Ultra-high energy particle collisions near black holes and singularities and super-Penrose process

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Two kinds of energies as a result of collisions

1) High (unbound) energy in the centre of mass frame E_c.m.

Particle moving towards horizon (BSW effect), Banados-Silk-West PRL 2009

Black holes, naked singularities, quasiblack holes, star-like configurations, wormholes

BHs: rotating or electrically charged

Collisions outside and inside BH

In magnetic field

Scalar field

Proximity to horizon
Ergoregion (high angular momentum),
Extremely rapid rotation

2) Possibility to get high (unbounded) energies E at infinity (debris after collision) – super-Penrose process

Physical explanation and properties of BSW effect

Universal character of BSW effect near BH

Kinematic nature of the BSW effect. Role of critical trajectories

BSW effect and acceleration horizons

Geometric explanation

Kinematic explanation for collisions inside BH

Extremal versus nonextremal BHs

Kinematic censorship

BSW effect versus Penrose process: what can be seen at infinity?

Role of self-force due to gravitational radiation

High energy processes near BHs

Key quantity: energy in centre of mass frame

1 particle $$m^2 = \left| P_\mu P^\mu \right|$$

2 particles colliding in some point

$$E^2{}_{cm} = \left| P_\mu P^\mu \right|$$

Total momentum $$P_\mu = p^{(1)}{}_\mu + p^{(2)}{}_\mu$$

$$P_a = (E_{c.m.}, 0, 0, 0)$$ $$u^\mu u_\mu = -1$$

Individual E finite, energy in CM frame unbounded (BSW)

Two different kinds of energy

Killing energy $E = -p_\mu \xi^\mu$ ξ^μ Killing vector

E conserved, integral of motion since metric is static or stationary

Energy in the CM frame $E_{c.m.}$

not conserved. Moreover, it is defined in one point only,
point of collision

Head-on collision

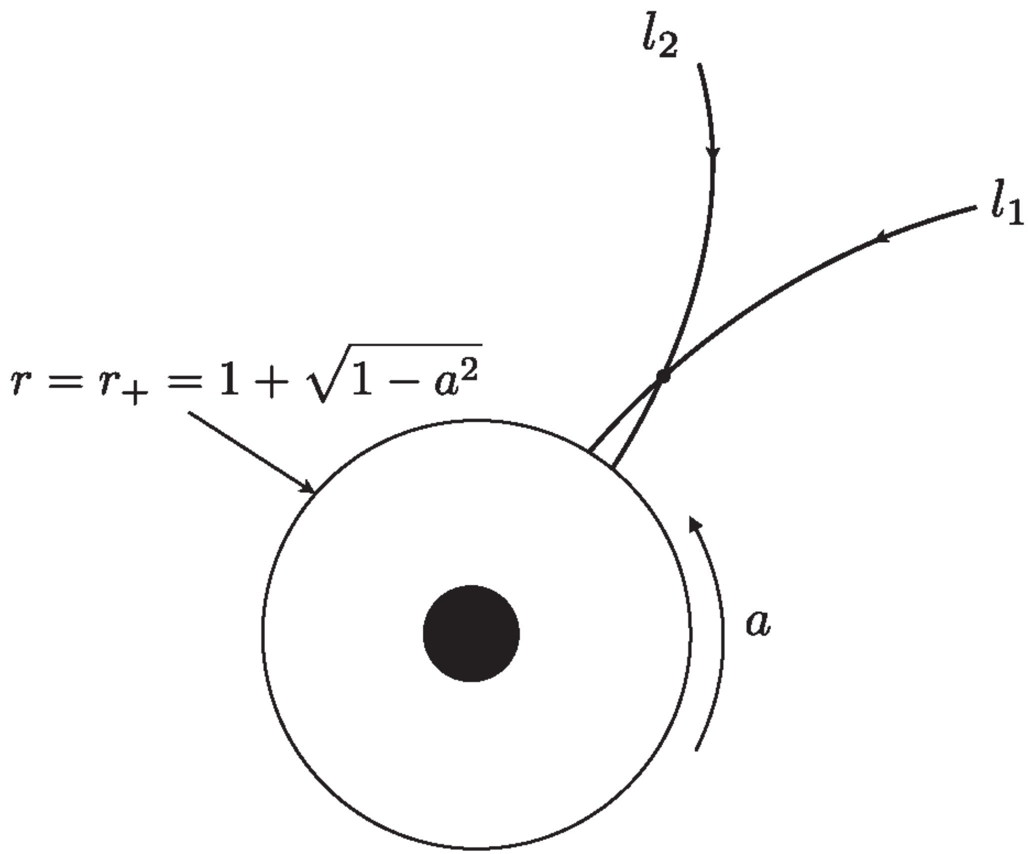
1975 - 1977 T. Piran, J. Katz and J. Shanam

Unbounded, if two particles move in opposite directions near BH (unphysical)
Almost infinite relative blue shift

E in CM frame almost diverges

Artifical scenario. Particle near black (not white) hole moving away from horizon and colliding with another particle

M. Banados, J. Silk, and S. M. West PRL 2009



$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

Both particles experience blue shift, centre of mass frame is in free fall.

Acceleration of particles as universal property of rotating black holes

O. Z., PRD 2010

Role of horizon

Universality of black hole physics

Unified approach to nonextremal versus extremal black holes

Energy in CM frame

$$\frac{E_{c.m.}^2}{2m^2} = \frac{X_1 X_2 - Z_1 Z_2}{N^2} + 1 - Y,$$

Three kinds of mechanism leading to unbound energy in CM frame

1) $N \rightarrow 0$ proximity to horizons BSW

2) $L_2 \rightarrow -\infty$ inside ergoregion, NOT near horizon Grib and Pavlov, Kerr metric

Generalization OZ

3) $\omega \rightarrow \infty$ rapid totation (wormholes)

BSW

$$\frac{E_{c.m.}^2}{2m^2} = \frac{X_1 X_2 - Z_1 Z_2}{N^2} + 1 - Y,$$

In general case, $E_{c.m.}^2$ remains bound in horizon limit $N \to 0$

Special conditions for unbound $E_{c.m.}^2$

Two kinds of particles (trajectories)

Usual $X_H \equiv E - \omega_H L \neq 0$

Critical $X_H \equiv E - \omega_H L = 0$

BSW effect near black holes

particle 1 is critical, particle 2 is usual

$$E^2_{c.m.} \sim N^{-1}$$

diverges

Extremal versus nonextremal

Problems with attaining extremality, a=0,998 (Thorne)

Jacobson et al, Berti at al: difficulties in realization

Grib and Pavlov: nonextremal Kerr

Extremal case: collision near horizon

$$E_{c.m.} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(L_H - L_2)}{1 - \sqrt{1 - a^2}}} \qquad L_1 = L_{(H)} - \delta$$

$$L_{(H)} = \frac{E}{\omega_H}$$

After collision. Collisional Penrose process

"Standard" Penrose process

Decay of particle $0 \to 1 + 2$

$$E_0 = E_1 + E_2 \qquad E_2 < 0 \qquad E_1 > E_0$$

Efficiency $\eta = \dfrac{E_1 - E_0}{E_0}$

ergoregion

Collisional Penrose process

$$1 + 2 \to 3 + 4$$

BSW process

Unbounded energy in the centre of mass (CM) frame

$E_{c.m.}$ versus E Killiing energy measured at infinity

Even in spite of unbounded $E_{c.m.}$

E is typically quite modest because of strong redshift

Equatorial plane

Kerr Excess less than 50 % Mejer et al, 2012
Harada et al 2012

Dirty black holes OZ 2012

Dirty = surrounded by matter, NOT Kerr BH

Standard scenario.
Particles 1 and 2 fall from infinity, collide
Particle 4 falls into a BH, particle 3 moves to infinity
Particle 3 either moves immediately after colliison towards BH and bounces back or moves to inifnity at once

Particle 1 is fine-tuned (critical)

Particle 2 is not fine-tuned (usual)

From analysis of conservation laws:
Particle 3 is critical or near-critical, particle 4 is usual

Special scenario J. Schnittman (2014), Kerr metric

Partcile 1 moves from BH, head-on collision with particle 2

Amplification, factor about 14 Kerr, numerics

Leiderschneider and Piran 2016, Ogasawara et al 2016 Kerr, analytically

O.Z. 2016 general approach, analytically

More radical result

If particle 1 (moves from BH) is usual

Unbounded efficiency (called super-Penrose process)

E. Berti, R. Brito and V. Cardoso, 2015 — Kerr, numerics

O. Z. 2015 — Dirty BH, analytically

Unfortunately, not realizable near BH

Near horizon, particle should move towards BH

White holes (Grib and Pavlov 2014)

Black holes. Attempt to arrange radical scenario with formally unbounded efficiency

We can try to prepare required state (usual particle moving from BH)

Is it possible to obtain it as a result of previous collision?

Full scenario

Step 1. Particle 1 and 2 fall from infinity and collide near BH

Step 2. They produce usual particle 3

Step 3. Particle 3 collides with particle 4 falling from inifnity (head-on collision)

Result: particle 5 with unbound energy moving to infinity

It turns out that one of particle falling from infinity has to have mass (N is lapse funciton)

$m_2 = O(N^{-2})$ Kerr metric, E. Leiderschneider and T. Piran 2015

General approach (O.Z., 2015)

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2$$

Equatorial plane, redefine radial coordinate

Effective metric

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{N^2}$$

Geodesic particles

Forward-in-time condition $X \geq 0$ $\sigma = \pm 1$

$X_H > 0$ usual "H" horizon, "c" collision

critical

but small near-critical $X_H = O(N_c)$

Conservation laws

$$E_{in} = E_{fin} \qquad L_{in} = L_{fin}$$

Consequence: $X_{in} = X_{fin}$

Let p particles collide and produce q new particles.

radial momentum

Conservation laws + forward-in-time conditions $\frac{dt}{d\tau} > 0$

Near-horizon limit, $N_c \to 0$

Statement. If in the initial configuration usual outgoing particles are absent, they cannot appear after collision.

Previous statement applis to case with finite masses, etc.

If we relax this condition, it is possible to obtain a usual outgoing particle, provided

$$m_2 = O(N^{-2})$$

Generalizes observation of E. Leiderschneider and T. Piran

Attempt to find loophole

Fractional degrees allow $X = O(N^s)$ $0 < s < 1$

Inconsistent with conservaiton laws

Instead of BH, we can consider head-on collision near would-be horizon, where N<<1

It turns out that there is no restriction on efficiency
(OZ, PRD 2013)

Detailed treatment: Kerr metric (Patil et al, 2016)
generic dirty black hole (Tanatarov and OZ, 2016)

Wald inequalities for collisional Penrose process

Now consider collisional Penrose process: when two particles with Killing energies E_1 and E_2 collide in ergosphere to produce two fragments, with Killing energies $E_3 < 0$ and $E_4 > E_1 + E_2$.

$$\frac{E}{\mu} - \sqrt{\frac{E^2}{\mu^2} + g_{tt}} \leq \frac{\nu_\infty}{\nu} \leq \frac{E}{\mu} + \sqrt{\frac{E^2}{\mu^2} + g_{tt}}$$

Substitute decaying particle for the effective compound particle with $\mu = M = 2\hbar\nu$

Then $$E - \sqrt{E^2 + \mu^2 g_{tt}} \leq 2\hbar\nu_\infty \leq E + \sqrt{E^2 + \mu^2 g_{tt}}$$

Thus $\hbar\nu_\infty$ can be large (diverge) only if M is large (diverging)

$M \rightarrow \infty$ $$\hbar\nu_\infty \approx \frac{M}{2}\sqrt{g_{tt}}$$

Particle in rotating stationary spacetime

Metric in equatorial plane

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{N^2}$$

$$g_t = g_{tt} = -N^2 + g_\phi \omega^2$$

Particle's velocity

$$u_\mu = (-E, L, \frac{Z}{N^2}) \qquad u^\mu = (\frac{X}{N^2}, \frac{\omega X}{N^2} + \frac{L}{g_\phi}, Z)$$

$$X = E - \omega L \qquad Z^2 = X^2 - N^2 (\frac{L^2}{g_\phi} + \varepsilon)$$

For massive particles $\varepsilon = 1$ (m=1) For massless $\varepsilon = 0$

Collision of two particles at small N

Consider two massive particles colliding in a region where lapse function N is small.

This can be near a BH horizon or in a place where horizon is almost formed, or near a wormhole's throat.

N is a small parameter, and we are interested in the formal limit $N \to 0$

thus use terms “unbounded", “diverge" etc.

The relative Lorentz factor is

$$\gamma = -u_{1\mu}u^{2\mu} = \frac{X_1 X_2 - \sigma Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi}$$

$$Z_i = \pm u_i^{\,r} = \sqrt{X^2 - N^2\left(\frac{L^2}{g_\phi} + 1\right)}$$

For m_1=m_2=1, $M^2 = -(u_1 + u_2)_\mu (u_1 + u_2)^\mu = 2(1+\gamma)$

BSW effect, particle 1 is critical

$$M \sim \frac{1}{\sqrt{N}}$$

However, $E_\infty < \infty$ Bejger at al, Harada et al, O.Z.

No horizon. One particle falls from the outside, and another moving from the inside after reflection from the potential barrier

$$M \sim \frac{1}{N}$$

Wald inequalities are directly applicable to collisional Penrose
process and are highly informative

Quantities registered at innity (E_3, L_3) are expressed in a simple
fashion through dynamic parameters of the collision (angle)

Registered energy at infinity can only (formally) be unbounded if M
(formally) is unbounded

For BSW the particles which escape to infinity are created in narrow
cones with small in the CM frame, so as to have finite energies;

Everything is obtained in very weak assumptions, for general
stationary rotating spacetimes with a region where lapse function N
is small; this can happen near BHs, naked singularities or wormholes.

It was assumed that parameters of metrics and angular momenta L are finite. Meanwhile, there are special scenarios.

ω Large. Rapid rotation (say, wormholes)

$$E^2_{c.m.} \approx \frac{4\omega^2 |L_1 L_2|}{N^2}$$

Head-on, nonzero L

Collisional Penrose process

Collision of 2 identical particles:

$$\frac{E_{\max}}{E_{c.m.}} \sim \omega \gg 1$$

Large L

Large $E_{c.m.}$

For Kerr -Grib and Pavlov, general – O.Z.

$$N = O(1)$$

Collisional SP OZ 2016

$$E_{\max} \sim |L|$$

Main conclusions

1) SP process near black holes is impossible

2) SP process is possible near systems without horizon: naked singularities, Wormholes, star-like configurations.

3) This happens if head-on collision occurs near would-be horizon ("almost formed"), N is small.

4) Special cases: rapid rotation, large angular momenta. Here, N=O(1).

Thank you!