Yang-Baxter deformations and generalized supergravity

Kentaroh Yoshida (Dept. of Phys., Kyoto U.)

The AdS/CFT correspondence

type IIB string on $AdS_5 \times S^5$ ⟺ 4D $\mathcal{N}=4$ SU(N) SYM $(N \to \infty)$

Recent progress: the discovery of integrability

[For a big review, Beisert et al., 1012.3982]

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Recent progress: the discovery of integrability

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Integrability is so powerful!

The integrability enables us to compute exactly physical quantities even at finite coupling, without relying on supersummetries.

EX anomalous dimensions, amplitudes etc.

Indeed, there are many directions of study with this integrability.

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Indeed, there are many directions of study with this integrability.

Here, among them, we are concerned with

the classical integrability on the string-theory side.

→ The existence of Lax pair (kinematical integrability)

The next issue

Integrable deformations of the $AdS_5 \times S^5$ superstring

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Integrable deformations

(as a 2D non-linear sigma model)

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Do the integrable deformations lead to solutions of type IIB SUGRA?

or, do they break the on-shell condition of type IIB SUGRA?

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Do the integrable deformations lead to solutions of type IIB SUGRA?

or, do they break the on-shell condition of type IIB SUGRA?

The main subject of my talk is to answer these questions

for a specific class of integrable deformations called

Yang-Baxter deformations

Yang-Baxter deformations

Yang-Baxter deformation

[Klimcik, 2002, 2008]

Integrable deformation!

An example

G-principal chiral model

Yang-Baxter sigma model

$$S = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}(J_\mu J_\nu) \quad \longrightarrow \quad S^{(\eta)} = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}\left(J_\mu \frac{1}{1-\eta R} J_\nu \right)$$

$$J_\mu = g^{-1}\partial_\mu g, \quad g \in G$$

η : a const. parameter

Yang-Baxter deformation

[Klimcik, 2002, 2008]

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What is R?

$$R \ : \ \mathfrak{g} \longrightarrow \mathfrak{g}$$

a linear op.

a classical r-matrix satisfying

the modified classical Yang-Baxter eq. (mCYBE)

An integrable deformation can be specified by a classical r-matrix.

Yang-Baxter deformation

[Klimcik, 2002, 2008]

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Strong advantage

Given a classical r-matrix, a Lax pair follows automatically.

No need to construct Lax pair in an intuitive manner case by case

Relation between R-operator and classical r-matrix

A linear R-operator ⟷ A skew-symmetric classical r-matrix

$R \ : \ \mathfrak{g} \longrightarrow \mathfrak{g}$

$r \in \mathfrak{g} \otimes \mathfrak{g}$

$$R(X) \equiv \langle r_{12}, 1 \otimes X \rangle = \sum_i a_i \langle b_i, X \rangle \qquad \text{for} \quad X \in \mathfrak{g}$$

$$r_{12} = \sum_i a_i \otimes b_i \qquad \text{with} \qquad a_i, b_i \in \mathfrak{g}$$

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Two sources of classical r-matrices

1) modified classical Yang-Baxter eq. (mCYBE) ⟵ the original work by Klimcik

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = \underline{-c^2 [X, Y]} \qquad (c \in \mathbb{C})$$

2) classical Yang-Baxter eq. (CYBE) $(c = 0)$ ⟵ a possible generalization

The list of generalizations of Yang-Baxter deformations (2 classes)

(i) modified CYBE (trigonometric class)

a) Principal chiral model [Klimcik, hep-th/0210095, 0802.3518]

b) Symmetric coset sigma model [Delduc-Magro-Vicedo, 1308.3581]

1) c) The $AdS_5 \times S^5$ superstring [Delduc-Magro-Vicedo, 1309.5850]

(ii) CYBE (rational class)

a) Principal chiral model [Matsumoto-KY, 1501.03665]

b) Symmetric coset sigma model [Matsumoto-KY, 1501.03665]

2) c) The $AdS_5 \times S^5$ superstring [Kawaguchi-Matsumoto-KY, 1401.4855]

NOTE bi-Yang-Baxter deformation [Klimcik, 0802.3518, 1402.2105]

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YB deformations of the $AdS_5 \times S^5$ superstring

[Delduc-Magro-Vicedo, 1309.5850]

[Kawaguchi-Matsumoto-KY, 1401.4855]

$$S = -\frac{1}{2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \; P_-^{\alpha\beta}\mathrm{Str}\left[A_\alpha d \circ \frac{1}{1-\eta\,[R]_g \circ d}(A_\beta)\right]$$

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R satisfies (m)CYBE.

The undeformed limit: $\eta \to 0$ ⇒ the Metsaev-Tseytlin action

[Metsaev-Tseytlin, hep-th/9805028]

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[Metsaev-Tseytlin, hep-th/9805028]

- Lax pair is constructed : classical integrability
- Kappa invariance : a consistency as string theory at classical level

NOTE

The difference between mCYBE and CYBE is reflected in some coefficients of some quantities such as Lax pair and kappa-transformation.

A group element: $g = g_{\rm b}\, g_{\rm f} \quad \in SU(2,2|4)$

$$g_{\rm b} = {g_{\rm b}}^{\rm AdS_5}\, {g_{\rm b}}^{\rm S^5}\,,$$

[For a big review, Arutyunov-Frolov, 0901.4937]

$$g_{\rm f} = \exp(\mathbf{Q}^I\theta_I)\,, \quad \mathbf{Q}^I\theta_I \equiv (\mathbf{Q}^{\check{\alpha}\hat{\alpha}})^I\,(\theta_{\check{\alpha}\hat{\alpha}})_I \quad (I=1,2;\ \check{\alpha},\hat{\alpha}=1,\ldots,4)$$

When we take a parametrization like

$$\begin{aligned} {g_{\rm b}}^{\rm AdS_5} &= \exp\Big[x^0\,P_0 + x^1\,P_1 + x^2\,P_2 + x^3\,P_3\Big]\,\exp\Big[(\log\,z)\,D\Big]\,, \\ {g_{\rm b}}^{\rm S^5} &= \exp\Big[\frac{i}{2}(\phi_1\,h_1 + \phi_2\,h_2 + \phi_3\,h_3)\Big]\,\exp\Big[\xi\,\mathbf{J}_{68}\Big]\,\exp\Big[-i\,r\,\mathbf{P}_6\Big]\,, \end{aligned}$$

the metric of AdS_5 x S^5 is given by

$$\begin{aligned} ds^2 &= ds^2_{\rm AdS_5} + ds^2_{\rm S^5}\,, \\ ds^2_{\rm AdS_5} &= \frac{-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{z^2} + \frac{dz^2}{z^2}\,, \\ ds^2_{\rm S^5} &= dr^2 + \sin^2 r\,d\xi^2 + \cos^2\xi\,\sin^2 r\,d\phi_1^2 + \sin^2 r\,\sin^2\xi\,d\phi_2^2 + \cos^2 r\,d\phi_3^2 \end{aligned}$$

An outline of supercoset construction

[Arutyunov-Borsato-Frolov, 1507.04239]
[Kyono-KY, 1605.02519]

The deformed action can be rewritten into the canonical form:

$$S = -\frac{\sqrt{\lambda_c}}{4}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \left[\gamma^{ab} G_{MN}\partial_a X^M \partial_b X^N - \epsilon^{ab} B_{MN}\partial_a X^M \partial_b X^N\right]$$
$$-\frac{\sqrt{\lambda_c}}{2}\, i\bar{\Theta}_I(\gamma^{ab}\delta^{IJ} - \epsilon^{ab}\sigma_3^{IJ})\, e_a^m \Gamma_m D_b^{JK}\Theta_K + \mathcal{O}(\theta^4)$$

This action is expanded w.r.t the fermions.

In general, the covariant derivative D is given by

[Cvetic-Lu-Pope-Stelle, hep-th/9907202]

$$D_a^{IJ} \equiv \delta^{IJ}\left(\partial_a - \frac{1}{4}\omega_a^{mn}\Gamma_{mn}\right) + \frac{1}{8}\sigma_3^{IJ} e_a^m H_{mnp}\Gamma^{np}$$
$$-\frac{1}{8}e^{\Phi}\left[\epsilon^{IJ}\Gamma^p F_p + \frac{1}{3!}\sigma_1^{IJ}\Gamma^{pqr}F_{pqr} + \frac{1}{2\cdot 5!}\epsilon^{IJ}\Gamma^{pqrst}F_{pqrst}\right] e_a^m \Gamma_m$$

From this expression, one can read off all of the fields of type IIB SUGRA.

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Summary of the resulting backgrounds

1) The mCYBE case [Delduc-Magro-Vicedo, 1309.5850]

η-deformation or standard q-deformation [Arutyunov-Borsato-Frolov, 1312.3542]

The background is not a solution of the usual type IIB SUGRA,

but satisfies the generalized type IIB SUGRA. [Arutyunov-Borsato-Frolov, 1507.04239]

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

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2) The CYBE case [Kawaguchi-Matsumoto-KY, 1401.4855]

- A certain class of classical r-matrices satisfying

The unimodularity condition [Borsato-Wulff, 1608.03570]

$$r^{ij}[b_i, b_j] = 0 \qquad \text{for a classical r-matrix} \qquad r = r^{ij} b_i \wedge b_j$$

⇒ Solutions of the standard type IIB SUGRA

EX Lunin-Maldacena-Frolov, Maldacena-Russo, Schrödinger spacetimes

[Kyono-KY, 1605.02519]

- The other ones lead to solutions of the ``generalized'' type IIB SUGRA

What is the generalized type IIB SUGRA?

The generalized eqns of type IIB SUGRA

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

$$R_{MN} - \frac{1}{4}H_{MKL}H_N{}^{KL} - T_{MN} + D_M X_N + D_N X_M = 0\,,$$

$$\frac{1}{2}D^K H_{KMN} + \frac{1}{2}F^K F_{KMN} + \frac{1}{12}F_{MNKLP}F^{KLP} = X^K H_{KMN} + D_M X_N - D_N X_M\,,$$

$$R - \frac{1}{12}H^2 + 4D_M X^M - 4X_M X^M = 0\,,$$

$$D^M \mathcal{F}_M - Z^M \mathcal{F}_M - \frac{1}{6}H^{MNK}\mathcal{F}_{MNK} = 0\,, \qquad I^M \mathcal{F}_M = 0\,, \qquad \mathcal{F}_{n_1 n_2 \ldots} = \mathrm{e}^{\Phi} F_{n_1 n_2 \ldots}$$

$$D^K \mathcal{F}_{KMN} - Z^K \mathcal{F}_{KMN} - \frac{1}{6}H^{KPQ}\mathcal{F}_{KPQMN} - (I \wedge \mathcal{F}_1)_{MN} = 0\,,$$

$$D^K \mathcal{F}_{KMNPQ} - Z^K \mathcal{F}_{KMNPQ} + \frac{1}{36}\epsilon_{MNPQRSTUVW}H^{RST}\mathcal{F}^{UVW} - (I \wedge \mathcal{F}_3)_{MNPQ} = 0$$

$$T_{MN} \equiv \frac{1}{2}\mathcal{F}_M \mathcal{F}_N + \frac{1}{4}\mathcal{F}_{MKL}\mathcal{F}_N{}^{KL} + \frac{1}{4\times 4!}\mathcal{F}_{MPQRS}\mathcal{F}_N{}^{PQRS} - \frac{1}{4}G_{MN}(\mathcal{F}_K \mathcal{F}^K + \frac{1}{6}\mathcal{F}_{PQR}\mathcal{F}^{PQR})$$

Modified Bianchi identities

$$(d\mathcal{F}_1 - Z \wedge \mathcal{F}_1)_{MN} - I^K \mathcal{F}_{MNK} = 0\,,$$

$$(d\mathcal{F}_3 - Z \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{MNPQ} - I^K \mathcal{F}_{MNPQK} = 0\,,$$

$$(d\mathcal{F}_5 - Z \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{MNPQRS} + \frac{1}{6}\epsilon_{MNPQRSTUVW}I^T \mathcal{F}^{UVW} = 0$$

The generalized eqns of type IIB SUGRA

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

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$$(d\mathcal{F}_5 - Z \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{MNPQRS} + \frac{1}{6} \epsilon_{MNPQRSTUVW} I^T \mathcal{F}^{UVW} = 0$$

New ingredients: $X, \quad I, \quad Z$ 3 vector fields

But $X_M \equiv I_M + Z_M$, so two of them are independent.

Then I & Z satisfy the following relations:

$$D_M I_N + D_N I_M = 0\,, \quad D_M Z_N - D_N Z_M + I^K H_{KMN} = 0\,, \quad I^M Z_M = 0$$

Assuming that I is chosen such that the Lie derivative

$$(\mathcal{L}_I B)_{MN} = I^K \partial_K B_{MN} + B_{KN} \partial_M I^K - B_{KM} \partial_N I^K$$

vanishes, the 2nd equation above can be solved by

$$Z_M = \partial_M \Phi - B_{MN} I^N \quad .$$

Thus only I is independent after all.

Note When $I = 0$, the usual type IIB SUGRA is reproduced.

A great progress on the Green-Schwarz string

Relation between kappa-symmetry and SUGRA

Old result: the on-shell condition of the standard type IIB SUGRA

⇒ kappa-invariant GS string theory [Grisaru-Howe-Mezincescu-Nilsson-Townsend, 1985]

The inverse was conjectured.

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This issue has been resolved after more than 30 years from the old work.

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Remarkably,

The generalized type II supergravities can be reproduced from DFT (or EFT).

[Sakatani-Uehara-KY, 1611.05856] [Baguet-Magro-Samtleben, 1612.07210] [Sakamoto-Sakatani--KY, 1703.09213]

What happens to the string world-sheet theory?

Pathology? [Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

Even if the background is not a solution of type IIB SUGRA, scale invariance is ensured, but Weyl invariance is not.

To resolve this issue, the DFT picture is very useful.

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By allowing the dilaton to depend on the dual coordinates, the appropriate counter-term can be constructed.

For the bosonic string case, see [Sakamoto-Sakatani--KY, 1703.09213]

In the case of superstring, the analysis should be very complicated, but the essential part of the proof of Weyl invariance has been resolved.

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A very nice application of DFT!

Summary

I have given an overview of the recent progress on

Yang-Baxter deformations of the $AdS_5 \times S^5$ superstring

- The mCYBE case

An η-deformation of $AdS_5 \times S^5$ is not a solution of type IIB SUGRA, but satisfies the generalized type IIB SUGRA.

There would be no solutions of the usual type IIB SUGRA.

- The CYBE case

1) The unimodular classical r-matrices lead to sols. of type IIB SUGRA.

EX Lunin-Maldacena-Frolov, Maldacena-Russo, Schrödinger spacetimes

2) The other non-unimodular ones lead to sols. of the generalized SUGRA.

Discussions

So far, we have considered the closed string picture (g_{MN}, B_{MN}, g_s).

But it would be much nicer to consider the open string picture $(G_{MN}, \Theta^{MN}, G_s)$.

The relations

$$G_{MN} = (g - Bg^{-1}B)_{MN} \qquad G_s = g_s \left(\frac{\det(g+B)}{\det g} \right)^{1/2}$$

$$\Theta^{MN} = -((g+B)^{-1}B(g-B)^{-1})^{MN}$$

Open string picture of YB deformations of AdS_5 with homogeneous CYBE:

G_{MN} : the undeformed $AdS_5 \times S^5$ $\qquad$ G_s : const.

Only the non-commutative parameter Θ^{MN} depends on the deformation.

→ Classical r-matrices determine non-commutativities

[van Tongeren, 1506.01023, 1610.05677]

[Araujo-Bakhmatov-O Colgain-Sakamoto-Sheikh Jabbari-KY, 1702.02861, 1705.02063]

The relation between SUGRA and noncommutativity

[Araujo-Bakhmatov-O Colgain-Sakamoto-Sheikh Jabbari-KY, 1702.02861, 1705.02063]

The on-shell condition of type IIB SUGRA (= the unimodularity condition)

$$\Rightarrow \quad \nabla_M \Theta^{MN} = 0$$

This condition is necessary for the cyclic property of star product

The relation between SUGRA and noncommutativity

[Araujo-Bakhmatov-O Colgain-Sakamoto-Sheikh Jabbari-KY, 1702.02861, 1705.02063]

The on-shell condition of type IIB SUGRA (= the unimodularity condition)

$$\Rightarrow \quad \nabla_M \Theta^{MN} = 0$$

This condition is necessary for the cyclic property of star product

For the generalized IIB SUGRA,

$$\Rightarrow \quad \nabla_M \Theta^{MN} = I^N$$

The extra vector field *I* has been related to the noncommutativity!

This relation implies an intimate relation to non-geometric flux (Q-flux), DFT (or EFT).

[Sakamoto-Sakatani-KY, 1705.07116, in progress]

Thank you!