Quantum radiation from a sandwich black hole

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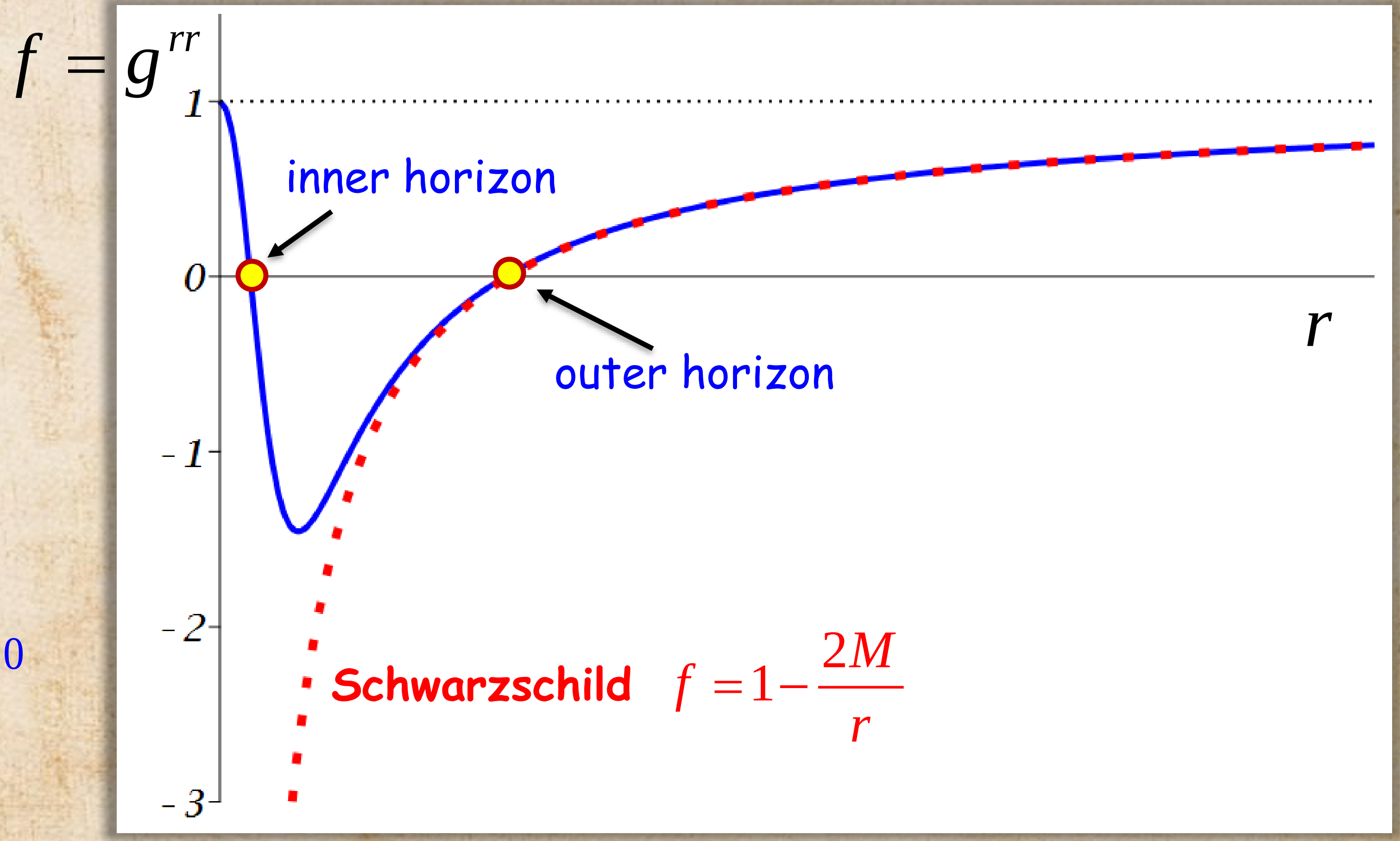
- Suppose there exists such a fundamental theory of gravity, in which black holes do not have singularities.
- What might be properties of black holes in such a theory? It is natural to expect, that the modified theory of gravity should include some fundamental length scale ℓ.

A natural requirement to nonsingular black hole metrics is that at large radius they correctly reproduce the Schwarzschild black hole solution of the general relativity. The curvature inside the black hole, being regular, nevertheless depends on the value of its mass, angular momentum, etc.. In a general case it may infinitely grow for a special limit of these parameters. The requirement, that it does not happen and the curvature always remains finite and is limited by some fundamental value (ℓ^{-2}), can be imposed as an additional principle, which restricts the variety of nonsingular black-hole models. The limiting curvature principle was first formulated by Markov (1982).

Inner and outer horizons

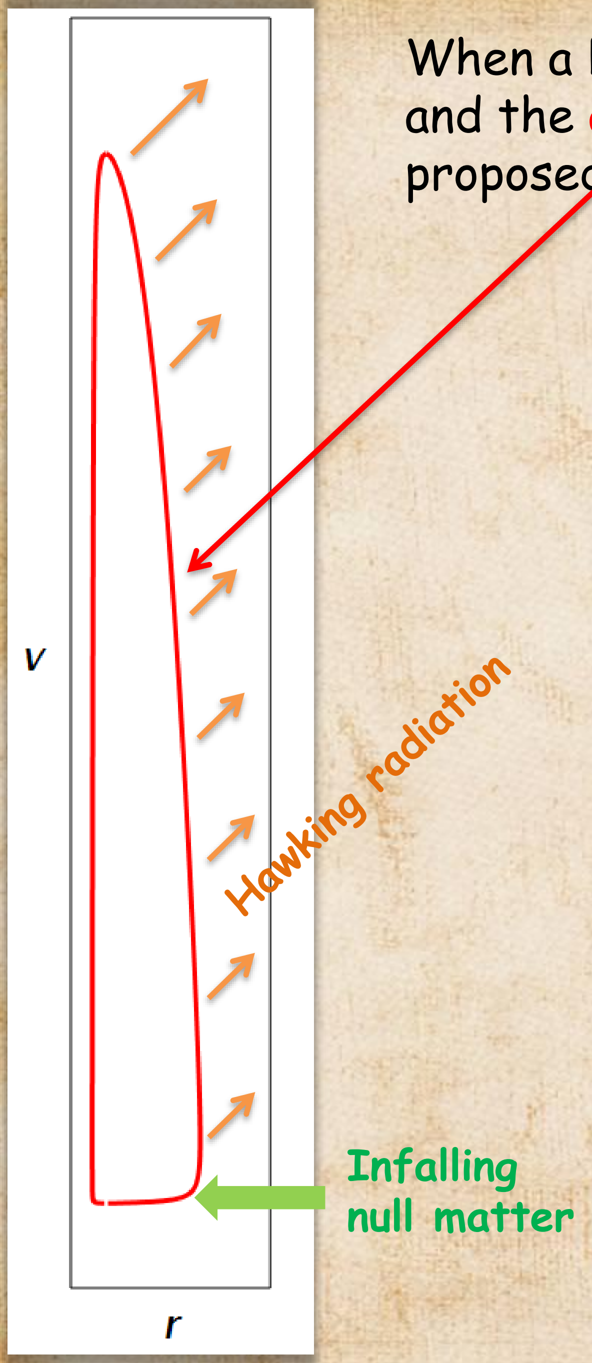
For a spherically symmetric black hole this principle implies that the apparent horizon cannot cross the center r = 0. In other words, besides the outer part of the apparent horizon there should also be an inner part.



Condition for an apparent horizon

$$\nabla^\alpha r \nabla_\alpha r = g^{rr} = 0$$

When a black hole evaporates, the event horizon does not exist and the apparent horizon is **closed**. Such a model was first proposed by V. Frolov and G. Vilkovisky (Phys. Lett. B 106, 307, 1981.)



$$ds^2 = -f(v,r)\,dv^2 + 2\,dv\,dr + r^2 d\omega^2 \,.$$

Condition for an apparent horizon $g^{rr} = f(v,r) = 0$

We study quantum radiation of massless particles from nonsingular black holes. To attack this problem we assume a number of simplifications.

To describe a spherically symmetric black hole which has finite life-time we assume that a black hole is formed a result of the collapse of the null shell of positive mass M and ends its existence as a result of collapse of another null shell with negative mass $-M$. We call it a **sandwich black hole**.

Certainly such a model is quite different from a "real" evaporating black hole. However some of its predictions are robust and remain valid for more realistic "smooth models".

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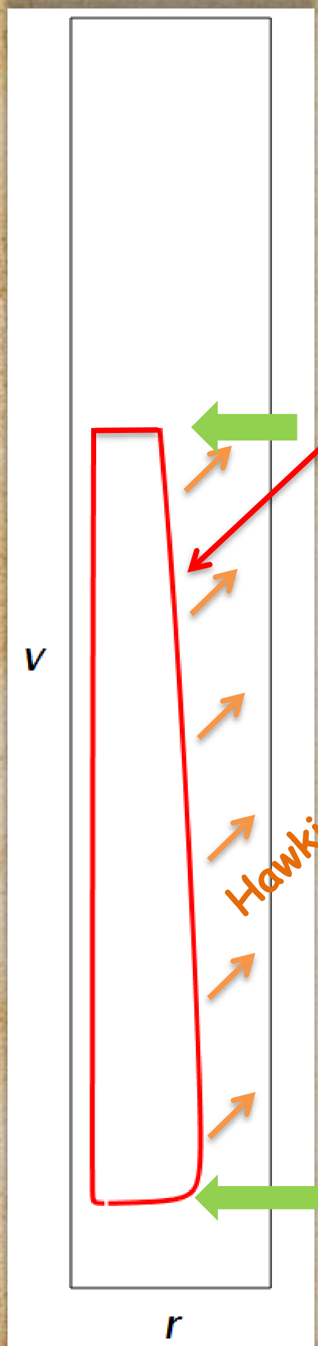

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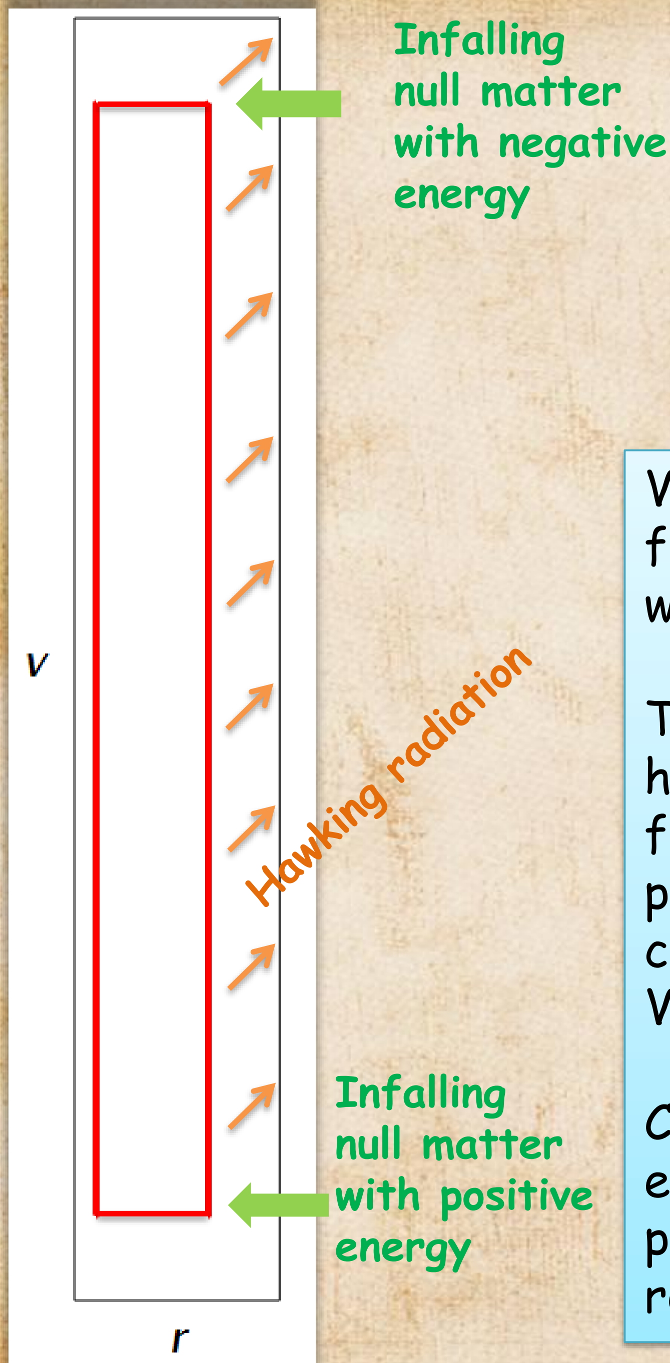
Sandwich black hole



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The most general spherically symmetric metric in the four-dimensional spacetime can be written in the form

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha\, dvdr + r^2 d\omega^2$$

The function f at spatial infinity must take the value 1 in order to escape a solid angle deficit

$$f(v,r)|_{r\to\infty} = 1$$

Using an ambiguity in the choice of v, we impose the (gauge fixing) condition

$$\alpha(v,r)|_{r\to\infty} = 1$$

Non-singular black holes imply the regularity of the metric at the center $r = 0$. It leads to the conditions

$$f(v,r) = 1 + f_2(v)\, r^2 + \cdots ,$$

$$\alpha(v,r) = \alpha_0(v) + \alpha_2(v)\, r^2 + \cdots .$$

In a general case, when $\alpha(v,r) \neq 1$, the rate of the proper time at the center differs from the rate of time

$$d\tau = \alpha_0(v) dv$$

Static black hole case

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha\, dvdr + r^2 d\omega^2$$

The Killing vector $\xi = \xi^\alpha \partial_\alpha = \partial_v$

$$\xi^2 = -\alpha^2 f$$

Surface gravity $\kappa = \frac{1}{2}(\alpha \partial_r f)|_H$

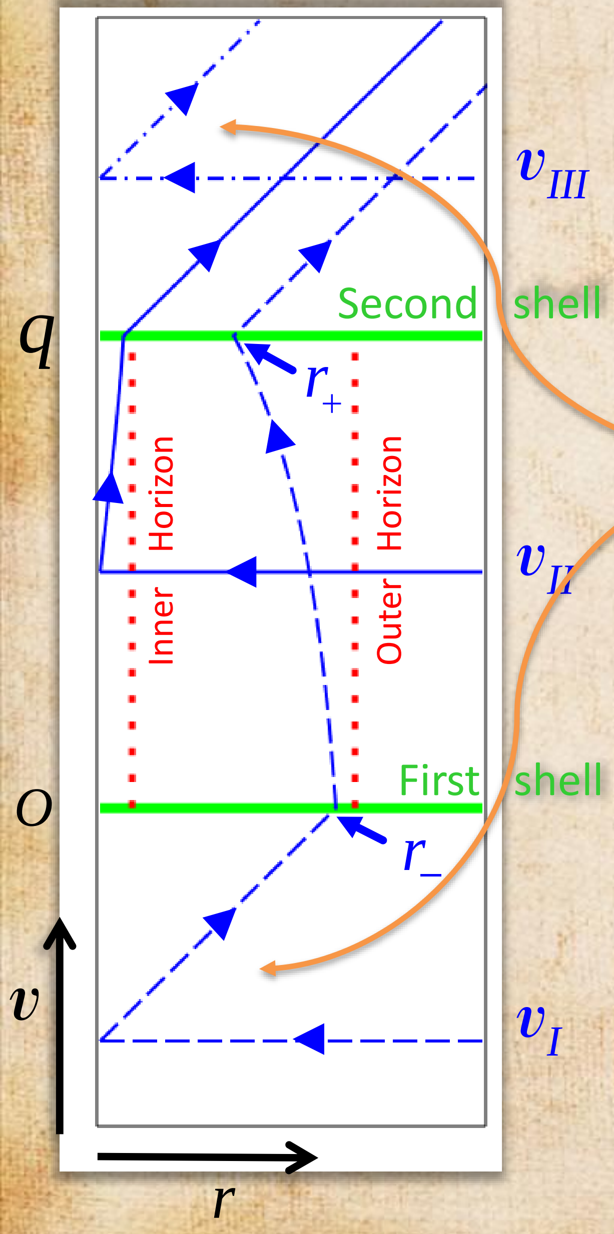
Nonsingular static black holes. The Hayward metric

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$$

Here ℓ is some fundamental scale larger than the Planck length.

$$\alpha = 1$$

Evaporating regular black holes



We assume now that a regular metric

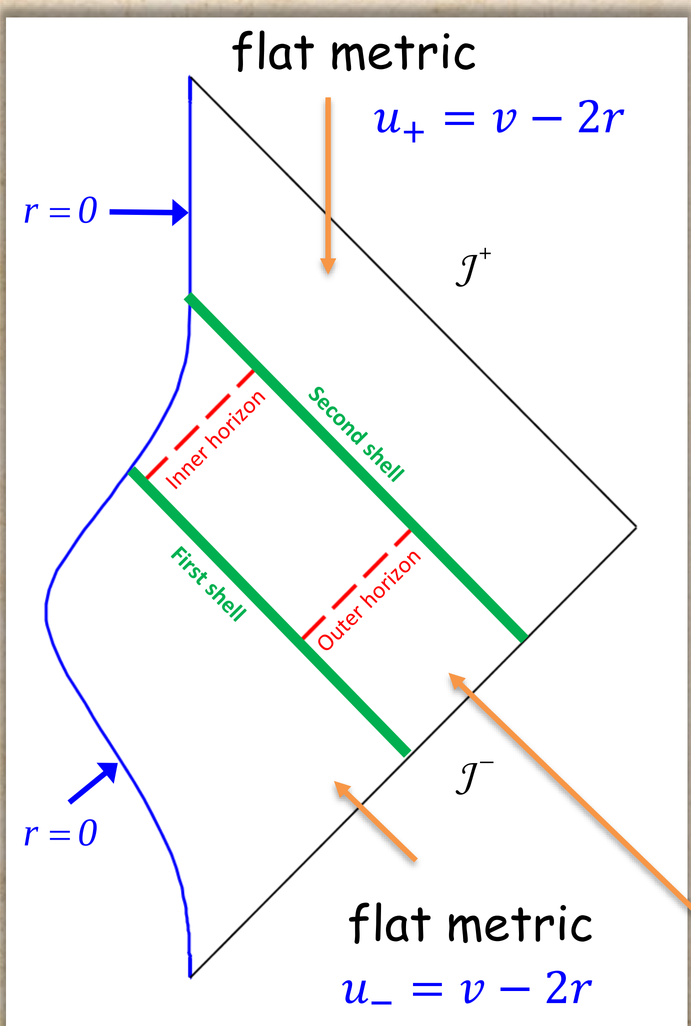
$$ds^2 = -\alpha^2 f\, dv^2 + 2\alpha\; dv dr + r^2\; d\omega^2$$

describes the black hole, which is created as a result of a spherical collapse at the moment $v = 0$, and which disappears after some finite time q.

$$f = \alpha = 1 \qquad \text{for } v < 0 \quad \text{and} \quad v > q$$

Consider an incoming radial null ray described by the equation $v = const$. It propagates from the past null infinity I^- and reaches the center $r = 0$. After passing the center, it becomes an outgoing radial null ray.

Penrose diagram of a sandwich black hole



nonsingular black hole

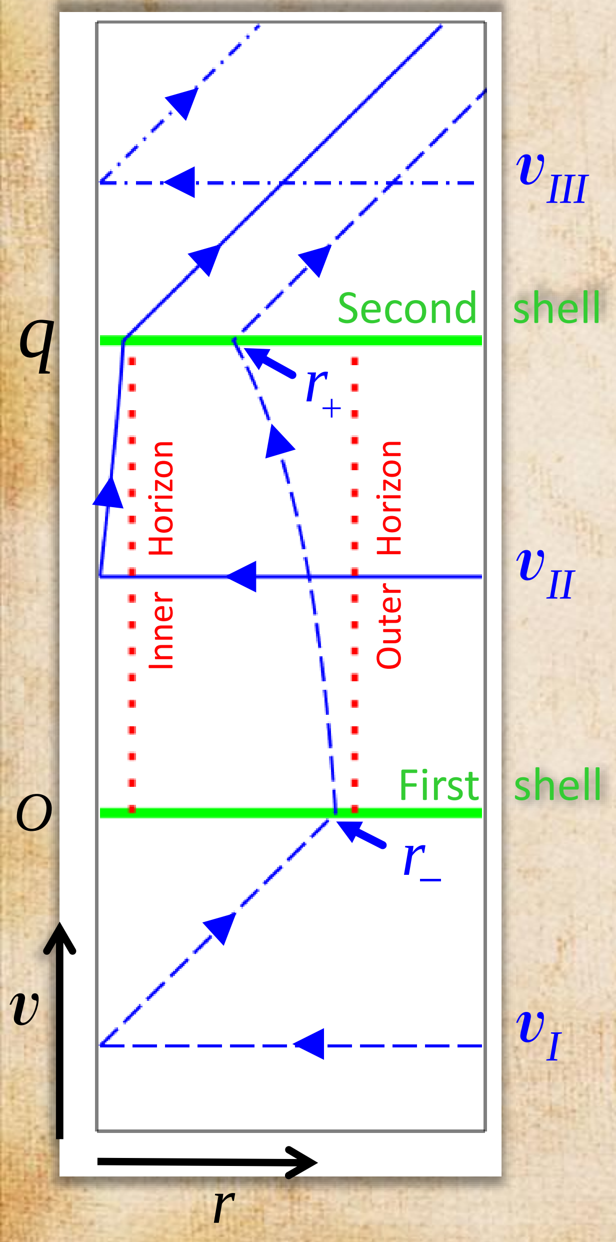
We choose the retarded null time parameter u_- so that at $r=0$ one has $u_- = v$. In the initial flat domain, where $v < 0$

$$u_- = v - 2r$$

However, in a general case, for $v > 0$ this relation between u_- and v is not valid. In particular, in the final flat domain, where $v > q$, the null coordinate $u_+ = v - 2r$ differs from u_-, and one has relations

$$u_+ = u_+(u_-), \qquad u_- = u_-(u_+)$$

Null rays



I) Consider first outgoing rays with $u_- < 0$.

They intersect the first null shell at $r_- = -\frac{1}{2}u_-$

Denote by $r(v)$ a solution on the differential equation

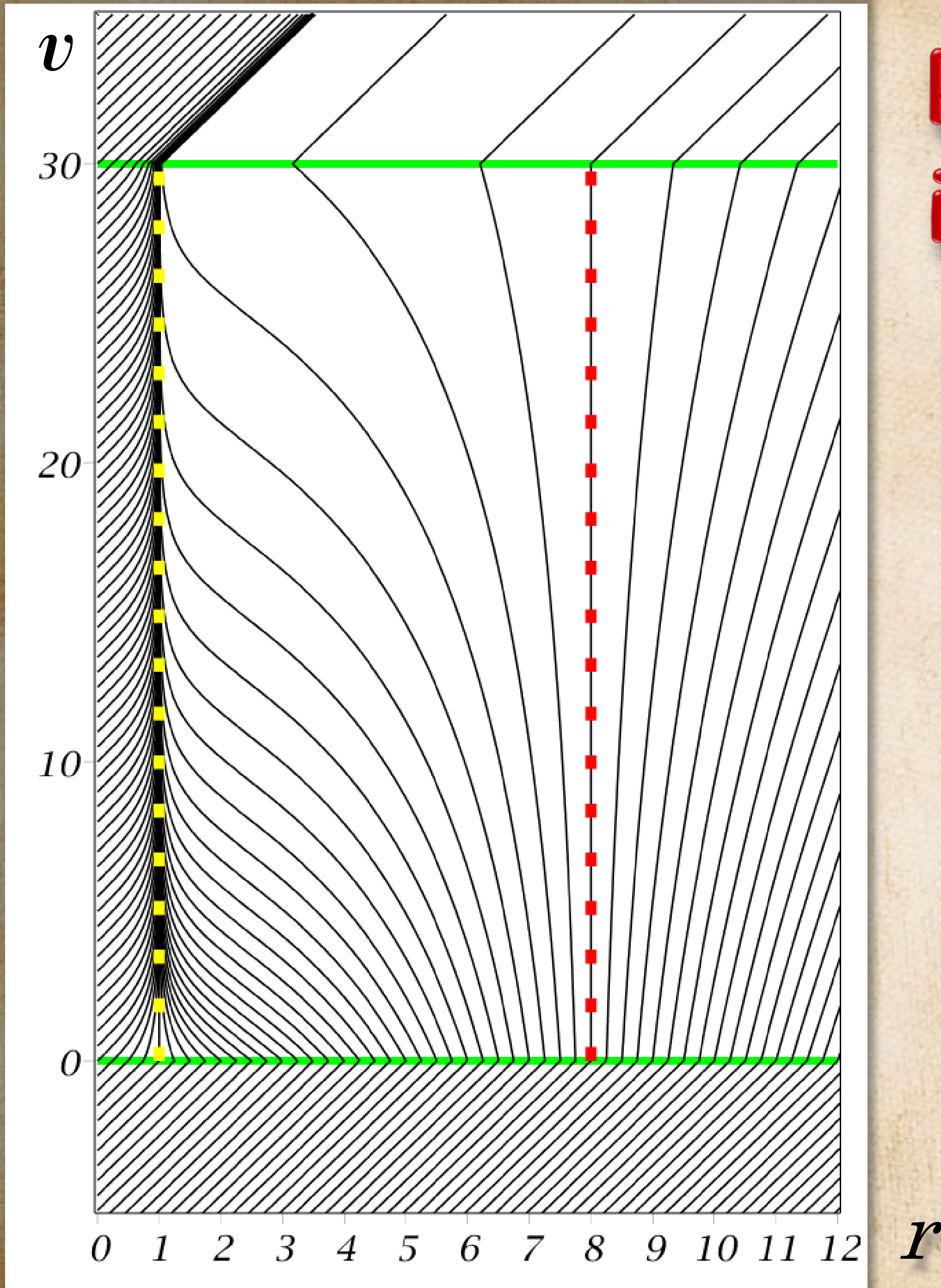
$$\frac{dr}{dv} = \frac{1}{2}\,\alpha(v,r)\,f(v,r)$$

with the initial condition

$$r(0) = r_- = -\frac{1}{2}u_-$$

This solution describes an outgoing null ray of type I.

II)-III) Similarly we define rays of type II and III.



Radial null rays in a nonsingular sandwich black hole

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha\, dv dr$$

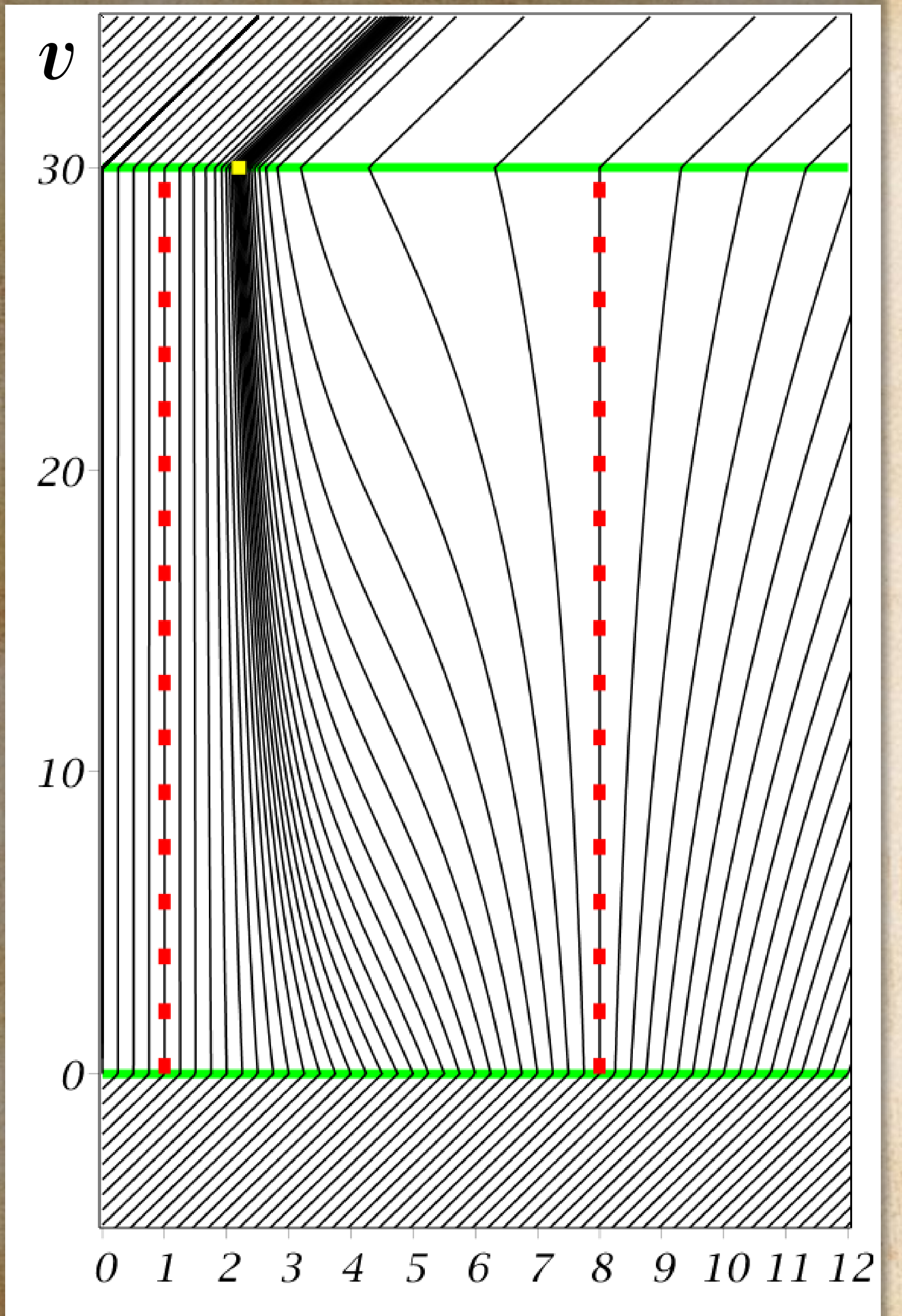
$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$$

$$f = \frac{(r-r_1)(r-r_2)(r-r_0)}{r^3 - r_1 r_2 r_0}, \qquad p = r_1/r_2$$

Standard model $\alpha = 1$

$p = 8, \quad q = 30, \quad \ell = 1$

Radial null rays in a nonsingular sandwich black hole



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Modified model $\alpha = \dfrac{r^n + 1}{r^n + 1 + p^k}$

$p = 8,\quad q = 30,\quad \ell = 1$

$n = 6,\quad k = 4$

Surface gravity

The dimensionless surface gravities $\kappa = \frac{1}{2}(\alpha \partial_r f)|_H$

calculated for α =1 at each horizon $r_1 = p\,, \quad r_2 = 1$ are

$$\kappa_1 = +\frac{(p-1)(p+2)}{2p(p^2+p+1)},$$

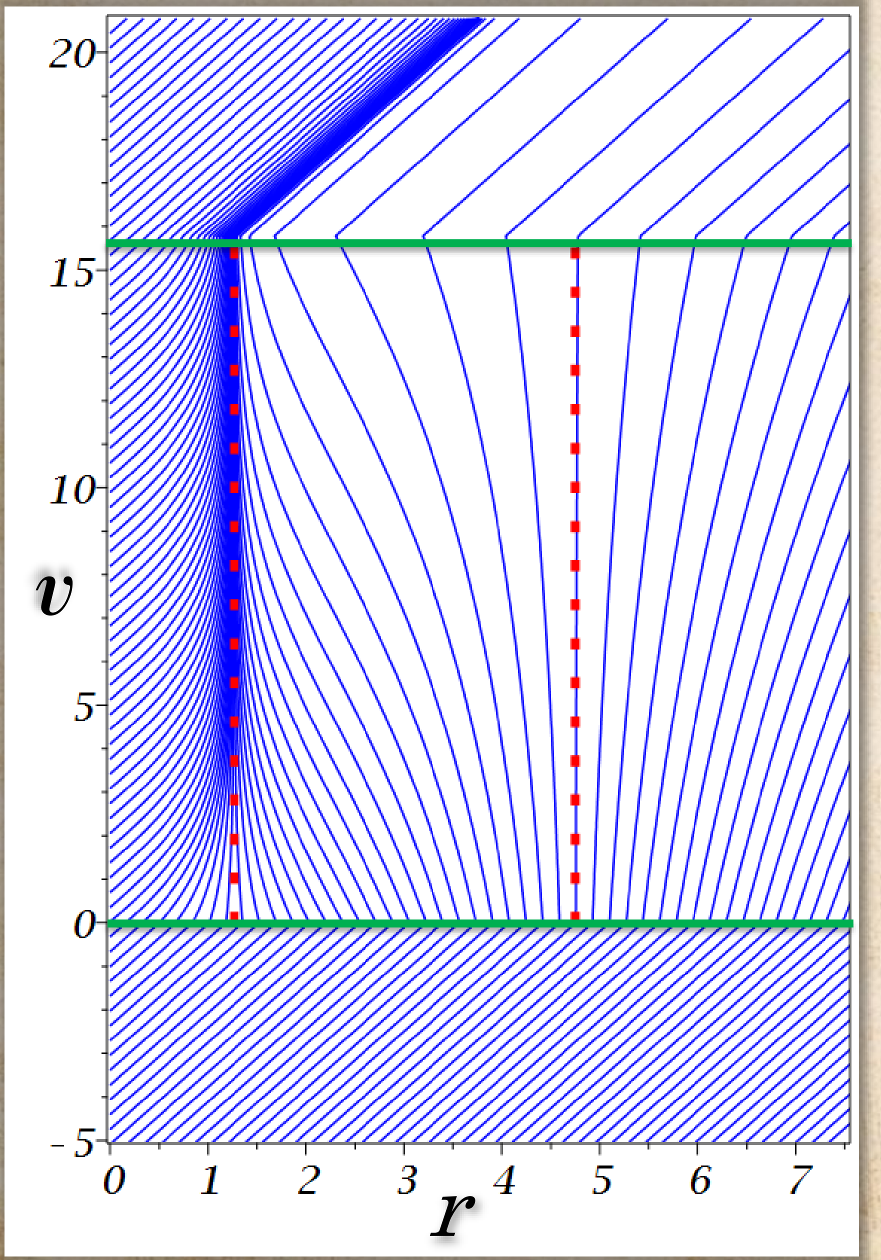
$$\kappa_2 = -\frac{(p-1)(2p+1)}{2(p^2+p+1)}$$

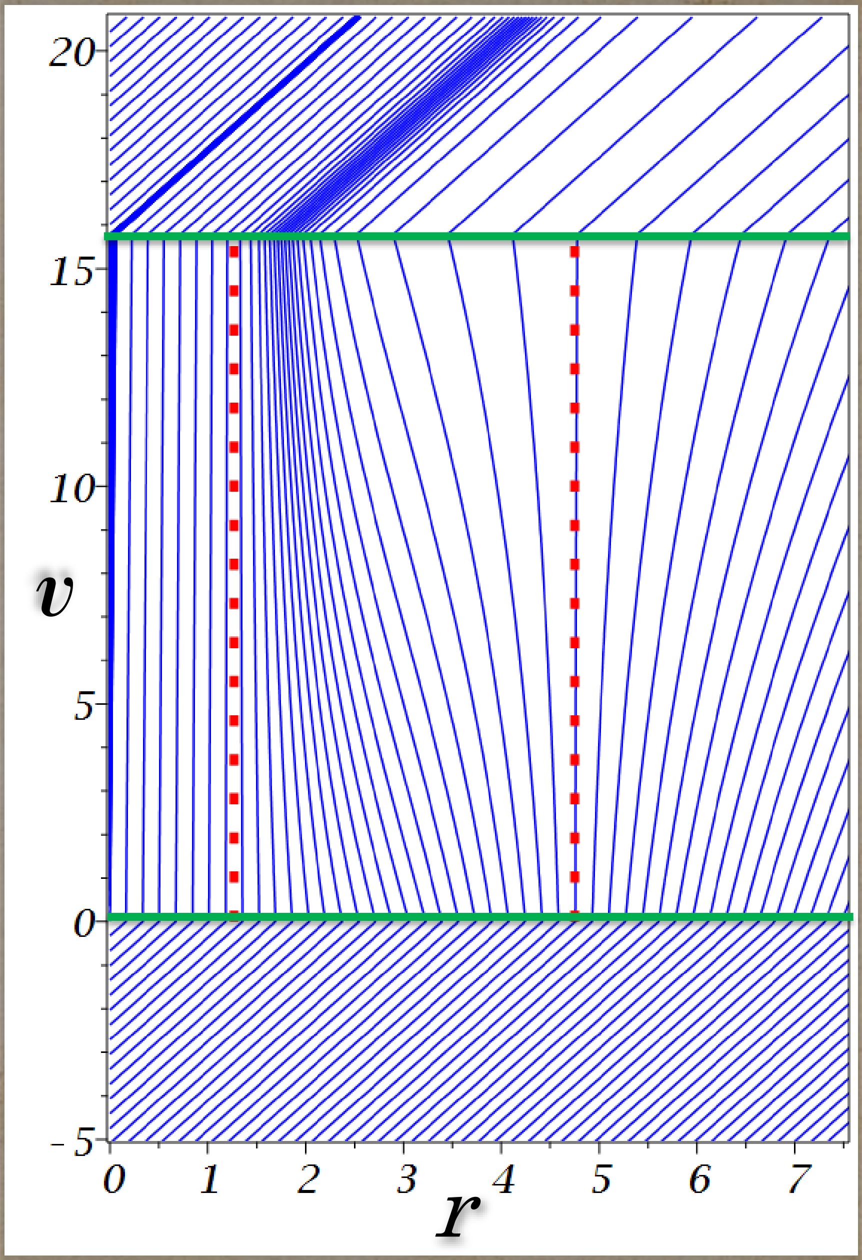
For the modified models $\kappa_{1,2}\,|_{\text{modified}} = \alpha(r_{1,2})\,\kappa_{1,2}\,|_{\text{standard}}\,.$

$$\alpha(r_1) \sim 1, \qquad \alpha(r_2) << 1.$$

- Outgoing radial null rays in the black-hole interior are accumulated in the vicinity of the inner horizon. This is a consequence of negative value of the surface gravity at the inner horizon. As a result one can expect that particles emitted from the inner horizon at the final stage of the black hole evaporation would have large blueshift.

Standard vs Modified





Quantum fluxes at $\mathcal{I}^+$

The action for a two-dimensional conformal scalar field

$$S = -\frac{1}{2}\int d^2 x\sqrt{-g}\,(\nabla\hat{\varphi})^2$$

The two-dimensional metric

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha\, dvdr$$

Let $\mathcal{E}$ be the dimensionless rate of the energy emission $T^{vv} = (\ell_{Pl}/\ell)^2\,\mathcal{E}$

$$\mathcal{E} = \frac{1}{48\pi}\left[-2\frac{d^2P}{du^2} + \left(\frac{dP}{du}\right)^2\right]$$

Where

$$P = ln\left|\frac{du_-}{du_+}\right| \qquad u_- = u_-(u_+)$$

Another representation in terms of the **Schwarzian derivative**

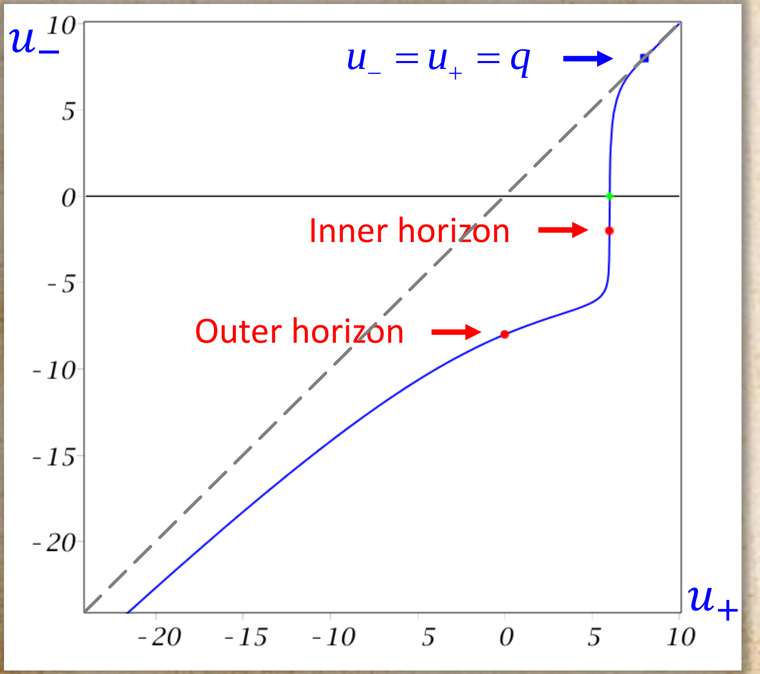
$$\mathcal{E} = -\frac{1}{24\pi}\{u_-, u_+\}, \qquad \{y, x\} = \frac{y'''}{y'} - \frac{3}{2}\left(\frac{y''}{y'}\right)^2$$

Gain function

To compute the energy fluxes $\mathcal{E}$ and the gain function $\beta = \frac{du_-}{du_+} = e^P$,
one needs to know the map $u_-(u_+)$.
The gain function β describes amplification of the particles energy, i.e.,
The ration of the final energy of a photon to its initial energy

$$\beta = \frac{E_+}{E_-} = \frac{1}{\alpha_0} \exp\left[-\int_{v_0}^{\infty} \kappa dv \right]. \qquad \kappa = \frac{1}{2} \partial_r (\alpha f).$$

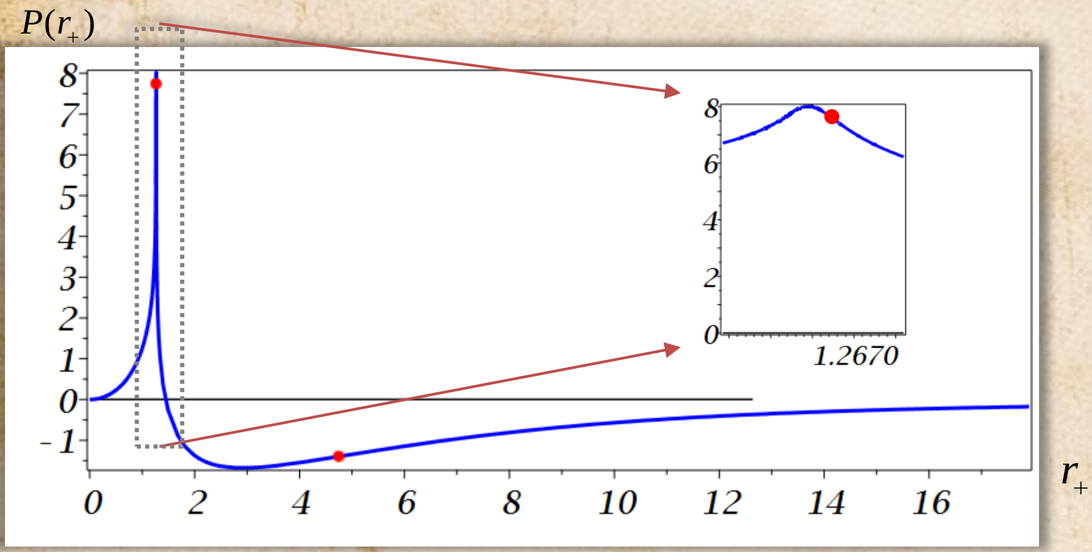
The map $u_-(u_+)$.



The total energy emitted during existence of the black hole is always positive

$$E_{tot} = \frac{1}{48\pi} \left[\int_{-\infty}^{\infty} du_+ \left(\frac{dP}{du_+} \right)^2 - 2 \frac{dP}{du_+} \Big|_{-\infty}^{\infty} \right].$$

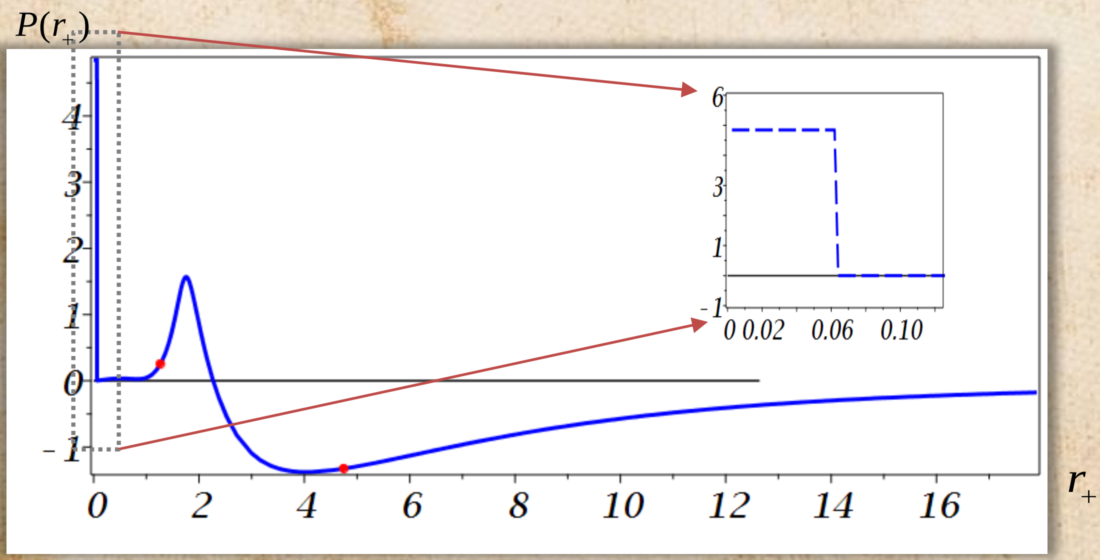
Gain function $\beta = e^P$



Standard model

$p = 3.75, \quad q = 16$

$r_1 = 4.75, \quad m \approx 2.5$



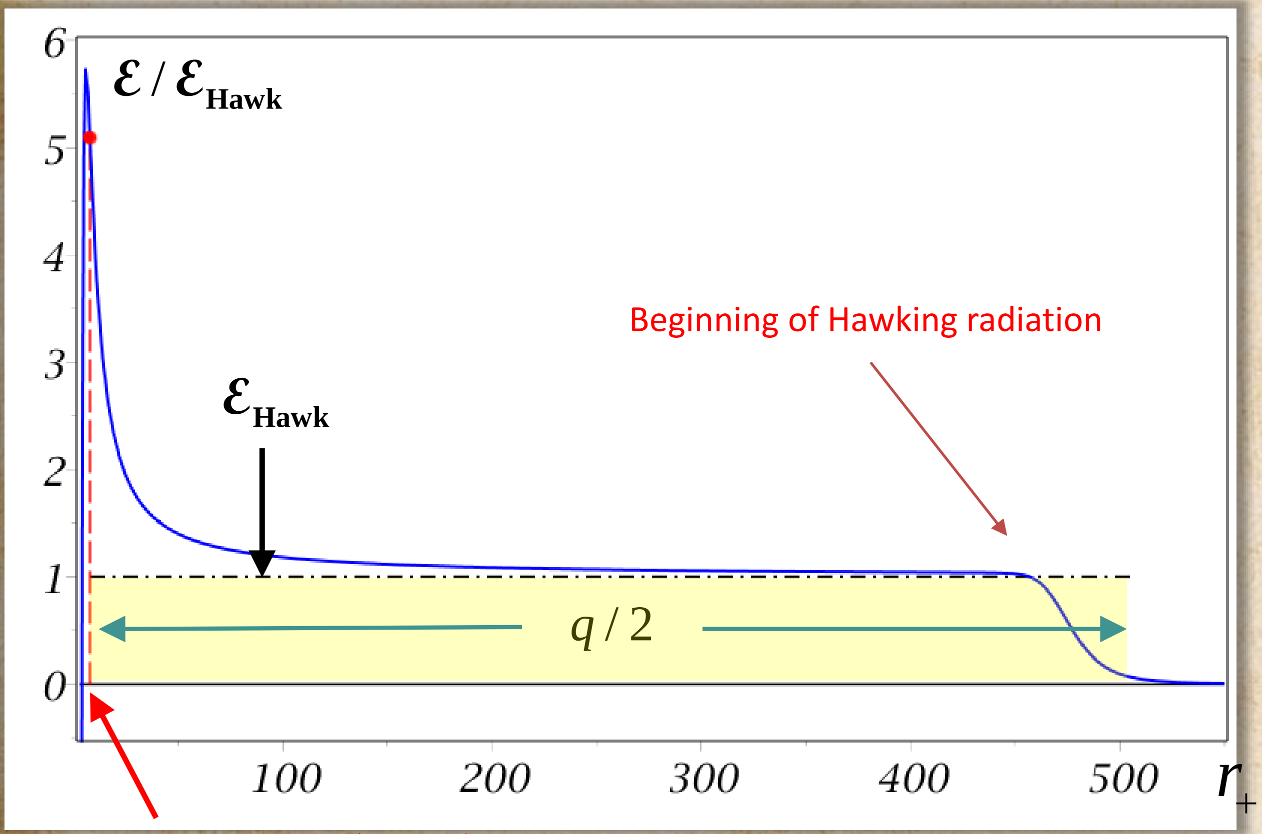
Modified model

$p = 3.75, \quad q = 16, \quad k = 3$

$r_1 = 4.75, \quad m \approx 2.5$

Hawking radiation

The Hawking result for the quantum energy flux from a black hole is correctly reproduced, when the mass parameter p and the lifetime of the blackhole q are large. The shape of the curve is almost the same for both standard and modified models. Duration of the almost constant tail of quantum radiation is approximately equal to q (lifetime of the black hole).

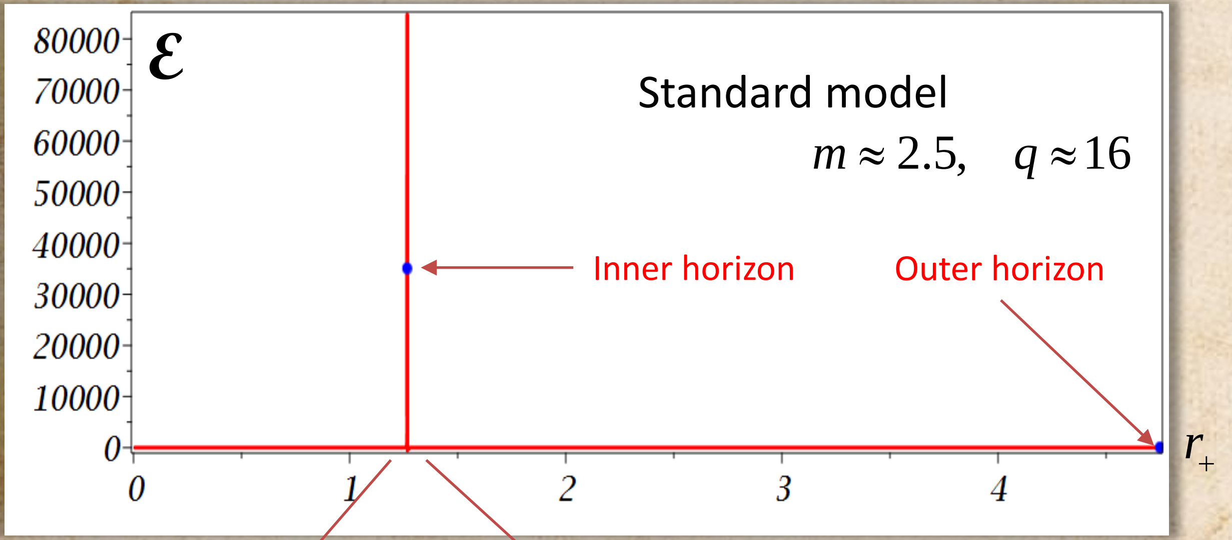


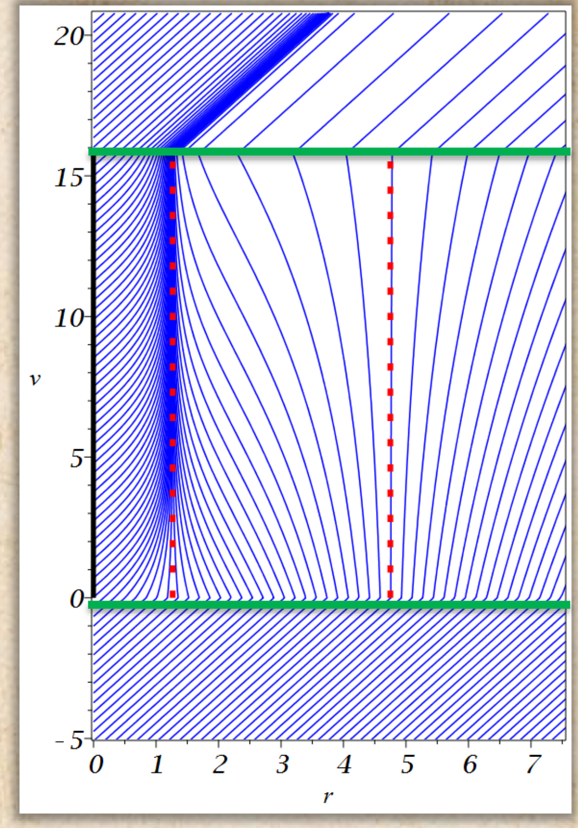
$$\mathcal{E}_{Hawk} = \frac{\kappa_1{}^2}{48\pi} = \frac{1}{192\,\pi\,p^2}$$

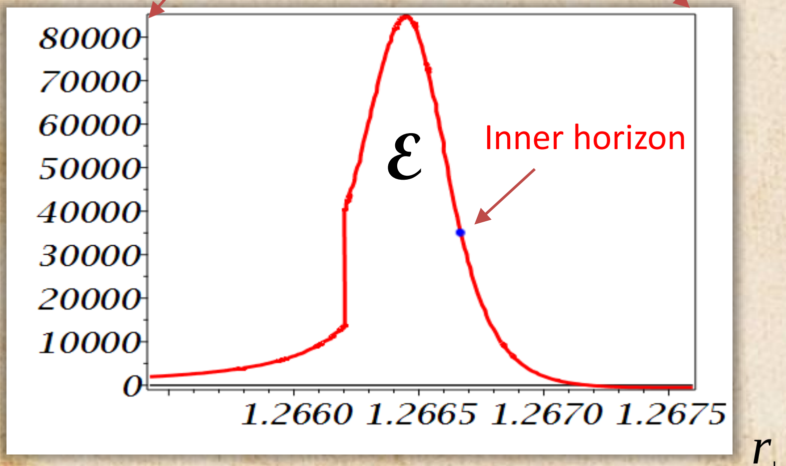
Outer horizon $(r_1 = 8)$

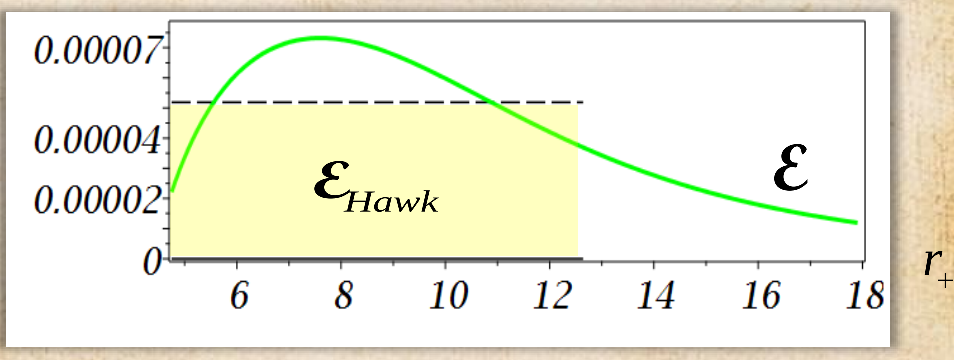
$q = 1000$

Quantum radiation: inside out









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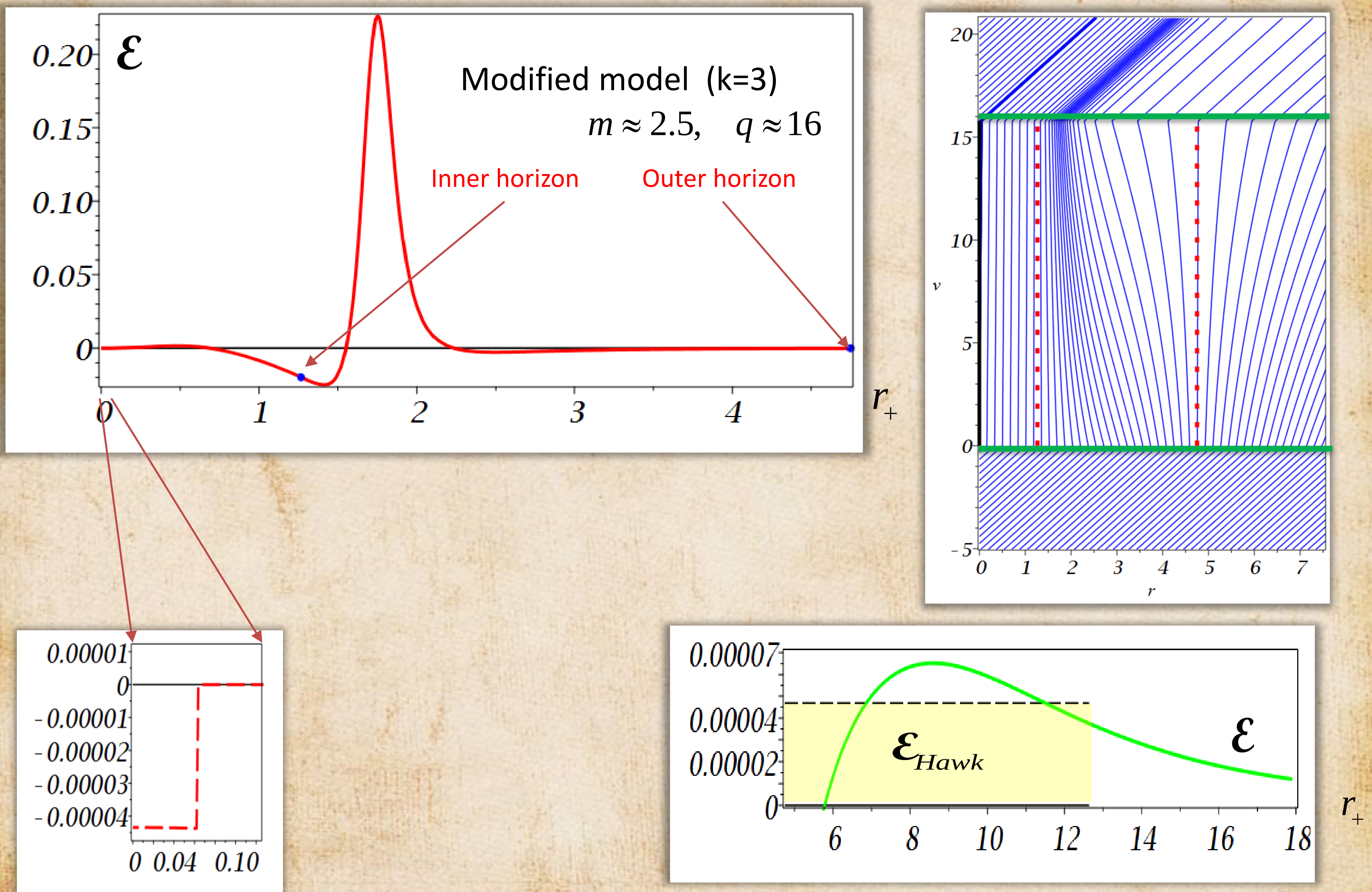

Summary

- The numerical calculations for large black holes are in agreement with the Hawking result for sandwich black holes, provided the duration parameter q is large enough. At the moment of time when the outer horizon crosses the second shell there exists a flash of the radiation. The radiation emitted between the outer- and inner-horizons remains relatively small. The very intensive outburst of the energy occurs near the inner horizon. We choose small value of q just to be able to show on the plot the radiation from all the domains. For higher value of the duration parameter q the amplitude of the burst radiation grows as $\sim exp(-2\kappa_2 q)$, while the width of the peak decreases as $\sim exp(\kappa_2 q)$ (keep in mind that as $\kappa_2 < 0$).

Conclusions

- For a standard sandwich black-hole model the metric between two null shell was chosen to coincide with the Hayward. A characteristic property of this geometry is that a falling photon, when it reaches the center has the same energy, as at the infinity. In other words, there is no red- or blue- shift for such photons. One of the consequences of this assumption is that the surface gravity at the inner horizon is high. As we demonstrated the quantum radiation from the inner horizon of such a black hole is high. For large duration parameter q the energy emitted from it is proportional to $exp(2q)$ and easily exceeds the mass of the black hole M. This property shows that such standard models are internally inconsistent.

- **Back reaction effects** of created particles on the background geometry are to be taken into account to restore self-consistency of the model.

- Certainly, the standard sandwich model is quite different from a "realistic" black hole, where the mass decrease is not abrupt, but is a smooth and continuous function of time. However qualitative conclusions, concerning role of quantum effects are robust and model independent.