On the resonances in positron scattering on a supercritical nucleus and e^+e^- pairs production

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WORK IN PROGRESS

Kuleshov V. M., Mur V. D., Narozhny N. B., Fedotov A. M.,
Lozovik Yu E. and Popov V. S.,

"Coulomb problem for a $Z > Z_{\rm cr}$ nucleus",
Phys. Usp., V 58, p. 785-791, 2015,

1. resonances in positron scattering on supercritical nucleus;

2. stability of the naked nucleus with arbitrary large Z; no e^+e^- pairs creation.

Point 1: New phenomena in Rutherford scattering - we confirm it;
Point 2: We do not agree.

Resonances were briefly discussed in Popov-Zeldovich UFN 1971 year review and later by Mur, Popov; Popov, Eletskii, Mur.

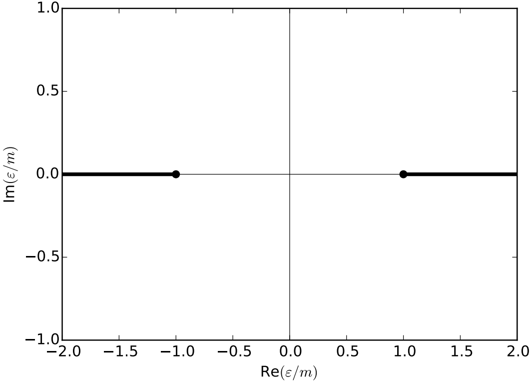

Fig 1. ε *plane.*

Dirac equation

$$\begin{cases} \dfrac{dF}{dr} + \dfrac{\varkappa}{r} F - (\varepsilon + m - V(r))\, G = 0, \\ \dfrac{dG}{dr} - \dfrac{\varkappa}{r} G + (\varepsilon - m - V(r))\, F = 0, \end{cases}$$

To deal with $Z\alpha > 1$ case:

$$V(r) = \begin{cases} -\dfrac{Z\alpha}{r}, & r > R, \\ -\dfrac{Z\alpha}{R}, & r < R. \end{cases}$$

Pomeranchuk, Smorodinsky (1945).

$$\left(\begin{array}{c} F \\ G \end{array}\right)\Big|_{r\to\infty} = A\cdot\left(\begin{array}{c} -i\sqrt{-m-\varepsilon} \\ -\sqrt{m-\varepsilon} \end{array}\right)\left\{e^{i(kr+\frac{Z\alpha\varepsilon}{k}\ln(2kr))}e^{2i\delta} \pm e^{-i(kr+\frac{Z\alpha\varepsilon}{k}\ln(2kr))}\right\},$$

where A is a constant and

$$e^{2i\delta} = \frac{\exp\left(\frac{\pi\tau}{2}+\frac{i\varphi}{2}\right)a^* - \exp\left(-\frac{\pi\tau}{2}-\frac{i\varphi}{2}\right)b}{\exp\left(\frac{\pi\tau}{2}-\frac{i\varphi}{2}\right)a - \exp\left(-\frac{\pi\tau}{2}+\frac{i\varphi}{2}\right)b^*},$$

where

$$\exp(i\varphi) = e^{2i\theta}\frac{(2k)^{-i\tau}\Gamma(2i\tau)}{(2k)^{i\tau}\Gamma(-2i\tau)},$$

$$a = \frac{Z\alpha\sqrt{m-\varepsilon}-(\tau+i\varkappa)\sqrt{-m-\varepsilon}}{\Gamma(1-i\tau-\frac{iZ\alpha\varepsilon}{k})},\quad b = \frac{Z\alpha\sqrt{m-\varepsilon}+(\tau+i\varkappa)\sqrt{-m-\varepsilon}}{\Gamma(1-i\tau+\frac{iZ\alpha\varepsilon}{k})}.$$

$\tau = \sqrt{Z^2\alpha^2-\varkappa^2}$, $e^{2i\theta}$ - Bessel functions, $\varkappa = -1$ for ground level.

Scattering phase 2

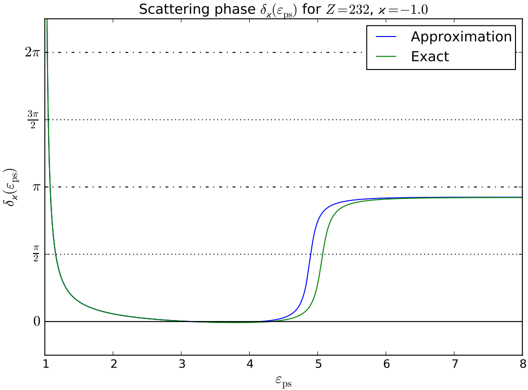

Fig 2. *Blue line corresponds to the approximation $Z\alpha mR << 1$ in matching, a'la Kuleshov et al.*

Resonance (Gamov) states

Resonances correspond to poles in
$S \equiv e^{2i\delta}$.
Where do resonances come from?
At $Z < Z_{cr}$ (ground state energy $\varepsilon > -m$) they are bound states, spectrum of which are given by zeroes of denominator of S.

At $Z > Z_{cr}$ pole goes to the second sheet of energy plane ABOVE left cut,

$$\varepsilon = -\varepsilon_0 + \frac{i}{2}\gamma$$

Unusual sign of γ.

Kuleshov et al.: at $Z > Z_{cr}$ state is at lower continuum - sign of energy should be reversed (negative electron energies at lower continuum describe positive positron energies).

BUT: resonance state originates from upper continuum...

Ground level trajectory

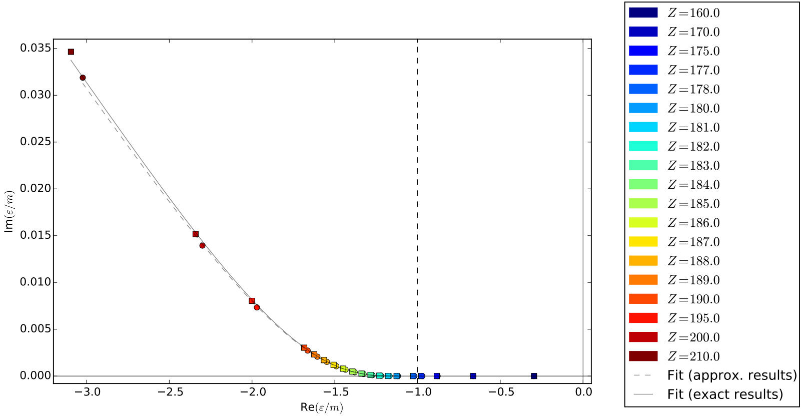

Fig 3. *Ground level energy and width.*

The Dirac equation for positron

$$Z\alpha \longrightarrow -Z\alpha$$

The Dirac equation for positron in the field of (positively charged) nucleus. Solving this equation we found scattering phase δ.

What about resonances? At first glance - no bound states, so no resonances.
Those who think so forget, that the Dirac equation describes BOTH electrons and positrons.
Thus "the Dirac equation for positron" describes BOTH positrons and electrons.

It describes bound states, which this time originates from lower continuum, where electrons are situated.
At small Z ground state energy equals

$$\varepsilon = -m + m(Z\alpha)^2/2$$

With growing Z energy crosses zero and at $Z > Z_{cr}$ first resonance (Gamov) state emerges.
It is situated on the second sheet of energy plane below right cut:

$$\varepsilon_{ps} = \varepsilon_0 - \frac{i}{2}\gamma$$

Proper place for the resonance state.

Naked nucleus with arbitrary large Z?

History of e^+e^- pairs production by supercritical nucleus:

Voronkov, V. V. and Kolesnikov, N. N.,JETP (1961);
Gershtein, S. S. and Zel'dovich, Ya. B., JETP (1969);
Pieper, W. and Greiner, W., Z. Phys. (1969);
Ya. B. Zeldovich and V. S. Popov, UFN (1971)...

Naked supercritical nucleus is screened by electrons, while positrons are emitted and can be detected.

Kuleshov et al.: Naked supercritical nucleus is stable, pairs are not produced.

No naked nucleus with $Z > Z_{cr}$

FOR OUR MIND:
0. Conservation of energy allows pairs production.

1. Resonance in the heavy nucleus - positron system just signal pair production. Without nucleus empty level in lower continuum is positron.
With nucleus the energy of state get imaginary part, so it decays with the lifetime $1/\gamma$. In this time electron which occupies the state with the same energy in the lower continuum "jumps" to this empty level creating "charged vacuum", while positron (hole in the lower continuum) is radiated to "infinity".

2. Kuleshov et al. wrote in Conclusions of their paper, that stable state of the supercritical nucleus consists of empty states of upper continuum, empty discrete levels and occupied states of lower continuum. It is just the picture I described. Occupied by electrons states of lower continuum which come from the upper continuum when Z grows and becomes larger that Z_{cr} form "charged vacuum" - its charge equals -n, where n is the number of these levels. Compensating charge +n was carried out by produced positrons.

Supercritical nucleus is not naked - its electric charge is partially screened by these electrons.