering



The gravity Compton scattering is expressed as the square of QED (abelian) Compton amplitudes [Bjerrum-Bohr, Donoghue, Vanhove]



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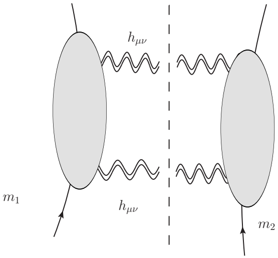
The one-loop amplitude between massive particles



We are only interested in the $1/\sqrt{-q^2}$ and $\log(-q^2)$ terms since the terms of $(q^2)^n/\sqrt{-q^2}$ and $(q^2)^n\log(-q^2)$ are negligible in the non-relativistic limit.

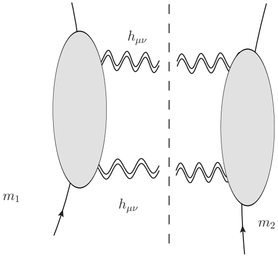
Only the massless graviton cut is enough.

The cut contributions

$$\mathfrak{M}^{1-\text{loop}}|_{\text{singlet cut}} = \int \frac{d^{4-2\epsilon}\ell}{\ell_1^2\ell_2^2\prod_{i=1}^4 \ell_1\cdot p_i}$$

$$\mathfrak{M}^{1-\text{loop}}|_{\text{non}-\text{singlet cut}} = \int d^{4-2\epsilon}\ell\,\frac{\Re\text{e}\left(\text{tr}_-(\not\ell_1\not p_1\not\ell_2\not p_2)\right)^4}{\ell_1^2\ell_2^2\prod_{i=1}^4 \ell_1\cdot p_i}$$

The one-loop amplitude between massive particles



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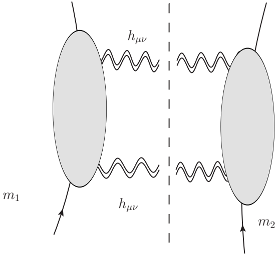
- In the non-relativistic limit the amplitude decomposes

$$\mathfrak{M}^{1-\text{loop}} \simeq G_N^2\,(m_1m_2)^4(I_4(s,t)+I_4(s,u)) + G_N^2(m_1m_2)^3 s(I_4(s,t)-I_4(s,u))$$
$$+\,G_N^2(m_1m_2)^2\,(I_3(s,m_1)+I_3(s,m_2))$$
$$+\,G_N^2(m_1m_2)^2 I_2(s)$$

The one-loop amplitude between massive particles



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Only the massless graviton cut is enough.

The result is given by

$$\mathfrak{M}^{1-\text{loop}} \simeq G_N^2 (m_1 m_2)^2 \left(\underbrace{6\pi}_{C} \frac{m_1+m_2}{\sqrt{-q^2}} \underbrace{-\frac{41}{5}}_{Q} \log(-q^2) \right)$$

Universality of the result

In the case of scattering of particles of different spin S_1 and S_2 the non-relativistic potential reads

$$\mathfrak{M}^{1-\text{loop}}(q^2) \simeq G_N^2 (m_1 m_2)^2 \left(C\, \frac{(m_1+m_2)}{\sqrt{-q^2}} + Q\hbar \log(-q^2) \right)$$

C and Q have a *spin-independent* and a *spin-orbit* contribution

$$C, Q = C, Q^{S-I} \langle S_1|S_1\rangle \langle S_2|S_2\rangle + C, Q^{S-O}_{1,2} \langle S_1|S_1\rangle \vec{S}_2 \cdot \frac{p_3 \vec{\times} p_4}{m_2} + (1 \leftrightarrow 2)$$

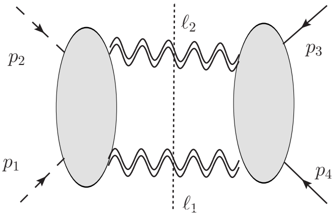
This expression is generic for all type of matter

the numerical coefficients are the same for all matter type

The universality of the coefficients with respect to the spin of the external states is a consequence of

- The reduction to the product of QED amplitudes
- the low-energy theorems of [Low, Gell-Mann, Goldberger] and [Weinberg]

The one-loop amplitude for massless particles



We consider the gravitational one-loop amplitude between a massless particle of spin S and a massive scalar

$$\begin{aligned}\kappa^{-4}\, i\mathfrak{M}_S^{1-\text{loop}} &= bo^S(s,t)\, I_4(s,t) + bo^S(s,u)\, I_4(s,u)\\ &+ t_{12}^S(s)\, I_3(s,0) + t_{34}^S(s)\, I_3(s,M^2)\\ &+ bu^S(s,0)\, I_2(s,0)\,.\end{aligned}$$

The coefficients satisfy interesting BCJ relations

$$\frac{bo^S(s,t)}{t-M^2} + \frac{bo^S(s,u)}{u-M^2} = t_{12}^S(s)$$

The amplitude

The low-energy approximation

$$
\begin{aligned}
i\mathfrak{M}_S^{\text{tree}+1-\text{loop}} &= \frac{\mathcal{N}^{(S)}}{\hbar}\left[\kappa^2\,\frac{(2M\omega)^2}{16q^2}\right.\\
&+ \hbar\frac{\kappa^4}{16}\Big(4(M\omega)^4(I_4(t,s)+I_4(t,u))+3(M\omega)^2 sI_3(t)\\
&- \left.\frac{15}{4}(M^2\omega)^2 I_3(t,M)+bu^S(M\omega)^2 I_2(t)\Big)\right]
\end{aligned}
$$

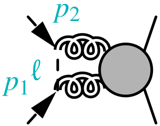
For photon scattering only the amplitudes with helicity $(++)$ and $(--)$ are non-vanishing.

Therefore there is no birefringence effects to contrary to case with electrons loops contributing to the interaction [Drummond, Hathrell;Berends, Gastmans]

The amplitude

$$
\begin{aligned}
i\mathfrak{M}_S^{\text{tree}+1-\text{loop}} &\simeq \frac{\mathcal{N}^{(S)}}{\hbar}\,\frac{(M\omega)^2}{4}\\
\times\quad & \Big[\frac{\kappa^2}{q^2} + \kappa^4 \frac{15}{512}\frac{M}{\sqrt{-q^2}}\\
+\quad & \hbar\kappa^4 \frac{15}{512\pi^2}\log\left(\frac{-q^2}{M^2}\right) - \hbar\kappa^4\frac{bu^S}{(8\pi)^2}\log\left(\frac{-q^2}{\mu^2}\right)\\
+\quad & \hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-q^2}{\mu^2}\right) + \kappa^4\frac{M\omega}{8\pi}\frac{i}{s}\log\left(\frac{-q^2}{M^2}\right)\Big]
\end{aligned}
$$

The last line contains the infrared divergences



$$
\propto \int_0 \frac{d^{4-2\epsilon}\ell}{\ell^2\, 2\ell\cdot p_1\, 2\ell\cdot p_2} \sim \frac{(t/\mu^2)^{-\epsilon}}{\epsilon^2\, t}
$$

The bending angle via Eikonal approximation

$$i\mathcal{M}(\boldsymbol{b}) \simeq 2(s-M^2)\left[e^{i(\chi_1+\chi_2)}-1\right]$$

$\chi_1(\boldsymbol{b})$ is the Fourier transform of the one graviton (tree-level) exchange

$$\chi_1(\boldsymbol{b}) = \frac{1}{2M2E}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \mathfrak{M}_S^{(1)}(\boldsymbol{q}) \simeq 4G_N M E\left[\frac{1}{d-2}-\log(b/2)-\gamma_E\right]$$

$\chi_2(\boldsymbol{b})$ is the Fourier transform of the two gravitons (one-loop) exchange

$$\chi_2(\boldsymbol{b}) = \frac{1}{2M2E}\int \frac{d^2\boldsymbol{q}}{(2\pi)^2}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \mathfrak{M}_X^{(2)}(\boldsymbol{q})$$

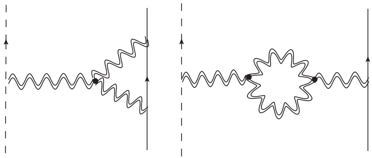
$$\simeq -G_N^2M^2E\frac{15\pi}{4b}-\frac{G_N^2M^2E}{2\pi b^2}\left(8bu^S+9-48\log\frac{b}{2b_0}\right).$$

The bending angle

The bending angle $\theta_S \simeq -\frac{1}{E}\frac{\partial}{\partial b}\left(\chi_1(b)+\chi_2(b)\right)$ is

$$\theta_S \simeq \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8bu^S + 9 - 48\log\frac{b}{2b_0}}{\pi}\,\frac{G^2\hbar M}{b^3}\,.$$

- The classical contribution including the 1rst Post-Newtonian correction is correctly reproduced
- The quantum corrections are new: not only from a quantum corrected metric



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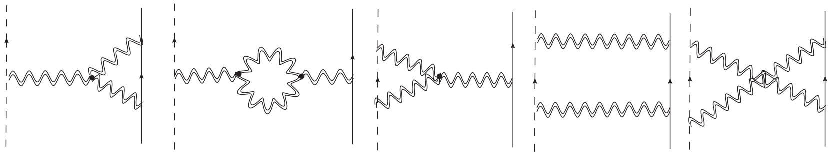

The difference between the bending angle for a massless photon and massless scalar

$$\theta_\gamma - \theta_\varphi = \frac{8(bu^\gamma - bu^\varphi)}{\pi}\,\frac{G^2\hbar M}{b^3}\,.$$

Outlook

Recent progresses from string theory technics, on-shell unitarity, double-copy formalism simplifies a lot perturbative gravity amplitudes computations

- The amplitudes relations discovered in the context of massless supergravity theories extend to the pure gravity case with massive matter
- The use of quantum gravity as an effective field theory allows to compute universal contributions from the long-range corrections
- We can reproduce the classical GR post-Newtonian corrections to the potential and understand some generic properties using low-energy theorems: hope to be able to simplify the computation of PPN corrections.